Topic 12: CP and Gecode
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Course 1DL441:
Combinatorial Optimisation and Constraint Programming,
whose part 1 is Course 1DL451:
Modelling for Combinatorial Optimisation

Outline

1. Constraint Programming (CP)
2. MiniZinc to Gecode
3. `linear`
4. `element`
5. MiniModel
6. `distinct`, `nvalues`, `count`
7. `binpacking`
8. `cumulative`, `unary`
9. `circuit`, `path`
10. `extensional`
11. `channel`
12. `precede`

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Reminder from Topic 1: Introduction

A solving technology offers methods and tools for:

what: **Modelling** constraint problems in declarative language.

and / or

how: **Solving** constraint problems intelligently:

- Search: Explore the space of candidate solutions.
- Inference: Reduce the space of candidate solutions.
- Relaxation: Exploit solutions to easier problems.

A solver is a software that takes a model as input and tries to solve the modelled problem.

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Constraint Programming (CP)

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Constraint Programming Technology

Constraint programming (CP) offers methods and tools for:

what: Modelling constraint problems in a high-level language.

and

how: **Solving** constraint problems intelligently by:

- either default systematic search upon pushing a button
- or systematic search guided by user-given strategies
- or local search guided by user-given (meta-)heuristics
- or hybrid search

plus inference, called propagation, but little relaxation.

Slogan of CP:

Constraint Program = Model [+ Search]

CP Solving = Propagation + Search

A CP solver conducts search interleaved with propagation:

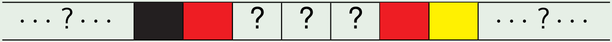
Each constraint has a propagator.

Propagation of one Constraint: Propagator

Example

Consider the constraint CONNECTED([$C_1, \ldots, C_n$]), which enforces max one stretch per colour among the n variables.

From



the CONNECTED([$C_1, \ldots, C_n$]) constraint infers

☞ Propagation is the elimination of ~~the~~ impossible values from the current domains of the variables, and thereby accelerates otherwise blind search.

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Procedural vs Declarative

With Gecode, which is a C++ library, one writes an imperative program that states (or: posts) — via sequential, conditional, iterative, or recursive composition — the declarative constraints, which are then given to the solver via propagators achieving chosen consistencies.

Reification

A MiniZinc reified constraint, such as `r <-> x < y`, where `r` is a variable of type `bool`, is modelled in Gecode by appending the reifying variable `r`, of type `Reify`, as an additional argument to the used constraint predicate:

```
rel(x,IRT_LE,y,r)
```

Careful: Not all constraints are reifiable, as in all CP solvers!

We will use the following definition and notation:

Definition

The reification of a constraint $\gamma(\dots)$ is the constraint $r \Leftrightarrow \gamma(\dots)$, where r is a "Boolean" variable, with the truth of $\gamma(\dots)$ represented by 1 and its falsity by 0.

Propagation may be poor: ☞ see Topic 16: Propagators.

Inference: Propagator and Consistency

A MiniZinc inference annotation (recall Topic 8: Inference & Search in CP & LCG) to a constraint, `bounds` or `domain`, is modelled in Gecode by appending the consistency as an additional argument to the used constraint predicate.

The options for integers are value consistency (`IPL_VAL`), bounds consistency (`IPL_BND`), and domain consistency (`IPL_DOM`), consistency being called integer propagation level (IPL), one of them being the default (`IPL_DEF`) in case no consistency is given.

For example:

```
distinct(X,IPL_DOM)
```

☞ For details, see Topic 13: Consistency.

Search: Variable/Value Selection Strategies

A MiniZinc search annotation (recall Topic 8: Inference & Search in CP & LCG) to an objective, such as `int_search(X,first_fail,indomain_min,...)`, is modelled in Gecode by specifying or writing a brancher.

For example:

```
branch(X,INT_VAR_SIZE_MIN(),INT_VAL_MIN())
```

☞ For details, see Topic 15: Search.

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The `linear` Predicate

A MiniZinc linear constraint, such as the linear equality constraint `sum(i in 1..n)(A[i]*X[i]) = d`, can be modelled in Gecode by using its reifiable `linear` predicate:

Definition

A `linear`$([a_1, \ldots, a_n], [x_1, \ldots, x_n], R, d)$ constraint, with

- $[a_1, \ldots, a_n]$ a sequence of non-zero integer constants,
- $[x_1, \ldots, x_n]$ a sequence of integer variables,
- R in $\{<, \leq, =, \neq, \geq, >\}$, and
- d an integer constant,

holds iff the linear relation $\left(\sum_{i=1}^{n} a_i \cdot x_i\right) \ R \ d$ holds.

Also, `linear`$([x_1, \ldots, x_n], R, d)$ holds iff $\left(\sum_{i=1}^{n} x_i\right) \ R \ d$.

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The `element` Predicate

A MiniZinc constraint on an array element at an unknown index `i`, such as `element(i,X,e)` or `X[i] = e` or a constraint involving the expression `X[i]`, must be modelled in Gecode by explicitly using its non-reifiable `element` predicate:

Definition (Van Hentenryck and Carillon, 1988)

An `element`$([x_1, \dots, x_n], i, e)$ constraint, where the x_j are variables, i is an integer variable, and e is a variable, holds if and only if $x_i = e$.

Several variants exist: see the Gecode documentation.

Example (Warehouse Location Problem)

Recall the one-way channelling constraint of Model 1 (in Topic 6: Case Studies) from the `Supplier` variables to the redundant `Open` variables:

```
constraint forall(s in Shops)
    (Open[Supplier[s]] = 1);
```

This must be modelled in Gecode as in the following MiniZinc reformulation:

```
constraint forall(s in Shops)
    (element(Supplier[s], Open, 1));
```

Example (Warehouse Location Problem, a last time)

Recall the objective of Model 1 in Topic 6: Case Studies:

```
solve minimize maintCost * sum(Open)
  + sum(s in Shops)(SupplyCost[s,Supplier[s]]);
```

This must be modelled in Gecode as in the following MiniZinc reformulation, by explicitly creating a `Cost[s]` variable and an `element` constraint for each implicit one:

```
array[Shops] of var int: Cost; % Cost[s] to supply s
constraint forall(s in Shops)
  (element(Supplier[s], SupplyCost[s,..], Cost[s]);
solve minimize maintCost * sum(Open) + sum(Cost);
```

Recall that we actually introduced these `Cost[s]` variables (in Topic 8: Inference & Search in CP & LCG) in order to state a maximal-regret search strategy on those variables.

Example (Job allocation at minimal salary cost)

Remember the model in Topic 3: Constraint Predicates:

```
1 array[Apps] of int: Salary;
2 array[Jobs] of var Apps: Worker; % job j by Worker[j]
3 solve minimize sum(j in Jobs)(Salary[Worker[j]]);
4 constraint ...;          % qualifications, workload, etc
```

Line 3 must be modelled in Gecode as in the following MiniZinc reformulation, by explicitly creating a `Cost[j]` variable and an `element` constraint for each implicit one:

```
array[Jobs] of var int: Cost; % job j costs Cost[j]
constraint forall(j in Jobs)
   (element(Worker[j], Salary, Cost[j]));
solve minimize sum(j in Jobs)(Cost[j]); % sum(Cost)
```

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MiniModel

- Using MiniModel, linear constraints can be formulated in Gecode like in MiniZinc: the appropriate `linear` constraints are generated by the Gecode toolchain. Another useful feature will be discussed at page 36.

- Gecode has no constrained functions: everything is modelled relationally, using only constraint predicates.

- Gecode does not eliminate common sub-expressions: a Gecode model automatically generated by the MiniZinc toolchain can outperform a handwritten Gecode model corresponding to the MiniZinc one.

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The `distinct` Predicate

A MiniZinc constraint of pairwise difference, such as `alldifferent(X)`, can be modelled in Gecode by using its non-reifiable `distinct` predicate:

Definition (Laurière, 1978)

A `distinct`$([x_1, \ldots, x_n])$ constraint holds if and only if all the variables x_i take different values.

This is equivalent to $\frac{n \cdot (n-1)}{2}$ disequality constraints:

$$\forall i, j \in 1..n \textbf{ where } i < j : x_i \neq x_j$$

Several variants exist: see the Gecode documentation.

The `nvalues` Predicate

A MiniZinc constraint on the number of distinct values within an array, such as `nvalue(m,X)`, can be modelled in Gecode by using its non-reifiable `nvalues` predicate:

Definition (Pachet and Roy, 1999)

An `nvalues`$([x_1, \ldots, x_n], R, m)$ constraint holds if and only if the number of distinct values taken by the elements of the sequence $[x_1, \ldots, x_n]$ of variables is in relation R with the variable m, where $R \in \{<, \leq, =, \neq, \geq, >\}$:

$$|\{x_1, \ldots, x_n\}| \;\; R \; m$$

Note that R is '$=$' for the `nvalue` predicate of MiniZinc.

Several variants exist: see the Gecode documentation.

The `count` Predicate

A MiniZinc constraint on value counts within an array, such as `global_cardinality(X,V,C)`, can be modelled in Gecode by using its non-reifiable `count` predicate:

Definition (Régin, 1996)

A `count`$([x_1, \ldots, x_n], [v_1, \ldots, v_m], [c_1, \ldots, c_m])$ constraint holds if and only if each variable c_j has the number of variables x_i that take the given value v_j.

Several variants exist: see the Gecode documentation.

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The `binpacking` Predicate

A MiniZinc bin-packing constraint, such as `bin_packing_load(L,B,V)`, can be modelled in Gecode by using its non-reifiable `binpacking` predicate:

Definition

Let item i have the given weight or volume v_i.
Let variable b_i denote the bin into which item i is put.
Let variable ℓ_j denote the load of bin j.
A `binpacking`$([\ell_1, \ldots, \ell_m], [b_1, \ldots, b_n], [v_1, \ldots, v_n])$ constraint holds iff each ℓ_j is the sum of the v_i where $b_i = j$.

There is No Knapsack Predicate in Gecode

A MiniZinc constraint on knapsack packing, such as `knapsack(V,P,X,v,p)`, can be modelled in Gecode by using two `linear` constraints:

$$\texttt{linear}(V, X, =, v)$$

$$\texttt{linear}(P, X, =, p)$$

Recall that `linear` is reifiable.

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The `cumulative` Predicate

A MiniZinc constraint on the bounded cumulative resource requirement of tasks, such as `cumulative(S,D,R,u)`, can be modelled in Gecode by using its non-reifiable `cumulative` predicate:

Definition (Aggoun and Beldiceanu, 1993)

A $\texttt{cumulative}(u, [s_1, \ldots, s_n], [d_1, \ldots, d_n], [r_1, \ldots, r_n])$ constraint, where each task T_i has a starting time s_i, a duration d_i, and a resource requirement r_i, holds if and only if the resource upper limit u is never exceeded when performing the tasks T_i.

Several variants exist: see the Gecode documentation.

The `unary` Predicate

A MiniZinc temporal non-overlap constraint on tasks, such as `disjunctive(S,D)`, can be modelled in Gecode by using its non-reifiable `unary` predicate, so called because it applies to tasks requiring a unary resource:

Definition (Carlier, 1982)

A `unary`$([s_1, \dots, s_n], [d_1 \dots, d_n])$ constraint, where each task T_i has a starting time s_i and a duration d_i, holds if and only if no two tasks T_i and T_j overlap in time.

Several variants exist: see the Gecode documentation.

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The `circuit` Predicate

A MiniZinc constraint on a Hamiltonian circuit, such as `circuit(S)`, can be modelled in Gecode by using its non-reifiable `circuit` predicate:

Definition (Laurière, 1978)

A `circuit`($[s_1, \ldots, s_n]$) constraint holds iff the arcs $i \rightarrow s_i$ form a Hamiltonian circuit in the graph defined by the domains of the variables s_i: each vertex is visited exactly once.

Several variants exist: see the Gecode documentation.

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No Subcircuit, but a Path Predicate

A MiniZinc constraint `subcircuit(S)` can be modelled in Gecode as in the MiniZinc default definition at http://www.minizinc.org/doc-lib/doc-globals.html, which is actually used by the Gecode backend to MiniZinc.

A MiniZinc constraint on a Hamiltonian path, such as `circuit(S) /\ S[t] = f`, can be modelled in Gecode by using its non-reifiable `path` predicate:

Definition

A $\texttt{path}([s_1, \ldots, s_n], f, t)$ constraint holds iff the arcs $i \to s_i$ form a Hamiltonian path from vertex f to vertex t in the graph defined by the domains of the variables s_i: each vertex is visited exactly once.

Several variants exist: see the Gecode documentation.

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The `extensional` Predicate

A MiniZinc constraint on membership in a table `T` or regular language, such as `table(X,T)` or `regular(X,R)`, where `R` is a regular expression or a deterministic finite automaton (DFA) defining a regular language, is modelled in Gecode by using its reifiable `extensional` predicate:

Definition

An `extensional`$([x_1, \ldots, x_n], \mathcal{R})$ constraint holds if and only if the values taken by the sequence $[x_1, \ldots, x_n]$ of variables form a row of the 2d table $\mathcal{R}$ of constants or form a string that belongs to the regular language accepted by the regular expression (when using MiniModel) or DFA $\mathcal{R}$.

Several variants exist: see the Gecode documentation.

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The `channel` Predicate

A MiniZinc constraint on two arrays representing a function and its inverse, such as `inverse(X,Y)`, can be modelled in Gecode by using its non-reifiable `channel` predicate:

Definition

A `channel`$([x_1, \ldots, x_n], [y_1, \ldots, y_n])$ constraint holds iff:

$$\forall i, j \in 1..n : x_i = j \Leftrightarrow y_j = i$$

Several variants exist: see the Gecode documentation.

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The `precede` Predicate

MiniZinc value symmetry-breaking constraints, such as `value_precede(v,w,X)` and its generalisation `value_precede_chain(V,X)`, can be modelled in Gecode by using its non-reifiable `precede` predicate:

Definition

A `precede`$([x_1, \ldots, x_n], v, w)$ constraint holds iff the first occurrence, if any, of value v precedes the first occurrence, if any, of value w among the variables x_i.

Definition

A `precede`$([x_1, \ldots, x_n], [v_1, \ldots, v_m])$ constraint holds iff the first occurrence, if any, of every value v_i precedes the first occurrence, if any, of value v_{i+1} among the variables x_i.