```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
      [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

Programming Languages Dan Grossman

Another Closure Idiom: Currying

Currying

- Recall every ML function takes exactly one argument
- Previously encoded n arguments via one n-tuple
- Another way: Take one argument and return a function that takes another argument and...
 - Called "currying" after famous logician Haskell Curry

Example

```
val sorted3 = fn x => fn y => fn z => z >= y andalso y >= x val t1 = ((sorted3 7) 9) 11
```

- Calling (sorted3 7) returns a closure with:
 - Code fn $y \Rightarrow$ fn $z \Rightarrow$ z >= y andalso y >= x
 - Environment maps x to 7
- Calling that closure with 9 returns a closure with:
 - Code fn $z \Rightarrow z \Rightarrow y$ and also $y \Rightarrow x$
 - Environment maps x to 7, y to 9
- Calling that closure with 11 returns true

Syntactic sugar, part 1

```
val sorted3 = fn x => fn y => fn z => z >= y andalso y >= x val t1 = ((sorted3 7) 9) 11
```

- In general, e1 e2 e3 e4 ...,
 means (... ((e1 e2) e3) e4)
- So instead of ((sorted3 7) 9) 11, can just write sorted3 7 9 11
- Callers can just think "multi-argument function with spaces instead of a tuple expression"
 - Different than tupling; caller and callee must use same technique

Syntactic sugar, part 2

```
val sorted3 = fn x => fn y => fn z => z >= y andalso y >= x val t1 = ((sorted3 7) 9) 11
```

- In general, fun f p1 p2 p3 ... = e,
 means fun f p1 = fn p2 => fn p3 => ... => e
- So instead of val sorted3 = fn x => fn y => fn z => ...
 or fun sorted3 x = fn y => fn z => ...,
 can just write fun sorted3 x y z = x >=y andalso y >= x
- Callees can just think "multi-argument function with spaces instead of a tuple pattern"
 - Different than tupling; caller and callee must use same technique

Final version

```
fun sorted3 x y z = z >= y andalso y >= x
val t1 = sorted3 7 9 11
```

As elegant syntactic sugar (even fewer characters than tupling) for:

```
val sorted3 = fn x => fn y => fn z => z >= y andalso y >= x val t1 = ((sorted3 7) 9) 11
```

Curried fold

A more useful example and a call to it

Will improve call next

```
fun fold f acc xs =
    case xs of
    [] => acc
    | x::xs' => fold f (f(acc,x)) xs'

fun sum xs = fold (fn (x,y) => x+y) 0 xs
```

Note: **fold1** in ML standard-library has **f** take arguments in opposite order