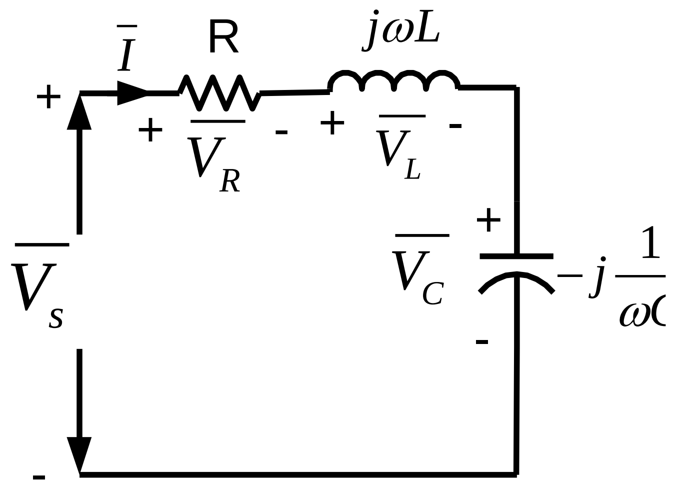
Introduction to Electrical Engineering
EE 103

Lecture 15

Phasor representation of circuits having sinusoidal forcing function under steady state operation

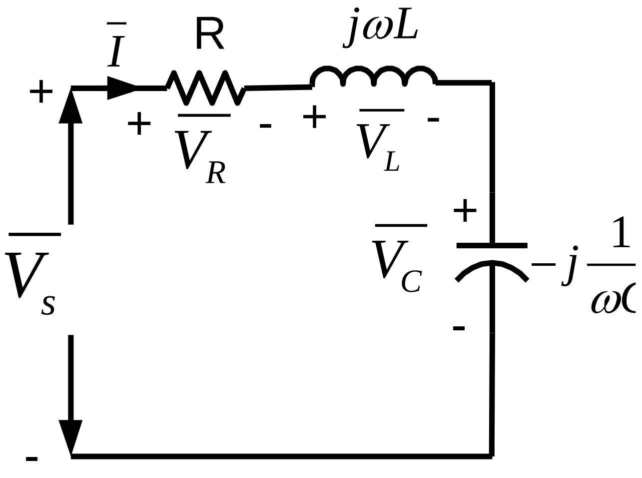


Applying KVL

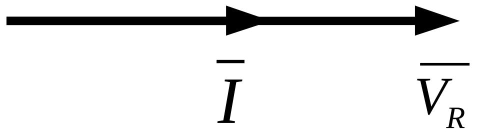
$$\overline{V_s} = \overline{V_R} + \overline{V_L} + \overline{V_C}$$

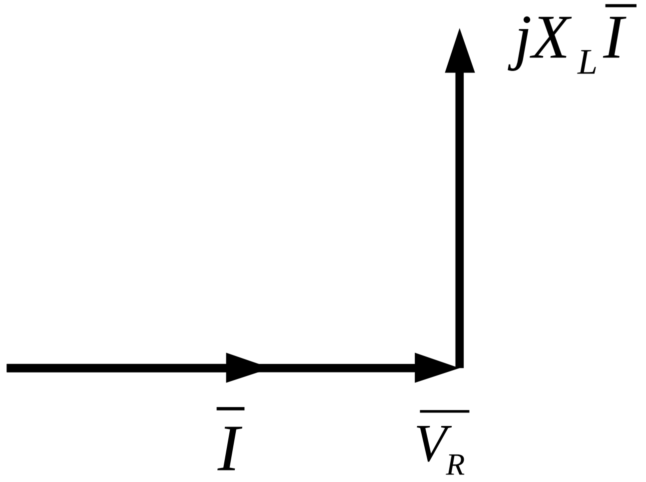
$$\overline{V_s} = R\bar{I} + jX_L\bar{I} - jX_C\bar{I}$$

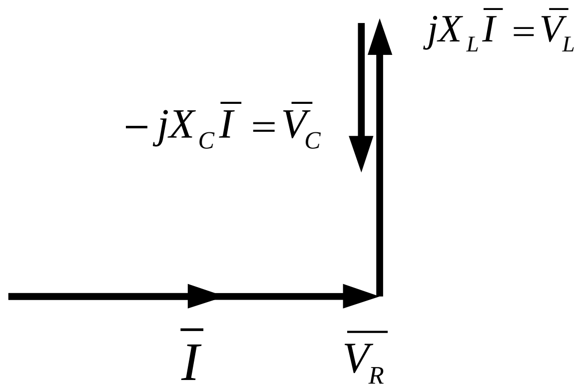
Technique for Phasor analysis of series circuits

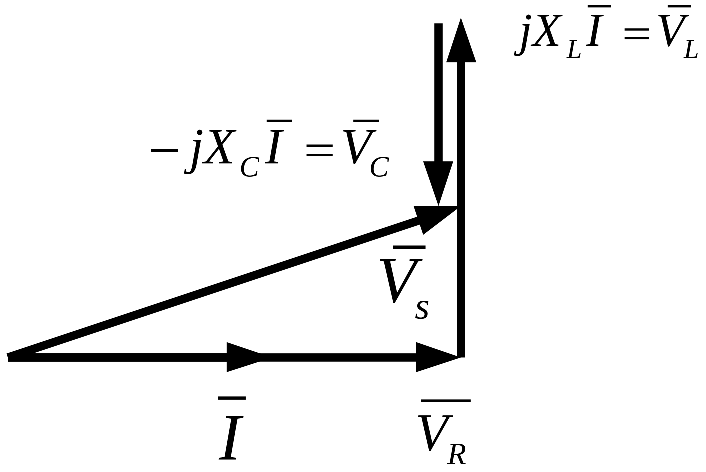


If $\bar{I}$ is chosen as a reference phasor, analysis becomes easier

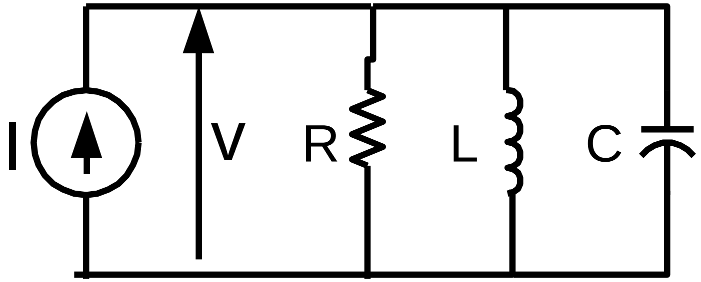


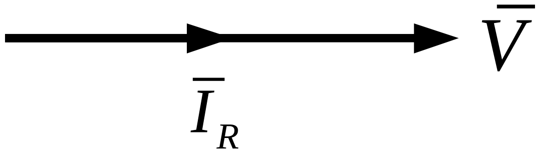


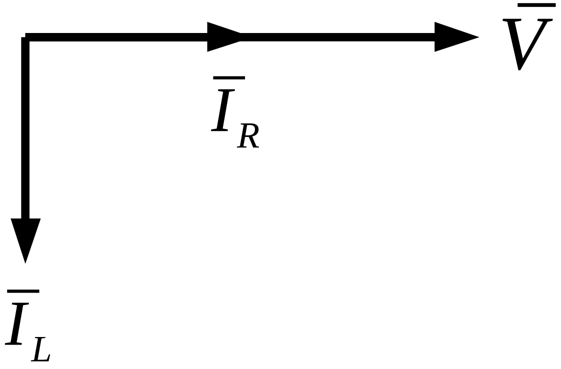




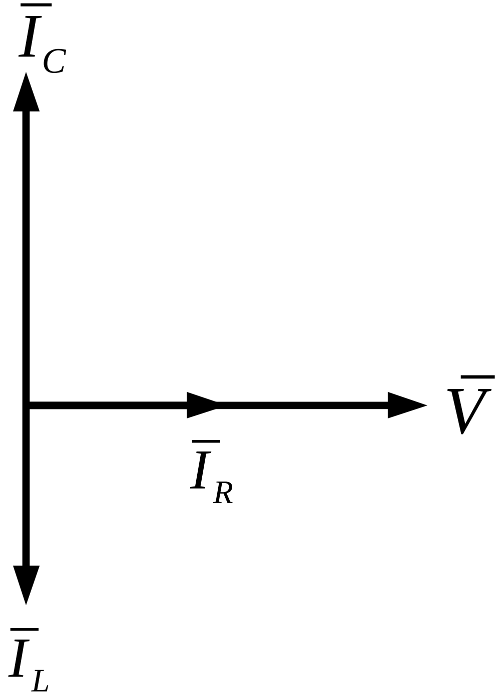
Technique for Phasor analysis of parallel circuits

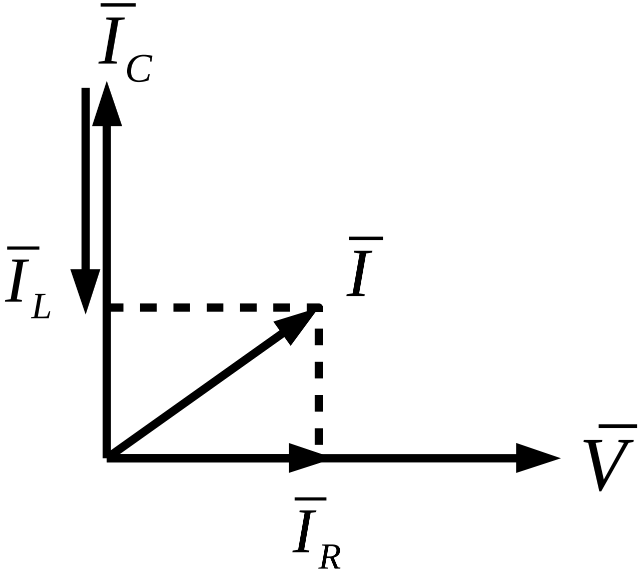




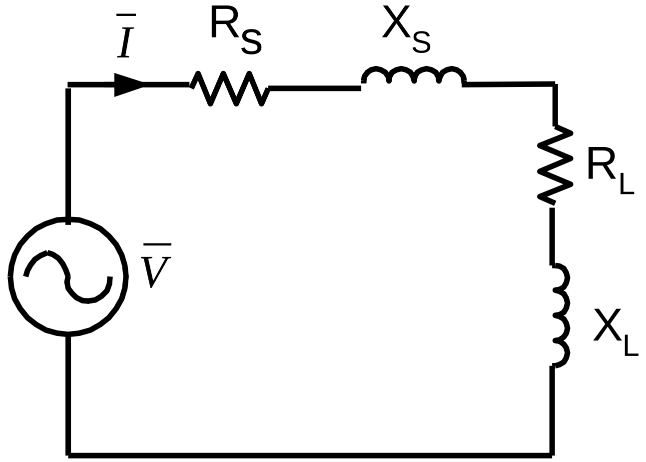


If $\bar{V}$ is chosen as a reference phasor, analysis becomes easier





Maximum Power Transfer Theorem



$$\bar{I} = \frac{\bar{V}}{(R_s + R_L) + j(X_s + X_L)}$$

$$\left|\bar{I}\right|^2 = \frac{V^2}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

Power dissipated in the load,

$$P = \frac{V^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

$$P = f(R_L, X_L)$$

$$P = \frac{V^2 R_L}{\left(R_s + R_L\right)^2 + (X_s + X_L)^2} \qquad P = f(R_L, X_L)$$

$$\frac{\partial P}{\partial R_L} = 0 \quad \Rightarrow \quad \frac{V^2\left[\left(R_s + R_L\right)^2 + \left(X_s + X_L\right)^2 - 2R_L\left(R_s + R_L\right)\right]}{\left[\left(R_s + R_L\right)^2 + \left(X_s + X_L\right)^2\right]^2} = 0 \qquad (1)$$

$$\frac{\partial P}{\partial X_L} = 0 \quad \Rightarrow \frac{V^2 R_L\left[-2R_L\left(X_s + X_L\right)\right]}{\left[\left(R_s + R_L\right)^2 + \left(X_s + X_L\right)^2\right]^2} = 0 \qquad (2)$$

From (2) $\quad X_L = -X_s \qquad (3)$

Putting (3) in (1)

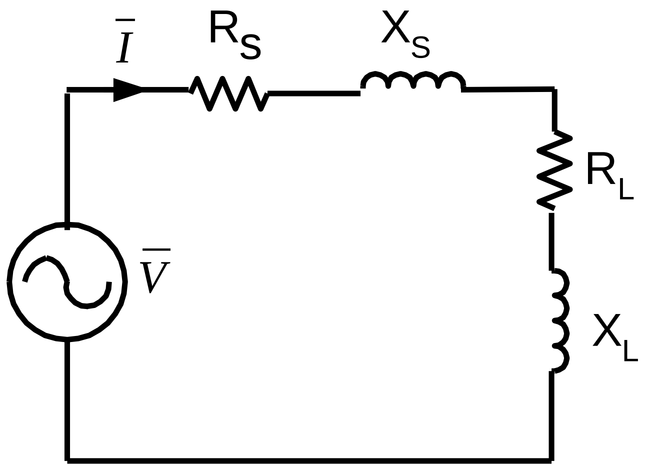
$$R_L = R_s$$

$$X_L = -X_s$$

$$R_L = R_s$$

Or

$$\overline{Z_L} = \overline{Z_s}^*$$

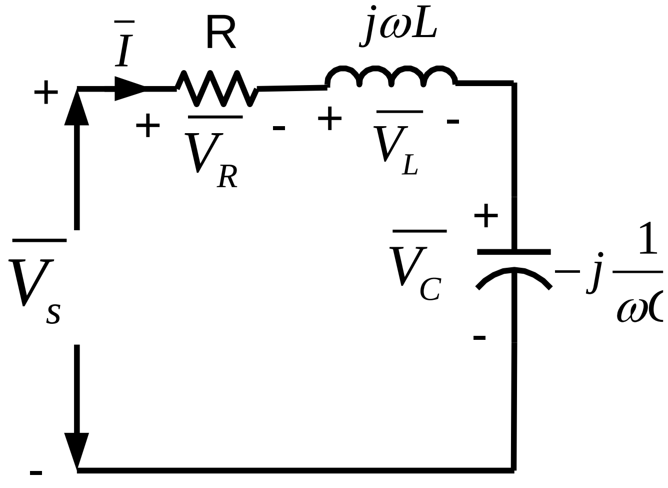


If only R_L is allowed to vary then from (1), keeping X_L and X_s fixed,

$$R_L = \sqrt{R_s^{\,2} + \left(X_s + X_L \right)^2}$$

Resonance: Steady state operation of a system (circuit) at that frequency for which the resultant response is in time phase with the source function.

Series Resonance



$$I = \frac{\overline{V_s}}{R + j\left(\omega_0 L - \frac{1}{\omega_0 C}\right)}$$

$$\omega_0 L - \frac{1}{\omega_0 C} = 0$$

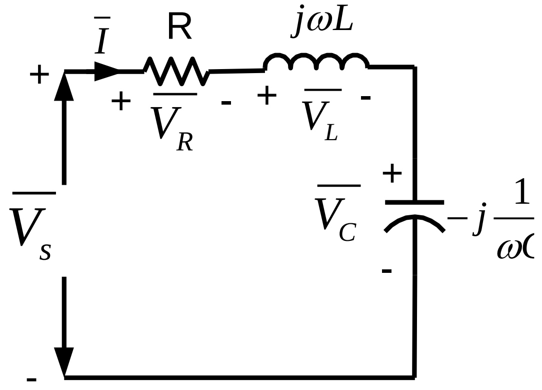
$$\omega_0{}^2 = \frac{1}{LC}$$

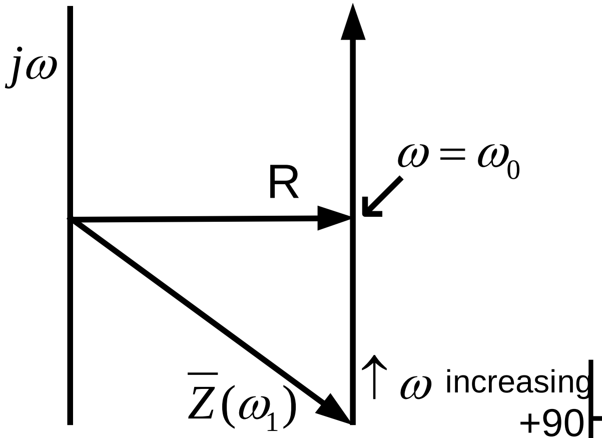
$$\omega_0 = \sqrt{\frac{1}{LC}}$$

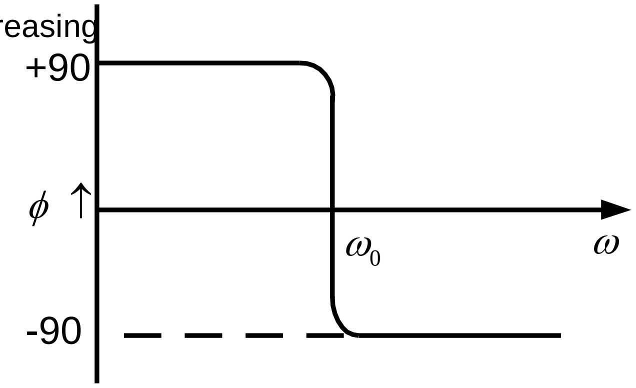
$$f_0 = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

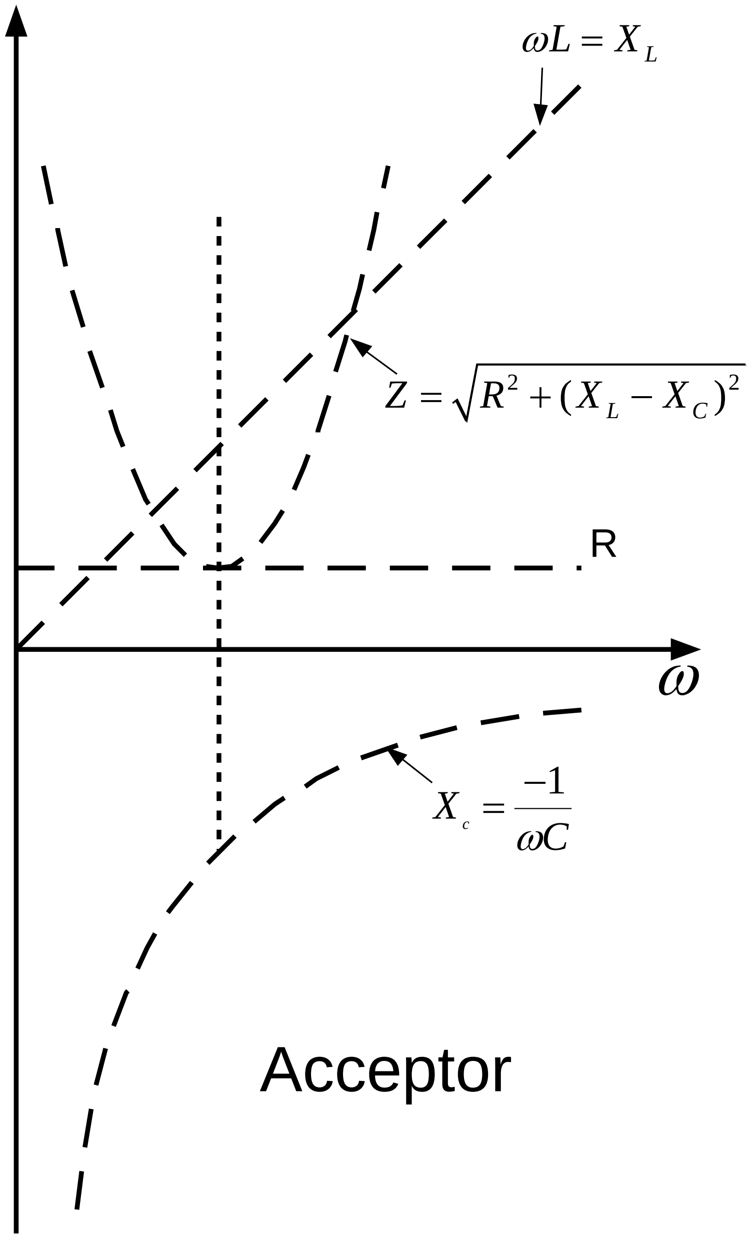
ω_0, resonant frequency decided by circuit parameters

Series Resonance

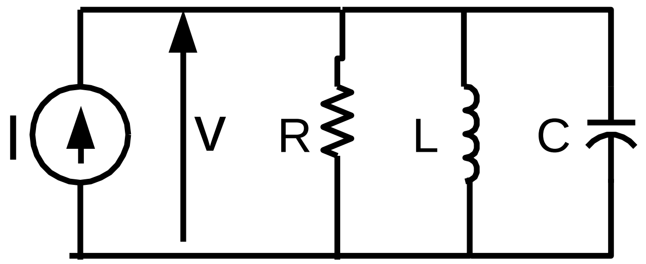








Parallel Resonance



$$\bar{Y} = G + jB$$

$$= \frac{1}{R} + j\left(-\frac{1}{\omega L} + \omega C\right)$$

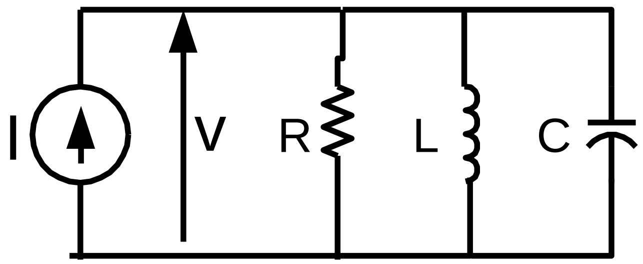
$\bar{I}$ and $\bar{V}$ will be in phase when

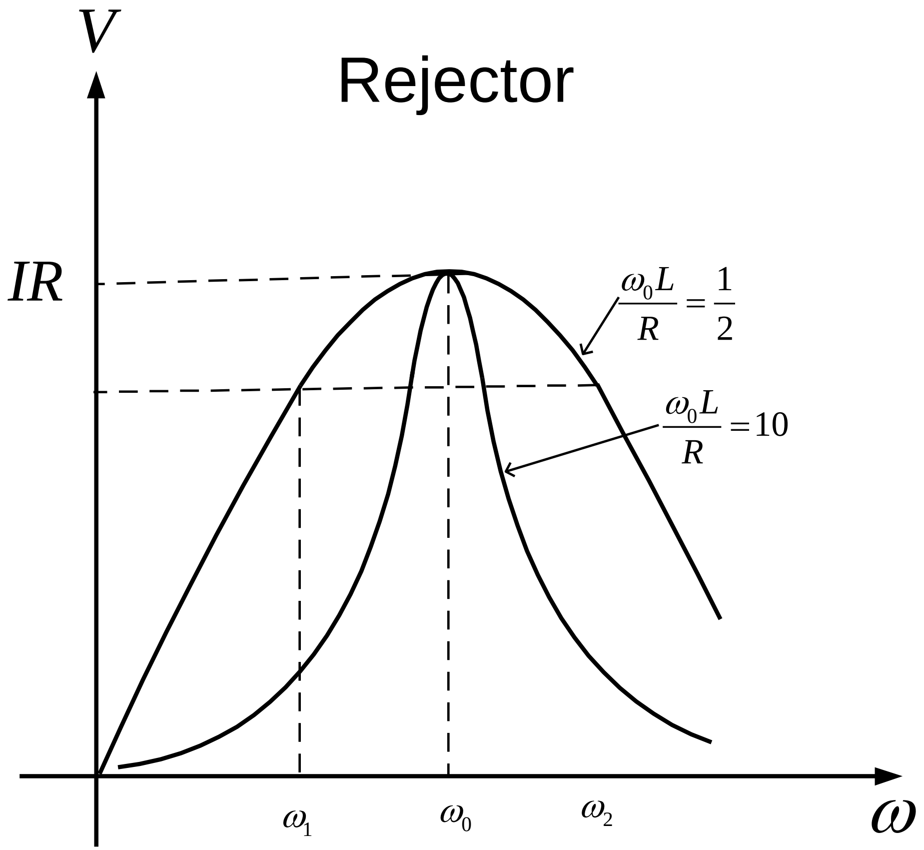
$$\omega L = \frac{1}{\omega C} \qquad \omega = \frac{1}{\sqrt{LC}}$$

$$\bar{I} = \bar{V}\left\{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)\right\}$$

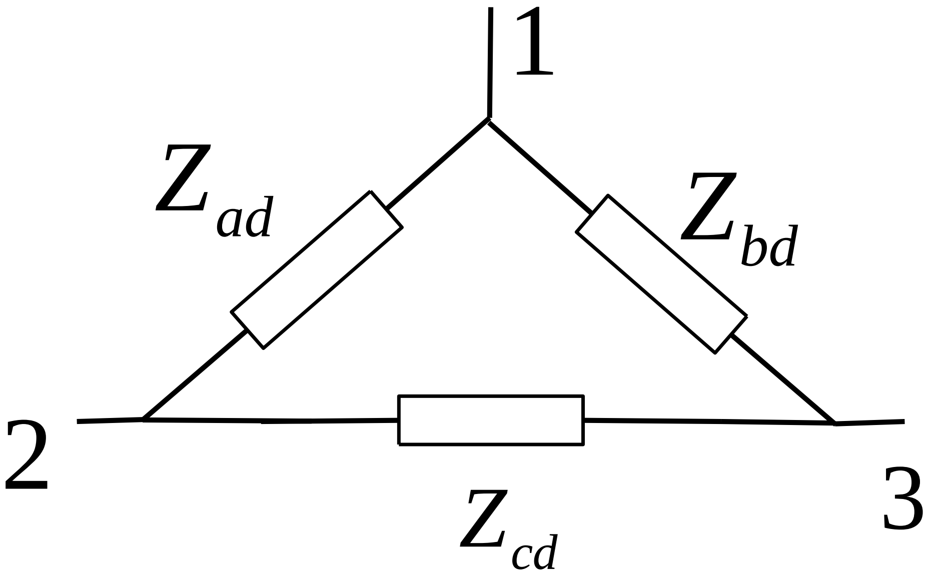
Power delivered at resonance $P_0 = V_0 I_0$

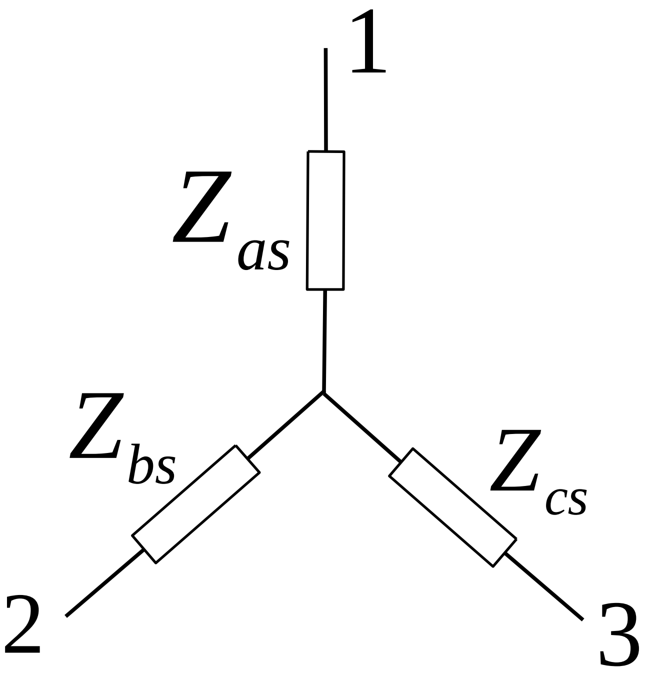
$$= \frac{V_0^{\ 2}}{R}$$





Star-Delta Transformation





$$\bar{Z}_{as} = \frac{\bar{Z}_{ad}\bar{Z}_{bd}}{\bar{Z}_{ad} + \bar{Z}_{bd} + \bar{Z}_{cd}}$$

$$\bar{Z}_{ad} = \frac{\bar{Z}_{as}\bar{Z}_{bs} + \bar{Z}_{bs}\bar{Z}_{cs} + \bar{Z}_{cs}\bar{Z}_{as}}{\bar{Z}_{cs}}$$

$$\bar{Z}_{bs} = \frac{\bar{Z}_{ad}\bar{Z}_{cd}}{\bar{Z}_{ad} + \bar{Z}_{bd} + \bar{Z}_{cd}}$$

$$\bar{Z}_{bd} = \frac{\bar{Z}_{as}\bar{Z}_{bs} + \bar{Z}_{bs}\bar{Z}_{cs} + \bar{Z}_{cs}\bar{Z}_{as}}{\bar{Z}_{bs}}$$

$$\bar{Z}_{cs} = \frac{\bar{Z}_{cd}\bar{Z}_{bd}}{\bar{Z}_{ad} + \bar{Z}_{bd} + \bar{Z}_{cd}}$$

$$\bar{Z}_{cd} = \frac{\bar{Z}_{as}\bar{Z}_{bs} + \bar{Z}_{bs}\bar{Z}_{cs} + \bar{Z}_{cs}\bar{Z}_{as}}{\bar{Z}_{as}}$$

Control of Systems: Control Engineering

Laplace Transformation

Pierre-Simon Laplace

文A 102 languages

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"Laplace" redirects here. For other uses, see Laplace (disambiguation).

Pierre-Simon, Marquis de Laplace (/ləˈplɑːs/; French: [pjɛʁ simɔ̃ laplas]; 23 March 1749 – 5 March 1827) was a French polymath, a scholar whose work has been instrumental in the fields of physics, astronomy, mathematics, engineering, statistics, and philosophy. He summarized and extended the work of his predecessors in his five-volume *Mécanique céleste* (*Celestial Mechanics*) (1799–1825). This work translated the geometric study of classical mechanics to one based on calculus, opening up a broader range of problems. Laplace also popularized and further confirmed Sir Isaac Newton's work.[2] In statistics, the Bayesian interpretation of probability was developed mainly by Laplace.[3]

Laplace formulated Laplace's equation, and pioneered the Laplace transform which appears in many branches of mathematical physics, a field that he took a leading role in forming. The Laplacian differential operator, widely used in mathematics, is also named after him. He restated and developed the nebular hypothesis of the origin of the Solar System and was one of the first scientists to suggest an idea similar to that of a black hole,[4] with Stephen Hawking stating that "Laplace essentially predicted the existence of black holes".[1] He originated Laplace's demon, which is a hypothetical all-predicting intellect. He also refined Newton's calculation of the speed of sound to derive a more

Pierre-Simon Laplace

Pierre-Simon Laplace as chancellor of the Senate under the First French Empire

Born	23 March 1749 Beaumont-en-Auge, Normandy, Kingdom of France
Died	5 March 1827 (aged 77)

$$\mathcal{L}[f(t)] = F(s) = \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt$$

$$s = \sigma + j\omega$$

Inverse Laplace transform

$$\mathcal{L}^{-1}\{\mathcal{L}[f(t)]\} = \mathcal{L}^{-1}[F(s)] = f(t)$$

Inverse Laplace transform

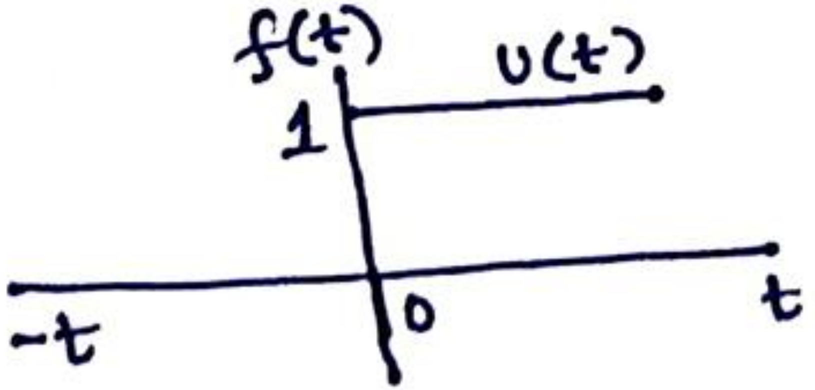
$$f(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(s)\, e^{st}\, ds$$

1) Tranform pair

$f(t) = 1$

$$\therefore \mathcal{L}[1] = \int_{0^-}^{\infty} e^{-st}\, dt = \left[-\frac{1}{s} e^{-st} \right]_0^{\infty} = \frac{1}{s}$$

For a unit step function



$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

$$\mathcal{L}\left[V u(t)\right] = \frac{V}{s} \quad \leftarrow \underline{\text{transform}}\ \underline{\text{pair}}$$

2) Another Transform Pair

$$f(t) = e^{at}$$

$$\mathcal{L}[e^{at}] = \int_{0^-}^{\infty} e^{at} \cdot e^{-st}\, dt = \int_{0^-}^{\infty} e^{-(s-a)t}\, dt$$

$$= \frac{1}{s-a}$$

$$e^{at} \underset{\text{transform pair}}{\Longleftrightarrow} \frac{1}{s-a}$$

3) $f(t) = \sin \omega t$ ← Another Transform Pair

$$\mathcal{L}[\sin \omega t] = \int_{0^-}^{\infty} \sin \omega t \, e^{-st} \, dt$$

$$= \left[\frac{e^{-st}(-s \sin \omega t - \omega \cos \omega t)}{s^2 + \omega^2} \right]_{0^-}^{\infty}$$

$$= \frac{\omega}{s^2 + \omega^2}$$

Table of Transform Pairs

$f(t)$	$F(s)$
$u(t)$	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$

Basic theorems :

(1) Transforms of linear Combinations

$$\mathcal{L}\left[a_1 f_1(t) + a_2 f_2(t)\right] = a_1 F_1(s) + a_2 F_2(s)$$

Applicability of KCL and KVL

Basic theorems :

(2) Transform of Derivatives

[i] $\mathcal{L}\left[\frac{d}{dt} f(t)\right] = \int_{0^-}^{\infty} \frac{d}{dt} f(t) e^{-st} \, dt$

Let $u = e^{-st}$ $\quad dv = df(t)$

$du = -s e^{-st} \, dt$ $\quad v = f(t)$

$\therefore \mathcal{L}\left[\frac{d}{dt} f(t)\right] = \left[e^{-st} f(t)\right]_{0^-}^{\infty} + s \int_{0^-}^{\infty} f(t) e^{-st} \, dt$

$= s F(s) - f(0^-)$

[ii] Transform of the second derivative

$$\frac{d^2}{dt^2} f(t) = \frac{d}{dt}\frac{d}{dt} f(t)$$

$$\mathcal{L}\left[\frac{d^2 f(t)}{dt^2}\right] = s\,\mathcal{L}\left[\frac{d}{dt} f(t)\right] - \frac{d}{dt} f(0-)$$

$$= s\left[sF(s) - f(0-)\right] - \frac{d}{dt} f(0-)$$

$$= s^2 F(s) - s f(0^-) - \frac{d}{dt} f(0^-)$$

$\frac{d}{dt} f(0-) \Rightarrow$ derivative of $f(t)$ evaluated at $t = 0$

The general expression for an n^{th} order derivative

$$\mathcal{L}\left[\frac{d^n f(t)}{dt^n}\right] = s^n F(s) - s^{n-1} f(0^-) - s^{n-2} \frac{d}{dt} f(0^-) - \cdots$$
$$\cdots\cdots - \frac{d^{n-1}}{dt^{n-1}} f(0^-)$$

(3) Transforms of integrals

$$\int_0^\infty f(t)\,e^{-st}\,dt = F(s)$$

integrating by parts $\Rightarrow$

$$F(s) = \underbrace{\left[e^{-st}\int f(t)\,dt\right]_{0^-}^{\infty}}_{A} + \underbrace{s\int_{0^-}^{\infty}\left[\int f(t)\,dt\right]e^{-st}\,dt}_{B}$$

$A = 0$ at $t = \infty$

at $t = 0^-$

$$A = \int f(t)\,dt\Big|_{t=0^-} = -f^{(-1)}(0^-)$$

= value of the integral evaluated at $t = 0^-$ $= \int_{-\infty}^{0^-} f(t)\,dt$

$f^{(-1)}$ indicates integration

the term B $\Rightarrow$ $s\mathcal{L}\int f(t)\,dt$

$\therefore$ Rearranging

$$F(s) = -f^{(-1)}(0^-) + s\mathcal{L}\int f(t)\,dt$$

$$\therefore \mathcal{L}\int f(t)\,dt = \frac{F(s)}{s} + \frac{f^{(-1)}(0^-)}{s}$$

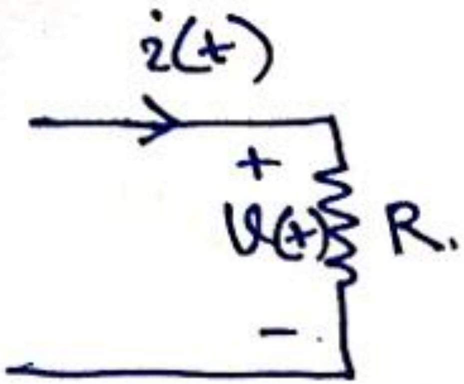
Similarly

$$\mathcal{L}\iint f(t)\,dt = \frac{F(s)}{s^2} + \frac{f^{-1}(0^-)}{s^2} + \frac{f^{(-2)}(0^-)}{s}$$

Similarly

$$\mathcal{L}\iint f(t)\,dt = \frac{F(s)}{s^2} + \frac{f^{-1}(0^-)}{s^2} + \frac{f^{(-2)}(0^-)}{s}$$

Basic Circuit Elements in the Laplace Domain

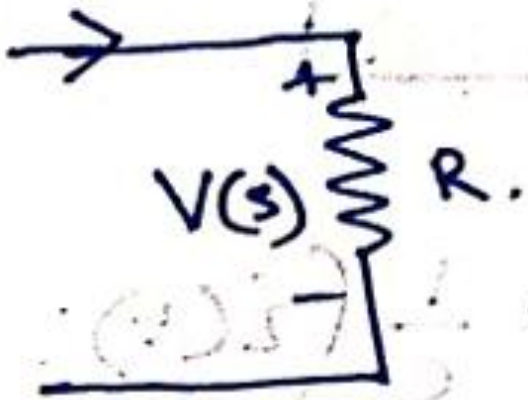


$$v(t) = R\,i(t)$$

$$\mathcal{L}\{v(t)\} = \mathcal{L}\{R\,i(t)\}$$

$$V(s) = R\,I(s)$$

$$i(t) \Rightarrow I(s)$$



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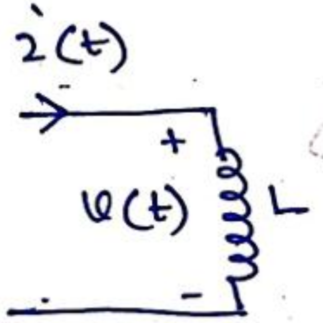
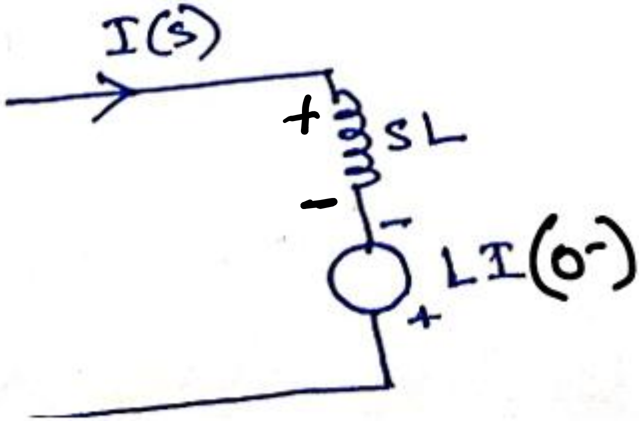


$$v(t) = L\,\frac{di(t)}{dt}$$

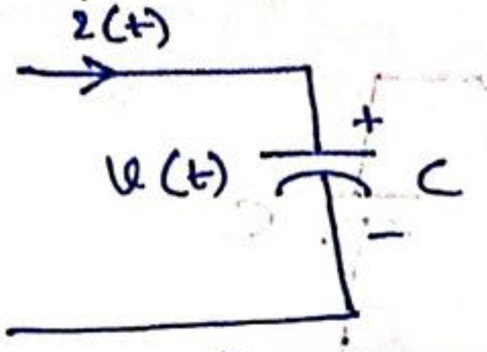
$$\mathcal{L}\{v(t)\} = L\,\mathcal{L}\left\{\frac{di(t)}{dt}\right\}$$

$$V(s) = L\{s\,I(s) - I(0^-)\}$$

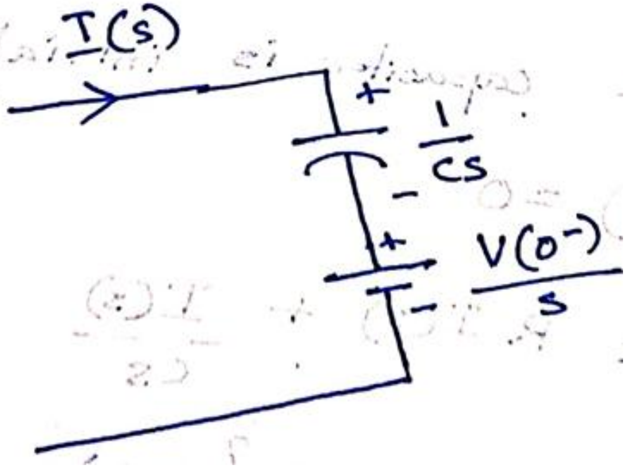
$$V(s) = sL\,I(s) - L\,I(0-)$$



Basic Circuit Elements in the Laplace Domain



$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau$$



$$\mathcal{L}\{v(t)\} = \mathcal{L}\left[\frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau\right]$$

$$= \frac{1}{Cs} I(s) + \frac{1}{Cs}\int_{-\infty} i_c(\tau)\,d\tau$$

$$V(0^-) = \frac{1}{C}\int_{-\infty}^{0^-} i_c(\tau)\,d\tau$$

$$V(s) = \frac{1}{Cs} I(s) + \frac{1}{s} V(0^-)$$