

TUTORIAL-04

MASTER THEOREM →

- ① If $f(n) = O(n^{\log_b a - \epsilon})$, $\epsilon > 0$, $T(n) = \Theta(n^{\log_b a})$
- ② If $f(n) = \Theta(n^{\log_b a} \log^k n)$, $k \geq 0$, $T(n) = \Theta(n^{\log_b a} \log^{k+1} n)$.
- ③ If $f(n) = \Omega(n^{\log_b a} \log^k n)$: $a_f(n/b) \leq c f(n)$

① $T(n) = 3T(n/2) + n^2$
 $a = 3, b = 2$
 $c = \log_2^3 = 1.5$
 $n^2 > n^{1.5}$
 $\therefore T(n) = \Theta(n^2)$

② $T(n) = 4T(n/2) + n^2$
 $a = 4, b = 2$
 $c = \log_2^4 = 2$
 $f(n) = n^c \therefore n^2 = n^2$
 $T(n) = \Theta(n^2 \log n)$

③ $T(n) = T(n/2) + 2^n$
 $a = 1, b = 2$
 $c = \log_2^1 = 0$
 $(n^c = 1) \quad 2^n > 1$
 $T(n) = \Theta(2^n)$

④ $T(n) = 2^n T(n/2) + 2^n$
 $a = 2^n, b = 2$
 $c = \log_2^{2^n}$
 $c = n \rightarrow n^c = n^n$
 $f(n) < n^c$
 $T(n) = \Theta(n^n)$

⑤ $T(n) = 16T(n/4) + n$
 $a = 16, b = 4$
 $c = \log_4^{16} = 2$
 $f(n) < n^c$
 $n < n^2$
 $T(n) = \Theta(n^2)$

⑥ $T(n) = 2T(n/2) + n \log n$
 $c = \log_2^2 = 1$
 $f(n) > n^c$
 $n \log n > n$
 $T(n) = \Theta(n \log n)$

$$(7) T(n) = 2T(n/2) + n \log n$$

$$a=2, b=2$$

$$c = \log_2^2 = 1$$

$$\frac{n}{\log n} < n$$

$$f(n) < n^c$$

$$T(n) = \Theta(n)$$

$$(9) T(n) = 0.5T(n/2) + 1/n$$

$$c = \log_2^{1/2} = -1$$

$$n! = n^1$$

$$T(n) = \Theta(n^{-1} \log n)$$

$$(11) T(n) = 4T(n/2) + \log n$$

$$a=4, b=2$$

$$c = \log_2^4 = 2$$

$$\log n < n^2$$

$$T(n) = \Theta(n^2)$$

$$(8) T(n) = 2T(n/4) + n$$

$$a=2, b=4$$

$$c = \log_4^2 = 1/2 = 0.5$$

$$f(n) = n^c$$

$$n^{0.5} = n^{0.5}$$

$$T(n) = \Theta(n^{0.5} \log n)$$

$$(10) T(n) = 16T(n/4) + n!$$

$$a=16, b=4$$

$$c = \log_4^{16} = 2$$

$$n! < n^2$$

$$n! > n^2$$

$$T(n) = \Theta(n!)$$

$$n! > n^2$$

$$T(n) = \Theta(n^2)$$

$$(12) T(n) = \sqrt{n} T(n/2) + \log n$$

$$a=\sqrt{n}, b=2$$

$$c = \log_2^{n^{1/2}}$$

$$(13) T(n) = 3T(n/2) + n$$

$$a=3, b=2$$

$$c = \log_2^3 = 1.5$$

$$f(n) < n^c$$

$$n < n^{1.5}$$

$$T(n) = \Theta(n^{1.5})$$

$$(14) T(n) = 3T(n/3) + \text{sq}(n)$$

$$a=3, b=3$$

$$c = \log_3^3 = 1$$

$$f(n) = n^{1/2}$$

$$n^c = n^1$$

$$f(n) < n^c$$

$$T(n) = \Theta(n)$$

$$(15) T(n) = 4T(n/2) + cn$$

$$a=4, b=2$$

$$c = \log_2 4 = 2$$

$$f(n) < n^c$$

$$n < n^2$$

$$T(n) = O(n^2)$$

$$(16) T(n) = 3T(n/3) + n/2$$

$$a=3, b=3$$

$$c = \log_3 3 = 1$$

$$f(n) = n/2$$

$$n^c = n$$

$$f(n) < n^c$$

$$T(n) = O(n)$$

$$(17) T(n) = 3T(n/4) + n \log n$$

$$a=3, b=4$$

$$c = \log_4 3$$

$$f(n) = n \log n$$

$$n^c = n^{0.79}$$

$$f(n) > n^c$$

$$T(n) = O(n \log n)$$

$$(18) T(n) = 6T(n/3) + n^2 \log n$$

$$a=6, b=3$$

$$c = \log_3 6 = 1.6$$

$$f(n) > n^c$$

$$T(n) = O(n^2 \log n)$$

$$(19) T(n) = 4T(n/2) + n \log n$$

$$a=4, b=2$$

$$c = \log_2 4 = 2$$

$$f(n) = n \log n$$

$$n^c = n^2$$

$$n \log n < n^2$$

$$T(n) = O(n^2)$$

$$(20) T(n) = 64T(n/8) - n^2 \log n$$

$$a=64, b=8$$

$$c = \log_8 64 = 2$$

$$f(n) > n^c$$

$$n^2 \log n > n^2$$

$$T(n) = O(n^2 \log n)$$

$$(21) T(n) = 7T(n/3) + n^2$$

$$a=7, b=3$$

$$c = \log_3 7 = 1.77$$

$$f(n) > n^c$$

$$n^2 > n^{1.77}$$

$$T(n) = \Theta(n^2)$$

$$(22) T(n) = T(n/2) + n(2 - \cos n)$$

$$a=1, b=2$$

$$\log_2 1 = 0$$

$$f(n) > n^c$$

$$n(2 - \cos n) > n^0$$

$$T(n) = \Theta(n(2 - \cos n))$$