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CBSE Class XII
Task–2 : IMO 2002 Problems

1. Let S be the set of all (h, k) with h, k non-negative integers such that

   $h + k < n$.

   Each element of S is colored red or blue, so that if (h, k) is red and

   $h' \leq h, \ k' \leq k$,

   then (h', k') is also red.

   A type 1 subset of S has n blue elements with different first members, and a type 2 subset of S has n blue elements with different second members.

   Show that there are the same number of type 1 and type 2 subsets.

2. BC is a diameter of a circle with center O. A is any point on the circle with

   $\angle AOC > 60°$.

   EF is the chord which is the perpendicular bisector of AO. D is the midpoint of the minor arc AB. The line through O parallel to AD meets AC at J.

   Show that J is the incenter of triangle CEF.

3. Find all pairs of integers $m > 2$, $n > 2$ such that there are infinitely many positive integers k for which

   $k^n + k^2 - 1 \mid k^m + k - 1$.

4. The positive divisors of the integer $n > 1$ are

   $d_1 < d_2 < \cdots < d_k,$

   so that $d_1 = 1$ and $d_k = n$.

   Let

   $d = d_1 d_2 + d_2 d_3 + \cdots + d_{k-1} d_k.$

   Show that

   $d < n^2$

   and find all n for which d divides n^2.

5. Find all real-valued functions on the reals such that

$$f(x) + f(y)\big(f(f(x)) + f(y)\big) = f(x - y) + f(xy + y)$$

for all real x, y.

6. $n > 2$ circles of radius 1 are drawn in the plane so that no line meets more than two of the circles.

Their centers are $O_1, O_2, \ldots, O_n$. Show that

$$\sum_{i<j} \frac{1}{O_i O_j} \le \frac{(n-1)\pi}{4}.$$