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CBSE Class XII

Task 2

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1. S is the set of all (h, k) with h, k non-negative integers such that $h+k < n$. Each element of S is colored red or blue, so that if (h, k) is red then (h', k') is also red for all $h' \leq h, k' \leq k$.

A type 1 subset of S has n blue elements with different first member and a type 2 subset of S has n blue elements with different second member. Show that there are the same number of type 1 and type 2 subsets.

2. BC is a diameter of a circle with center O . A is any point on the circle with $\angle AOC > 60^\circ$. EF is the chord which is the perpendicular bisector of AO . D is the midpoint of the minor arc AB . The line through O parallel to AD meets AC at J . Show that J is the incenter of triangle CEF .
3. Find all pairs of integers $m > 2, n > 2$ such that there are infinitely many positive integers k for which

$$k^m + k^2 - 1 \mid k^n + k - 1.$$

4. The positive divisors of the integer $n > 1$ are

$$d_1 < d_2 < \cdots < d_k,$$

so that $d_1 = 1, d_k = n$. Let

$$d = d_1 d_2 + d_2 d_3 + \cdots + d_{k-1} d_k.$$

Show that $d < n^2$ and find all n for which d divides n^2 .

5. Find all real-valued functions on the reals such that

$$f(x+y)(f(x) + f(y)) = f(xy) + f(x) + f(y)$$

for all real numbers x, y .

6. $n > 2$ circles of radius 1 are drawn in the plane so that no line meets more than two of the circles. Their centers are O_1, O_2, \dots, O_n . Show that

$$\sum_{1 \leq i < j \leq n} \frac{1}{O_i O_j} \leq \frac{(n-1)\pi}{4}.$$