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*CBSE Class XII*

*Task 2*

*44<sup>th</sup> International Mathematical Olympiad  
IMO 2003*

1.  $S$  is the set  $\{1, 2, 3, \dots, 1000000\}$ . Show that for any subset  $A$  of  $S$  with 101 elements we can find 100 distinct elements  $x_i$  of  $S$  such that the sets  $\{a + x_i \mid a \in A\}$  are all pairwise disjoint.
2. Find all pairs  $(m, n)$  of positive integers such that

$$\frac{m^2}{2m^2 - n^2 + 1}$$

is a positive integer.

3. A convex hexagon has the property that for any pair of opposite sides the distance between their midpoints is  $\sqrt{3}/2$  times the sum of their lengths. Show that all the hexagon's angles are equal.
4.  $ABCD$  is cyclic. The feet of the perpendiculars from  $D$  to the lines  $AB, BC, CA$  are  $P, Q, R$  respectively. Show that the angle bisectors of  $\angle ABC$  and  $\angle CDA$  meet on the line  $AC$  if and only if

$$RP = RQ.$$

5. Given  $n > 2$  and reals  $x_1 \leq x_2 \leq \dots \leq x_n$ , show that

$$\sum_{i < j} |x_i - x_j|^2 \leq \frac{n^2 - 1}{3} \sum_{i < j} (x_i - x_j)^2.$$

Show that equality holds if and only if the sequence is an arithmetic progression.

6. Show that for each prime  $p$ , there exists a prime  $q$  such that

$$n^p - p$$

is not divisible by  $q$  for any positive integer  $n$ .