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CBSE Class XII

Task 2

44th International Mathematical Olympiad

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Each problem is worth seven points.

1. S is the set $\{1, 2, 3, \dots, 1000000\}$. Show that for any subset A of S with 101 elements we can find 100 distinct elements x_i of S such that the sets $\{a + x_i \mid a \in A\}$ are all pairwise disjoint.
2. Find all pairs (m, n) of positive integers such that $\frac{m^2}{2m^2 - n^2 + 1}$ is a positive integer.
3. A convex hexagon has the property that for any pair of opposite sides the distance between their midpoints is $\sqrt{3}/2$ times the sum of their lengths. Show that all the hexagon's angles are equal.
4. $ABCD$ is cyclic. The feet of the perpendiculars from D to the lines AB, BC, CA are P, Q, R respectively. Show that the angle bisectors of $\angle ABC$ and $\angle CDA$ meet on the line AC if and only if $RP = RQ$.
5. Given $n > 2$ and reals $x_1 \leq x_2 \leq \dots \leq x_n$, show that $\sum_{i < j} |x_i - x_j|^2 \leq \frac{n^2 - 1}{3} \sum_{i < j} (x_i - x_j)^2$. Show that equality holds if and only if the sequence is an arithmetic progression.
6. Show that for each prime p , there exists a prime q such that $n^p - p$ is not divisible by q for any positive integer n .