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CBSE Class XII

EXERCISE 4.6

Examine the consistency of the system of equations in Exercises 1 to 6.

1. $x + 2y = 2$
   $2x + 3y = 3$

2. $2x - y = 5$
   $x + y = 4$

3. $x + 3y = 5$
   $2x + 6y = 8$

4. $x + y + z = 1$
   $2x + 3y + 2z = 2$
   $ax + ay + 2az = 4$

5. $3x - y - 2z = 2$
   $2y - z = -1$
   $3x - 5y = 3$

6. $5x - y + 4z = 5$
   $2x + 3y + 5z = 2$
   $5x - 2y + 6z = -1$

Solve system of linear equations using matrix method in Exercises 7 to 14.

7. $5x + 2y = 4$
   $7x + 3y = 5$

8. $2x - y = -2$
   $3x + 4y = 3$

9. $4x - 3y = 3$
$3x - 5y = 7$

10. $5x + 2y = 3$
$3x + 2y = 5$

11. $2x + y + z = 1$
$x - 2y - z = \dfrac{3}{2}$
$3y - 5z = 9$

12. $x - y + z = 4$
$2x + y - 3z = 0$
$x + y + z = 2$

13. $2x + 3y + 3z = 5$
$x - 2y + z = -4$
$3x - y - 2z = 3$

14. $x - y + 2z = 7$
$3x + 4y - 5z = -5$
$2x - y + 3z = 12$

15. If $A = \begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$, find A^{-1}. Using A^{-1} solve

$$\begin{aligned} 2x - 3y + 5z &= 11 \\ 3x + 2y - 4z &= -5 \\ x + y - 2z &= -3 \end{aligned}$$

16. The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs. 60.
The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs. 90.
The cost of 6 kg onion, 2 kg wheat and 3 kg rice is Rs. 70.
Find the cost of each item per kg by matrix method.

Summary

- Determinant of a matrix $A = [a_{11}]_{1\times 1}$ is given by $|a_{11}| = a_{11}$.
- Determinant of a matrix $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ is given by $|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$.
- Determinant of a matrix $A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$ is given by (expanding along R_1)

  $|A| = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$.
- For any square matrix A, the determinant $|A|$ satisfies the following properties.
- $|A'| = |A|$, where A' is the transpose of A.
- If we interchange any two rows (or columns), then the sign of determinant changes.
- If any two rows or any two columns are identical or proportional, then the value of determinant is zero.
- If we multiply each element of a row or a column of a determinant by a constant k, then the value of determinant is multiplied by k.
- Multiplying a determinant by k means multiplying elements of only one row (or one column) by k.
- If $A = [a_{ij}]_{3\times 3}$, then $|kA| = k^3|A|$.
- If elements of a row or a column of a determinant can be expressed as sum of two or more elements, then the determinant can be expressed as sum of two or more determinants.
- If to each element of a row or a column of a determinant the equimultiples of corresponding elements of other rows or columns are added, then the value of determinant remains same.

- Area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by $\Delta = \dfrac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$.

- Minor of an element a_{ij} of the determinant of matrix A is the determinant obtained by deleting i^{th} row and j^{th} column and is denoted by M_{ij}.

- Cofactor of a_{ij} is given by $A_{ij} = (-1)^{i+j} M_{ij}$.

- Value of determinant of a matrix A is obtained by sum of product of elements of a row (or a column) with corresponding cofactors. For example, $|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$.

- If elements of one row (or column) are multiplied with cofactors of elements of any other row (or column), then their sum is zero. For example, $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} = 0$.

- If $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$, then $adj\, A = \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$, where A_{ij} is the cofactor of a_{ij}.

- $A(adj\, A) = (adj\, A)A = |A|I$, where A is a square matrix of order n.

- A square matrix A is said to be singular or non-singular according as $|A| = 0$ or $|A| \neq 0$.

- If $AB = BA = I$, where B is a square matrix, then B is called inverse of A. Also $A^{-1} = B$ or $B^{-1} = A$ and hence $(A^{-1})^{-1} = A$.

- A square matrix A has inverse if and only if A is non-singular.

- $A^{-1} = \dfrac{1}{|A|}(adj\, A)$.

- If $a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$, $a_3x + b_3y + c_3z = d_3$, then these equations can be written as $AX = B$, where $A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $B = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$.

- Unique solution of equation $AX = B$ is given by $X = A^{-1}B$, where $|A| \neq 0$.
- A system of equation is consistent or inconsistent according as its solution exists or not.
- For a square matrix A in matrix equation $AX = B$:

  (i) If $|A| \neq 0$, there exists a unique solution.

  (ii) If $|A| = 0$ and $(adj\ A)B \neq 0$, then there exists no solution.

  (iii) If $|A| = 0$ and $(adj\ A)B = 0$, then the system may or may not be consistent.