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CBSE Class XII
Task-2 : IMO 2001 Problems

1. Let ABC be an acute-angled triangle with circumcentre O . Let P be the foot of the altitude from A on BC .

Suppose that

$$\angle BCA \geq \angle ABC + 30^\circ.$$

Prove that

$$\angle CAB + \angle COP < 90^\circ.$$

2. Prove that

$$\frac{a}{\sqrt{a^2 + 8bc}} + \frac{b}{\sqrt{b^2 + 8ca}} + \frac{c}{\sqrt{c^2 + 8ab}} \geq 1$$

for all positive real numbers a, b, c .

3. Twenty-one girls and twenty-one boys took part in a mathematical contest.

- Each contestant solved at most six problems.
- For each girl and each boy, at least one problem was solved by both of them.

Prove that there was a problem that was solved by at least three girls and at least three boys.

4. Let n be an odd integer greater than 1 and k_1, k_2, \dots, k_n be integers.

For each permutation

$$a = (a_1, a_2, \dots, a_n)$$

of $1, 2, \dots, n$, define

$$S(a) = \sum_{i=1}^n k_i a_i.$$

Prove that there exist two distinct permutations b and c such that

$$n! \mid (S(b) - S(c)).$$

5. In triangle ABC , let AP bisect $\angle BAC$ and BQ bisect $\angle ABC$.

Given that

$$\angle BAC = 60^\circ \quad \text{and} \quad AB + BP = AQ + QB,$$

find the possible angles of triangle ABC .

6. Let a, b, c, d be integers such that

$$a > b > c > d > 0$$

and

$$ac + bd = (b + d + a - c)(b + d - a + c).$$

Prove that $a + b + c + d$ is not prime.