IIITB COMET Foundation

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CBSE Class XII
Task–2 : IMO 2001 Problems

1. Let ABC be an acute–angled triangle with circumcentre O. Let P be the foot of the altitude from A on BC. Suppose that

$$\angle BCA \geq \angle ABC + 30°.$$

   Prove that

$$\angle CAB + \angle COP < 90°.$$

2. Prove that

$$\frac{a}{\sqrt{a^2 + 8bc}} + \frac{b}{\sqrt{b^2 + 8ca}} + \frac{c}{\sqrt{c^2 + 8ab}} \geq 1$$

   for all positive real numbers a, b, c.

3. Twenty–one girls and twenty–one boys took part in a mathematical contest.

   - Each contestant solved at most six problems.
   - For each girl and each boy, at least one problem was solved by both of them.

   Prove that there was a problem that was solved by at least three girls and at least three boys.

4. Let n be an odd integer greater than 1 and let $k_1, k_2, \ldots, k_n$ be integers. For each permutation

$$a = (a_1, a_2, \ldots, a_n)$$

   of $1, 2, \ldots, n$, define

$$S(a) = \sum_{i=1}^{n} k_i a_i.$$

   Prove that there exist two distinct permutations b and c such that

$$n! \mid (S(b) - S(c)).$$

5. In triangle ABC, let AP bisect $\angle BAC$ and BQ bisect $\angle ABC$. Given that

$$\angle BAC = 60° \quad \text{and} \quad AB + BP = AQ + QB,$$

   find the possible angles of triangle ABC.

6. Let a, b, c, d be integers such that

$$a > b > c > d > 0$$

and

$$ac + bd = (b + d + a - c)(b + d - a + c).$$

Prove that $a + b + c + d$ is not prime.