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*Task 2*

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*Each problem is worth seven points.*

1. Let  $ABC$  be an acute-angled triangle with circumcentre  $O$ . Let  $P$  on  $BC$  be the foot of the altitude from  $A$ .

Suppose that  $\angle BCA \geq \angle ABC + 30^\circ$ .

Prove that  $\angle CBA + \angle COP < 90^\circ$ .

2. Prove that

$$\frac{a}{\sqrt{a^2 + 8bc}} + \frac{b}{\sqrt{b^2 + 8ca}} + \frac{c}{\sqrt{c^2 + 8ab}} \geq 1$$

for all positive real numbers  $a$ ,  $b$ , and  $c$ .

3. Twenty-one girls and twenty-one boys took part in a mathematical contest.

- Each contestant solved at most six problems.
- For each girl and each boy, at least one problem was solved by both of them.

Prove that there was a problem that was solved by at least three girls and at least three boys.

4. Let  $n$  be an odd integer greater than 1, and let  $k_1, k_2, \dots, k_n$  be given integers. For each of the  $n!$  permutations

$$a = (a_1, a_2, \dots, a_n)$$

of  $1, 2, \dots, n$ , let

$$S(a) = \sum_{i=1}^n k_i a_i.$$

Prove that there are two permutations  $b$  and  $c$ ,  $b \neq c$ , such that  $n!$  divides  $S(b) - S(c)$ .

5. In a triangle  $ABC$ , let  $AP$  bisect  $\angle BAC$ , with  $P$  on  $BC$ , and let  $BQ$  bisect  $\angle ABC$ , with  $Q$  on  $CA$ .

It is known that  $\angle BAC = 60^\circ$  and that

$$AB + BP = AQ + QB.$$

What are the possible angles of triangle  $ABC$ ?

6. Let  $a, b, c, d$  be integers with  $a > b > c > d > 0$ . Suppose that

$$ac + bd = (b + d + a - c)(b + d - a + c).$$

Prove that  $a + b + c + d$  is not prime.