IIITB COMET FOUNDATION

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CBSE Class XII
Task–2 : IMO 2002 Problems

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1. Let S be the set of all (h, k) with h, k non–negative integers such that

   $h + k < n$.

   Each element of S is colored red or blue, so that if (h, k) is red and

   $h' \leq h, \quad k' \leq k,$

   then (h', k') is also red.

   A type–1 subset of S has n blue elements with different first members, and a type–2 subset of S has n blue elements with different second members.

   Show that there are the same number of type–1 and type–2 subsets.

2. BC is a diameter of a circle with center O. A is any point on the circle such that

   $\angle AOC > 60°$.

   EF is the chord which is the perpendicular bisector of AO. D is the midpoint of the minor arc AB. The line through O parallel to AD meets AC at J.

   Show that J is the incenter of triangle CEF.

3. Find all pairs of integers $m > 2$ and $n > 2$ such that there are infinitely many positive integers k for which

   $k^n + k^2 - 1 \mid k^m + k - 1$.

4. Let the positive divisors of an integer $n > 1$ be $d_1 < d_2 < \cdots < d_k$, where $d_1 = 1$ and $d_k = n$. Let

   $d = d_1 d_2 + d_2 d_3 + \cdots + d_{k-1} d_k$.

   Show that $d < n^2$ and determine all n for which d divides n^2.

5. Find all real–valued functions f defined on $\mathbb{R}$ such that

   $f(x) + f(y)\big(f(f(x)) + f(y)\big) = f(x - y) + f(xy + y)$

   for all real x, y.

6. Let a, b, c, d be integers such that

   $a > b > c > d > 0$

   and

   $ac + bd = (b + d + a - c)(b + d - a + c)$.

   Prove that $a + b + c + d$ is not prime.