DAA Tutorial

DATE ____________
PAGE ____________

Name :- Harshit Dhiman.
Section :- D
Roll no :- 7
University Roll no :- 2016768

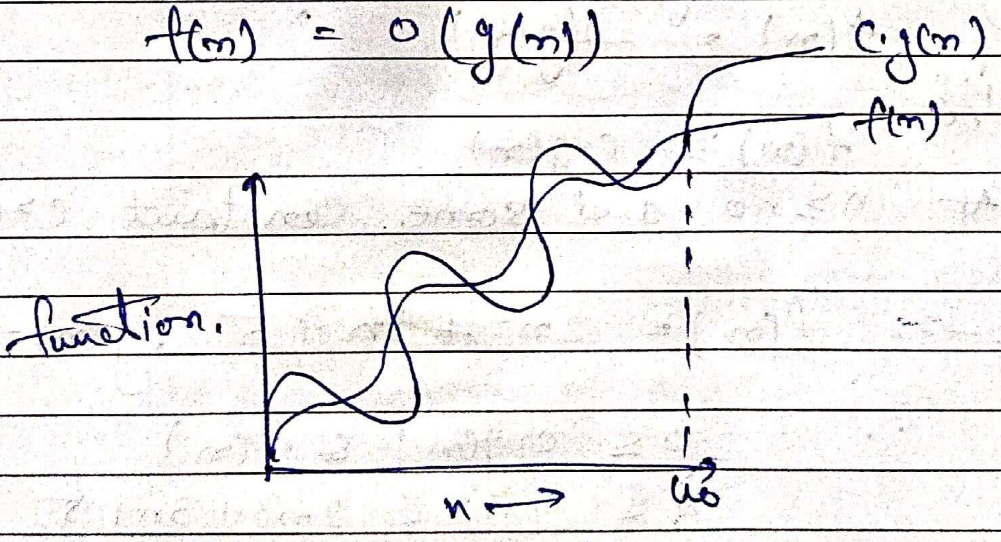
Soln :- 1. -

Asymptotic Notations :- Asymptotic mean tending to infinite or very large. These are used to tell the complexity for very large input.

Diff. Asymptotic notations :-

Big O (O)

$f(n) = O(g(n))$



$g(n)$ is "tight" upper bound

$f(n) = O(g(n))$

iff

$f(n) \leq c.g(n)$

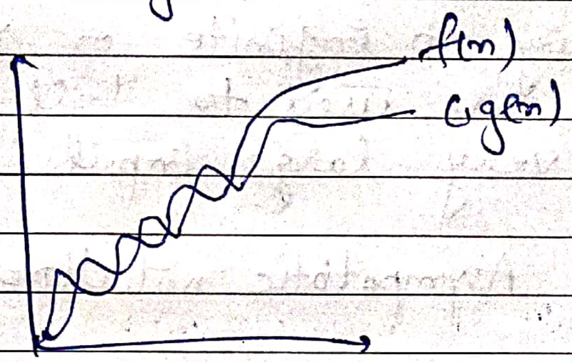
$\forall$ $n \geq n_0$ and some const., $c > 0$

Eg:- for (i=1 ; i<=n; i++){
print (i); —— O(1)
}

$\Rightarrow T(n) = O(n)$

(ii) Big Omega (Ω):

$f(n) = \Omega g(n)$



g(n) is "tight" lower bound

$f(n) = \Omega(g(n))$

iff

$f(n) \geq c.g(n)$

$\forall$ $n \geq n_0$ and some constant $c > 0$

eg:- $f(n) = 2n^2 + 3n + 5$, $g(n) = n^2$

$\because$ $0 \leq c.g(n) \leq f(n)$

$0 \leq c.n^2 \leq 2n^2+3n+5$

$c \leq 2 + \frac{3}{n} + \frac{5}{n^2}$

on putting $n = \infty$, $\Rightarrow \frac{3}{n} \to 0$, $\frac{5}{n^2} \to 0$

$\Rightarrow c = 2$;

DATE

PAGE

$$2n^2 \le 2n^2 + 3n + 5$$

on putting $n=1$

$$2 \le 2+3+5$$

$$2 \le 10 \text{ , True}$$

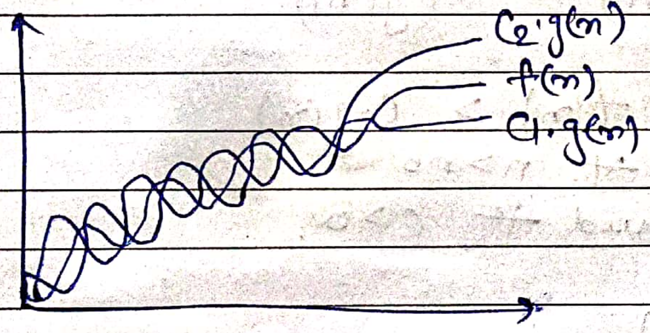
$$\boxed{c=2 \text{ , } n = n_0 = 1}$$

$$0 \le 2n^2 \le 2n^2 + 3n + 5$$

$$f(n) = \Omega(n^2)$$

(iii) Big Thita (θ) :

$$f(n) = \theta(g(n))$$



g(n) is both "tight" upper and lower bound of f(n)

$$f(n) = \theta(g(n))$$

iff

$$c_1 \cdot g(n) \le f(n) \le c_2 \cdot g(n)$$

$\forall$ $n \ge \max(n_1, n_2)$

and some const. $c_1 > 0$, $c_2 > 0$.

(iv) Small oh (o) :-

$$f(n) = o(g(n))$$

$g(n)$ is the upper bound of the function $f(n)$

$$f(n) = o(g(n))$$

when, $f(n) \le c \cdot g(n)$

$\forall$ $n > n_0$

and $\forall$ constant, $c > 0$

(v) Small Omega (ω):—

$$f(n) = \omega(g(n))$$

$g(n)$ is lower bound of the function $f(n)$

$$f(n) = \omega(g(n))$$

when

$$f(n) > c \cdot g(n)$$

$\forall$ $n > n_0$

and $\forall$ $c > 0$,

Q Solution 2 :—

$$i = \underbrace{1, 2, 4, 8, 16 \; - - - - \; n.}_{k \text{ terms}}$$

$\Rightarrow$ $a = 1$, $r = 2$

$\Rightarrow$ k^{th} term.

$$t_k = a r^{k-1}$$

$$n = 1 \cdot 2^{k-1}$$

$$n = 2^{k-1}$$

DATE
PAGE

taking log both side

$$\log_2 n = \log_2 2^{k-1}$$

$$\log_2 n = (k-1)\log_2 2$$

$$\log_2 n = k-1 \quad [\because \log_a a = 1]$$

$$\Rightarrow k = 1 + \log_2 n$$

$$\Rightarrow T(n) = O(k)$$
$$= O(1+\log_2 n)$$
$$= O(\log_2 n)$$

Solution (3) :-

$$T(n) = 3T(n-1) \quad \text{—— (1)}$$

put $n = n-1$ in eqn (1)

$$T(n-1) = 3T(n-2) \quad \text{—— (2)}$$

put this value in (1)

$$T(n) = 3[3T(n-2)] \quad \text{—— (3)}$$

put $n = n-2$

$$T(n-2) = 3T(n-3) \quad \text{—— (4)}$$

put (4) in (3)

$$T(n) = 9[3T(n-3)]$$
$$T(n) = 27\,T(n-3)$$

Scanned by CamScanner

DATE
PAGE

Generalised form !—

$$T(n) = 3^k T(n-k)$$

put $n-k=0$

$$\Rightarrow T(n) = 3^n T(0)$$
$$T(0) = 1$$
$$T(n) = 3^n$$
$$\Rightarrow O(3^n)$$

Solution (5) !—

```
int i=1 , s=1 ;
  while ( s<=n){
    i++ ;  s=s+i ;
    print ("#") ;
```
$\longrightarrow O(1)$

$$\Rightarrow s = \underbrace{1, 3, 6, 10, 15 \; --- \; n.}_{k\ \text{term.}}$$

Kth term
$$t_k = t_{n-1} + k$$
$$k = t_n - t_{k-1} \quad — (1)$$

Loop runs k times

$$T.C = O(1+1+1+ n --- t_{k-1})$$
but $t_{n-1} = c$ (const.)
$$T.C = O(\cancel{1} + n - \cancel{c})$$
$$= O(n).$$

Scanned by CamScanner

DATE
PAGE

Solution (6) :-

Time Complexity of :-

```
void function (int n) {          — O(1).
int i, count = 0;                — O(1)
for (i=1 ; i<=n; i++) {
    count ++;                    — O(1).
}
```

$i*i = \underbrace{1^2, 2^2, 3^2, 4^2, 5^2 - - - n.}_{n \text{ terms}}$

Kth term

$t_K = K^2$

$K^2 = n$

$K = n^{1/2}$

$T.C = O(1+1+1+ n^{1/2} +1)$

$= O(n^{1/2}) = O(\sqrt{n}).$

Solution (7) :—

```
void function (int n) {                          — O(1)
    int i, j, k, count = 0;                      — O(1)
for ( i = n/2 ; i <= n ; i++ )
  for ( j = 1 ; j <= n ; j = j*2 )               log2 n
    for ( k = 1 ; k <= n ; k = k*2 )             log2 n
      count ++ ;                                 — O(1)
}.
```

$T.C = (\log_2 n)^2.$

DATE

PAGE

Solution 8 :-

```
function (int n){
    if (n==1) return ;          —— O(1)
  for (i=1 to n) {              —— O(n)
   for (j=1 to n) {             —— O(n)
      printf ("*") ;            —— O(1)
   }
  }
  function (n-3);
}
```

for function call,

$n, \; n-3, \; n-6, \; n-9 \; ---- \; 1$

A.P with $d = -3$

$\Rightarrow \; l = a + (k-1)d$

$1 = n + (k-1)(-3)$

$\frac{1-n}{(-3)} = k-1$

$k = \frac{n-1+3}{3}$

$$\boxed{k = \frac{n+2}{3}}$$

function gives a recursive call $\frac{n+2}{3}$ times

Time complexity $= \frac{(n+2)}{3}(n)(n)$

$= n^3$

$\Rightarrow O(n^3).$

DATE
PAGE

$i = n/2, \frac{n+2}{2}, \frac{n+4}{2}, \frac{n+6}{2}$ --- upto n.

$\rightarrow \frac{n+0\times2}{2}, \frac{n+1\times2}{2}, \frac{n+2\times2}{2}, \frac{n+3\times2}{2}$ --- n.

general term.

$$t_k = \frac{n+k\times2}{2}$$

total terms = k+1

$$t_{k+1} = n$$

$$\frac{n+(k+1)\times2}{2} = n$$

$$n+2k+2 = 2n$$

$$2k = n-2$$

$$k = \frac{n}{2} - 1$$

i	j	k
$n/2$	$\log_2 n$ times	$(\log_2 n)^2$
$n+2/2$	"	$(\log_2 n)^2$
$n+4/6$	"	$(\log_2 n)^2$
⋮	⋮	⋮
n	$\log_2 n$ times	$(\log_2 n)^2$
$\left(\frac{n}{2}-1\right)$ times		

$$\rightarrow \left(\frac{n}{2}-1\right)(\log_2 n)^2$$

$$= O\left(\frac{n}{2}\log^2 n - \log^2 n\right)$$

$$\Rightarrow O(n\log^2 n)$$

DATE
PAGE

Solution – 9

for $i = 1 \longrightarrow j = 1, 2, 3, 4 \text{ ---- } n = n.$
for $i = 2 \longrightarrow j = 1, 3, 5 \text{ ---- }, n = n/2$
for $i = 3 \longrightarrow j = 1, 4, 7 \text{ ---- }, n = n/3$

for $i = n \Rightarrow j = 1, \text{ ---- }, n = 1$

$\rightarrow$ for $i = n \Rightarrow j = 1, \text{ ---- } n = 1$

$\sum_{j=n}^{1} n + n/2 + n/3 \text{ ---- } + 1$

$\sum_{j=n}^{1} n \left[1 + 1/2 + 1/3 + \text{ --- } + 1/n \right]$

$\sum_{j=n}^{1} n \log n$

$T(n) = [n \log n]$
$T(n) = \underline{O(n \log n)}$

Solution – 10

As given, n^k & c^n
Relation b/w n^k & c^n is $n^k = O(c^n)$
as $n^k \leq a c^n \quad \forall$
$n \geq n_0$, for (a) Constant $(a \geq 0)$

for $n_0 = 1$
$c = 2$
$\Rightarrow 1^k \leq a 2^1$

$\boxed{\therefore n_0 = 1 \text{ \& } c = 2}$