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Section :- D
Roll no. :- 03

Tutorial - 4

DATE
PAGE

1. $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

$T(n) = aT\left(\frac{n}{2}\right) + f(n)$

$a \geq 1 \; , \; b > 1$

On comparing,

$a = 3 \; , \; b = 2 \; , \; f(n) = n^2$

Now,

$c = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$\therefore \; f(n) > n^c$

$\therefore \; T(n) = \theta(n^2)$

2. $T(n) = 4T(n/2) + n^2$

$a \geq 1 \; , \; b > 1$

$a = 4 \; , \; b = 2 \; , \; f(n) = n^2$

$c = \log_2 4 = 2$

$\therefore \; n^c = n^2 = f(n) = n^2$

$\therefore \; T(n) = \theta(n^2 \log_2 n)$

3. $T(n) = T(n/2) + 2^n$

$a = 1 \; , \; b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \theta(2^n)$

4. $T(n) = 2^n T(n/2) + n^n$

$a = 2^n$

$b = 2, \quad f(n) = n^n.$

$c = \log_b a = \log_2 2^n = n.$

$n^c \Rightarrow n^n$

$\therefore f(n) = n^c$

$T(n) = \Theta(n^2 \log_2 n)$

5. $T(n) = 16T\left(\frac{n}{4}\right) + n$

Solu!- $a = 16, \ b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2$

$n^c = n^2$

$f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

6. $T(n) = 2T(n/2) + n\log n)$

Solu!- $a = 2, \ b = 2$

$f(n) = n\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$n\log n > n$

$\therefore f(n) > n^c$

$T(n) = \Theta(n\log n)$

7. $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}.$

Solu!- $a = 2, \ b = 2, \ f(n) = n/\log n.$

$c = \log_2 2 = 1$

$\therefore n^c = n^1 = n.$

since, $\frac{n}{\log n} < n$.

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n)$

8. $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

Soln:- $a=2$, $b=4$, $f(n) = n^{0.51}$

$C = \log_b a = \log_4 2 = 0.5$

$\therefore n^c = n^{0.5}$

since, $n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \Theta(n^{0.51})$

9. $T(n) = 0.5\,T\left(\frac{n}{2}\right) + \frac{1}{n}$

Soln:- $a=0.5$, $b=2$

acc. to masters method, $a \geq 1$, but here a is 0.5. so we cannot apply master theorem

10. $T(n) = 16T\left(\frac{n}{4}\right) + n!$

Soln:- $a=16$, $b=4$, $f(n) = n!$

$\therefore C = \log_b a = \log_4 16 = 2$

Now, $n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \Theta(n!)$

DATE

PAGE

11. $4T(n/2) + \log n$.

$a = 4$, $b = 2$, $f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$

$f(n) = \log n$

since $\log n < n^2$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n^c)$

$= \Theta(n^2)$

12. $T(n) = \text{sqrt}(n) T(n/2) + \log n$

Soln $a = \sqrt{n}$, $b = 2$

$c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2} \log_2 n$

$\therefore \frac{1}{2} \log_2 n < \log(n)$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(f(n))$

$= \Theta(\log(n))$

13. $T(n) = 3T\left(\frac{n}{2}\right) + n$.

$a = 3$, $b = 2$, $f(n) = n$.

$c = \log_b a = \log_2 3 = 1.5849$.

$= n^c = n^{1.5849}$.

$\therefore n < n^{1.5849}$.

$f(n) < n^c$

$T(n) = \Theta(n^{1.5849})$

DATE

PAGE

14: $T(n) = 3T(n/3) + \text{Sqrt}(n)$

$a = 3$, $b = 3$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

As $\text{Sqrt}(n) < n$

$\therefore f(n) < n^c$

$T(n) = \Theta(n)$

15. $T(n) = 4T(n/2) + (n)$

$a = 4$, $b = 2$

$c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$

$c^n < n^2$

$f(n) < n^c$

$T(n) = \Theta(n^2)$

16 $T(n) = 3T(n/4) + n\log n$

$a = 3$, $b = 4$, $f(n) = n\log n$.

$c = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$\therefore n^{0.792} < n\log n$

$\therefore T(n) = \Theta(n\log n)$

17. $T(n) = 3T(n/3) + n/2$

$a = 3$, $b = 3$.

$c = \log_b a = \log_3 3 = 1$

$f(n) = n/2$

DATE

PAGE

$\therefore n^c = n^1 = n.$

As $n/2 < n$

$f(n) < n^c$

$\therefore T(n) = \Theta(n)$

18. $T(n) = 6T(n/3) + n^2 \log n$

$a=6, b=3$

$c = \log_b a = \log_3 6 = 1.6309.$

$n^c = n^{1.6309}$

As, $n^{1.6309} < n^2 \log n.$

$\therefore T(n) = \Theta(n^2 \log n)$

19. $T(n) = 4T(n/2) + n \log n.$

$a=4, b=2, f(n) = \frac{n}{\log n}.$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\therefore \frac{n}{\log n} < n^2$

$\therefore T(n) = \Theta(n^2)$

20. $T(n) = 64T(n/8) - n^2 \log n.$

$a=64, b=8$

$c = \log_b a = \log_8 64 = \log_8 (8)^2$

$c=2$

$\therefore n^c = n^2$

$n^2 \log n > n^2$

$\therefore T(n) = \Theta(n^2 \log n)$

DATE
PAGE

21. $T(n) = 7T\left(n/3\right) + n^2$

$a=7$, $b=3$, $f(n) = n^2$
$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$
$\Rightarrow n^{1.7712} < n^2$
$\therefore T(n) = \theta(n^2)$

22. $T(n) = T\left(n/2\right) + n\left(2 - \cos n\right)$

Soln :- $a=1$, $b=2$
$C = \log_b a = \log_2 1 = 0$

$\therefore n^c = n^0 = 1$
$\therefore n(2 - \cos n) > n^c$
$\therefore T(n) = \theta\left(n(2 - \cos n)\right)$