Communication Systems Lab
Assignment - 2

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September 1, 2022

1 FIR(Finite Impulse Response) Filter

It is a filter whose impulse response (or response to any finite length input) is of finite duration.
FIR filters are easy to design in discrete time and we use a simple method call window method to design our filter.

2 Window method

If the desired ideal frequency response is $Hd(e^{j\omega})$, we take IFFT of this to get hd[n].
But hd[n] is not of finite length, so we truncate it using a finite length window function w[n] to get h[n].
Now , h[n] = hd[n] $\times$ w[n]
In our code we use the hamming window function ,

$$w[n] = \begin{cases} 0.54 - 0.46cos(2n/M) & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$$

3 LPF

We know that the ideal low pass filter has a magnitude response which looks like a rectangle and if we take IFFT of it we get a sinc function i.e

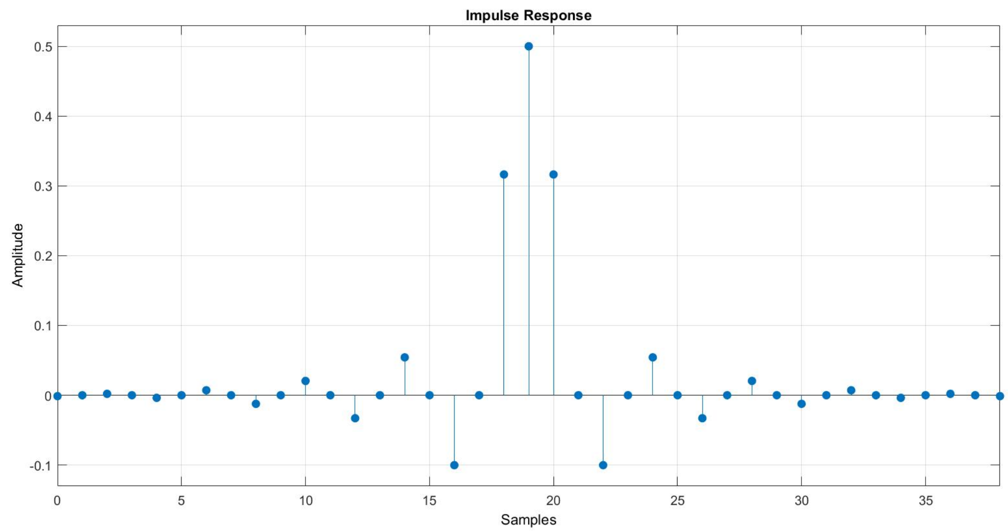
$$hd[n] = \frac{sin(\omega_c n)}{\pi n}, \text{where } \omega_c = \frac{2\pi f_c}{f_s}$$

Since this is of infinite length(IIR) we multiply it with a window function w[n] to get an FIR filter.
If we take w[n] with N taps then the resulting LPF impulse response,h[n] = hd[n] $\times$ w[n].

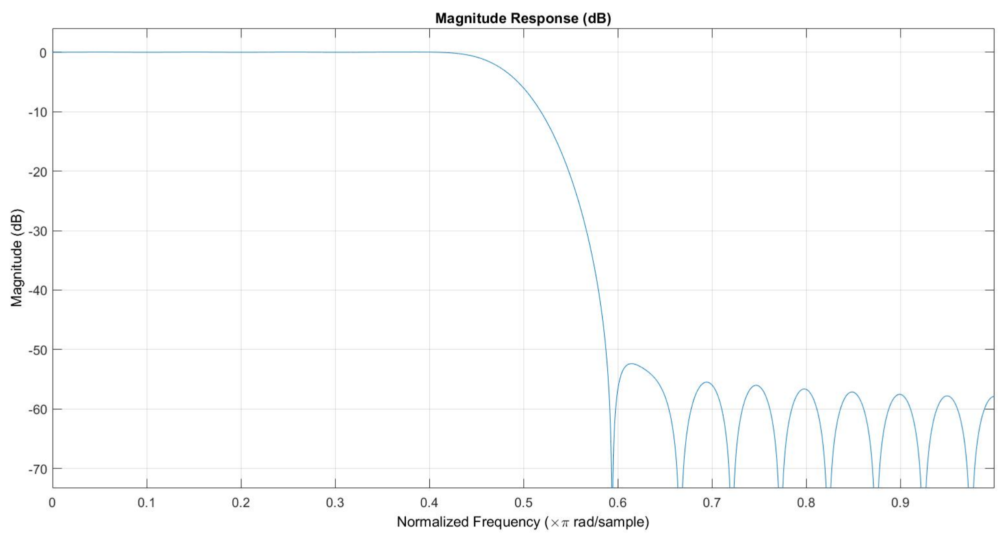
$$h_d[n] = \begin{cases} \frac{sin(\omega_c n)}{\pi n} & \text{if } -(N-1)/2 \leq n \leq (N-1)/2 \\ \frac{\omega_c}{\pi}, & \text{if } n = 0 \end{cases}$$

4 LPF 1

Given $f_c = 400$ Hz, $\omega_c = \pi/2$ and N = 39
So $f_s = 2\pi f_c/\omega_c = 1600Hz$
This is a special case of LPF called as half band filter where $f_c = f_s/4$
Impulse response of half band filter is 1/2 when n = 0 and 0 otherwise.
We get the following plots from the code -



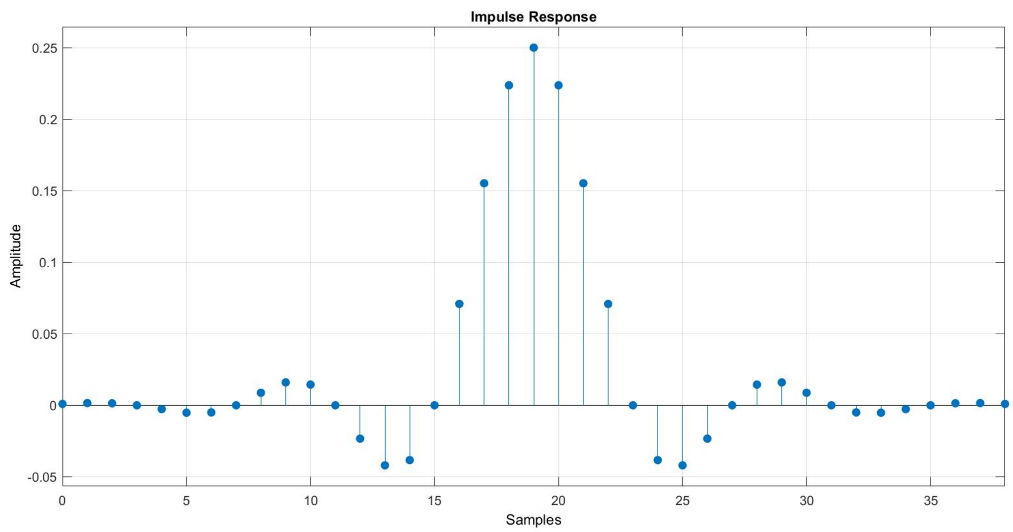
Since we multiplied a w[n] we don't get an ideal impulse response which we can see in above plot but it is close to the ideal halfband impulse response.



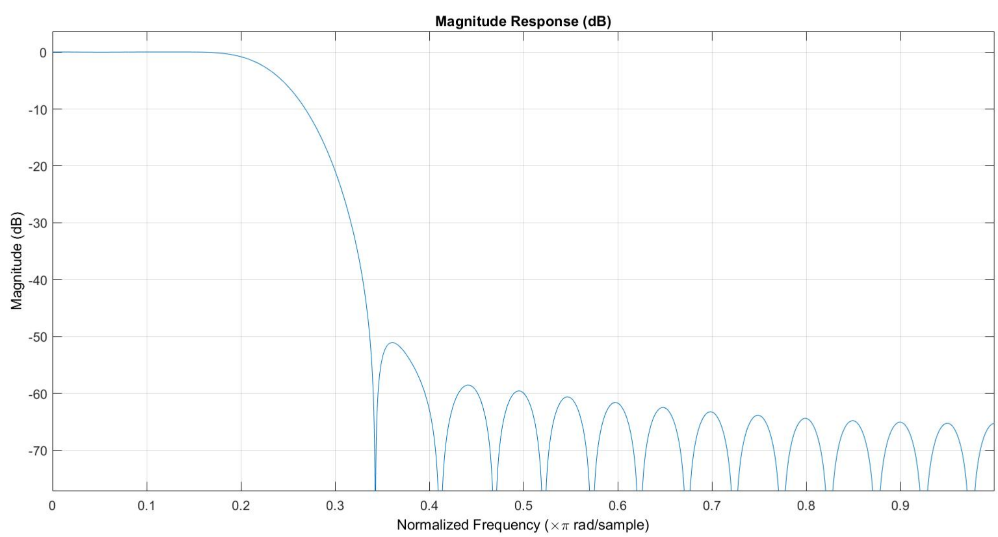
We can see that there are some stopband ripples in our magnitude response and **since we used the hamming window function we get very less ripples compared to other window functions.**

5 LPF 2

Given $f_c = 400$ Hz, $\omega_c = \pi/4$ and N = 39
So $f_s = 2\pi f_c/\omega_c = 3200Hz$
We can generalise a halfband filter to an M-band filter and this one is a 4-band filter.
We get the following plots from the code -



We see that at n = 0, the amplitude of impulse response is 1/4 as expected.



We can see that there are some stopband ripples in our magnitude response same as above filter but the bandwidth is reduced since ω_c is smaller

6 BPF

To get the magnitude response of BPF we subtract the magnitude responses of LPFs with cutoff frequencies ω_{c_1} and frequency ω_{c_2}.

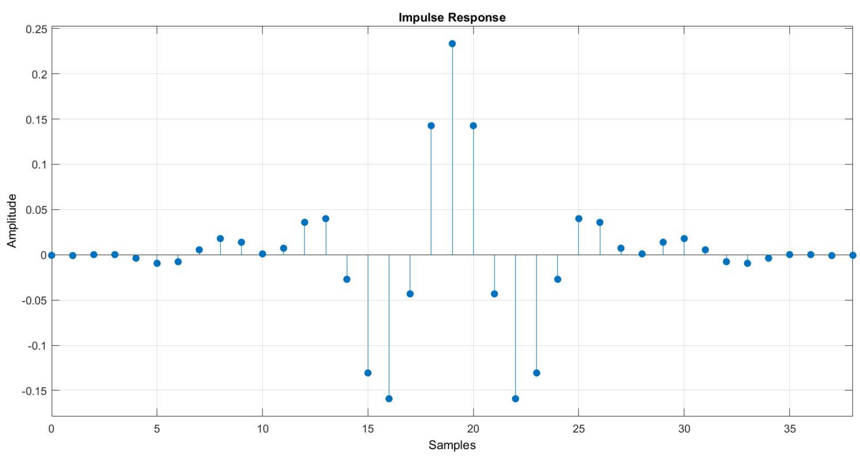
$$hd[n] = \frac{sin(\omega_{c_1} n)}{\pi n} - \frac{sin(\omega_{c_2} n)}{\pi n}$$

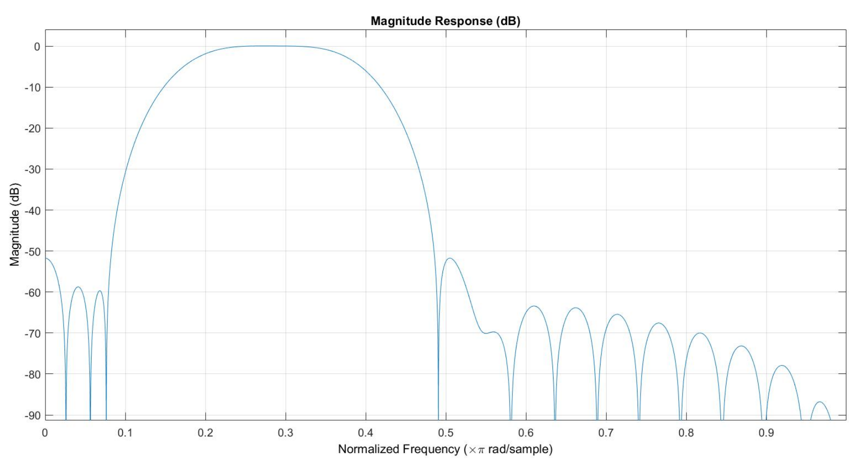
$$\text{For n} = 0,\ hd[n] = \frac{\omega_{c_1} - \omega_{c2}}{\pi n}$$

For a practical BPF we take w[n] with N taps, then the resulting BPF impulse response,h[n] = hd[n] $\times$ w[n].

$$h_d[n] = \begin{cases} \frac{sin(\omega_{c_1} n)}{\pi n} - \frac{sin(\omega_{c_2} n)}{\pi n}, & \text{if } -(N-1)/2 \leq n \leq (N-1)/2 \\ \frac{\omega_c n}{\pi n}, & \text{if } n = 0 \end{cases}$$

Given f_{c_1} = 500 Hz, f_{c_2} = 1200 Hz, f_s = 6000 Hz and N = 39





As we see from the above plots that the filter only allows frequencies in a range.