DSP LAB
Assignment - 2

S.V.Harshith - EE19BTECH11018

February 5, 2022

1 Decimator

This process consists of a low pass filter followed by a downsampler.

1.1 LPF

- The job of LPF here is to prevent aliasing. Hence it is known as anti-aliasing filter.
- Gain = 1
- Cutoff frequency is π/M.

1.2 Downsampler

- It is used to make a digital audio signal smaller by lowering its sampling rate or sample size (bits per sample).
- Downsampling is done to decrease the bit rate when transmitting over a limited bandwidth or to convert to a more limited audio format.
- Time domain relation between input and output -

$$y[n] = x[Mn]$$

  Where M is the factor of downsampling.
- We can see that it is a Linear Time Varying (LTV) system.
- Frequency domain relation between input and output -

$$Y(e^{j\omega}) = \frac{1}{M}\sum_{k=0}^{N-1} X(e^{j\frac{(\omega-2\pi k)}{M}})$$

- The input signal bandwidth must be less than 2π/M so that there is no aliasing.

2 Interpolator

This process consists of an upsampler followed by a low pass filter.

2.1 LPF

- The job of LPF here is to remove unwanted image of $X(e^{j\omega})$. Hence it is known as anti-imaging filter.
- Gain = L
- Cutoff frequency is π/L.

2.2 Upsampler

- To make a digital audio signal higher quality by increasing the sample rate, and interjecting new samples in between existing samples.
- The sample size is also increased for finer granularity. The objective is to have a smoother digital wave going into the digital-to-analog converter
- Time domain relation between input and output -

  $$y[n] = \begin{cases} x[n/L] & \text{, if n is multiple of L} \\ 0 & \text{, otherwise} \end{cases}$$

  Where L is the factor of upsampling.
- We can see that it is a Linear Time Varying (LTV) system.
- Frequency domain relation between input and output -

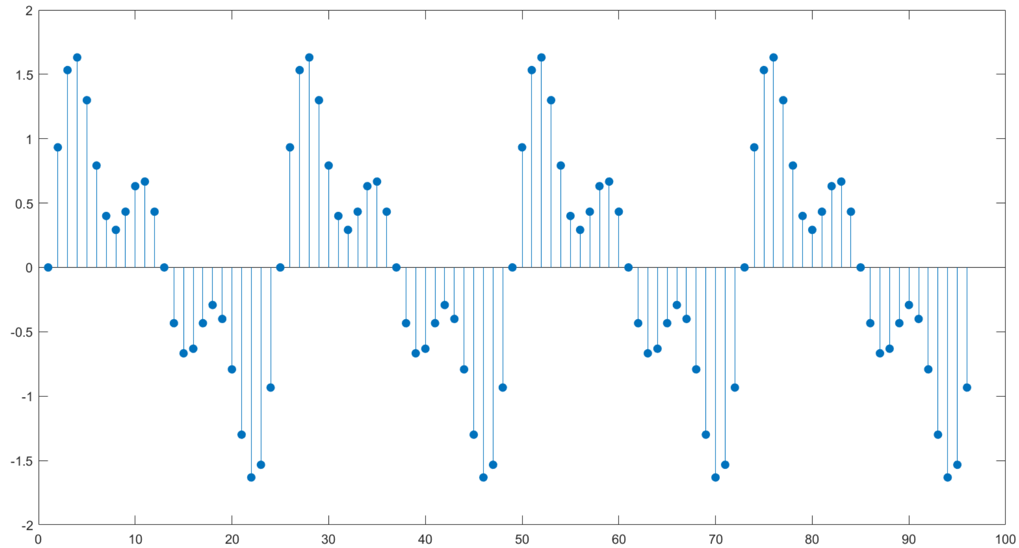
  $$Y(e^{j\omega}) = X(e^{j\omega L})$$

3 Practical implementation

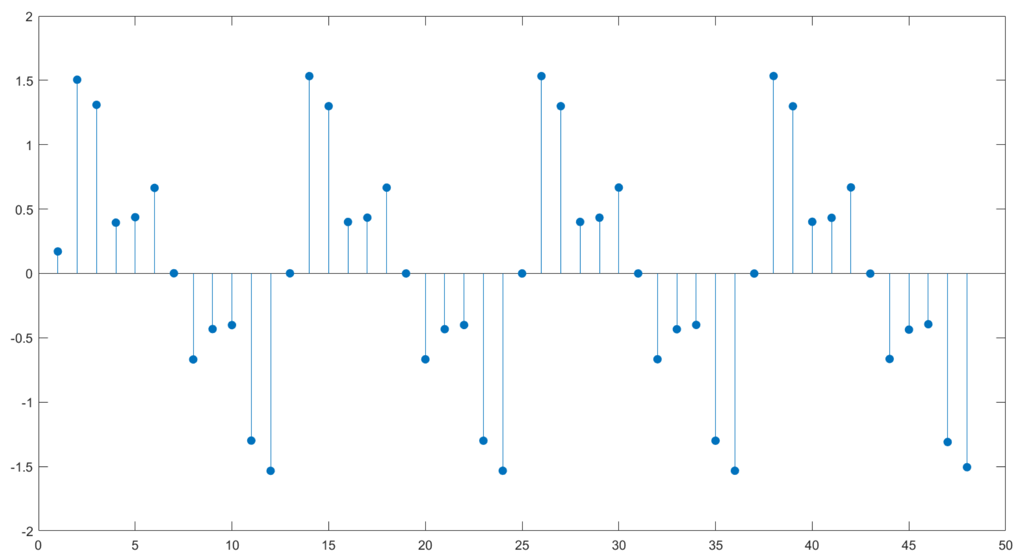
- In decimator after convolving the input signal(x[n]) with the low pass filter(h[n]) the length of output signal is $l_x + l_h - 1$ so we discard first and last $(l_h - 1)/2$ samples from xf[n] , i.e. take only middle l_x samples of xf[n] and then do downsampling.
- Similarly in interpolator also we discard first and last $(l_h - 1)/2$ samples from y[n] , i.e. take only middle l_{xu} samples of y[n].

4 M = L = 2

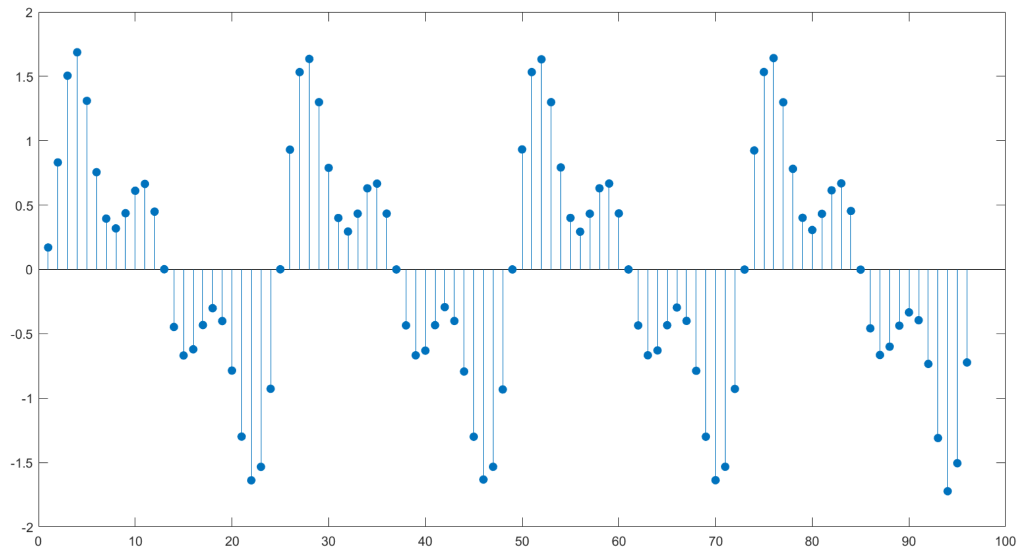
4.1 Input signal x[n]



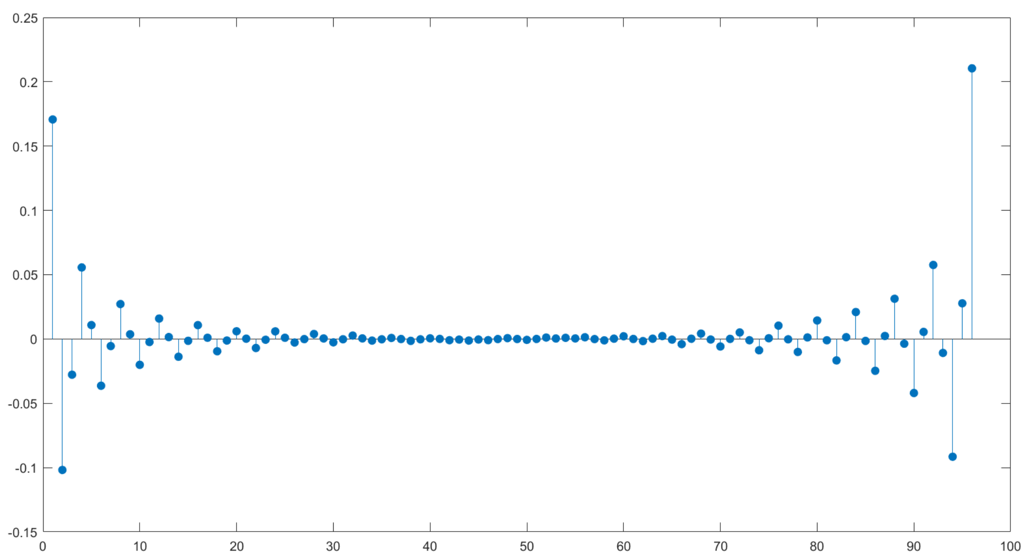
4.2 Decimated output $x_d[n]$



4.3 Interpolated output y[n]



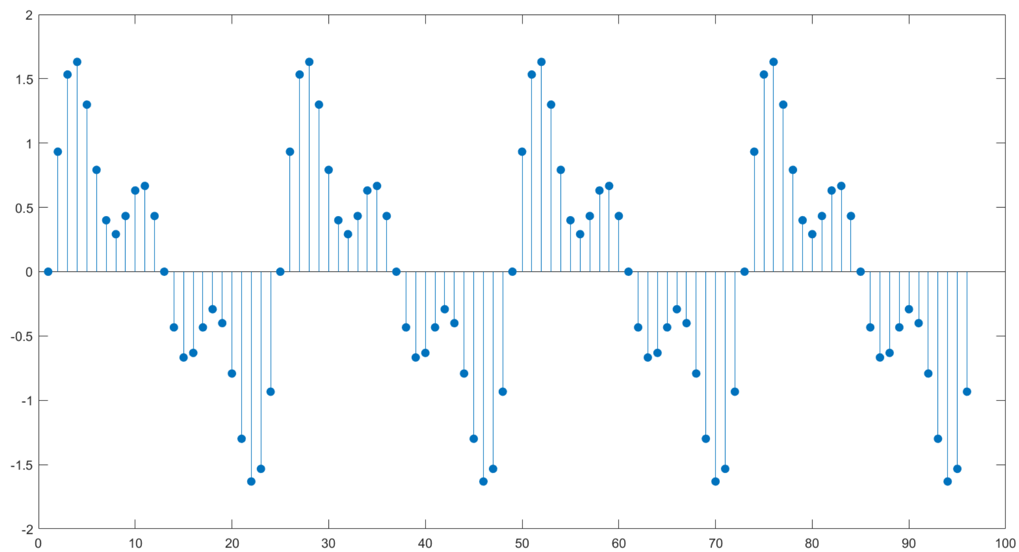
4.4 Error vector e[n]



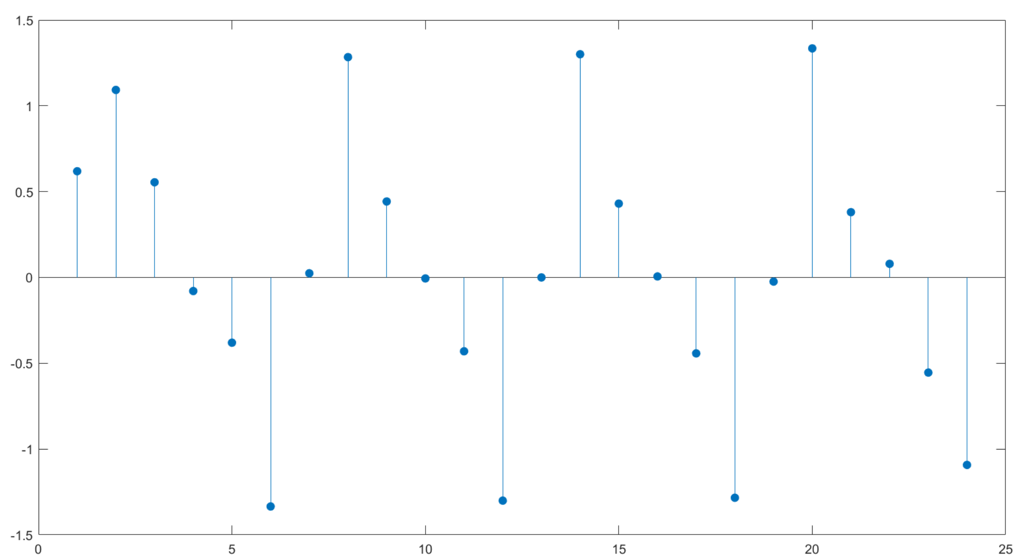
Average Error = 0.0124

5 $\mathbf{M = L = 4}$

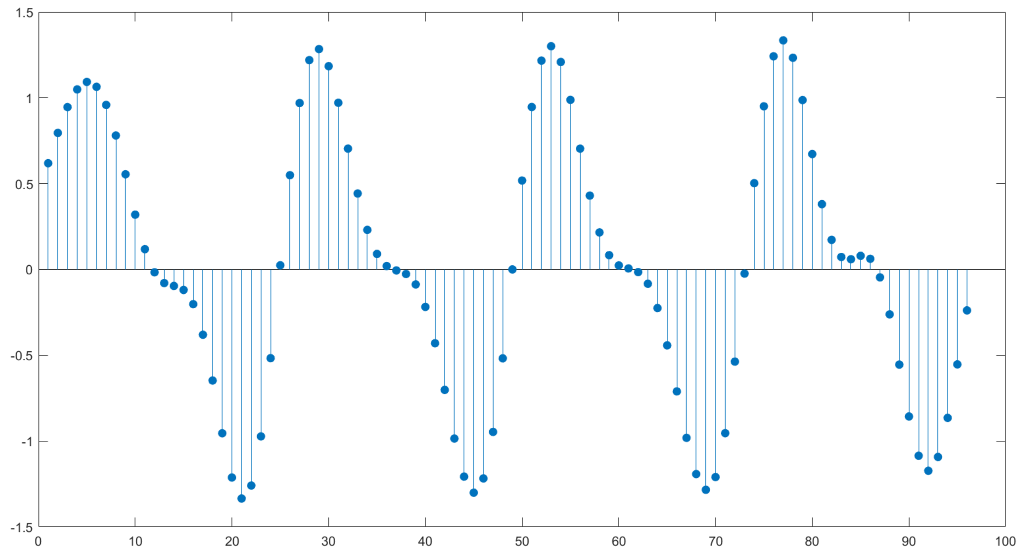
5.1 Input signal x[n]



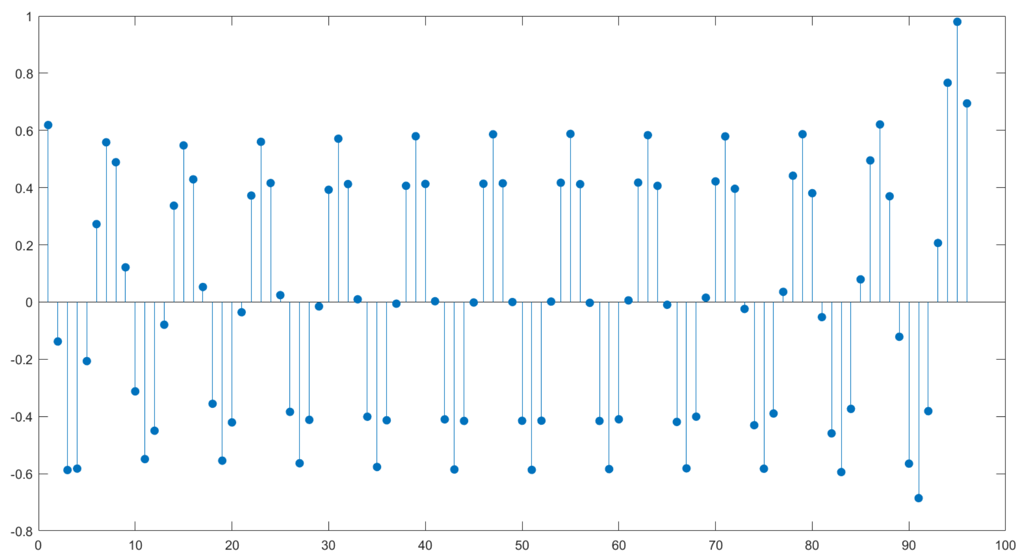
5.2 Decimated output $x_d[n]$



5.3 Interpolated output y[n]



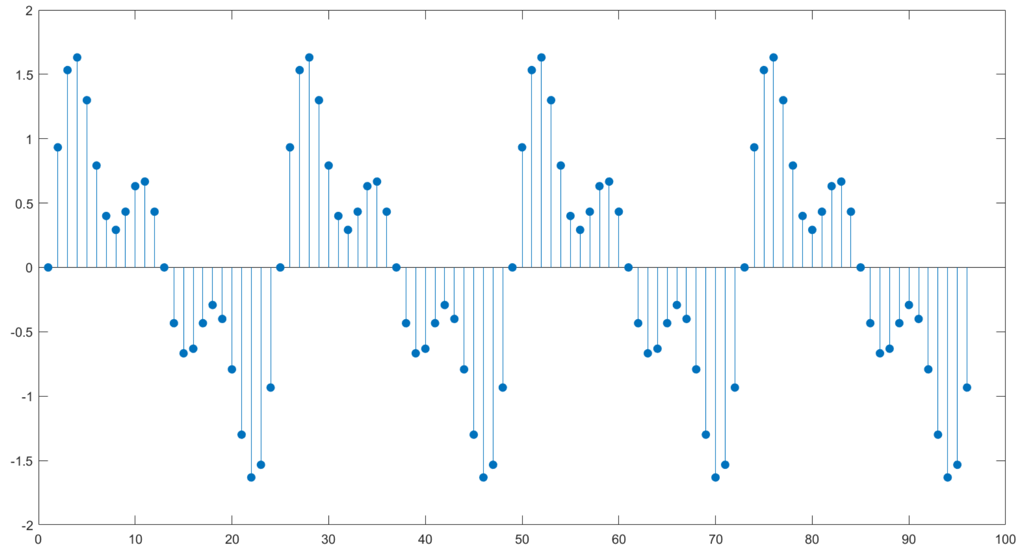
5.4 Error vector e[n]



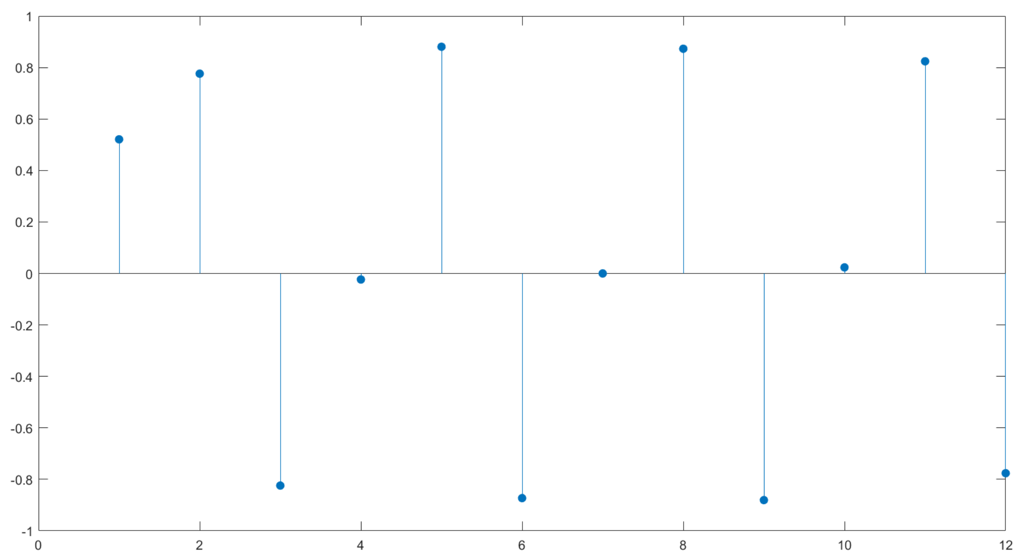
Average Error = 0.3777

6 M = L = 8

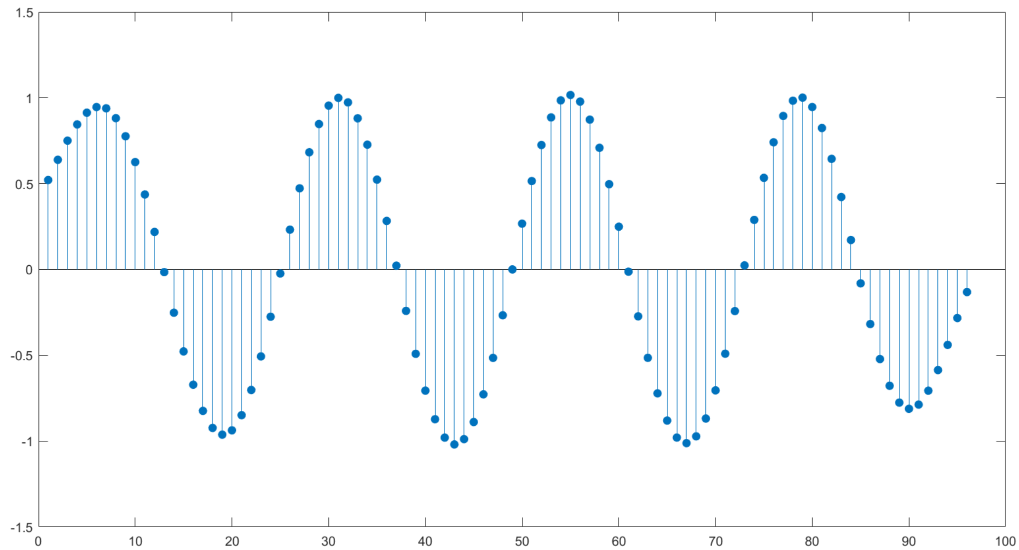
6.1 Input signal x[n]



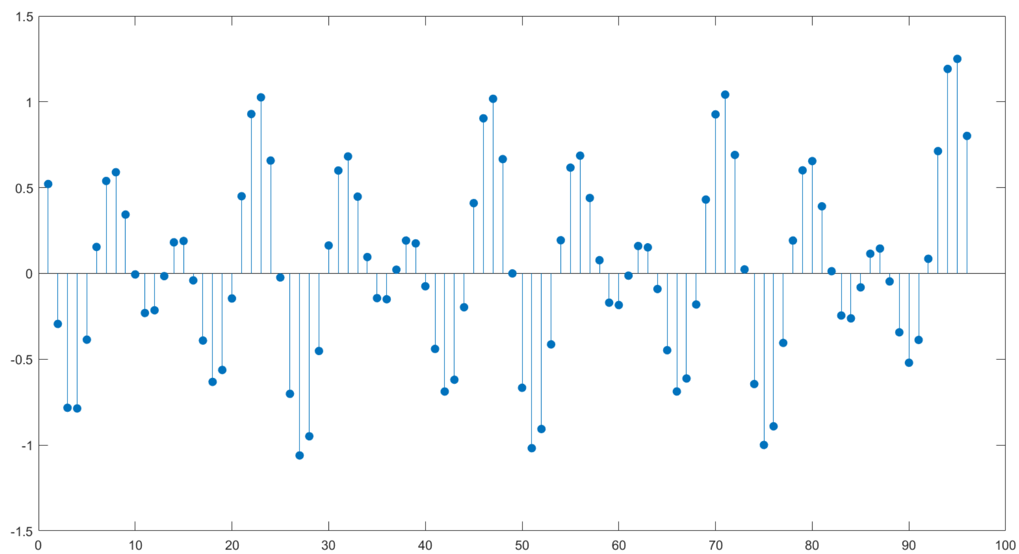
6.2 Decimated output $x_d[n]$



6.3 Interpolated output y[n]



6.4 Error vector e[n]



Average Error = 0.4452