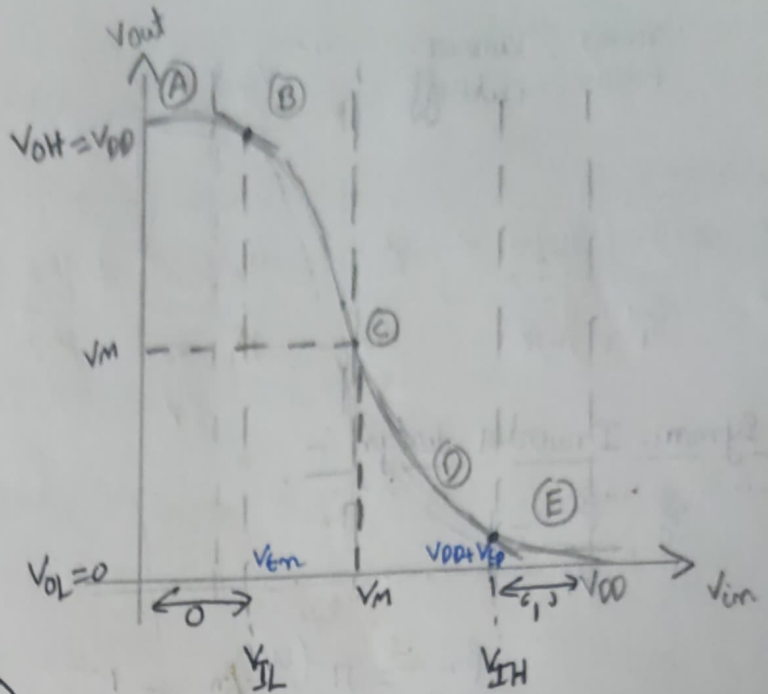
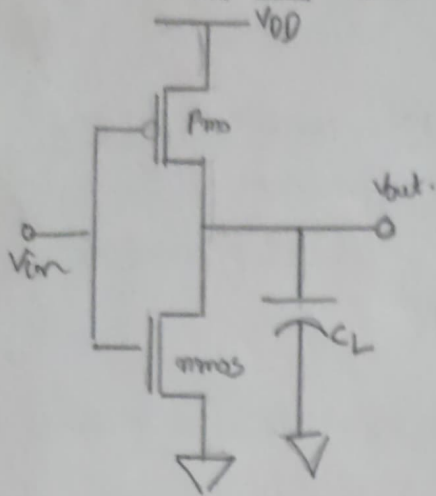


Inverter :-

DC characteristics :-



Region A :- ($0 \leq V_{in} \leq V_{tn}$)

nmos : cut off

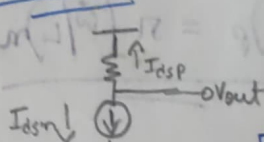
Pmos : linear region

$$V_{out} = V_{DD}$$

Region B :- ($V_{tn} \leq V_{in} \leq V_{DD}/2$)

nmos : saturation

Pmos : linear region



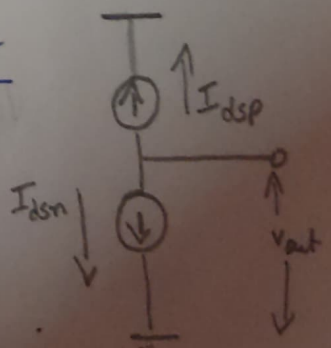
$$V_{out} = (V_{in} - V_{tp}) + \sqrt{(V_{in} - V_{tp})^2 - 2(V_{in} - \frac{V_{DD}}{2} - V_{tp}) - \frac{\beta_n}{\beta_p}(V_{in} - V_{tn})^2}$$

Region C :- ($V_{in} = \frac{V_{DD}}{2}$)

nmos : Saturation

Pmos : Saturation

$$V_m = \frac{V_{DD} - |V_{tp}| + \sqrt{\frac{\beta_n}{\beta_p} V_{tn}}}{1 + \sqrt{\frac{\beta_n}{\beta_p}}}$$

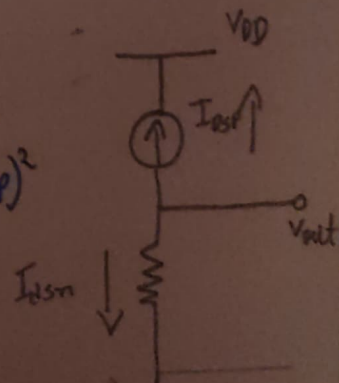


Region D :- ($V_{DD}/2 \leq V_{in} \leq V_{DD} - V_{tp}$)

nmos : Saturation

Pmos : non-saturation

$$V_{out} = (V_{in} - V_{tn}) - \sqrt{(V_{in} - V_{tn})^2 - \frac{\beta_p}{\beta_n}(V_{in} - V_{DD} - V_{tp})^2}$$



Region E: $(V_{in} \geq V_{DD} - V_{tp})$

nmos: linear
pmos: cut off

$$V_{out} = 0$$

$$\frac{\beta_n}{\beta_p} = \frac{K_n' (W/L)_n}{K_p' (W/L)_p}, \quad \frac{K_n'}{K_p'} = \frac{\mu_n C_{ox}}{\mu_p C_{ox}} = \frac{\mu_n}{\mu_p} = \pi$$

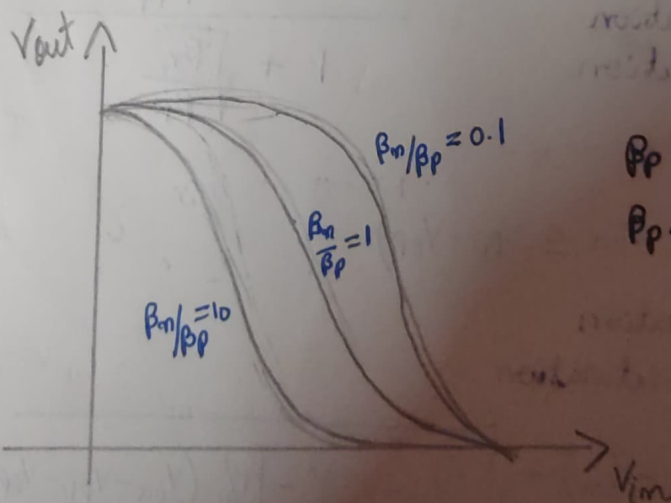
For Symm. Inverter design:-

$$\frac{\beta_n}{\beta_p} = 1$$

$$\frac{\beta_n}{\beta_p} = \pi \frac{(W/L)_n}{(W/L)_p} = 1$$

$$(W/L)_p = \pi (W/L)_n \rightarrow \text{aspect ratio}$$

$$\frac{\beta_n}{\beta_p} = \left(\frac{V_{DD}/2 - |V_{tp}|}{V_{DD}/2 - V_{tn}} \right)^2$$



$\beta_p > \beta_n$: High Skewed Inv

$\beta_p < \beta_n$: Low Skewed Inv.

Noise Margin:-

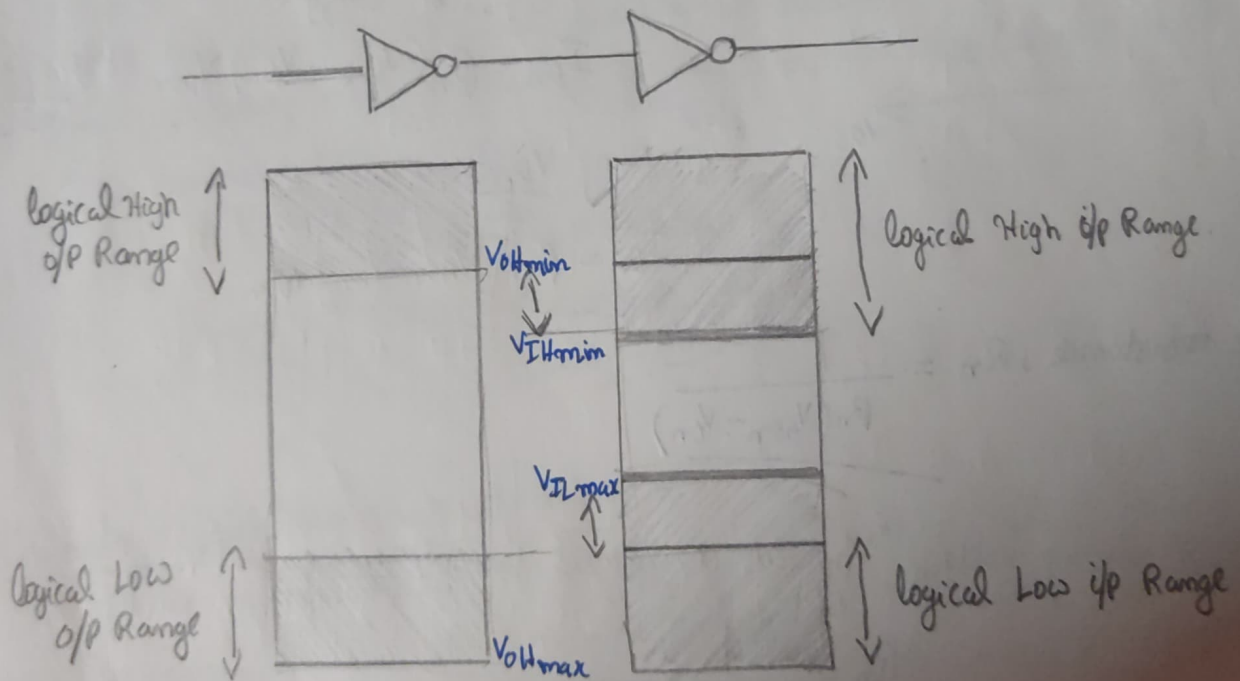
This parameter allows us to determine the allowable noise voltage on the input of a gate so that the o/p will not be affected.

• Low noise Margin, NM_L :- difference in magnitude b/w the maximum Low o/p voltage of the driving gate & the max i/p Low voltage recognized by the driven gate

$$NM_L = |V_{OLmax} - V_{ILmax}|$$

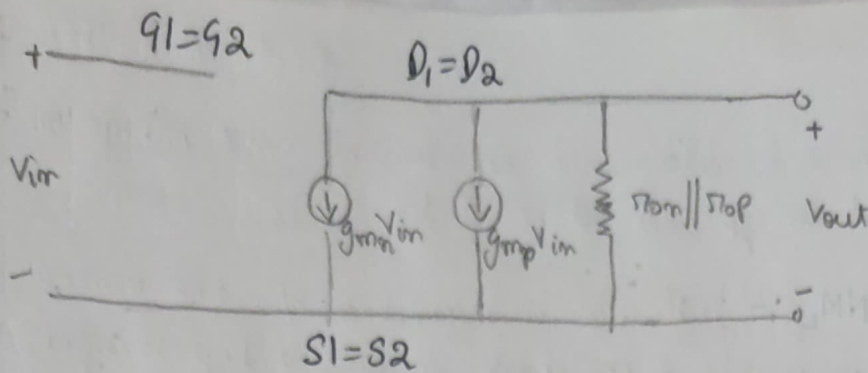
• High noise Margin, NM_H :- difference in magnitude b/w the min. HIGH o/p voltage of the driving gate & the min. i/p HIGH voltage recognized by the receiving gate.

$$NM_H = |V_{OHmin} - V_{IHmin}|$$



$$NM_L = V_{inv} - \frac{V_{DD} - V_{IN}}{A_v}$$

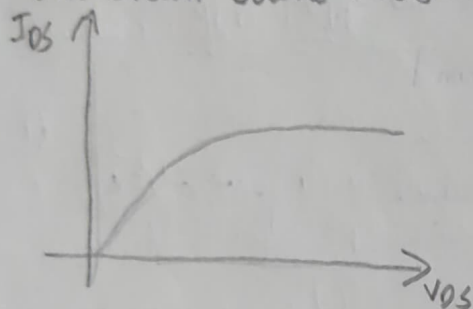
$$NM_H = V_{DD} - V_{inv} \left(1 + \frac{1}{|A_v|} \right)$$



$$A_v = -(g_{mn} + g_{mp})(r_{on} \parallel r_{op})$$

① Drain-Source FET Resistance :-

The drain-source resistance at any point on curve $R_n = \frac{V_{DSn}}{I_{Dn}}$



Current in non-saturated region

$$I_{Dn} = \beta_n \left[(V_{GSn} - V_{tn}) V_{DSn} - \frac{V_{DSn}^2}{2} \right]$$

*** ignoring $\frac{V_{DSn}^2}{2}$

$$I_{Dn} = \beta_n [V_{GSn} - V_{tn}] V_{DSn}$$

$$\text{The resistance, } R_n = \frac{1}{\beta_n (V_{GSn} - V_{tn})}$$

The power supply voltage V_{DD} as the largest possible value for V_{GSn}

$$R_n = \frac{1}{\beta_n (V_{DD} - V_{tn})}$$

$$R_n \propto \frac{1}{\beta_n}$$

large β_n conducts more current than small β_n

Capacitances :-

(3)

i) gate capacitance C_g :- i.e. due to the MOS structure. Since this is the region that has a gate oxide thickness of t_{ox} .

$$C_g = C_{ox} A_g$$

$C_g = C_{ox} wL$, gate capacitance \propto width of the channel

ii) gate source capacitance & gate drain capacitance :-

Their value change with the voltage due to the changing shape of the channel region, the capacitance is said to be non-linear.

$$C_{gs} = \frac{1}{2} C_g \approx C_{gd}$$

iii) Junction Capacitance :-

Pn junction automatically exhibits capacitance due to the opposite polarity charge involved. This is called junction or depletion capacitance and is found at every drain or source region of a FET. C_{SB} (source-bulk) & C_{DB} (drain-bulk)

$$C_n = C_{bot} + C_{sw}$$

$$= C_j A_{bot} + C_{jsw} P_{sw}$$

Inverter Switching Characteristics :-

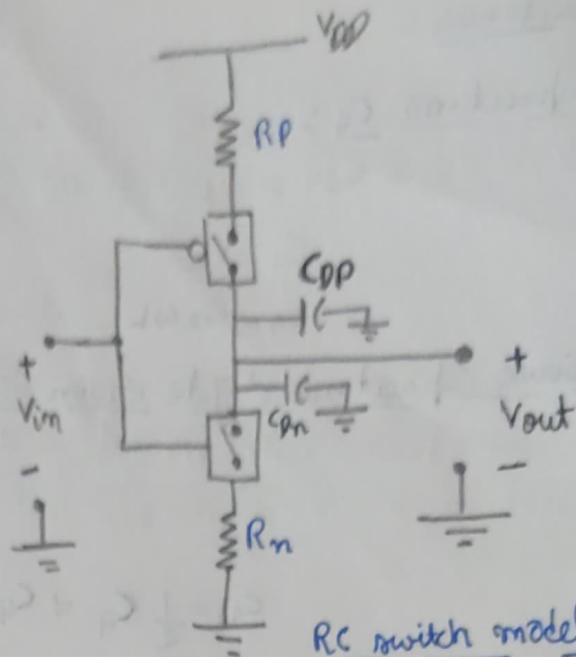
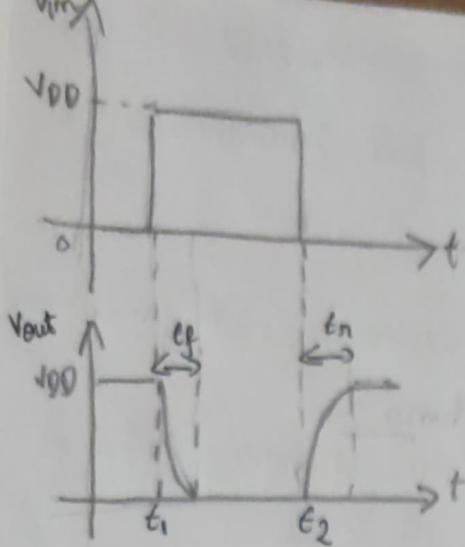
The o/p waveform reacts to the i/p, but o/p voltage cannot change instantaneously. The o/p 1-to-0 transition introduces fall time delay of t_f ; while the 0-to-1 change at the o/p is described by rise time t_r .

The rise & fall time delays are due to parasitic resistance and capacitances of the transistors.

resistance, $R_n = \frac{1}{P_n (V_{DD} - V_{tn})}$

$$R_n = \frac{1}{\frac{\partial I_{DS}}{\partial V_{DS}}}$$

$$R_p = \frac{1}{P_p (V_{DD} - |V_{tp}|)}$$



RC switch model Equivalent

① Capacitance C_{in} & C_{DP} :-

$\rightarrow C_{gs} \approx C_{os} = \frac{1}{2} C_g \rightarrow$ gate capacitance

i) $C_{Dn} = C_{gsn} + C_{DBn}$

$$C_{Dn} = \underbrace{\frac{1}{2} C_{ox} L' W_n}_{\text{gate-source/drain cap.}} + \underbrace{\frac{C_{jn} A_n + C_{jsw} P_n}{L_{bottom}}}_{\text{Drain-Bulk cap/Junction capacitance.}}$$

Area Perimeter
Side wall

ii) $C_{DP} = C_{gsP} + C_{DBP} = \frac{1}{2} C_{ox} L' W_p + C_{jp} A_p + C_{jsw} P_p$

② In a logic chain, every logic gate must drive another gate or set of gates to be useful. The no. of gates is specified by the fan-out (Fo) of a gate.

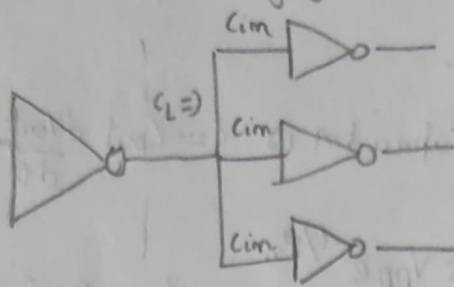
The fan-out gates acts a load to the driving ckt because of their input capacitance C_{in} .

*** Input capacitance is the sum of the FET capacitance i.e. gate capacitance of nmos & pmos

$$C_{in} = C_{gp} + C_{gn} \quad \text{---} (*) (*)$$

The i/p capacitance to each gate adds as an external load capacitance ' C_L ' to the driving gate.

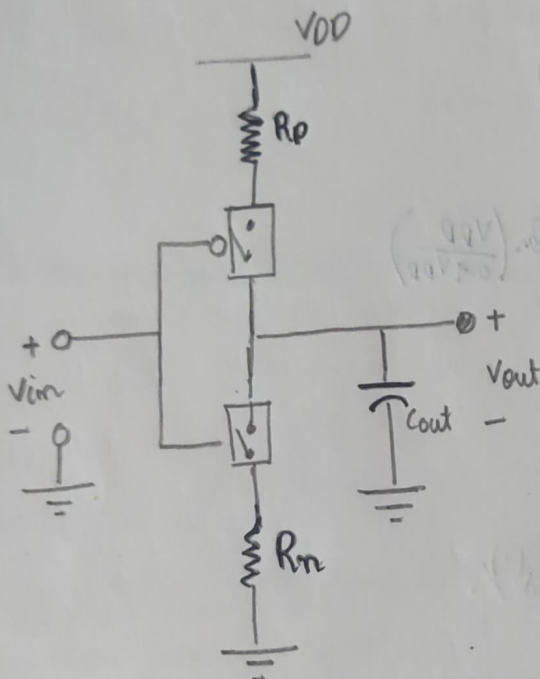
(7)



$C_L = 3C_{in}$ (cap are in parallel)

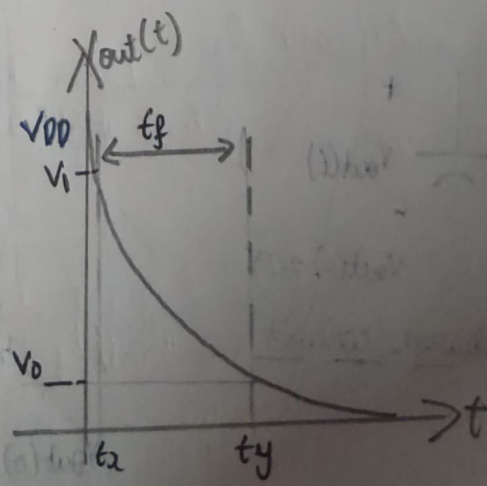
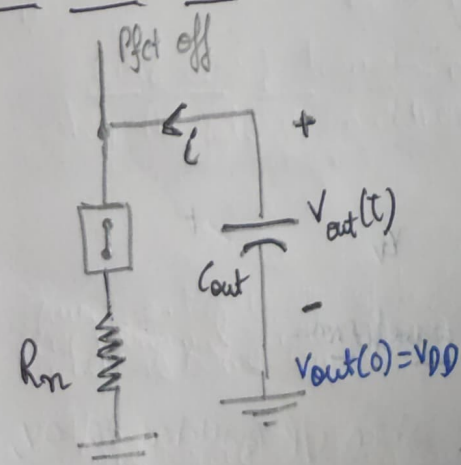
Total output capacitance $C_{out} = C_{FET} + C_L$

$C_{FET} = C_{pn} + C_{dp}$



$\left(\frac{9.1V}{1.0V}\right) m^2 n^3 = 3$
 $\left(\frac{9.1V}{1.0V}\right) m^2 n^3 - \left(\frac{1.0V}{1.0V}\right) m^2 n^3 = 3$
 $(9.1 - 1.0) m^2 n^3 = 3$
 $8.1 m^2 n^3 = 3$
 $m^2 n^3 = \frac{3}{8.1} = 0.37$

① Fall Time :-



a) Discharge circuit

b) o/p waveform

The initial condition at the o/p is $V_{out}(0) = V_{DD}$. When the i/p is switched, the nFET goes active while pFET is driven into cutoff. The capacitor C_{out} initially charged to voltage V_{DD} .

The current leaving the capacitor is $i = -C_{out} \frac{dV_{out}}{dt} = \frac{V_{out}}{R_n}$

$$V_{out}(t) = V_{DD} e^{-t/\tau_n} \quad \tau_n = R_n C_{out}$$

The fall time is defined to be the time interval from 90% V_{DD} to 10% V_{DD}

$$t_f = \tau_n \ln\left(\frac{V_{DD}}{V_{out}}\right)$$

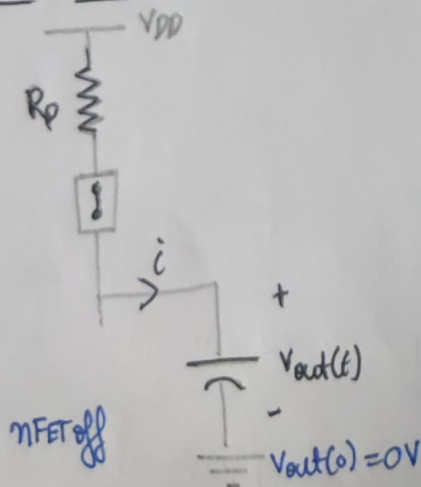
$$t_f = t_y - t_x$$

$$t_f = \tau_n \ln\left(\frac{V_{DD}}{0.1V_{DD}}\right) - \tau_n \ln\left(\frac{V_{DD}}{0.9V_{DD}}\right)$$

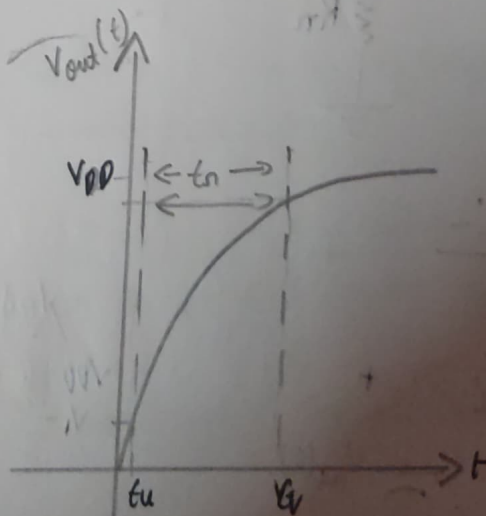
$$t_f = \tau_n \ln(9)$$

$$t_f = 2.2 \tau_n = t_{HL}$$

Rise Time :-



a) charge circuit



b) o/p waveform

The initial condition at the o/p is $V_{out}(0) = 0V$. When i/p switched to 0V, nFET turns off & pFET turns on. The capacitor C_{out} is at 0V, at $t=0$

charging current is given by

(5)

$$i = C_{out} \frac{dV_{out}}{dt} = \frac{V_{DD} - V_{out}}{R_n}$$

$$V_{out} = V_{DD} (1 - e^{-t/\tau_n})$$

$$\tau_p = R_p C_{out}$$

The rise time is taken b/w 10% & 90%.

$$t_r = t_v - t_u$$

$$t_r = 2.2 \tau_p$$

$$\text{maximum signal frequency } f_{max} = \frac{1}{t_{HL} + t_{LH}} = \frac{1}{t_r + t_f}$$

① Propagation delay :-

The propagation delay time t_p is often used to estimate the "reaction" delay time from i/p to o/p

$$t_p = \frac{t_{pf} + t_{pr}}{2}$$

t_{pf} is the o/p fall time from the max. level to 50% voltage i.e V_{DD} to $\frac{V_{DD}}{2}$

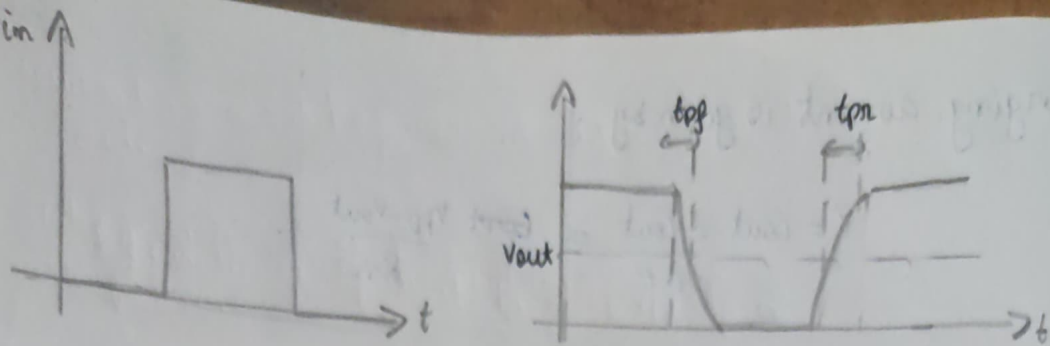
t_{pr} is the propagation rise time from 0V to $(V_{DD}/2)$.

$$t_{pf} = \ln(2) \tau_n = 0.693 \tau_n$$

$$t_{pr} = \ln(2) \tau_p = 0.693 \tau_p$$

$$t_p = 0.35 (\tau_n + \tau_p)$$

** The propagation delay is a useful estimate of the basic delay, but does not provide detailed info. on the rise & fall times as individual quantities.



$$C_{out} = C_{FET} + C_L$$

rise time $t_r = 2.2 R_n C_{out}$

$$t_r = 2.2 R_n (C_{FET} + C_L)$$

$$t_r = t_{r0} + \alpha_n C_L$$

$$t_{r0} = 2.2 R_n C_{FET}$$

$$\alpha_n = 2.2 R_n$$

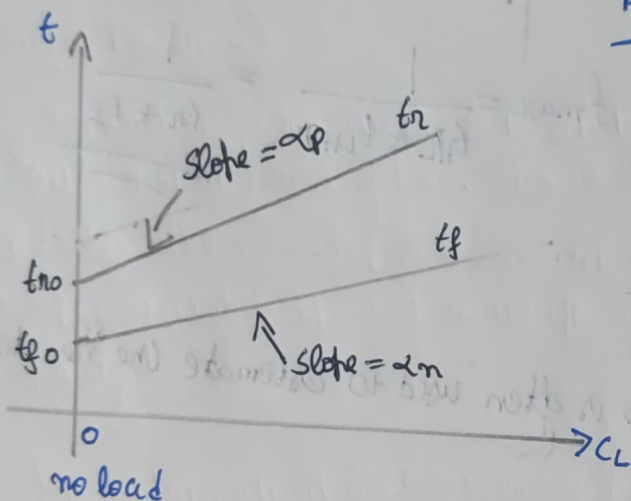
fall time $t_f = 2.2 R_p C_{out}$

$$t_f = 2.2 R_p (C_{FET} + C_L)$$

$$t_f = t_{f0} + \alpha_p C_L$$

$$t_{f0} = 2.2 R_p C_{FET}$$

$$\alpha_p = 2.2 R_p$$



Speed versus area trade off

Power dissipation :-

i) Static dissipation :- In CMOS inverter, one transistor is always OFF when the gate is in either of the logic states. Since no current flows into the terminal, there is no DC current path from \$V_{DD}\$ to \$V_{SS}\$, the resultant quiescent (steady-state) current, hence power \$P_s\$ is zero.

There is some small static dissipation due to reverse bias leakage b/w

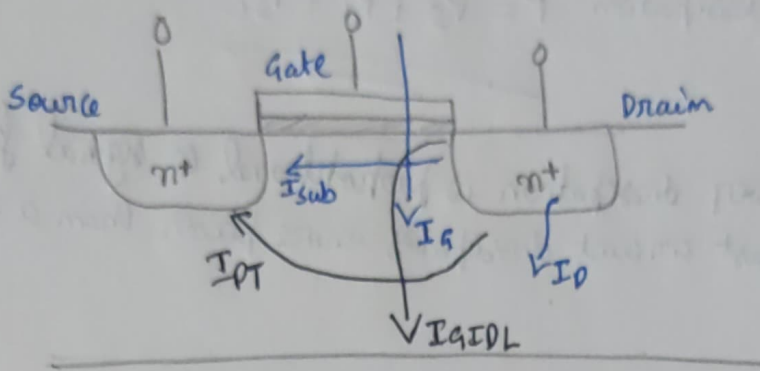
The source-drain diffusions & n-well diffusion from parasitic diodes. The parasitic diodes is shown b/w n-well & substrate. Since parasitic diodes are reverse biased, only their leakage current contributes to static power dissipation.

leakage current $I_0 = I_s (e^{qV/kT} - 1)$

Static power dissipation.

$P_s = V_{DD} I_0$

③ leakage power:-

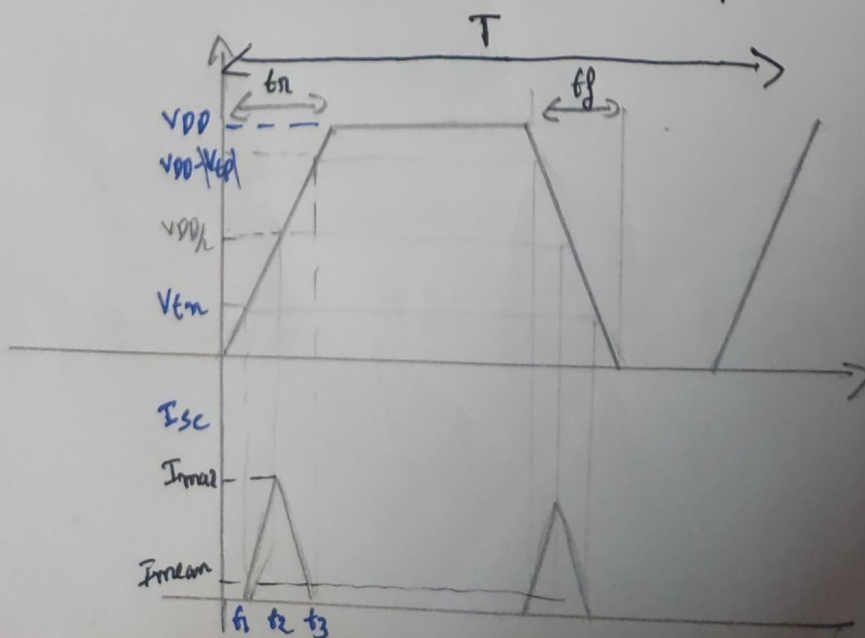


③ Dynamic Power:-

During transition from either 0 to 1, or 1 to 0, both n & p transistor are "ON" for short period of time. This results in short current pulse from V_{DD} to V_{SS} . The current pulse from V_{DD} to V_{SS} results in "short circuit" dissipation that is dependent on the clp rise & fall time, the load capacitance - Cl & gate design.

For inverter without load, assuming that $t_r = t_f$ (t_{nf})

$$P_{sc} = \beta/12 (V_{DD} - 2V_t)^3 \frac{t_{nf}}{T}$$



current is also required to charge & discharge the o/p capacitance load.

Dynamic dissipation $P_d = C_L V_{DD}^2 f_p \propto$

Total Power dissipation $P = P_s + P_d + P_{sc}$

- ***
 * The dynamic power dissipation is proportional to signal frequency
 , other words a fast circuit dissipates more power than a slow circuit

$$\frac{f_{avg}}{T} = \frac{1}{10} \left(\frac{1000 - 1000}{1000} \right) = 0.1$$

