Q6. Multi-Input Feedforward

Consider a neural network with 3 inputs x1=2, x2=1, x3=3, one hidden layer of 2 sigmoid units, and one sigmoid output.
Weights:

$$A = \begin{bmatrix} 0.2 & -0.5 \\ 0.1 & 0.3 \\ -0.2 & 0.8 \\ 0.4 & -0.6 \end{bmatrix}, \quad B = \begin{bmatrix} 0.7 \\ -1.2 \\ 0.5 \end{bmatrix}$$

- A includes biases (last row).
- B includes bias (last element).

Tasks

a) Compute hidden pre-activations and activations.

$z_h = \tilde{x} \cdot A$

- $z_1 = 2(0.2) + 1(0.1) + 3(-0.2) + 1(0.4) = 0.3$
- $z_2 = 2(-0.5) + 1(0.3) + 3(0.8) + 1(-0.6) = 1.1$

$h_1 = \sigma(z_1) = \sigma(0.3) \approx \mathbf{0.5744}$
$h_2 = \sigma(z_2) = \sigma(1.1) \approx \mathbf{0.7503}$

b) Compute the output activation y.

Output pre-activation (include bias):
$z_o = 0.7 \cdot h_1 + (-1.2) \cdot h_2 + 0.5$
$= 0.7(0.5744) - 1.2(0.7503) + 0.5$
$\approx 0.4021 - 0.9004 + 0.5 \approx \mathbf{0.0018}$

$y = \sigma(z_o) = \sigma(0.0018) \approx \mathbf{0.5004}$

c) If the true label is t=1, calculate the binary cross-entropy loss.

$L = -[\, t \cdot \ln(y) + (1-t) \cdot \ln(1-y) \,] = -\ln(y)$
$= -\ln(0.5004) \approx \mathbf{0.6923}$