Q8. Decision Boundary and Misclassification

You train a perceptron with decision rule:

$$y = \begin{cases} 1 & \text{if } w_1x_1 + w_2x_2 + b > 0 \\ 0 & \text{otherwise} \end{cases}$$

with parameters w1=1, w2=−2, b=1.

Dataset:
$(2,1)\rightarrow1$, $(1,3)\rightarrow0$, $(3,2)\rightarrow1$, $(0,1)\rightarrow0$.

Tasks

a) Write the decision boundary equation and sketch it.

Set the activation to zero:
$$x_1 - 2x_2 + 1 = 0$$

Rearranging for x_2:
$$x_2 = (x_1 + 1)/2$$

- Slope = ½
- Intercept = 0.5
- Region for class 1 → $x_1 - 2x_2 + 1 > 0$ → below the line

Decision boundary: $x_2 = (x_1 + 1)/2$

b) Classify each point using the perceptron rule. Which ones are misclassified?

Input (x_1, x_2)	Calculation	$s = x_1 - 2x_2 + 1$	Prediction $(\hat{y})$	True Label	Result
(2, 1)	2 − 2×1 + 1	1	1	1	Correct
(1, 3)	1 − 6 + 1	-4	0	0	Correct
(3, 2)	3 − 4 + 1	0	0	1	Misclassified
(0, 1)	0 − 2 + 1	-1	0	0	Correct

Misclassified point: (3, 2)

c) Compute the perceptron loss (number of mistakes).

Loss = 1 mistake

Total loss = 1

d) If we apply one weight update with learning rate η=0.5 on the first mistake, what are the new weights?

Learning rate $(\eta) = 0.5$
First mistake: $(3, 2) \rightarrow y = 1, \hat{y} = 0$

Update rule:
   $w \leftarrow w + \eta(y - \hat{y})x$
   $b \leftarrow b + \eta(y - \hat{y})$

Compute updates:
   $\Delta w_1 = 0.5 \times 3 = 1.5$
   $\Delta w_2 = 0.5 \times 2 = 1.0$
   $\Delta b = 0.5 \times 1 = 0.5$

New parameters:
   $w_1 = 1 + 1.5 = 2.5$
   $w_2 = -2 + 1.0 = -1.0$
   $b = 1 + 0.5 = 1.5$

Updated weights:
$w_1 = 2.5$
$w_2 = -1.0$
$b = 1.5$