

Q3. Cosine Similarity Between Word Vectors

Given word vectors:

- $\vec{a}=[3,4]$
- $\vec{b}=[4,3]$

Tasks:

- (a) Compute the dot product $\vec{a} \cdot \vec{b}$

$$\begin{aligned}\vec{a} \cdot \vec{b} &= (3)(4) + (4)(3) \\ &= 12 + 12 \\ &= 24\end{aligned}$$

$$\vec{a} \cdot \vec{b} = 24$$

- (b) Compute magnitudes $\|\vec{a}\|$ and $\|\vec{b}\|$

$$\begin{aligned}\|\vec{a}\| &= \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \\ \|\vec{b}\| &= \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5 \\ \|\vec{a}\| &= 5, \|\vec{b}\| = 5\end{aligned}$$

- (c) Compute cosine similarity:

$$\cos(\theta) = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|}$$

$$\begin{aligned}\cos(\theta) &= (\vec{a} \cdot \vec{b}) / (\|\vec{a}\| \times \|\vec{b}\|) \\ &= 24 / (5 \times 5) \\ &= 24 / 25 \\ &= 0.96\end{aligned}$$

$$\text{Cosine similarity} = 0.96$$