Neuro Syllabus

$\boxed{L0 - L10}$

$e^{i(2\pi f t + \theta)} = \cos(2\pi f t + \theta) + i \sin(2\pi f t + \theta)$

$2\pi f t + \theta$

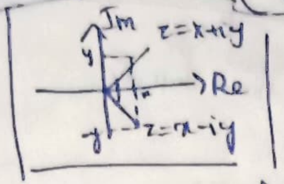
$e^{i\alpha} = \cos\alpha + i\sin\alpha$

$\boxed{\text{Fourier Coeff}}$

- $\boxed{\text{Complex sine waves}}$
  - $\boxed{\text{Sine waves}}$
    - $x(t) = a\sin(2\pi f t + \theta)$
  - $\boxed{\text{Complex NO'S}}$
- $\boxed{\text{Complex dot product}}$
  - $\boxed{\text{Complex no's}}$
  - $\boxed{\text{Dot prod}}$

$\frac{15}{\times 8}$ 4

120



$\vec{a} = [a_1, a_2, a_3]$

$\vec{b} = [b_1, b_2, b_3]$

$z = a + bi$

$z^* = a - bi$

$z \times z^* = (a+bi)(a-bi)$

$= a^2 + b^2$

$|z|^2 = z \cdot z^*$

Projection on one vector onto another

How much of one vector lies in the dir of the other.

$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

To compare similarity b/w 2 signals

- If dot prod → Too large → they're similar.

Fourier transform → computes how much of each complex sine wave (freq) is present in your signal

Fourier coeff at freq f. ⟵ $X(f) = \sum_t x(t) \cdot e^{-j2\pi f t}$

$15 \times 8 - \left(\frac{1}{2} \times 15 \times 8 + \right.$ 4

15, 17, 8

$\frac{\pi \times 2^2}{2}$

225
$+64$
289

$\frac{11}{7} \times \frac{(8.5)^2}{2}$

64 60+

120 − (60+

$\boxed{\text{Dot product} \rightarrow \text{Convolution}}$

Sliding dot product

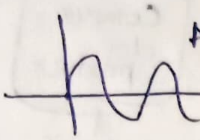
$S(t)$ $K(t)$

$(S * K)t = \sum S(\tau) K(t - \tau)$

Fourier Tranform → Transforms signal from time domain to the frequency domain.

$x(t)$
↓
function of time
how signal changes over time

But ? what frequencies the signal is made of with?
How strong is each one?



Amplitude → how strong
frequency → how fast it osciliates
phase → how shifted it is in time

Fourier Transform
$$\boxed{X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt}$$
$x(t)$ → signal in time domain.

$$e^{-i\theta} = \cos\theta - i\sin\theta$$
$$e^{-j2\pi f t} = \cos(2\pi f t) - i\sin(2\pi f t)$$

Inverse FT
$$\boxed{x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\, dt}$$

$$X(f)\cdot X(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X(f)\cdot x(t)\cdot e^{-j2\pi f t}\, e^{j2\pi f t}\, dt$$

DFT → Discrete Fourier Tranform

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-i2\pi \frac{k}{N} n} \qquad O(N^2)$$

$$x_n = \frac{1}{N}\sum_{n=0}^{N-1} X_k \cdot e^{i2\pi \frac{k}{N} n}$$

FFT → Fast Fourier Transform

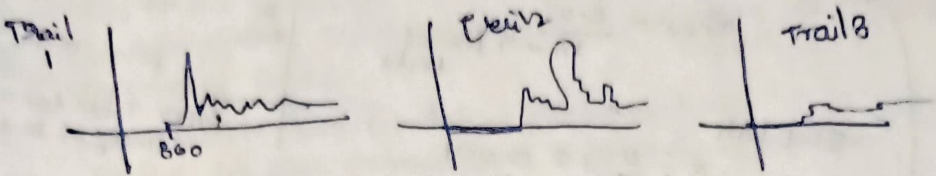
$O(N \log N)$ fft(x).

STFT | Wavelet Transform
→ To Analyse freq over time

If our signal is non statinary
⇓
Single FT will mix up all time varying freqs.

→ Why do we need statistics;

EEG Data → Extremely noisy



So, we avg across trails or subjects
& use statistics to test whether
an observed effect < robust ? / or due to chance?

→ At what level should we apply statistics,

① Within Subject level
many trails
You can test condition A vs B within each subject

② Across Subject (Group) level.
Once we have subject level results (like avg alpha power diff) we can test if it is consistent across subjects

If <8 Subjects ⇒ prefer within subject ⇒ use trails
many subjects ⇒ use across subject statistics ⇒ use subject means

→ The multiple Comparisons prblm:

Type1 Error: Falsely claiming an Effect when there isn't one.
: Over Estimation.

Prob of making T1 Error:

$\alpha = 0.05 = 5\%$ FP rate / test

False positive
⇓
Exgarration

eg: Pregnant Men

20 Independent Tests
Each with 5% FP rate

Chance of atleast one FP = $1 - (1-0.05)^{20}$
≈ 0.64

Significant result ✗ Luck ✓

T1 — Effect exists when there isn't — α

T2 — No Effect when there is — β

One test at $\alpha = 0.05$ fine
But 1000 tests
It skyrockets

So, we need corrections for multiple comparisons → to keep overall FWER around 0.05.

3 Hypothesis

0.05 - FP chance
0.95 - being correct

P(all 3 are correct) = $(0.95)^3 = 0.857$

P(atleast one FP) = $1 - (0.857)$
$= 0.143$

5% ⇒ 14%

We need to adjust α

- Bonferroni Correction

Reject Null Hyp $P \leq 0.05$

	Reject H_0	Fail to Reject H_0	
H_0 True	FP	TN	m_0
H_0 False	TP	FN	$m-m_0$
	R	m-R	m

Error Table for a family of m tests

$$\alpha_{new} = \frac{0.05}{m} \rightarrow \text{No. of tests}$$

50 t-tests

$$\alpha = \frac{0.05}{50} = 0.001$$

Each Individual test must be $p < 0.001$ to be significant.

Chance of Increasing T2 Errors → missing real Effects

→ when 1000+ tests like EEG

ok to apply when we have small set of Comparisons · 3 electrodes × 1 Freq band

- Non parametric permutation Testing

→ modern & much better way to handle EEG/MEG Data.

⊛ Instead of assuming theoretical distributions like t or F → we build empirical null distribution directly from the data.

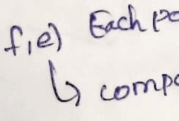
2 conditions

(A) Real movement (100 trails)

(B) Imaginary movement (100 trails)

- (t, f, e) Each point
  → compute real t-statistic for A vs B
- Shuffle the condition labels randomly
- Recompute t-statistic for shuffled data
- Repeat this 1000 times.
- Compare real t-value to this Null distribution.

If: more extreme than 95% of those significant $p < 0.05$)



Map level thresholding

- For each permutation take the maximum t-value across all pixels (t, f, e)
- Gives you a distribution of max statistics

Cluster level thresholding

1. Compute t-statistics for all pixels.
2. Apply ~~a~~ pre threshold
3. Identify clusters of continuous Significant pixels.
4. For each cluster sum t-values → cluster statistic
5. Do 2-4 for each permutation shuffle & record the max cluster statistic
6. Build a null distribution of max cluster sums
7. Take 95%ile as our cluster th
8. keep only clusters in your real data that exceeds this

- one Isolated significant pixel ⇒ Noise
- But many clusters ⇒ Real effect.