

COMPETITIVE PROGRAMMING

Assignment-02

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B-07

Assignment I: Fibonacci using Memoization

Imagine a financial planning software that predicts future savings growth based on a recursive formula similar to the Fibonacci sequence. If the software calculates Fibonacci numbers using plain recursion, it repeats the same calculations many times, making it slow for large inputs.

Example:

To compute $F(10)$:

- $F(10)$ calls $F(9)$ and $F(8)$
- $F(9)$ again calls $F(8)$ and $F(7)$
- $F(8)$ is computed multiple times

Using memoization, once $F(8)$ is computed, it is stored and reused, making the program fast and efficient.

Sample Input

10

Expected Output:

55

Explanation:

The 10th Fibonacci number is

0 1 1 2 3 5 8 13 21 34 55

Algorithm: Fibonacci using Memoization

1. Read integer **n**.
2. Create an array (or dictionary) **memo** to store already computed Fibonacci values.
3. Define a recursive function **fib(n)**:
 - a. If $n == 0$, return 0.
 - b. If $n == 1$, return 1.
 - c. If **memo[n]** is already computed, return **memo[n]**.
 - d. Otherwise:
 - i. Compute **fib(n-1) + fib(n-2)**.
 - ii. Store the result in **memo[n]**.
 - iii. Return the stored value.
4. Call **fib(n)**.
5. Print the result.

Code and output:

```
ass_2..1.py > ...
1  n=int(input())
2  memo=[-1]*(n+1)
3
4  def fib(n):
5      if n==0:
6          return 0
7      if n==1:
8          return 1
9      if memo[n]!=-1:
10         return memo[n]
11     memo[n]=fib(n-1)+fib(n-2)
12     return memo[n]
13
14 print(fib(n))
15
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PO

```
PS C:\AAC_> & C:/Users/harsh/AppData/Local/Programs/Python/3.8.5/python.exe ass_2..1.py
10
55
PS C:\AAC_> █
```

Assignment 2: 0/1 Knapsack Problem

Algorithm:

1. Read integers **n** (number of items) and **W** (maximum capacity).
2. Read **n** pairs of integers (**value**, **weight**) for each item.
3. Create a 2D DP table **dp** of size (**n+1**) × (**W+1**).
 - a. $dp[i][w]$ = maximum value using first **i** items with capacity **w**.
4. Initialize:
 - a. $dp[0][w] = 0$ for all **w** (no items \rightarrow no value).
5. Fill the DP table:
 - a. For **i** from 1 to **n**:
 - i. For **w** from 0 to **W**:
 1. If $weight[i-1] \leq w$:
 - a. $dp[i][w] = \max(dp[i-1][w], value[i-1] + dp[i-1][w - weight[i-1]])$
 2. Else:
 - a. $dp[i][w] = dp[i-1][w]$
 6. The answer is $dp[n][W]$.
 7. Print the result.

Code and output:

```
⌚ ass_2.2.py > ...
1  n,W=map(int,input().split())
2  values=[]
3  weights=[]
4  for i in range(n):
5      v,w=map(int,input().split())
6      values.append(v)
7      weights.append(w)
8  dp=[[0]*(W+1) for _ in range(n+1)]
9  for i in range(1,n+1):
10     for w in range(W+1):
11         if weights[i-1]<=w:
12             dp[i][w]=max(dp[i-1][w],values[i-1]+dp[i-1][w-weights[i-1]])
13         else:
14             dp[i][w]=dp[i-1][w]
15
16 print(dp[n][W])
17
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS

```
PS C:\AAC_> & C:/Users/harsh/AppData/Local/Programs/Python/Python314/python.exe c:/AAC
3 50
60 10
100 20
120 30
220
```

Assignment 3: Longest Common Subsequence (LCS)

Algorithm:

1. Read two strings **X** and **Y**.
2. Let **m = length of X, n = length of Y**.
3. Create a 2D DP table **dp** of size $(m+1) \times (n+1)$.
 - a. $dp[i][j]$ stores the length of the LCS of $X[0...i-1]$ and $Y[0...j-1]$.
4. Initialize:
 - a. $dp[i][0] = 0$ for all **i**
 - b. $dp[0][j] = 0$ for all **j**
5. Fill the table:
 - a. For **i** from 1 to **m**:
 - i. For **j** from 1 to **n**:
 1. If $X[i-1] == Y[j-1]$:
 - a. $dp[i][j] = dp[i-1][j-1] + 1$
 2. Else:
 - a. $dp[i][j] = \max(dp[i-1][j], dp[i][j-1])$
 6. The value $dp[m][n]$ is the length of the LCS.
 7. Print $dp[m][n]$.

Code and output:

```
ass_2.3.py > ...
4     m=len(x)
5     n=len(y)
6
7     dp=[[0]*(n+1) for _ in range(m+1)]
8
9     for i in range(1,m+1):
10        for j in range(1,n+1):
11            if x[i-1]==y[j-1]:
12                dp[i][j]=dp[i-1][j-1]+1
13            else:
14                dp[i][j]=max(dp[i-1][j],dp[i][j-1])
15
16    print(dp[m][n])
17
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS

```
PS C:\AAC-> & C:/Users/harsh/AppData/Local/Programs/Python/Pyt
AGGTAB
GXTXAYB
4
PS C:\AAC->
```

Assignment 4: Coin Change – Minimum Coins

Algorithm:

1. Read integer **n** (number of coin denominations).
2. Read **n** integers representing the coin values.
3. Read integer **amount** (total value to be made).
4. Create a DP array **dp** of size **amount + 1**.
 - a. **dp[i]** stores the minimum number of coins needed to make amount **i**.
5. Initialize:
 - a. **dp[0] = 0**
 - b. For all **i > 0**, set **dp[i] = ∞** (a very large value).
6. For each amount **i** from 1 to **amount**:
 - a. For each coin value **c**:
 - i. If **c ≤ i**:
 1. **dp[i] = min(dp[i], dp[i - c] + 1)**
7. After filling the array, **dp[amount]** contains the minimum number of coins.
8. Print **dp[amount]**.

Code and output:

```
ass_2.4.py > ...
1  n=int(input())
2  coins=list(map(int,input().split()))
3  amount=int(input())
4
5  dp=[10**9]*(amount+1)
6  dp[0]=0
7
8  for i in range(1,amount+1):
9      for c in coins:
10         if c<=i:
11             dp[i]=min(dp[i],dp[i-c]+1)
12
13 print(dp[amount])
14
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS

```
PS C:\AAC_> & C:/Users/harsh/AppData/Local/Programs/Python/Python38-32/ass_2.4.py
3
1 3 4
6
2
PS C:\AAC_>
```

Assignment 5: Matrix Chain Multiplication

Algorithm: Matrix Chain Multiplication (Dynamic Programming)

1. Read integer n (number of matrices).
2. Read n integers representing matrix dimensions in an array p ,
where matrix A_i has dimensions $p[i-1] \times p[i]$.
3. Create a 2D DP table dp of size $n \times n$.
 - a. $dp[i][j]$ stores the minimum number of scalar multiplications
needed to multiply matrices from A_i to A_j .
4. Initialize:
 - a. $dp[i][i] = 0$ for all i (single matrix \rightarrow no multiplication).
5. Consider chain lengths from 2 to $n-1$:
 - a. For each starting index i :
 - i. Let $j = i + \text{length} - 1$.
 - ii. Set $dp[i][j] = \infty$.
 - iii. For each possible split point k from i to $j-1$:
 1. Compute cost
$$\text{cost} = dp[i][k] + dp[k+1][j] + p[i-1] * p[k] * p[j]$$
 2. Update
$$dp[i][j] = \min(dp[i][j], \text{cost})$$
6. The answer is stored in $dp[1][n-1]$.
7. Print the result.

Code and output:

```
ass_2.5.py > ...
1  n=int(input())
2  p=list(map(int,input().split()))
3  dp=[[0]*n for _ in range(n)]
4  for length in range(2,n):
5    for i in range(1,n-length+1):
6      j=i+length-1
7      dp[i][j]=10**18
8    for k in range(i,j):
9      cost=dp[i][k]+dp[k+1][j]+p[i-1]*p[k]*p[j]
10     dp[i][j]=min(dp[i][j],cost)
11
12 print(dp[1][n-1])
13
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS

```
PS C:\AAC-> & C:/Users/harsh/AppData/Local/Programs/Python/Python37-32/ass_2.5.py
4
10 20 30 40
18000
PS C:\AAC-> 
```