

COMPETITIVE PROGRAMMING

Assignment-02

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B-07

Assignment I: Fibonacci using Memoization

Imagine a financial planning software that predicts future savings growth based on a recursive formula similar to the Fibonacci sequence. If the software calculates Fibonacci numbers using plain recursion, it repeats the same calculations many times, making it slow for large inputs.

Example:

To compute $F(10)$:

- $F(10)$ calls $F(9)$ and $F(8)$
- $F(9)$ again calls $F(8)$ and $F(7)$
- $F(8)$ is computed multiple times

Using memoization, once $F(8)$ is computed, it is stored and reused, making the program fast and efficient.

Sample Input

10

Expected Output:

55

Explanation:

The 10th Fibonacci number is

0 1 1 2 3 5 8 13 21 34 55

Algorithm: Fibonacci using Memoization

1. Read integer **n**.
2. Create an array (or dictionary) **memo** to store already computed Fibonacci values.
3. Define a recursive function **fib(n)**:
 - a. If $n == 0$, return 0.
 - b. If $n == 1$, return 1.
 - c. If `memo[n]` is already computed, return `memo[n]`.
 - d. Otherwise:
 - i. Compute `fib(n-1) + fib(n-2)`.
 - ii. Store the result in `memo[n]`.
 - iii. Return the stored value.
4. Call `fib(n)`.
5. Print the result.

Code and output:

```
ass_2..1.py > ...
1  n=int(input())
2  memo=[-1]*(n+1)
3
4  def fib(n):
5      if n==0:
6          return 0
7      if n==1:
8          return 1
9      if memo[n]!=-1:
10         return memo[n]
11     memo[n]=fib(n-1)+fib(n-2)
12     return memo[n]
13
14 print(fib(n))
15
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PO

```
PS C:\AAC_> & C:/Users/harsh/AppData/Local/Prog
10
55
PS C:\AAC_> █
```

Assignment 2: 0/1 Knapsack Problem

Algorithm:

1. Read integers **n** (number of items) and **W** (maximum capacity).
2. Read **n** pairs of integers (**value**, **weight**) for each item.
3. Create a 2D DP table **dp** of size $(n+1) \times (W+1)$.
 - a. $dp[i][w]$ = maximum value using first **i** items with capacity **w**.
4. Initialize:
 - a. $dp[0][w] = 0$ for all **w** (no items \rightarrow no value).
5. Fill the DP table:
 - a. For **i** from 1 to **n**:
 - i. For **w** from 0 to **W**:
 1. If $weight[i-1] \leq w$:
 - a. $dp[i][w] = \max(dp[i-1][w], value[i-1] + dp[i-1][w - weight[i-1]])$
 2. Else:
 - a. $dp[i][w] = dp[i-1][w]$
6. The answer is $dp[n][W]$.
7. Print the result.

Code and output:

```
ass_2.2.py > ...
1  n,W=map(int,input().split())
2  values=[]
3  weights=[]
4  for i in range(n):
5      v,w=map(int,input().split())
6      values.append(v)
7      weights.append(w)
8  dp=[[0]*(W+1) for _ in range(n+1)]
9  for i in range(1,n+1):
10     for w in range(W+1):
11         if weights[i-1]<=w:
12             dp[i][w]=max(dp[i-1][w],values[i-1]+dp[i-1][w-weights[i-1]])
13         else:
14             dp[i][w]=dp[i-1][w]
15
16  print(dp[n][W])
17
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS

```
PS C:\AAC_ > & C:/Users/harsh/AppData/Local/Programs/Python/Python314/python.exe c:/AAC
3 50
60 10
100 20
120 30
220
```

Assignment 3: Longest Common Subsequence (LCS)

Algorithm:

1. Read two strings **X** and **Y**.
2. Let **m = length of X**, **n = length of Y**.
3. Create a 2D DP table **dp** of size $(m+1) \times (n+1)$.
 - a. **dp[i][j]** stores the length of the LCS of **X[0...i-1]** and **Y[0...j-1]**.
4. Initialize:
 - a. **dp[i][0] = 0** for all **i**
 - b. **dp[0][j] = 0** for all **j**
5. Fill the table:
 - a. For **i** from 1 to **m**:
 - i. For **j** from 1 to **n**:
 1. If **X[i-1] == Y[j-1]**:
 - a. **dp[i][j] = dp[i-1][j-1] + 1**
 2. Else:
 - a. **dp[i][j] = max(dp[i-1][j], dp[i][j-1])**
6. The value **dp[m][n]** is the length of the LCS.
7. Print **dp[m][n]**.

Code and output:

```
ass_2.3.py > ...
4  m=len(x)
5  n=len(y)
6
7  dp=[[0]*(n+1) for _ in range(m+1)]
8
9  for i in range(1,m+1):
10     for j in range(1,n+1):
11         if x[i-1]==y[j-1]:
12             dp[i][j]=dp[i-1][j-1]+1
13         else:
14             dp[i][j]=max(dp[i-1][j],dp[i][j-1])
15
16  print(dp[m][n])
17
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS

```
PS C:\AAC_> & C:/Users/harsh/AppData/Local/Programs/Python/Pyt
AGGTAB
GXTXAYB
4
PS C:\AAC_> █
```

Assignment 4: Coin Change – Minimum Coins

Algorithm:

1. Read integer **n** (number of coin denominations).
2. Read **n** integers representing the coin values.
3. Read integer **amount** (total value to be made).
4. Create a DP array **dp** of size **amount + 1**.
 - a. **dp[i]** stores the minimum number of coins needed to make amount **i**.
5. Initialize:
 - a. **dp[0] = 0**
 - b. For all **i > 0**, set **dp[i] = ∞** (a very large value).
6. For each amount **i** from 1 to **amount**:
 - a. For each coin value **c**:
 - i. If **c ≤ i**:
 1. **dp[i] = min(dp[i], dp[i - c] + 1)**
7. After filling the array, **dp[amount]** contains the minimum number of coins.
8. Print **dp[amount]**.

Code and output:

```
ass_2.4.py > ...
1  n=int(input())
2  coins=list(map(int,input().split()))
3  amount=int(input())
4
5  dp=[10**9]*(amount+1)
6  dp[0]=0
7
8  for i in range(1,amount+1):
9      for c in coins:
10         if c<=i:
11             dp[i]=min(dp[i],dp[i-c]+1)
12
13  print(dp[amount])
14
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS

```
PS C:\AAC_> & C:/Users/harsh/AppData/Local/Programs/Python/Pyt
3
1 3 4
6
2
PS C:\AAC_> █
```

Assignment 5: Matrix Chain Multiplication

Algorithm: Matrix Chain Multiplication (Dynamic Programming)

1. Read integer **n** (number of matrices).
2. Read **n** integers representing matrix dimensions in an array **p**, where matrix **A_i** has dimensions $p[i-1] \times p[i]$.
3. Create a 2D DP table **dp** of size $n \times n$.
 - a. **dp[i][j]** stores the minimum number of scalar multiplications needed to multiply matrices from **A_i** to **A_j**.
4. Initialize:
 - a. **dp[i][i] = 0** for all **i** (single matrix \rightarrow no multiplication).
5. Consider chain lengths from 2 to $n-1$:
 - a. For each starting index **i**:
 - i. Let $j = i + \text{length} - 1$.
 - ii. Set **dp[i][j] = ∞** .
 - iii. For each possible split point **k** from **i** to **j-1**:
 1. Compute cost
$$\text{cost} = \text{dp}[i][k] + \text{dp}[k+1][j] + p[i-1] * p[k] * p[j]$$
 2. Update
$$\text{dp}[i][j] = \min(\text{dp}[i][j], \text{cost})$$
6. The answer is stored in **dp[1][n-1]**.
7. Print the result.

Code and output:

```
ass_2.5.py > ...
1  n=int(input())
2  p=list(map(int,input().split()))
3  dp=[[0]*n for _ in range(n)]
4  for length in range(2,n):
5      for i in range(1,n-length+1):
6          j=i+length-1
7          dp[i][j]=10**18
8          for k in range(i,j):
9              cost=dp[i][k]+dp[k+1][j]+p[i-1]*p[k]*p[j]
10             dp[i][j]=min(dp[i][j],cost)
11
12  print(dp[1][n-1])
13
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS

```
PS C:\AAC_> & C:/Users/harsh/AppData/Local/Programs/Python/Python3
4
10 20 30 40
18000
PS C:\AAC_> 
```