CASE STUDY 15:

Work Experience

TEAM MEMBERS:

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ABSTRACT:

In this work, to show that real life cases scenario when tested satisfied queuing theory, we obtain data from Kalinga restaurant, Jodhpur. We then calculated the arrival rate, service rate, utilization rate, waiting time and the probability of potential customers to balk based on the collected data, little's theorem and M/M/I queuing model. The arrival rate during busiest period of the day is 1.42 customer per minute. The average number of customers in the restaurant is 70 and utilisation rate of 0.9859. We conclude the work by given the concluding remarks.

Instructor Observations:

- 1.The students, most of whom work, had no trouble identifying probabilistic problems at work. The students spent a reasonable amount of time on this project.
- One group missed opportunities in the work environments of its members.
- Groups were reluctant to have a non-expert be the presenter. The primary outcome of this experiment was that the students learned to statistically analyze situations within their own technical job environments. In previous classes, the students had been able to learn the statistical concepts such as the idea of "trials and successes" in the binomial distribution, and viewing data through histograms, etc. They also learned to recognize instances of uncertainty intheir field of engineering such as beam failures and construction specification tolerance limits. This project helped the students to make the connection between the statistical tools and the engineering applications.

Queuing Theory:-

In 1903 FrJohannsen was appointed copehengen telephone company director and he observed that the manual switch were not dimensioned in the right way. FrJohannsen published some work using the mathematical theory of probability from an economic point of view he stated that in one of his work "The overloading of the subscribers resulted in considerable extra expenses on account of the telephone operators having to make repeated attempt to establish a connection". In order to develop more precise dimensioning FrJohannsen established scientific laboratory where he engaged the mathematician AgnerKrarupErlang. Erlang stated to work on the holding time in a telephone switch and then identified that the telephone conversations satisfied a Poisson distribution as well as the telephone holding time was exponentially distributed. Erlang work is the origin of the current network optimisation and queuing theory .

Little's Theorem:-

Little's theorem [3, 4] describes the relationship throughout arrival and service rate, cyclic time and work process. The theorem states that the expected numbers of customers L for a system in steady state can be derive from little's theorem;

$$L\ T = \lambda \ ...\ (1\ )$$

Where λ is the average customers arrival rate and T is the average service time for a customer. Three fundamental relationships can be derive from little's theorem

- L Increase if λ or T increase
- λ Increase if L increaseor T decrease
- T Increase if L increase or λ decrease

M/M/1 Queuing Model:

M/M/I queuing model means that arrival and service time are exponentially distributed or Poisson process. The following variables will be investigated;

- λ : The mean customer's arrival rate
- μ : The mean service rate $\rho\ \lambda\ \mu =$: Utilization factor ... 2( )
- Pn : The probability of having n customers $(1\ )$ n Pn $= - \rho\ \rho$ ... 3( )
  - $*$ Probability of zero customers 0P $= -1\ \rho$ ... 4( )
- L : The average number of customers 1 L $\rho\ \lambda\ \rho\ \mu\lambda = = - -$ ... 5( )
- Lq : The average number of customers in the queue 2 1 L L q $\rho\ \rho\lambda\ \rho\ \rho\ \mu\lambda = \times = = - -$ ... 6( )
- Wq : The average waiting time in the queue

$$W_q = \frac{L_q}{\lambda} = \frac{1}{\mu - \lambda} \qquad ...(7)$$

- * W : The average time spent, including the waiting time

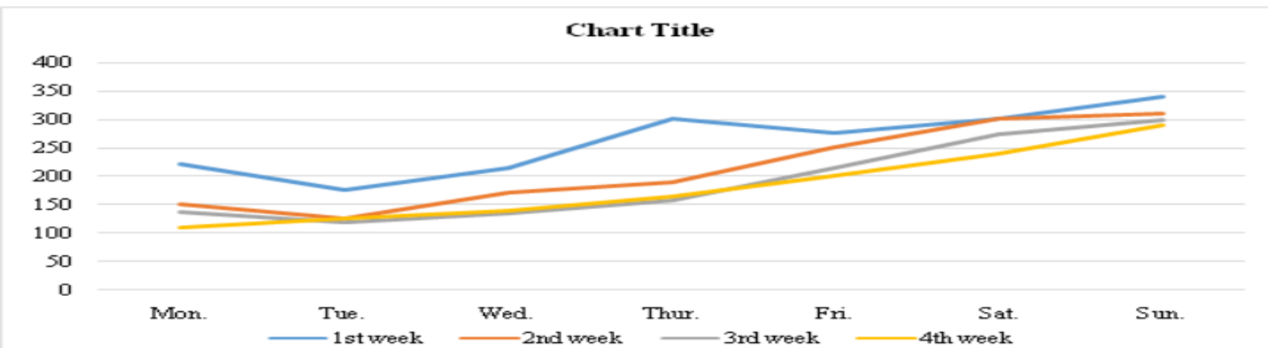
$$W = \frac{L}{\lambda} = \frac{1}{\mu - \lambda} \qquad ...(8)$$

Observations

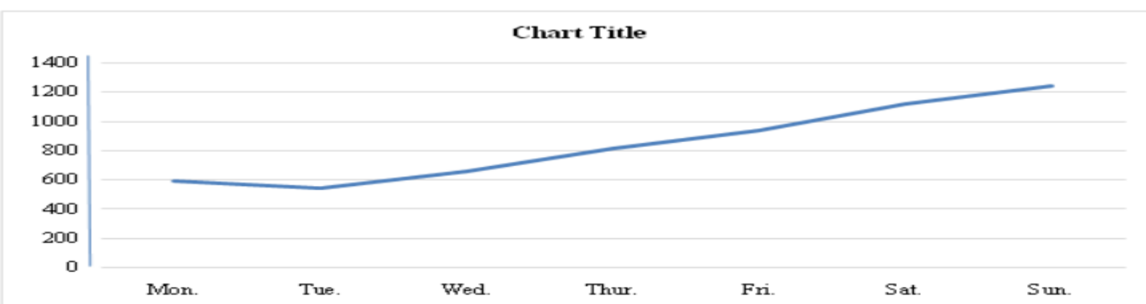
Below is one month daily customer data which was collected from the restaurant

Table One month customer count

	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Sun.
1st week	200	175	215	300	275	300	340
2nd week	150	125	170	190	250	300	310
3rd week	137	119	134	157	215	275	300
4th week	110	125	140	165	200	240	290
Total	597	544	659	812	940	1115	1240



One month daily customers count



One month total Customers Count

Calculations

It was investigated that, after Friday, during the weekend there are on average 210 people coming to the restaurant in 2(1/2) hours' time of dinner. From this we obtain the arrival rate as;

$$\lambda = 210/150 = 1.4 \ \text{Customers/minute}$$

We also found out from observations and discussions with the manager that each customer spend 50 minutes on average in the restaurant (W)the queue length is around 25 people (Lq) on average and waiting time is around 15 minutes. It can be shown that by (7)that the observed actual waiting time does not differ much when compared with to the theoretical waiting time as shown below

$$\begin{aligned} W_q &= \frac{25 customers}{1.4 cpm} \\ &= 17.85714286 \end{aligned}$$

Next, we will calculate the average number of people in the restaurant using (6)

$$L \ cpm = \times \ 1.4 \ 50\text{min} = 70\text{customers}$$

Using (2) & (5) we can also calculate the service and utilization rate as shown below;

$$\begin{aligned} \mu &= \frac{1.4(1+70)}{70} \\ &= 1.42 \end{aligned}$$

Probability of customers going away

=P (more than 10 people in the queue)

=P (more than110 people in the restaurant)

$$\begin{aligned} \therefore P_{111-130} &= \sum_{n=111}^{130} P_n \\ &= 0.01408 \sum_{n=111}^{130} (0.9859)^n \\ &= 0.051157132 \\ &= 5.1157\% \end{aligned}$$

Conclusion

The utilization rate at the restaurant is very large (0.9859). This is only utilization rate during lunch and dinner on Saturday and Sunday.

- In case the customer waiting time is less than 15mins, the number of customers that will be served per minute will increase. When the service rate is higher the utilization rate will be lower.
- The queue model formulation is much simpler than to provide simulation model.
- This research can help to increase the quality of service by anticipating if there are many customers in the queue.

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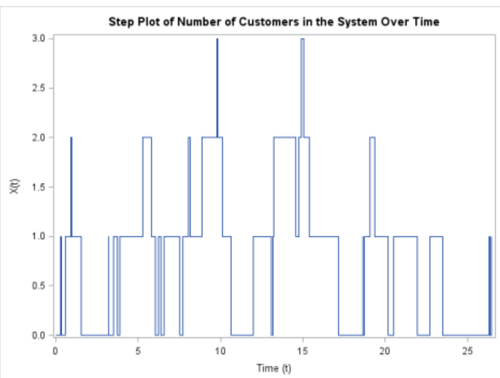
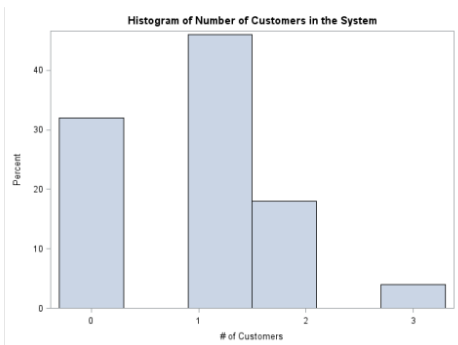
CODE:

```
/* M/M/1 Queue */
data mm1;
  * Specify parameters;
  lambda = 10; * arrival rate ;
  mu = 20; * service rate ;

  * Specify # of jumps for simulation ;
  Njumps = 50;

  * Start simulation at time 0 ;
  Xt = 0; * number of customers in time t;
  cumsum_Wn = 0; * cumsum_Wn is the cumulative waiting time for job nth ;
  p = 0; * 1 if server is busy and 0 otherwise ;

  output;

  * Start simulation in a do-until loop until we reach the limit ;
  do n = 2 to Njumps;
    t_rexp = rand("Exponential") * lambda;

      /* This if-else statement is based on the transition probability matrix
         and we randomly sample either -1 or 1 */
    if rand("Uniform") <= mu / (lambda+mu) then
      event = -1;
    else
      event = 1;

      /* If there is no customer in time t, then immediately move the process from
        0 to 1 customer according to transition rate matrix  */
    if Xt = 0 then
      do;
        Xt = Xt + 1;
       cumsum_Wn = cumsum_Wn + (t_rexp / lambda);
       p = 0;
      end;
    else
      do;
        Xt = Xt + event;
       cumsum_Wn = cumsum_Wn + (t_rexp / (lambda + mu));
       p = 1;
      end;

    output;
  end;

  drop lambda mu mins event hours;
run;

data mm1;
  set mm1;
  Wn = dif(cumsum_Wn); * Take the difference between waiting time

    * Create a variable called Lq where it measures # of customers waiting in the queue;
    if Xt = 0 then
      Lq = 0;
    else
      Lq = Xt - 1;
  run;

  data mm1;
    set mm1;
```

```
      Wn_Lq = abs(dif(Wn)); * Take the difference between waiting time
                              of the nth job and (n-1)th job in the QUEUE;
   run;

   /* M/D/1 Queue */
   data md1;
     * Specify parameters;
     D = 20;   * Deterministic time ;
     lambda = 15; * arrival rate ;
     mu = D; * service rate ;

     * Specify time for simulation ;
     Njumps = 50;

     * Start simulation at time 0 ;
     Xt = 0; * number of customers in time t;
     cumsum_Wn = 0; * cumsum_Wn is the cumulative waiting time for job nth ;
     p = 0;
     output;

     do n = 2 to Njumps;
       t_rexp = rand("Exponential") * lambda;

          /* This if-else statement is based on the transition probability matrix
             and we randomly sample either -1 or 1 */
       if rand("Uniform") <= mu / (lambda + mu) then
         event = -1;
       else
         event = 1;

          /* If there is no customer in time t, then immediately move the process from
            0 to 1 customer according to transition rate matrix  */
       if Xt = 0 then
            do;
           Xt = Xt + 1;
                event = 1;
             cumsum_Wn = cumsum_Wn + (t_rexp / lambda);
            end;
run;
```





Display 2. Histogram of number of customers in the M/M/1 system.

M/M/1 Result

Operating Characteristics	Mean
Average Number of Customers in the Queue	0.26
Average Number of Customers in the System	0.94
Average Waiting Time in the Queue	0.42
Average Time in the System	0.54
Average Server Utilization	0.68

Table 2. M/M/1 result with performance metrics.