

# Mathematics

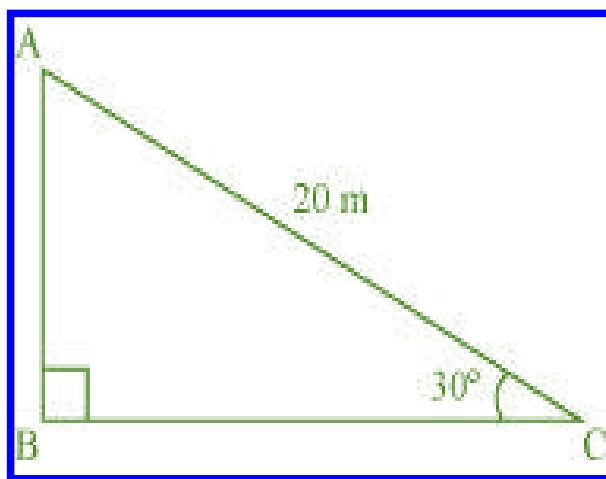
## (Chapter – 9) (Some Applications of Trigonometry)

(Class – X)

### Exercise 9.1

#### Question 1:

A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is  $30^\circ$ .



#### Answer 1:

It can be observed from the figure that AB is the pole.

In  $\triangle ABC$ ,

$$\frac{AB}{AC} = \sin 30^\circ$$

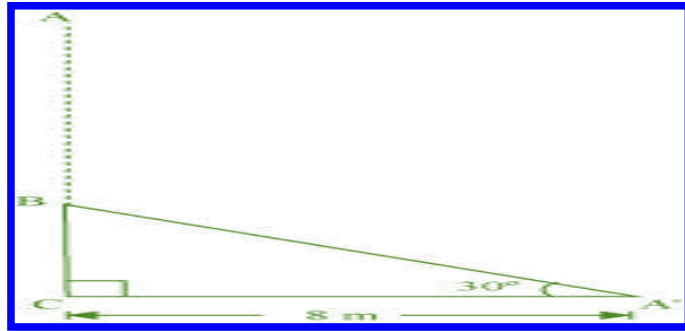
$$\frac{AB}{20} = \frac{1}{2}$$

$$AB = \frac{20}{2} = 10$$

Therefore, the height of the pole is 10 m.

**Question 2:**

A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle  $30^\circ$  with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

**Answer 2:**

Let AC was the original tree. Due to storm, it was broken into two parts. The broken part  $A'B$  is making  $30^\circ$  with the ground.

$$\begin{aligned} \text{In } \triangle A'BC, \quad \frac{BC}{A'C} &= \tan 30^\circ \\ \frac{BC}{8} &= \frac{1}{\sqrt{3}} \\ BC &= \left( \frac{8}{\sqrt{3}} \right) \text{ m} \\ \frac{A'C}{A'B} &= \cos 30^\circ \\ \frac{8}{A'B} &= \frac{\sqrt{3}}{2} \\ A'B &= \left( \frac{16}{\sqrt{3}} \right) \text{ m} \end{aligned}$$

Height of the tree =  $A'B + BC$

$$\begin{aligned} &= \left( \frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}} \right) \text{ m} = \frac{24}{\sqrt{3}} \text{ m} \\ &= 8\sqrt{3} \text{ m} \end{aligned}$$

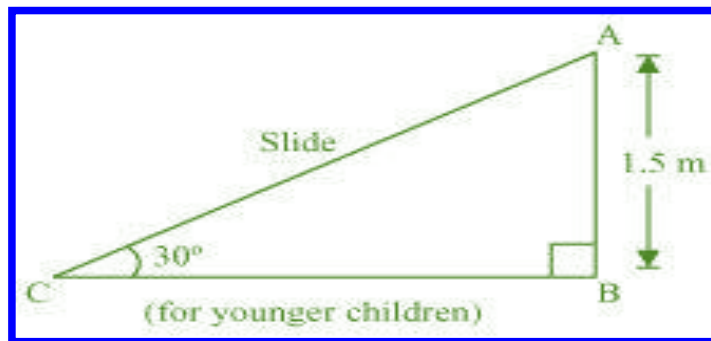
Hence, the height of the tree is  $8\sqrt{3}$  m.

**Question 3:**

A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of  $30^\circ$  to the ground, where as for the elder children she wants to have a steep side at a height of 3 m, and inclined at an angle of  $60^\circ$  to the ground. What should be the length of the slide in each case?

**Answer 3:**

It can be observed that AC and PR are the slides for younger and elder children respectively.

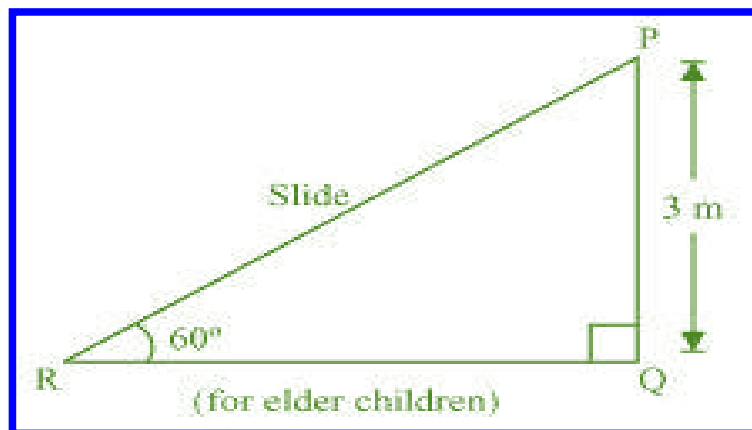


In  $\triangle ABC$ ,

$$\frac{AB}{AC} = \sin 30^\circ$$

$$\frac{1.5}{AC} = \frac{1}{2}$$

$$AC = 3 \text{ m}$$



In  $\triangle PQR$ ,

$$\frac{PQ}{PR} = \sin 60$$

$$\frac{3}{PR} = \frac{\sqrt{3}}{2}$$

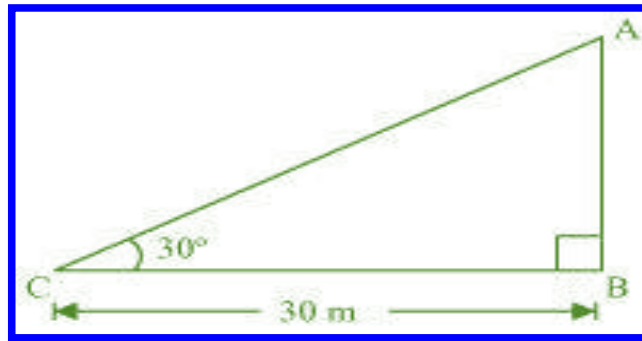
$$PR = \frac{6}{\sqrt{3}} = 2\sqrt{3} \text{ m}$$

Therefore, the lengths of these slides are 3 m and  $2\sqrt{3}$  m.

**Question 4:**

The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower is  $30^\circ$ . Find the height of the tower.

**Answer 4:**



Let AB be the tower and the angle of elevation from point C (on ground) is  $30^\circ$ .

In  $\triangle ABC$ ,

$$\frac{AB}{BC} = \tan 30^\circ$$

$$\frac{AB}{30} = \frac{1}{\sqrt{3}}$$

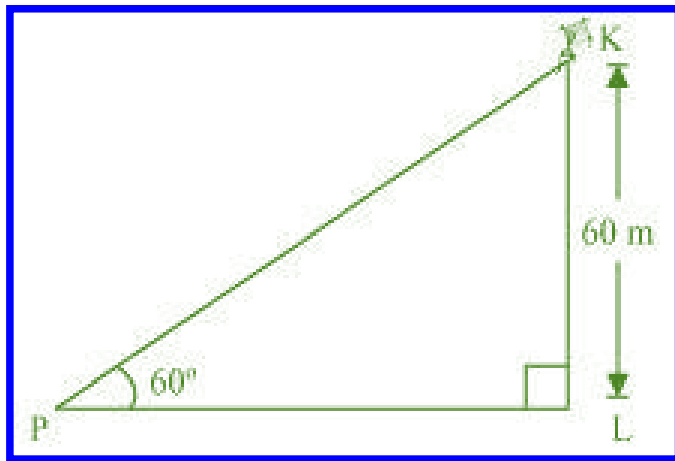
$$AB = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m}$$

Therefore, the height of the tower is  $10\sqrt{3}$  m.

**Question 5:**

A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is  $60^\circ$ . Find the length of the string, assuming that there is no slack in the string.

**Answer 5:**



Let K be the kite and the string is tied to point P on the ground.

In  $\triangle KLP$ ,

$$\frac{KL}{KP} = \sin 60^\circ$$

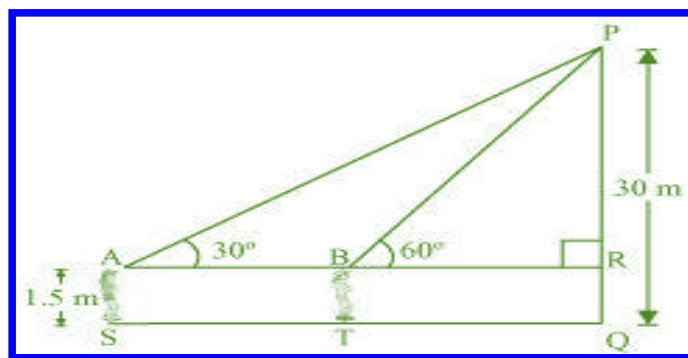
$$\frac{60}{KP} = \frac{\sqrt{3}}{2}$$

$$KP = \frac{120}{\sqrt{3}} = 40\sqrt{3} \text{ m}$$

Hence, the length of the string is  $40\sqrt{3}$  m.

**Question 6:**

A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from  $30^\circ$  to  $60^\circ$  as he walks towards the building. Find the distance he walked towards the building.

**Answer 6:**

Let the boy was standing at point S initially. He walked towards the building and reached at point T. It can be observed that

$$\begin{aligned} PR &= PQ - RQ \\ &= (30 - 1.5) \text{ m} = 28.5 \text{ m} = \frac{57}{2} \text{ m} \end{aligned}$$

In  $\triangle PAR$ ,

$$\frac{PR}{AR} = \tan 30^\circ$$

$$\frac{57}{2AR} = \frac{1}{\sqrt{3}}$$

$$AR = \left( \frac{57}{2} \sqrt{3} \right) \text{ m}$$

In  $\triangle PRB$ ,

$$\frac{PR}{BR} = \tan 60^\circ$$

$$\frac{57}{2BR} = \sqrt{3}$$

$$BR = \frac{57}{2\sqrt{3}} = \left( \frac{19\sqrt{3}}{2} \right) \text{ m}$$

$$ST = AB$$

$$= AR - BR = \left( \frac{57\sqrt{3}}{2} - \frac{19\sqrt{3}}{2} \right) \text{m}$$

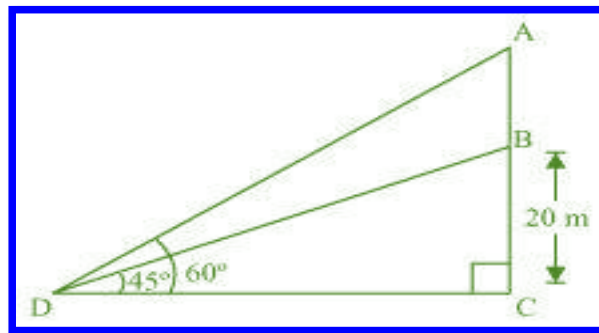
$$= \left( \frac{38\sqrt{3}}{2} \right) \text{m} = 19\sqrt{3} \text{ m}$$

Hence, he walked  $19\sqrt{3}$  m towards the building.

### Question 7:

From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are  $45^\circ$  and  $60^\circ$  respectively. Find the height of the tower.

### Answer 7:



Let BC be the building, AB be the transmission tower, and D be the point on the ground from where the elevation angles are to be measured.

In  $\triangle BCD$ ,

$$\frac{BC}{CD} = \tan 45^\circ$$

$$\frac{20}{CD} = 1$$

$$CD = 20 \text{ m}$$

In  $\triangle ACD$ ,

$$\frac{AC}{CD} = \tan 60^\circ$$

$$\frac{AB + BC}{CD} = \sqrt{3}$$

$$\frac{AB + 20}{20} = \sqrt{3}$$

$$AB = (20\sqrt{3} - 20) \text{ m}$$

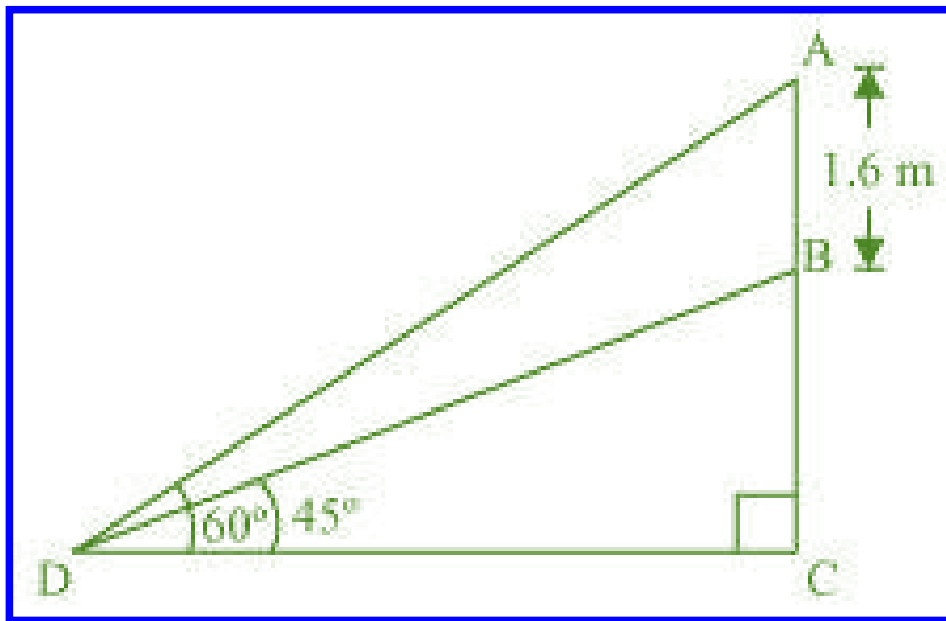
$$= 20(\sqrt{3} - 1) \text{ m}$$

Therefore, the height of the transmission tower is  $20(\sqrt{3} - 1)$  m.

**Question 8:**

A statue, 1.6 m tall, stands on a top of pedestal, from a point on the ground, the angle of elevation of the top of statue is  $60^\circ$  and from the same point the angle of elevation of the top of the pedestal is  $45^\circ$ . Find the height of the pedestal.

**Answer 8:**





Let AB be the statue, BC be the pedestal, and D be the point on the ground from where the elevation angles are to be measured.

In  $\triangle BCD$ ,

$$\frac{BC}{CD} = \tan 45^\circ$$

$$\frac{BC}{CD} = 1$$

$$BC = CD$$

In  $\triangle ACD$ ,

$$\frac{AB + BC}{CD} = \tan 60^\circ$$

$$\frac{AB + BC}{BC} = \sqrt{3}$$

$$1.6 + BC = BC\sqrt{3}$$

$$BC(\sqrt{3} - 1) = 1.6$$

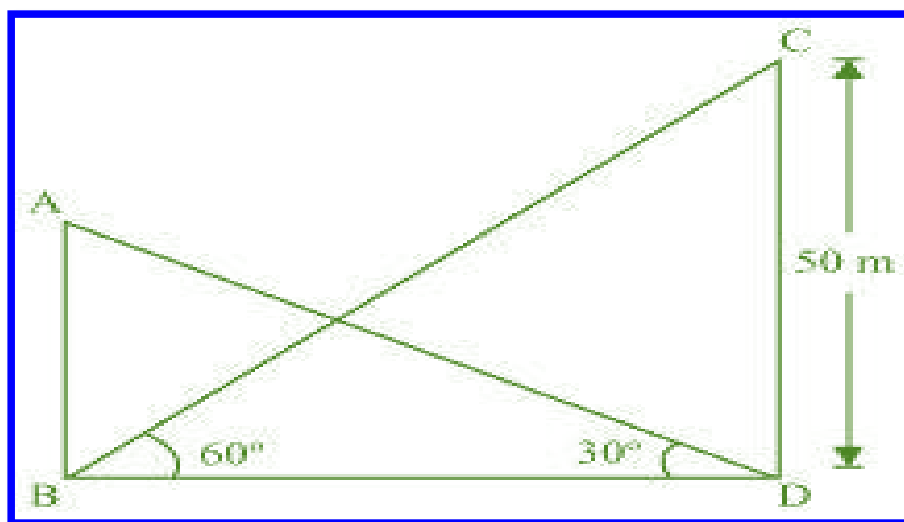
$$\begin{aligned} BC &= \frac{(1.6)(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} \\ &= \frac{1.6(\sqrt{3} + 1)}{(\sqrt{3})^2 - (1)^2} \\ &= \frac{1.6(\sqrt{3} + 1)}{2} = 0.8(\sqrt{3} + 1) \end{aligned}$$

Therefore, the height of the pedestal is  $0.8(\sqrt{3} + 1)$  m.

### Question 9:

The angle of elevation of the top of a building from the foot of the tower is  $30^\circ$  and the angle of elevation of the top of the tower from the foot of the building is  $60^\circ$ . If the tower is 50 m high, find the height of the building.

**Answer 9:**



Let AB be the building and CD be the tower.

In  $\triangle CDB$ ,

$$\frac{CD}{BD} = \tan 60^\circ$$

$$\frac{50}{BD} = \sqrt{3}$$

$$BD = \frac{50}{\sqrt{3}}$$

In  $\triangle ABD$ ,

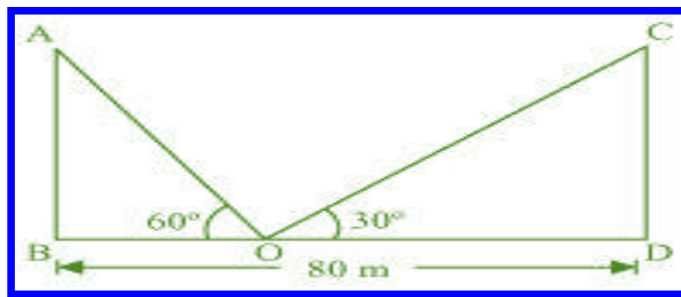
$$\frac{AB}{BD} = \tan 30^\circ$$

$$AB = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{50}{3} = 16\frac{2}{3}$$

Therefore, the height of the building is  $16\frac{2}{3}$  m.

**Question 10:**

Two poles of equal heights are standing opposite each other and either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are  $60^\circ$  and  $30^\circ$ , respectively. Find the height of poles and the distance of the point from the poles.

**Answer 10:**

Let AB and CD be the poles and O is the point from where the elevation angles are measured.

In  $\triangle ABO$ ,

$$\frac{AB}{BO} = \tan 60^\circ$$

$$\frac{AB}{BO} = \sqrt{3}$$

$$BO = \frac{AB}{\sqrt{3}}$$

In  $\triangle CDO$ ,

$$\frac{CD}{DO} = \tan 30^\circ$$

$$\frac{CD}{80 - BO} = \frac{1}{\sqrt{3}}$$

$$CD\sqrt{3} = 80 - BO$$

$$CD\sqrt{3} = 80 - \frac{AB}{\sqrt{3}}$$

$$CD\sqrt{3} + \frac{AB}{\sqrt{3}} = 80$$

Since the poles are of equal heights,

$$CD = AB$$

$$CD \left[ \sqrt{3} + \frac{1}{\sqrt{3}} \right] = 80$$

$$CD \left( \frac{3+1}{\sqrt{3}} \right) = 80$$

$$CD = 20\sqrt{3} \text{ m}$$

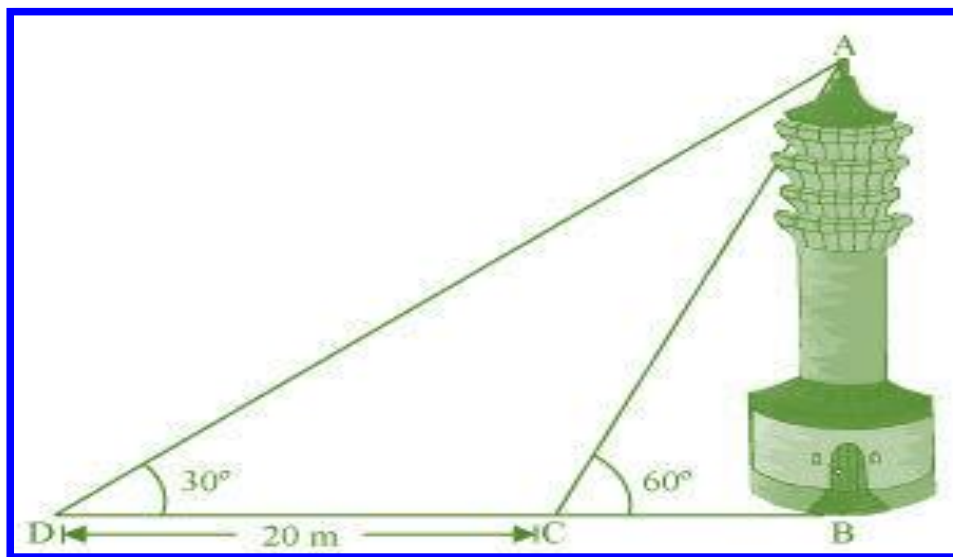
$$BO = \frac{AB}{\sqrt{3}} = \frac{CD}{\sqrt{3}} = \left( \frac{20\sqrt{3}}{\sqrt{3}} \right) \text{ m} = 20 \text{ m}$$

$$DO = BD - BO = (80 - 20) \text{ m} = 60 \text{ m}$$

Therefore, the height of poles is  $20\sqrt{3}$  m and the point is 20 m and 60 m far from these poles.

### Question 11:

A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower the angle of elevation of the top of the tower is  $60^\circ$ . From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is  $30^\circ$ . Find the height of the tower and the width of the canal.



**Answer 11:**

In  $\triangle ABC$ ,

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\frac{AB}{BC} = \sqrt{3}$$

$$BC = \frac{AB}{\sqrt{3}}$$

In  $\triangle ABD$ ,

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\frac{AB}{BC + CD} = \frac{1}{\sqrt{3}}$$

$$\frac{AB}{\frac{AB}{\sqrt{3}} + 20} = \frac{1}{\sqrt{3}}$$

$$\frac{AB\sqrt{3}}{AB + 20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$3AB = AB + 20\sqrt{3}$$

$$2AB = 20\sqrt{3}$$

$$AB = 10\sqrt{3} \text{ m}$$

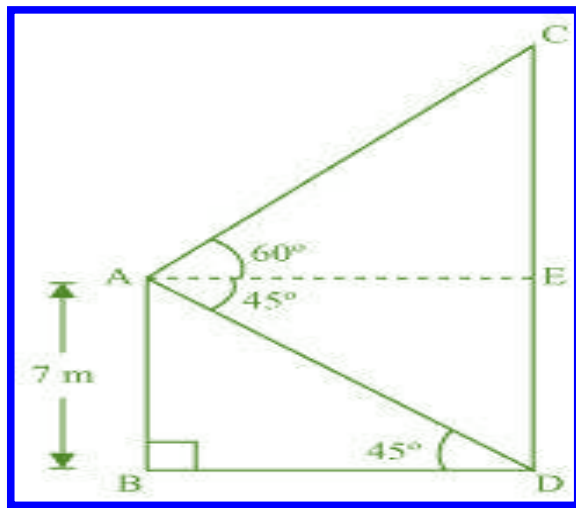
$$BC = \frac{AB}{\sqrt{3}} = \left( \frac{10\sqrt{3}}{\sqrt{3}} \right) \text{ m} = 10 \text{ m}$$

Therefore, the height of the tower is  $10\sqrt{3}$  m and the width of the canal is 10 m.

**Question 12:**

From the top of a 7 m high building, the angle of elevation of the top of a cable tower is  $60^\circ$  and the angle of depression of its foot is  $45^\circ$ . Determine the height of the tower.

**Answer 12:**



Let AB be a building and CD be a cable tower.

In  $\triangle ABD$ ,

$$\frac{AB}{BD} = \tan 45^\circ$$

$$\frac{7}{BD} = 1$$

$$BD = 7 \text{ m}$$

In  $\triangle ACE$ ,

$$AC = BD = 7 \text{ m}$$

$$\frac{CE}{AE} = \tan 60^\circ$$

$$\frac{CE}{7} = \sqrt{3}$$

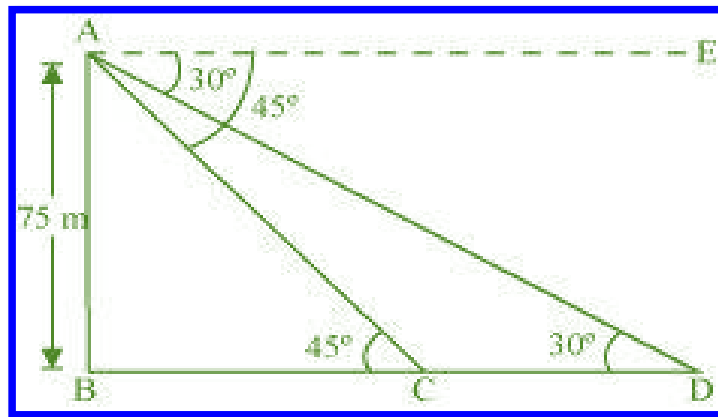
$$CE = 7\sqrt{3} \text{ m}$$

$$\begin{aligned} CD &= CE + ED = (7\sqrt{3} + 7) \text{ m} \\ &= 7(\sqrt{3} + 1) \text{ m} \end{aligned}$$

Therefore, the height of the cable tower is  $7(\sqrt{3} + 1) \text{ m}$ .

**Question 13:**

As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are  $30^\circ$  and  $45^\circ$ . If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

**Answer 13:**

Let AB be the lighthouse and the two ships be at point C and D respectively.

In  $\triangle ABC$ ,

$$\frac{AB}{BC} = \tan 45^\circ$$

$$\frac{75}{BC} = 1$$

$$BC = 75 \text{ m}$$

In  $\triangle ABD$ ,

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\frac{75}{BC + CD} = \frac{1}{\sqrt{3}}$$

$$\frac{75}{75 + CD} = \frac{1}{\sqrt{3}}$$

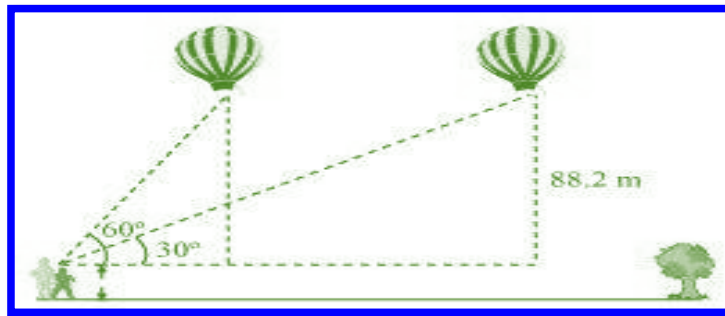
$$75\sqrt{3} = 75 + CD$$

$$75(\sqrt{3} - 1) \text{ m} = CD$$

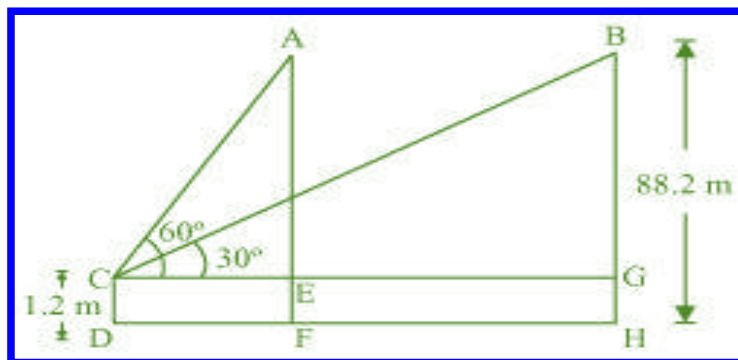
Therefore, the distance between the two ships is  $75(\sqrt{3} - 1) \text{ m}$ .

**Question 14:**

A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is  $60^\circ$ . After some time, the angle of elevation reduces to  $30^\circ$ . Find the distance travelled by the balloon during the interval.



**Answer 14:**



Let the initial position A of balloon change to B after some time and CD be the girl.  
In  $\triangle ACE$ ,



$$\frac{AE}{CE} = \tan 60^\circ$$

$$\frac{AF - EF}{CE} = \tan 60^\circ$$

$$\frac{88.2 - 1.2}{CE} = \sqrt{3}$$

$$\frac{87}{CE} = \sqrt{3}$$

$$CE = \frac{87}{\sqrt{3}} = 29\sqrt{3} \text{ m}$$

In  $\triangle BCG$ ,

$$\frac{BG}{CG} = \tan 30^\circ$$

$$\frac{88.2 - 1.2}{CG} = \frac{1}{\sqrt{3}}$$

$$87\sqrt{3} \text{ m} = CG$$

Distance travelled by balloon =  $EG = CG - CE$

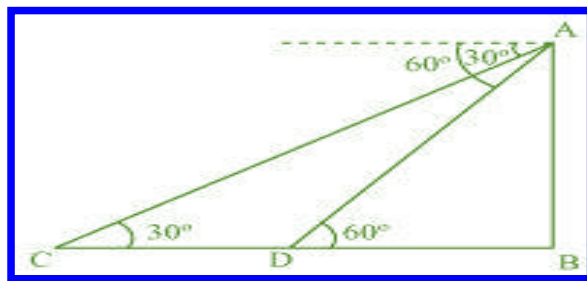
$$= (87\sqrt{3} - 29\sqrt{3}) \text{ m}$$

$$= 58\sqrt{3} \text{ m}$$

### Question 15:

A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car as an angle of depression of  $30^\circ$ , which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be  $60^\circ$ . Find the time taken by the car to reach the foot of the tower from this point.

### Answer 15:



Let AB be the tower.

Initial position of the car is C, which changes to D after six seconds.

In  $\triangle ADB$ ,

$$\frac{AB}{DB} = \tan 60^\circ$$

$$\frac{AB}{DB} = \sqrt{3}$$

$$DB = \frac{AB}{\sqrt{3}}$$

In  $\triangle ABC$ ,

$$\frac{AB}{BC} = \tan 30^\circ$$

$$\frac{AB}{BD + DC} = \frac{1}{\sqrt{3}}$$

$$AB\sqrt{3} = BD + DC$$

$$AB\sqrt{3} = \frac{AB}{\sqrt{3}} + DC$$

$$DC = AB\sqrt{3} - \frac{AB}{\sqrt{3}} = AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$
$$= \frac{2AB}{\sqrt{3}}$$

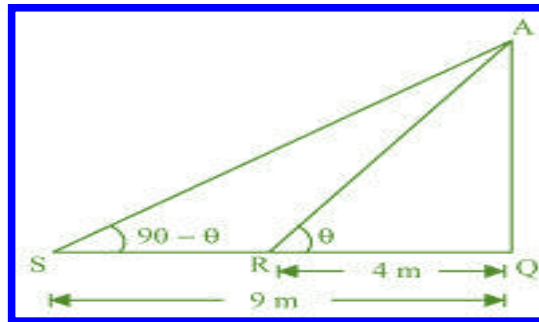
Time taken by the car to travel distance  $DC = \left(\text{i.e., } \frac{2AB}{\sqrt{3}}\right)$  6 seconds Time taken by

the car to travel distance  $DB \left(\text{i.e., } \frac{AB}{\sqrt{3}}\right) = \frac{6}{\frac{2AB}{\sqrt{3}}} \times \frac{AB}{\sqrt{3}}$

$$= \frac{6}{2} = 3 \text{ seconds}$$

**Question 16:**

The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m. from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

**Answer 16:**

Let AQ be the tower and R, S are the points 4m, 9m away from the base of the tower respectively.

The angles are complementary. Therefore, if one angle is  $\theta$ , the other will be  $90^\circ - \theta$ .

In  $\triangle AQR$ ,

$$\frac{AQ}{QR} = \tan \theta$$

$$\frac{AQ}{4} = \tan \theta \quad \dots (i)$$

In  $\triangle AQS$ ,

$$\frac{AQ}{SQ} = \tan(90^\circ - \theta)$$

$$\frac{AQ}{9} = \cot \theta \quad \dots (ii)$$

On multiplying equations (i) and (ii), we obtain

$$\left( \frac{AQ}{4} \right) \left( \frac{AQ}{9} \right) = (\tan \theta) \cdot (\cot \theta)$$

$$\frac{AQ^2}{36} = 1$$

$$AQ^2 = 36$$

$$AQ = \sqrt{36} = \pm 6$$

However, height cannot be negative.

Therefore, the height of the tower is 6 m.