WRITE ALGORITHM TO SOLVE SYSTEM OF LINEAR EQUATION

Introduction

Numeric algorithms for solving the linear systems of tridiagonal type have already existed. The well-known Thomas algorithm is an example of such algorithms. The current paper is mainly devoted to constructing symbolic algorithms for solving tridiagonal linear systems of equations via transformations. The new symbolic algo-rithms remove the cases where the numeric algorithms fail. The computational cost of these algorithms is given.MAPLE procedures based on these algorithms are presented.

Some illustrative examples are given.Linear systems of equations of tridiagonal type arise in solving problems in a wide variety of disciplines including physics [1,2], mathematics [3-8], engineering [9,10] and others. Many researchers have been devoted to dealing with such systems (see [11-27]).

When a system of linear equations has a coefficient matrix of special structure, it is recommended to use a tailor-made algorithm for such systems of equations. The tailor-made algorithms are not only more efficient in terms of computational time and computer memory, but also accumulate smaller round-off errors. As a matter of fact, many problems arising in practice lead to the solution of linear system of equations with special coefficient matrices. The current paper is mainly devoted to developing new algorithms for solving linear system of equations of tridiagonal type of the form:

This is always a good habit in computation in order to save memory space. Of course, the non-singularity of the coefficient matrix should be checked firstly to make sure that the system

Every conceivable physical system or scientific problem involves at its core a system of linear algebraic equations. For instance, fitting a curve to a set of datapoints, optimizing a cost function, analyzing electrical networks etc, requires solving linear equations.

Some of the fundamental tasks in computational linear algebra are:

- Solving a system of linear equations
- Finding the inverse of a square matrix
- Computing the determinant of a matrix

An applied introduction to the algorithms for performing such fundamental tasks follows.

In this tutorial, we'll learn, how a matrix can be decomposed into simpler lego blocks. Operating on these smaller lego blocks is often easier, than working with the original matrix.

Gaussian Elimination With Partial Pivoting

Consider an elementary system of three linear equations :in three unknowns . We can write this system in the matrix form as :

The high-school algorithm to solve a system of linear equations consists of performing elementary row operations on the equations. Since performing operations on the equations also affects their right-hand sides, keeping track of everything is most easily done using the augmented matrixWe shall call the entry of the coefficient matrix , the first pivot, the entry as the second pivot, and so on. In regular Gaussian Elimination, the basic idea is to make all entries in the column below the th pivot equal to .

A key requirement, therefore, is that, a pivot element must always be nonzero. We sometimes need to interchange rows and , to ensure that the pivot is non-zero. The most common practice is to choose , such that is maximal. This is called partial pivoting.

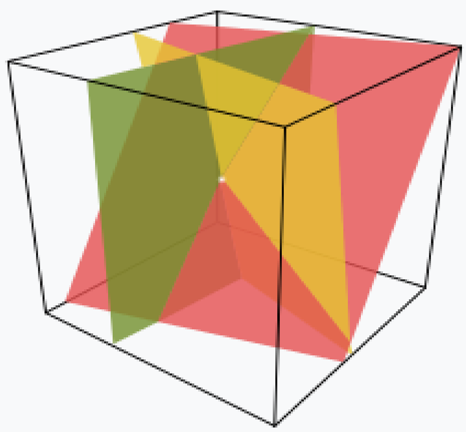
Since we have performed elementary operations on both the left and righthand sides of the system, the resulting system of equations , is equivalent to the original system . and have the same solution vector .

Note that, is an upper-triangular matrix with all non-zero elements – the pivots on the diagonal. A square matrix is called non-singular if it can be reduced to an upper triangular form by elementary row operations.

The Gaussian Elimination algorithm with partial pivoting in the form of psuedocode is given below:

System of linear equation

In mathematics,a system of linear equations (or linear system) is a collection of one or more linear equation involving the same variable.



A linear system in three variables determines a collection of plans The intersection point is the solution.

For example is a system of three equations in the three variables x, y, z. A solution to a

linear system is an assignment of values to the variables such that all the equations are simultaneously satisfied.
A soluation to the system above is given by the triple

since it makes all three equations valid. The word "system" indicates that the equations are to be considered collectively, rather than individually.

In mathematics, the theory of linear systems is the basis and a fundamental part of linear algebra, a subject which is used in most parts of modern mathematics.

Computational algorithms for finding the solutions are an important part of numerical linear algebra, and play a prominent role in engineering, physics, chemistry, computer science, and economics. A system of non-linear equation can often be appoximated by a linear system (see linearization), a helpful technique when making a mathematical model or computer simulation of a relatively complex system.

Very often, and in this article, the coefficients of the equations are real or complex number and the solutions are searched in the same set of numbers, but the theory and the algorithms apply for coefficients and solutions in any field For solutions in an integral domain like the ring of the integers, or in other algebric struct, other theories have been developed, see linear equation over a ring. Integer linear programming is a collection of methods for finding the "best" integer solution (when there are many).

Grobner basic theory provides algorithms when coefficients and unknowns are polynomials Also tropical geometry is an example of linear algebra in a more exotic structure.

Elementary examples

Trivial example

The system of one equation in one unknown has the solution

However, a linear system is commonly considered as having at least two equations.

Simple nontrivial example

The simplest kind of nontrivial linear system involves two equations and two variables:

One method for solving such a system is as follows. First, solve the top equation for in terms of :

Now substitute this expression for *x* into the bottom equation:

This results in a single equation involving only the variable . Solving gives , and substituting this back into the equation for yields . This method generalizes to systems with additional variables (see "elimination of variables" below, or the article on elementry algebra.)

General form

A general system of *m* linear equations with and coefficients can be written as

where are the unknowns, are the coefficients of the system, and are the constant terms.

Often the coefficients and unknowns are real or complex number, but integers and rational number are also seen, as are polynomials and elements of an abstract algebric structure.

Vector equation

One extremely helpful view is that each unknown is a weight for a column veator in a linear combination.

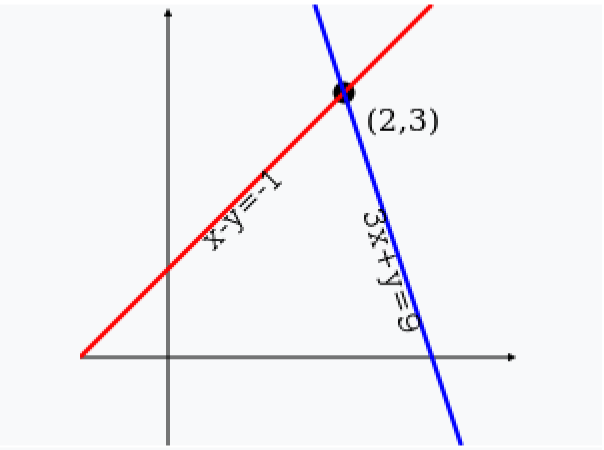
This allows all the language and theory of vector space (or more generally, moduals) to be brought to bear. For example, the collection of all possible linear combinations of the vectors on the left-hand side is called their span, and the equations have a solution just when the righthand vector is within that span. If every vector within that span has exactly one expression as a linear combination of the given left-hand vectors, then any solution is unique. In any event, the span has a basis of linearly independent vectors that do guarantee exactly one expression; and the number of vectors in that basis (its dimention) cannot be larger than *m* or *n*, but it can be smaller.

This is important because if we have *m* independent vectors a solution is guaranteed regardless of the right-hand side, and otherwise not guaranteed.

Matrix equation

The vector equation is equivalent to a matrix equation of the form where *A* is an *m*×*n* matrix, **x** is a column vector with *n* entries, and **b** is a column vector with *m* entries.
The number of vectors in a basis for the span is now expressed as the rank of the matrix.

Solution set



The solution set for the equations $x - y = -1$ and $3x + y = 9$ is the single point (2, 3).

A **solution** of a linear system is an assignment of values to the variables $x_1, x_2, ..., x_n$ such that each of the equations is satisfied. The set of all possible solutions is called the soluation set A linear system may behave in any one of three possible ways:

1. The system has *infinitely many solutions*.
2. The system has a single *unique solution*.
3. The system has *no solution*.

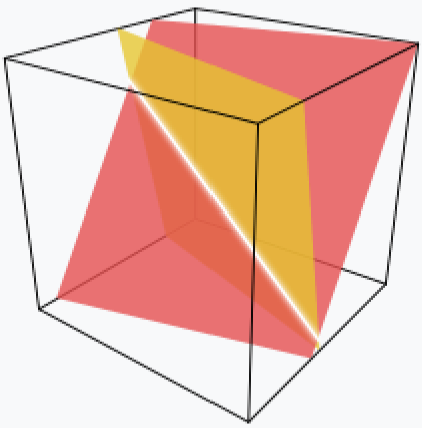
Geometric interpretation

For a system involving two variables (*x* and *y*), each linear equation determines a line on the *xy*-plane. Because a solution to a linear system must satisfy all of the equations, the solution set is the intersection of these lines, and is hence either a line, a single point, or the empty set.

For three variables, each linear equation determines a plane in three dimentional space , and the solution set is the intersection of these planes. Thus the solution set may be a plane, a line, a single point, or the empty set. For example, as three parallel planes do not have a common point, the solution set of their equations is empty; the solution set of the equations of three planes intersecting at a point is single point; if three planes pass through two points, their equations have at least two common solutions; in fact the solution set is infinite and consists in all the line passing through these points.

For *n* variables, each linear equation determines a hyperplane in ndimensional. The solution set is the intersection of these hyperplanes, and is a flat, which may have any dimension lower than *n*.

General behavior

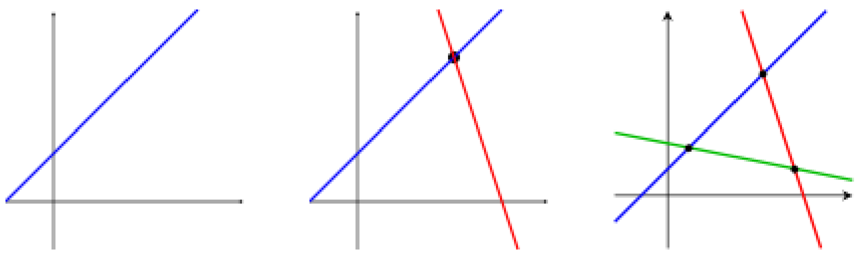


The solution set for two equations in three variables is, in general, a line.

- In general, the behavior of a linear system is determined by the relationship between the number of equations and the number of unknowns. Here, "in general" means that a different behavior may occur for specific values of the coefficients of the equations.

- In general, a system with fewer equations than unknowns has infinitely many solutions, but it may have no solution. Such a system is known as an underdetermined system.
- In general, a system with the same number of equations and unknowns has a single unique solution.
- In general, a system with more equations than unknowns has no solution. Such a system is also known as an overdetermined system.
- In the first case, the dimention of the solution set is, in general, equal to $n - m$, where n is the number of variables and m is the number of equations.

The following pictures illustrate this trichotomy in the case of two variables:



One equation Two equations Three equations

The first system has infinitely many solutions, namely all of the points on the blue line. The second system has a single unique solution, namely the intersection of the two lines. The third system has no solutions, since the three lines share no common point.

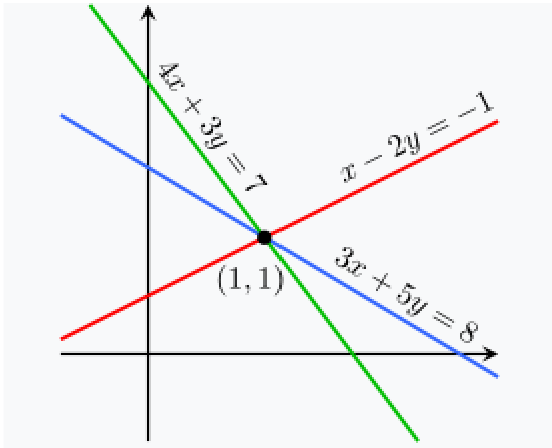
It must be kept in mind that the pictures above show only the most common case (the general case). It is possible for a system of two equations and two unknowns to have no solution (if the two lines are parallel), or for a system of three equations and two unknowns to be solvable (if the three lines intersect at a single point).

A system of linear equations behave differently from the general case if the equations are linearly dependent , or if it is inconsistent and has no more equations than unknowns.

Properties

Independence

The equations of a linear system are **independent** if none of the equations can be derived algebraically from the others. When the equations are independent, each equation contains new information about the variables, and removing any of the equations increases the size of the solution set. For linear equations, logical independence is the same as linear independence.



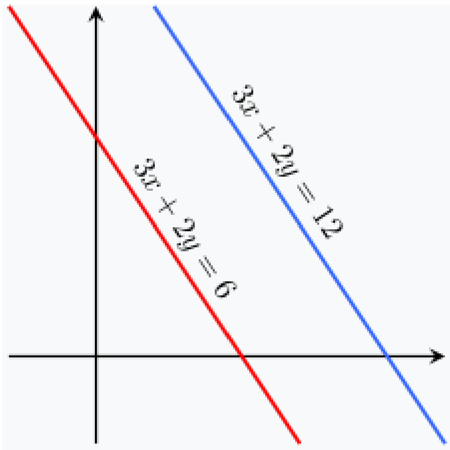
The equations $x - 2y = -1$. $3x + 5y = 8$, and $4x + 3y = 7$ are linearly dependent.

For example, the equations are not independent — they are the same equation when scaled by a factor of two, and they would produce identical graphs. This is an example of equivalence in a system of linear equations.

For a more complicated example, the equations are not independent, because the third equation is the sum of the other two. Indeed, any one of these equations can be derived from the other two, and any one of the equations can be removed without affecting the solution set. The graphs of these equations are three lines that intersect at a single point.

Consistency

See also: consistent and inconsistent equations



The equations 3*x* + 2*y* = 6 and 3*x* + 2*y* = 12 are inconsistent.

A linear system is inconsistent if it has no solution, and otherwise it is said to be consistent. When the system is inconsistent, it is possible to derive a contradiction from the equations, that may always be rewritten as the statement 0 = 1. For example, the equations are inconsistent. In fact, by subtracting the first equation from the second one and multiplying both sides of the result by 1/6, we get 0 = 1. The graphs of these equations on the *xy*-plane are a pair of parallel lines.It is possible for three linear equations to be inconsistent, even though any two of them are consistent together. For example, the equations are inconsistent. Adding the first two equations together gives 3*x* + 2*y* = 2, which can be subtracted from the third equation to yield 0 = 1. Any two of these equations have a common solution. The same phenomenon can occur for any number of equations.

In general, inconsistencies occur if the left-hand sides of the equations in a system are linearly dependent, and the constant terms do not satisfy the dependence relation. A system of equations whose left-hand sides are linearly independent is always consistent.

Putting it another way, according to the rauche-capelli theorem , any system of equations (overdetermined or otherwise) is inconsistent if the rank of the augmented matrix is greater than the rank of the coefficient matrix. If, on the other hand, the ranks of these two matrices are equal, the system must have at least one solution. The solution is unique if and only if the rank equals the number of variables. Otherwise the general solution has *k* free parameters where *k* is the difference between the number of variables and the rank; hence in such a case there are an infinitude of solutions. The rank of a system of equations (that is, the rank of the augmented matrix) can never be higher than [the number of variables] + 1, which means that a system with any number of equations can always be reduced to a system that has a number of independent equations that is at most equal to [the number of variables] + 1.

Equivalence

Two linear systems using the same set of variables are equivalent if each of the equations in the second system can be derived algebraically from the equations in the first system, and vice versa. Two systems are equivalent if either both are inconsistent or each equation of each of them is a linear combination of the equations of the other one. It follows that two linear systems are equivalent if and only if they have the same solution set.

Solving a linear system

There are several algorithms for solving a system of linear equations.

Describing the solution

When the solution set is finite, it is reduced to a single element. In this case, the unique solution is described by a sequence of equations whose left-hand sides are the names of the unknowns and right-hand sides are the corresponding values,

for example .

When an order on the unknowns has been fixed, for example the alphabetical order the solution may be described as a vector of values, like for the previous example.

To describe a set with an infinite number of solutions, typically some of the variables are designated as free (or independent, or as parameters), meaning that they are allowed to take any value, while the remaining variables are dependent on the values of the free variables. For example, consider the following system:The solution set to this system can be described by the following equations:

Here z is the free variable, while x and y are dependent on z. Any point in the solution set can be obtained by first choosing a value for z, and then computing the corresponding values for x and y.

Each free variable gives the solution space one degree of freedom, the number of which is equal to the dimesion of the solution set. For example, the solution set for the above equation is a line, since a point in the solution set can be chosen by specifying the value of the parameter z. An infinite solution of higher order may describe a plane, or higher-dimensional set.

Different choices for the free variables may lead to different descriptions of the same solution set. For example, the solution to the above equations can alternatively be described as follows:

Here x is the free variable, and y and z are dependent.

Elimination of variables

The simplest method for solving a system of linear equations is to repeatedly eliminate variables. This method can be described as follows:

1. In the first equation, solve for one of the variables in terms of the others.
2. Substitute this expression into the remaining equations. This yields a system of equations with one fewer equation and unknown.

Repeatsteps 1 and 2 until the system is reduced to a single linear equation.

Solve this equation, and then back-substitute until the entire solution is found. For example, consider the following system:

Solving the first equation for x gives $x = 5 + 2z - 3y$, and plugging this into the second and third equation yields

Since the LHS of both of these equations equal y, equating the RHS of the equations. We now have:

Substituting $z = 2$ into the second or third equation gives $y = 8$, and the values of y and z into the first equation yields $x = -15$. Therefore, the solution set is the ordered triple .

Row reduction

Main article*:* gaussian elimination

In row reduction (also known as Gaussian elimination), the linear system is represented as an augmented matrix:

This matrix is then modified using elementary row operations until it reaches reduced row echelon form. There are three types of elementary row operations:

Type 1: Swap the positions of two rows.
Type 2: Multiply a row by a nonzero scalar.
Type 3: Add to one row a scalar multiple of another.

Because these operations are reversible, the augmented matrix produced always represents a linear system that is equivalent to the original.

There are several specific algorithms to row-reduce an augmented matrix, thesimplest of which are gaussian elimination and gauss-jordan elimation. The following computation shows Gauss–Jordan elimination applied to the matrix above:

The last matrix is in reduced row echelon form, and represents the system $x = -15$, $y = 8$, $z = 2$. A comparison with the example in the previous sectionon the algebraic elimination of variables shows that these two methods are in fact the same; the difference lies in how the computations are written down.

CONCLUSIONS

The graph-theoretical approach to the problem of solving sparse systems of linear equations was first used by Parter in 1961 [4]. Since then the subject has been treated extensively in the literature (e.g., [3, 5, 61). In many algorithms, the edges of a graph are used to represent the nonzero structure of the matrix A, rather than the explicit equations.

Some properties of the graph are then explored in order to minimize the amount of computation when Gaussian elimination is used. We have shown that for systems of linear equations with at most two variables per equation, the edges of a graph are well suited to represent the explicit equations. Similar situations occur in, for example,

linear programming and logic when the number of variables per constraint, or respectively per clause, is at most two [2]. For the LE(2) problem, the key observation is that a given system can be partitioned into independent subsystems with at most one free parameter.

It would be interesting to determine whether an approach similar to the presented one can be used for systems with at most three variables per 124 BENGT ASPVALL AN-D YOSSI SHILOACH equation. Can this problem be partitioned into subproblems with relatively few free parameters using some combinatorial structure? Are k-trees interesting generalizations in this context? Any dense n X n system of linear equations can, by the introduction of new variables, be transformed into a sparse n2 x n2 system with at most three variables per equation. It will therefore probably be difficult to find a method using only O(d) free parameters for some small positive E, but an O(fi) bound on the number of free parameters would be helpful in developing an efficient algorithm for sparse systems without any apparent structure