Physical and Mathematical modelling of steelmaking processes (MSE629A)

Final Project

Turbulent Flow in Tundish and Shroud System

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Aim:

To numerically compute the steady turbulent velocity and concentration profile of the tracer at the outlet.

Objective:

1. Estimation of the volume of liquid in the tundish at a steady state and hence, calculation of theoretical or nominal residence time.
2. Plotting the concentration (exit) as a function of time and calculating the minimum breakthrough time as well as the time at which the probe registers maximum concentration.
3. Determination of numerical mean residence time by integrating the curve suitably.
4. Calculation of the proportions of dead, well-mixed and dispersed plug flow volumes.
5. Inferences on the metallurgical performance of the industrial scale tundish system.

Formulation of Problem:

1. <u>Governing Equations</u>

The continuity Equation

$$\frac{\partial(\rho)}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

X component of the incompressible Navier stoke equation

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho uu)}{\partial x}+\frac{\partial(\rho uv)}{\partial y}+\frac{\partial(\rho uw)}{\partial x}=\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial w}{\partial z}\right)+\rho g_x$$

Y component

$$\frac{\partial(\rho v)}{\partial t}+\frac{\partial(\rho vu)}{\partial x}+\frac{\partial(\rho vv)}{\partial y}+\frac{\partial(\rho vw)}{\partial x}=\frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial v}{\partial z}\right)+\rho g_y$$

Z component

$$\frac{\partial(\rho w)}{\partial t}+\frac{\partial(\rho wu)}{\partial x}+\frac{\partial(\rho wv)}{\partial y}+\frac{\partial(\rho ww)}{\partial x}=\frac{\partial}{\partial x}\left(\mu\frac{\partial w}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial w}{\partial z}\right)+\rho g_z$$

Here, $\mu = \mu_t$ (Turbulent viscosity) + μ_l (Laminar viscosity) $\approx \mu_t$ ($\mu_t >> \mu_l$), and *u, v* and *w* are the mean velocity

Turbulent (k and ε) flow equation:

$\mu_t = C_\mu \rho k^2/\varepsilon$, and $\varepsilon = \frac{dk}{dt}$

C_μ is the universal dissipation rate constant, k is specific kinetic energy and ε is the dissipation rate.

2. Boundary Conditions

Since the given question is in a steady state

$0 = \frac{\partial(\rho)}{\partial t} = \frac{\partial(\rho u)}{\partial t} = \frac{\partial(\rho v)}{\partial t} = \frac{\partial(\rho w)}{\partial t} = \rho g_x = \rho g_y = \rho g_z$ (No gravity considered)

At the free surface, the gradient of the velocity will be zero i.e.

$$\frac{\partial(v,w)}{\partial x}=\frac{\partial(u,w)}{\partial y}=\frac{\partial(u,v)}{\partial z}=0$$

At the walls, At the inlet

$u = v = w = 0$ $v = 0.15306$ m/sec

for velocity at the inlet, $V = V_0 \lambda^{1/2} = 0.15306$ m/sec

And for any length, $L = L_0 \lambda$, $\lambda = 0.15$.

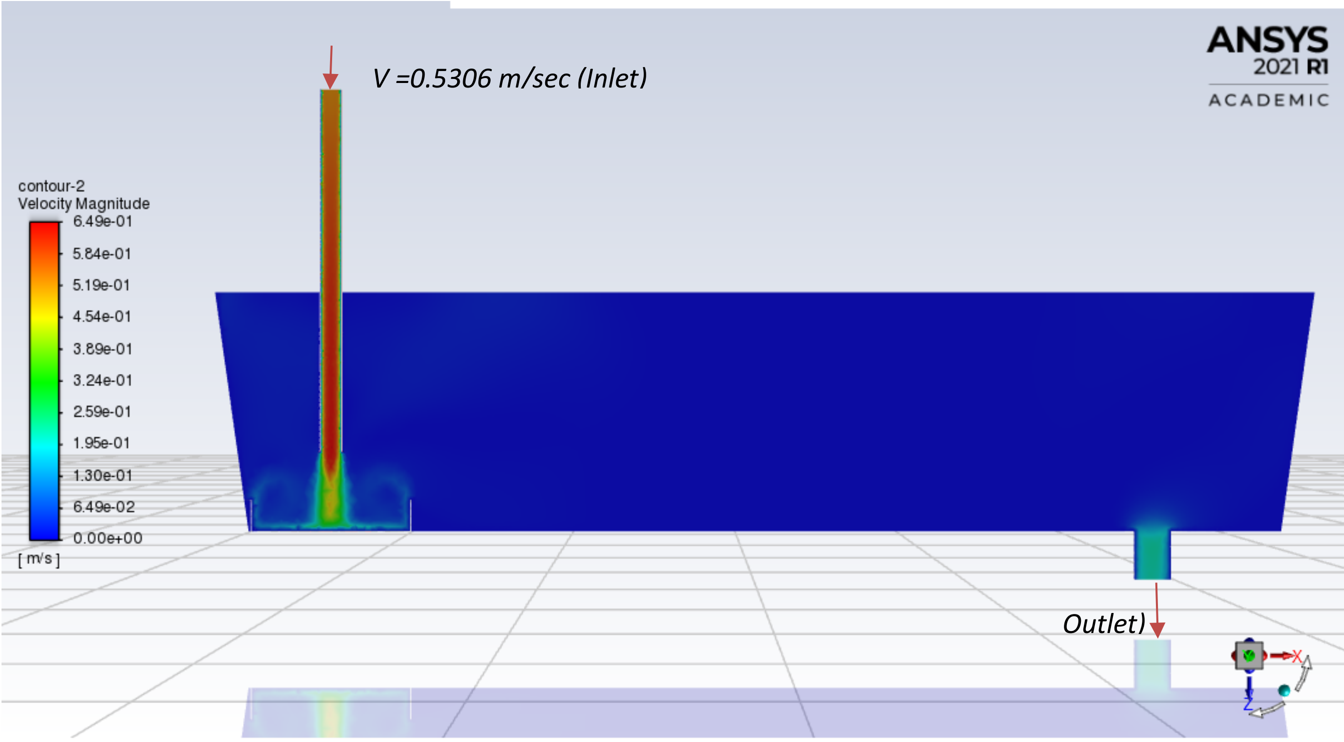


Fig: The contour velocity profile on the central vertical plane of the tundish

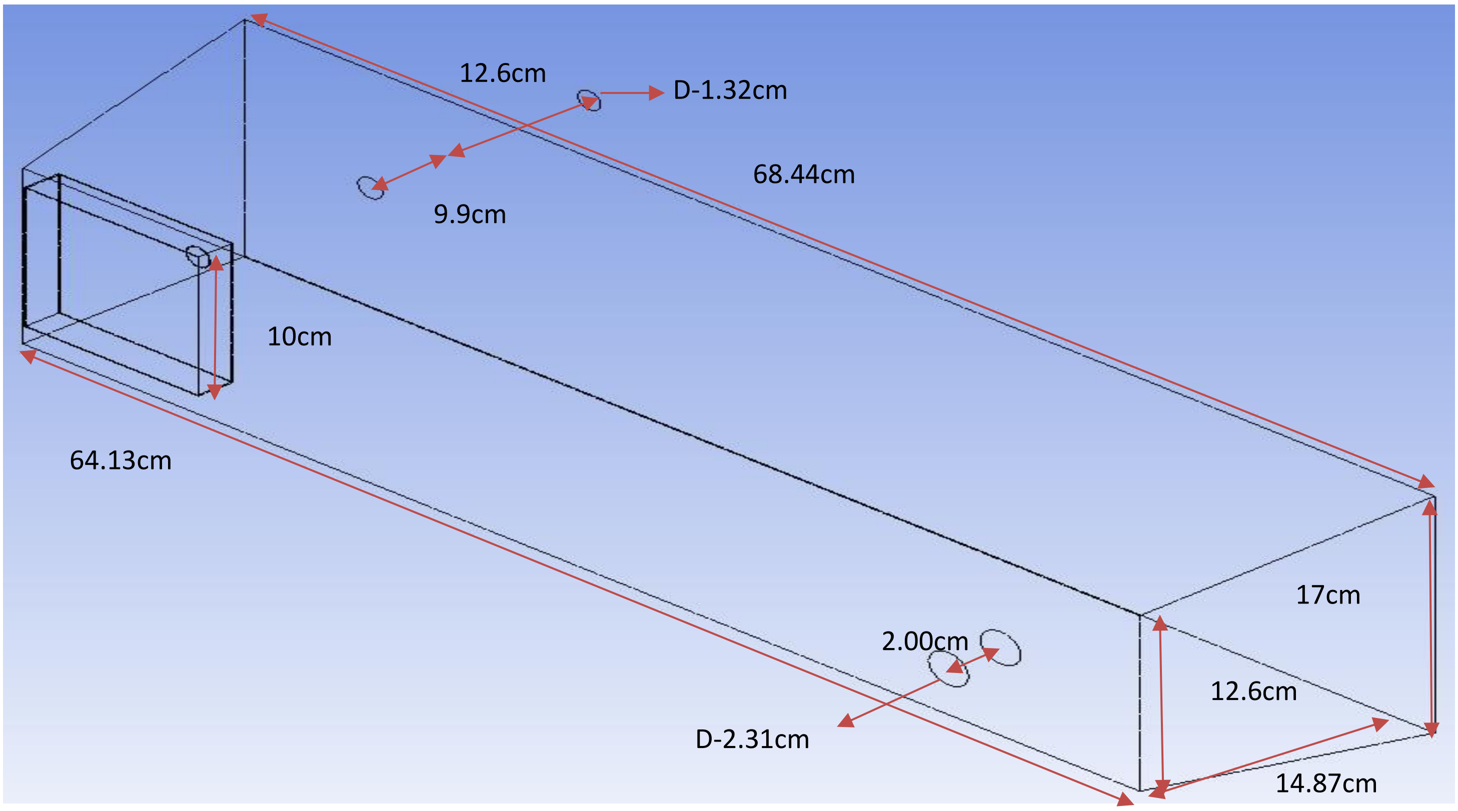


Fig: Dimensions of the tundish (in cm)

Analysis:

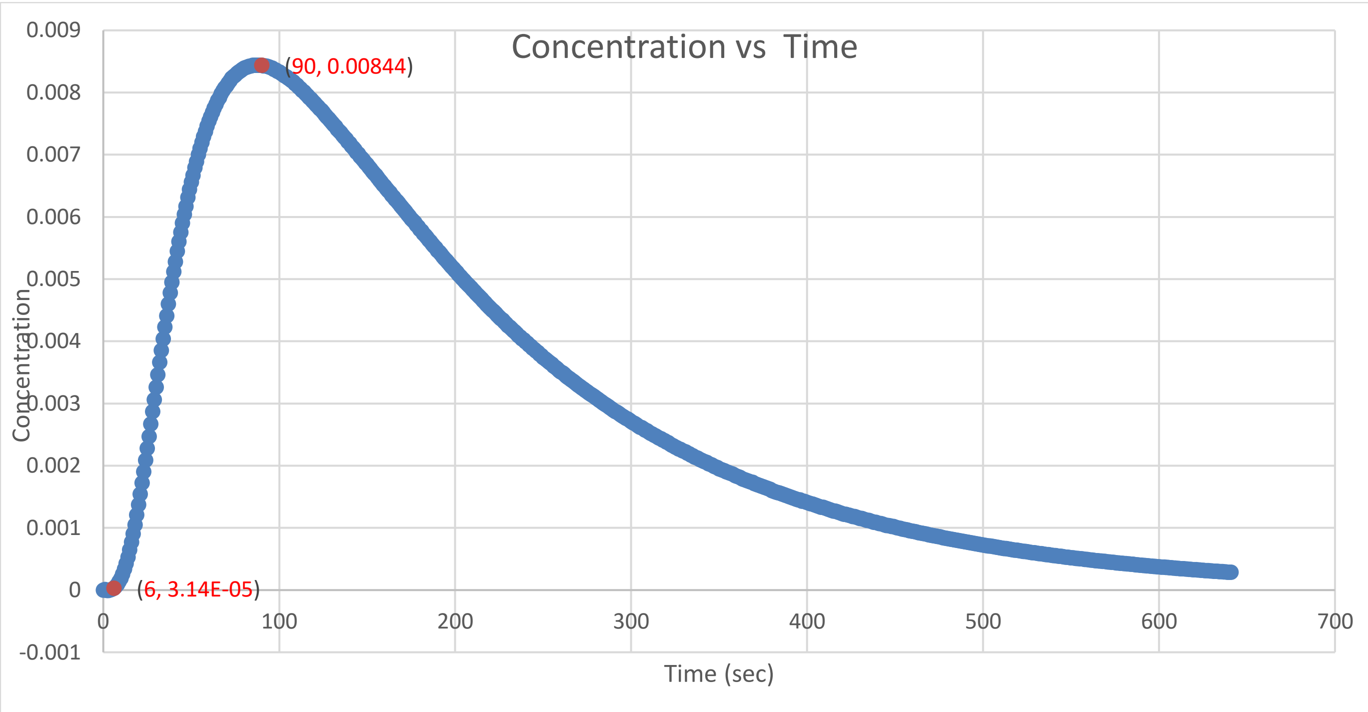
*The concentration (C_0) which was put at the inlet as a scaler value was **two** and that was hold for **one** second then it was closed by putting scaler value to **zero.***

1. Nominal residence time

Nominal residence time (T) is calculated by the volume of filled tundish (V) at the steady state divided by the volumetric flow rate (Q) at the inlet,

$$T = V/\,Q \;=\; V/\,(A.V) \;=\; 14514.2/(53.06 * 1.36) \;=\; 200 \text{ sec}$$

2. Break through and maximum concentration time



Graph.1: concentration profile of scaler value at outlet

- The breakthrough time is **6** sec after which the plot starts to show significant concentration and hence start rising (plotted in orange dot).
- The maximum concentration time is at **90** sec on which the concentration reached to its peak and that concentration is **0.00844** (again plotted in orange dot).

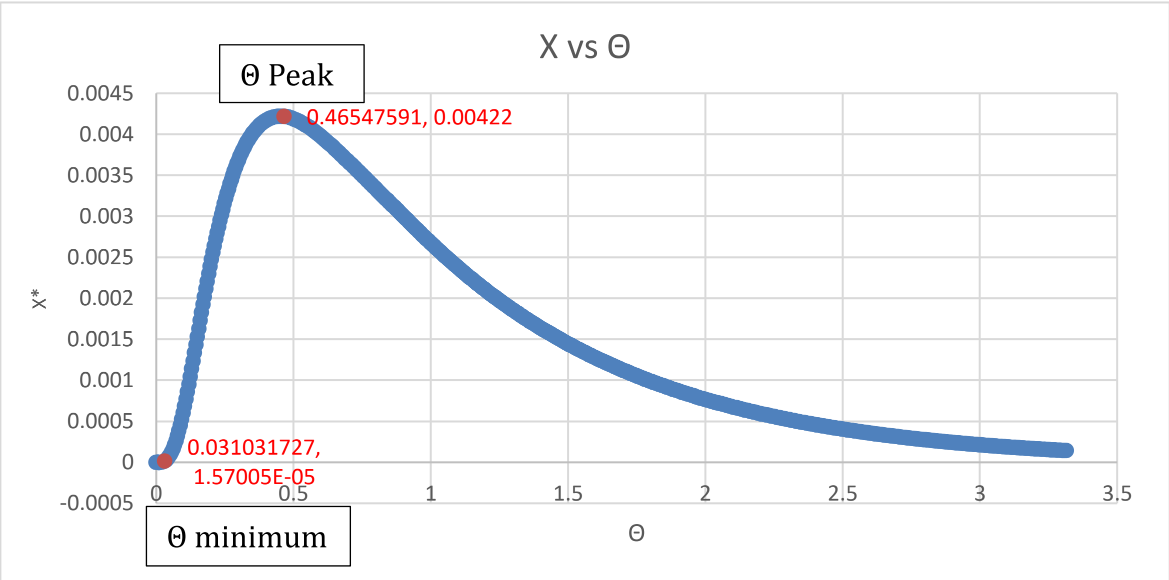
3. Mean residence time

- The formula which is used to calculate the *mean residence time* is given below

(Mean residence time) $\mathrm{T} = \dfrac{\int_0^\infty Ctdt}{\int_0^\infty Cdt}$

- The calculated Ƭ is **193.35**, integrated upto 650 sec that is upto whole time as represented in the graph.2

4. Proportions of volume



Graph.2: Non-dimensional concentration vs non-dimensional residence time

- The above graph (Graph.2) is of X^* *and* Θ, which are calculated as follows

  $X = C / C_0 = C / 2$ *and* $\Theta = t / T = t / 193.35$

- Let the Plug flow volume, Dead flow volume and Mixed volume flow be V_p, V_d and V_m respectively.

$$\Theta_{avg} = \frac{\int_0^2 X\Theta d\Theta}{\int_0^2 X d\Theta}$$

Θ Type	Θ_{Avg}	Θ_{peak}	Θ_{min}
Θ Values	0.843	0.465	0.031

And

Proportion volm	V_d (Dead)	V_p (Plug)	V_m (Mixed)
Formula	$1 - \Theta_{Avg}$	$(\Theta_{peak} + \Theta_{min}) / 2$	$1 - V_d - V_p$
Calculated value	0.157	0.248	0.595

5. Overall Inference
    - The dead flow value is least i.e 0.157 that means our model is quite good enough as the maximum volume of tundish is being utilised.
    - The value of plug flow is also at good value as very low value of plug flow volume is not good for inclusion removal and very high value is also not good as it stops well mixing and hence it may not allow steel to get perfectly homogenised.
    - The well mixed flow volume is highest which about 0.6 which seems good enough for well mixing so that our steel is homogenised at all places properly and can show property isotopically.

Conclusion:

- For turbulent flow we need to think different than laminar flow, i.e with laminar flow properties we need to apply turbulent flow properties in mathematical equations as well.
- To calculate the steady state turbulent flow, we may apply K- ε equation to calculate the velocities and other parameters.
- Calculation of scaler value concentration at outlet gives us an appropriate calculation of tundish performance.
- Through the analysis of different fractional values of flow volume, we can estimate and adjust tundish model to an ideal value so that it can be good in mixing and inclusion removal.