Physical and Mathematical modelling of steelmaking processes (MSE629A)

Initial Project

Steady Laminar Flow and heat transfer in a circular cross section pipe

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Aim:

To numerically compute steady, laminar velocity and temperature field in circular cross section pipe with constant wall temperature

Objective:

1. Computation of the number of iterations and the time of convergence.
2. Plotting of cross-sectional average temperature as a function of length of the pipe and comment on thermal equilibrium between wall and liquid temperature
3. Calculation of Nusselt number at different locations on the wall from x=0 to x=10m and then plotting it to show the variation as a function of length of the pipe.
4. Comparison of entrance length estimation with our numerical/modelling solution with the theoretical value i.e. entrance length=0.05* Re*

Formulation of Problem:

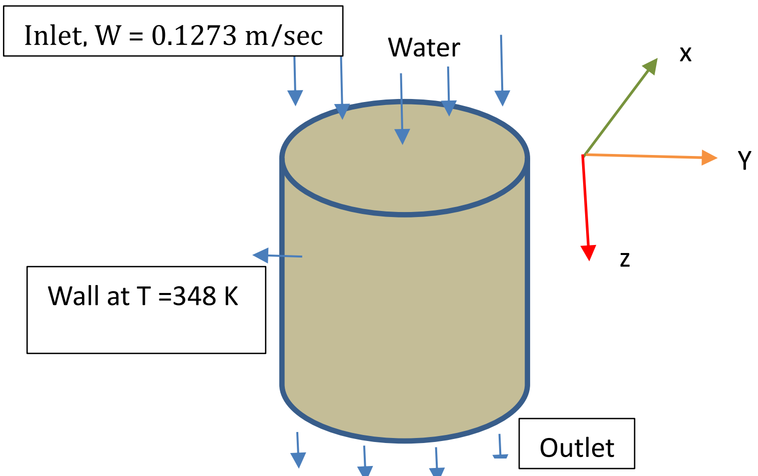
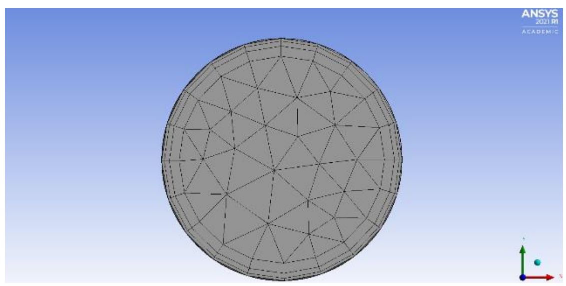


Fig: Schematic of pipe flow



Fix: The messing profile of the cylinder

Now, as per the given problem statement, the geometry is cylindrical, so we will apply the Navier-Stoke equation in a cylindrical coordinate system.

*Let **u**, **v**, and **w** be the velocity vectors along the **r**, **Θ**, and **z** directions respectively.*

Therefore,
The continuity equation:

$$\frac{\partial(\rho)}{\partial t}+\frac{\partial(\rho r u)}{r\partial r}+\frac{1\partial(\rho v)}{r\partial \Theta}+\frac{\partial(\rho w)}{\partial z}=0$$

'r' component of the incompressible Navier Stoke equation:

$$\rho\left(\frac{\partial(u)}{\partial t}+\frac{u\partial(u)}{\partial r}+\frac{v\partial(u)}{r\partial\Theta}+\frac{w\partial(u)}{\partial z}-\frac{v^2}{r}\right)=\mu\left[\frac{\partial}{\partial r}\left(\frac{\partial(ru)}{r\partial r}\right)+\frac{\partial^2(u)}{r^2\partial\Theta^2}-\frac{2\partial(v)}{r^2\partial\Theta}+\frac{\partial^2(u)}{\partial z^2}\right]-\frac{\partial(p)}{\partial r}-\rho g_r$$

'Θ' component of the incompressible Navier Stoke equation:

$$\rho\left(\frac{\partial(v)}{\partial t}+\frac{u\partial(v)}{\partial r}+\frac{v\partial(v)}{r\partial\Theta}+\frac{w\partial(v)}{\partial z}+\frac{vu}{r}\right)=\mu\left[\frac{\partial}{\partial r}\left(\frac{\partial(rv)}{r\partial r}\right)+\frac{\partial^2(v)}{r^2\partial\Theta^2}-\frac{2\partial(u)}{r^2\partial\Theta}+\frac{\partial^2(v)}{\partial z^2}\right]-\frac{\partial(p)}{r\partial\Theta}+\rho g_\Theta$$

'z' component of the incompressible Navier Stoke equation:

$$\rho\left(\frac{\partial(w)}{\partial t}+\frac{u\partial(w)}{\partial r}+\frac{v\partial(w)}{r\partial\Theta}+\frac{w\partial(w)}{\partial z}\right)=\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial(w)}{\partial r}\right)+\frac{\partial^2(w)}{r^2\partial\Theta^2}+\frac{\partial^2(w)}{\partial z^2}\right]-\frac{\partial(p)}{\partial z}+\rho g_z$$

The thermal energy equation:

$$\left(\boldsymbol{u}\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{r}}+\frac{\boldsymbol{v}}{\boldsymbol{r}}\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{\theta}}+\boldsymbol{w}\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{z}}\right)=\frac{\boldsymbol{k}}{\boldsymbol{\rho c_p}}\left[\frac{\mathbf{1}}{\boldsymbol{r}}\frac{\partial}{\partial \boldsymbol{r}}\left(\boldsymbol{r}\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{r}}\right)+\frac{\mathbf{1}}{\boldsymbol{r}^2}\frac{\partial^2 \boldsymbol{T}}{\partial \boldsymbol{\theta}^2}+\frac{\partial^2 \boldsymbol{T}}{\partial \boldsymbol{z}^2}\right]+\boldsymbol{S_T}$$

Now, as per the problem statement, the following **boundary conditions** would be applicable:

- Since the fluid is incompressible, is in a steady state, no source term, axis symmetry and there is no gravity along the r and Θ directions, therefore:

$$\frac{\partial(\rho)}{\partial t}=\frac{\partial(v)}{\partial t}=\frac{\partial(u)}{\partial t}=\frac{\partial(w)}{\partial t}=\rho g_r=\rho g_{\boldsymbol{\Theta}}=\boldsymbol{S_T}=0$$

- As per give in the problem statement, the temperature boundary conditions would be

Tw (Wall temperature) = 348 K and
At Z = 0 and Z = 10m, (Fluid temperature) T_f = 298 K

And at any point, $\frac{\partial T}{\partial \theta} = 0$

- The velocity boundary will be
  At the boundary of the cylinder i.e. at

$$\boxed{\begin{array}{l} R = 0.005\text{m},\ u = v = w = 0 \\ Z = 0,\ W = 0.1273\ \text{m/sec} \end{array}}$$

And at any point in the fluid,
$u = v = \frac{\partial (v)}{\partial \Theta} = 0$

Properties of fluid:

Property	Density	Cp (Specific Heat)	Thermal Conductivity	Viscosity	Thermal Expansion Co-efficient
Value	1000 Kg/m^3	4182 J/(kg k)	0.6 W/m K	0.001 Kg/m.sec	0

Analysis:

1. General Profiles of different parameters

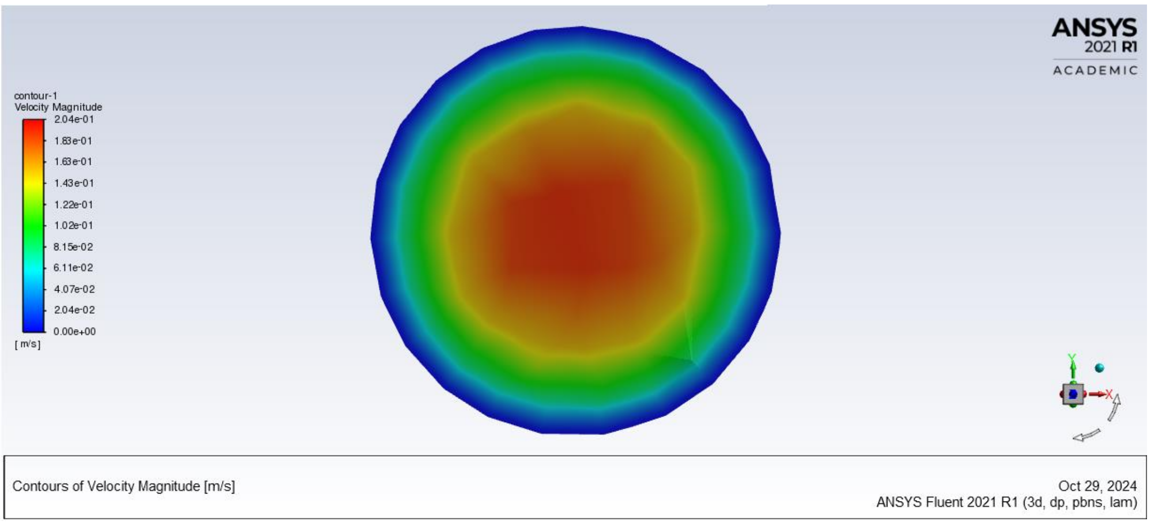


Fig: The contour of velocity profile on the central horizontal plane

- The maximum velocity will be at centre of the cylinder at any particular Z.

- The minimum velocity will be zero at the surface of the wall.
- After the fluid become fully developed the velocity profile become parabola and become constant at any particular R.

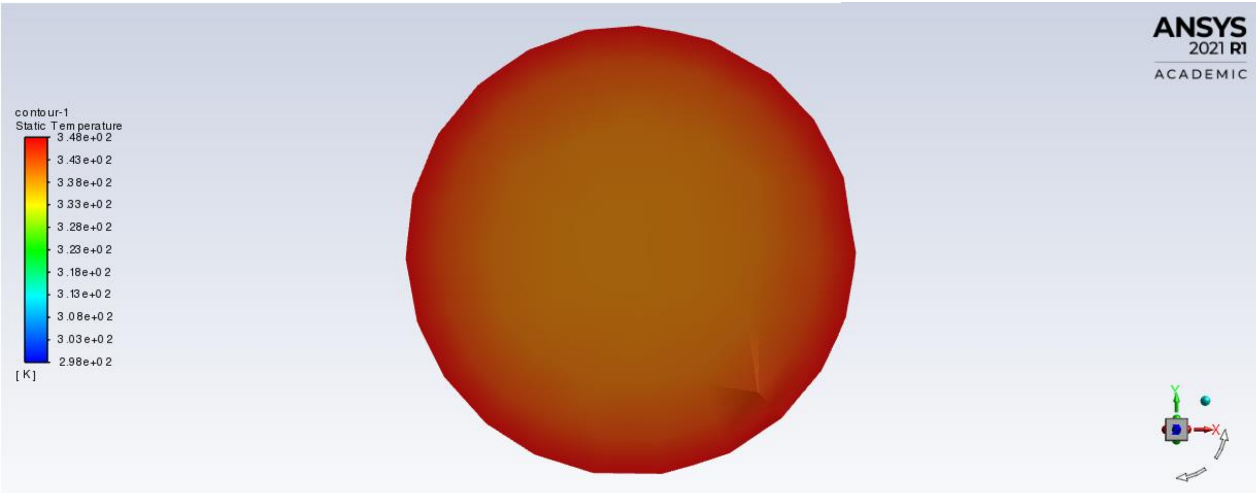


Fig: The contour of temperature profile on the central horizontal plane

- With the distance cover the temperature profile of whole water keep on tending to wall temperature i.e. 348K.
- From this modelling we can conclude that for very long cylindrical pipe, when the liquid comes out of the cylinder, it's all particle would have attained the wall temperature.

2. Iterations and computational time

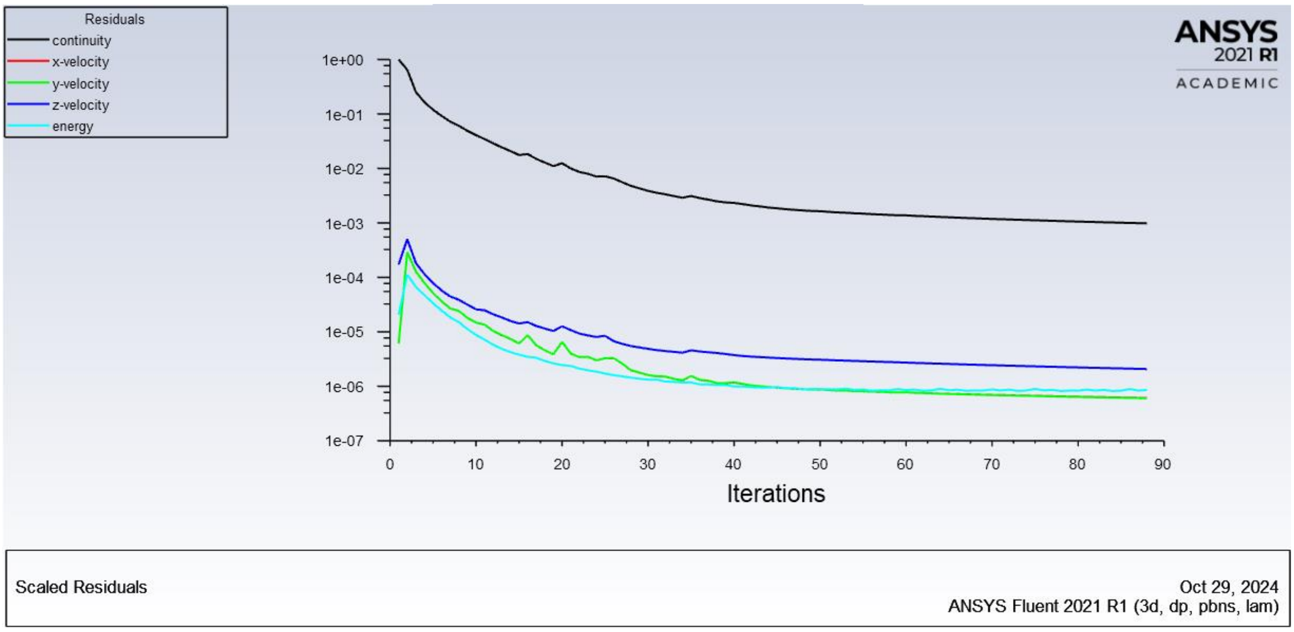


Fig: Residual vs Iteration plot

- As seen in the figure, it took 87 iterations to reach all the parameter to its residual value.

- The plot of X and Y velocities overlapping each other, explaining that they have quite similar/symmetrical conditions during the application of numerical analysis by Ansys.
- X and Y velocity component reach fastest at the residual value followed by z component and continuity term.

Parameters	X-velocity	Y-velocity	Z-velocity	Continuity	Energy
Residual limit	0.001m/sec	0.001m/sec	0.001m/sec	0.001Pa	0.000001K

Table: Residual limit corresponding to their equations

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3. Computational data:

Convergence (Iteration) time	**Case**	**Viscosity Type**	**Cells**	**Nodes**	**Element Size**	**Scheme**
334.14 sec	Fully developed	Laminar	1820198	641428	0.02m	SIMPLE

4. Entrance length

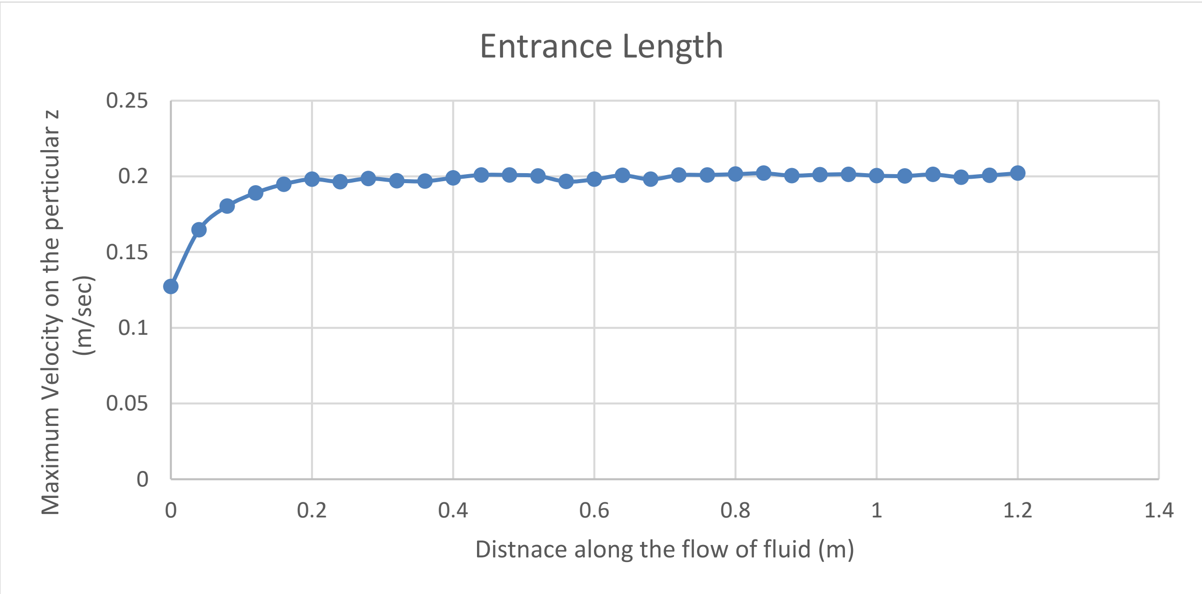


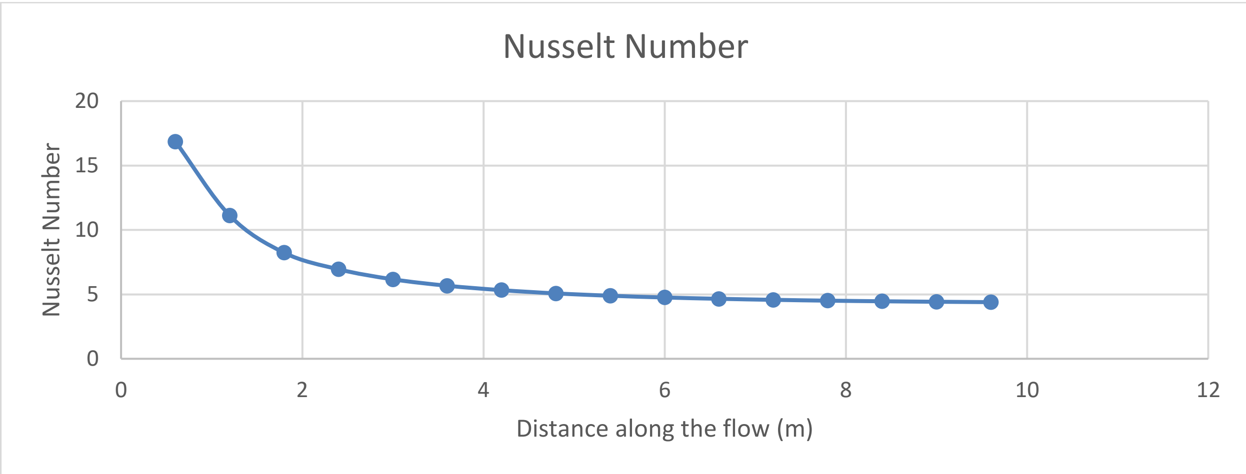
Fig: The velocity vector profile of the fluid on the central horizontal plane.

- The maximum velocity profile more or less seems constant right from at the distance of 20cm but its value gets very stable at the distance of approximately 72 cm, so taking its average value for the entrance length would be more suitable and hence that would be 46 cm.

- According to theoretical value i.e. entrance length=0.05* Re* pipe = 0.05*1273*0.02, diameter would come out as about 63 cm, therefore

Theoretical Length	46.00 cm
Calculated Length	63.65 cm

5. Nusselt Number



Distance(m)	Nusselt Number
0.6	16.86207
1.2	11.1112
1.8	8.231981
2.4	6.945302
3	6.167681
3.6	5.669229
4.2	5.325353
4.8	5.073271
5.4	4.892363
6	4.756672
6.6	4.654596
7.2	4.573911
7.8	4.511328
8.4	4.46297
9	4.425687
9.6	4.39553

- The closest value I could get to the 3.56 is 4.39553 which is farthest point i.e. at the 9.6 m.
- As the fluid is becoming the thermally fully developed the Nusselt number seems closer to the 3.56.
- The temperature profile which I got till the last was not fully developed and probably, the Nusselt number is not very close to the 3.56.
- Other factors that might have influenced the Nusselt number Such as convergence criteria, Number of mess elements, we could have closer value if we had deeper residual value and more mess elements

$$h(T_w - T_m) = \ q_w$$

$$\boldsymbol{Nu} \ = \frac{\boldsymbol{q_w \ D}}{(\boldsymbol{T_w - T_m})\boldsymbol{k}}$$

Where,

$$q_w \ = Wall\ tempreture$$

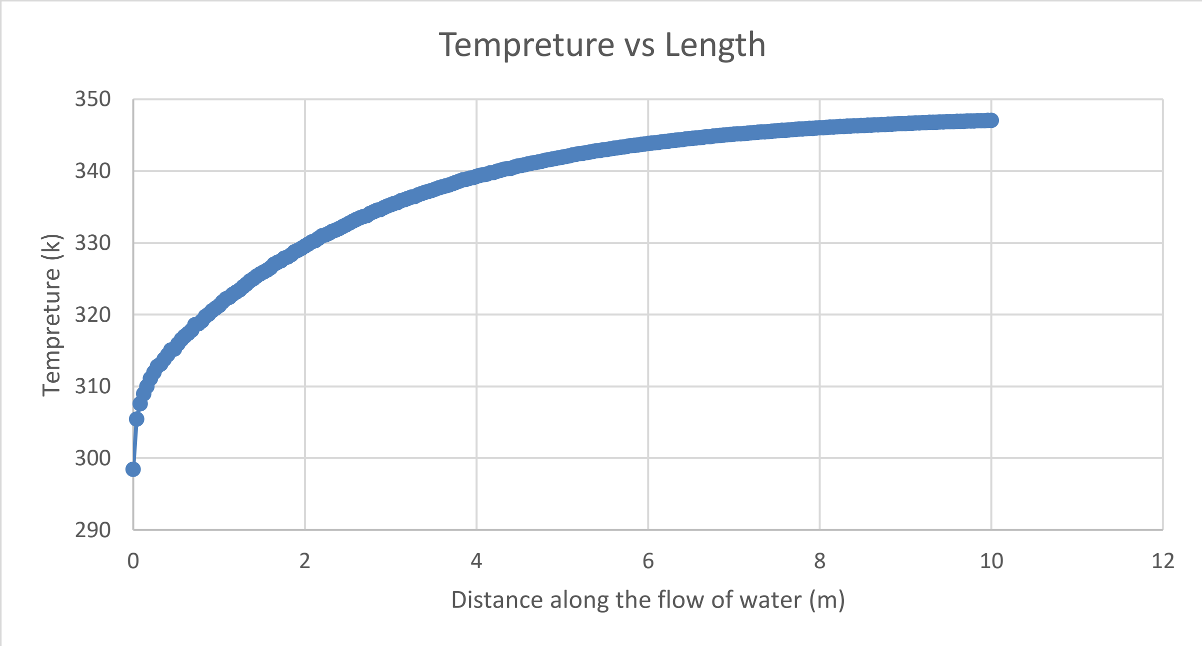
T_m =Average mass temperature

$$\boldsymbol{T_w \ = wall\ tempreture}$$

$$\boldsymbol{D = Diameter}$$

$$\boldsymbol{k = Thermal\ conductivity}$$

6. Temperature as a function of length



- The exit temperature I got is 347.0411 which quite close to the wall temperature but not the wall temperature which shows that that the till the end fluid is not fully thermally developed.
- As the fluid is still not fully thermally developed hence the system is yet to come to the thermal equilibrium, it will reach the thermal equilibrium when at every point the temperature equals to wall temperature.

Conclusion:

1. For the convergence of thermal energy equation, we need to give very smaller residual value as comparison to continuity and momentum equation.
2. The Nusselt number obtained from the simulation, approximately 4.4 near the outlet, deviated from the expected value of 3.56. which again shows that there could be a potential scope to better our model using different ways.
3. For the convergence of thermal energy equation, we need to give very smaller residual value as comparison to continuity and momentum equation.