Homework - 1

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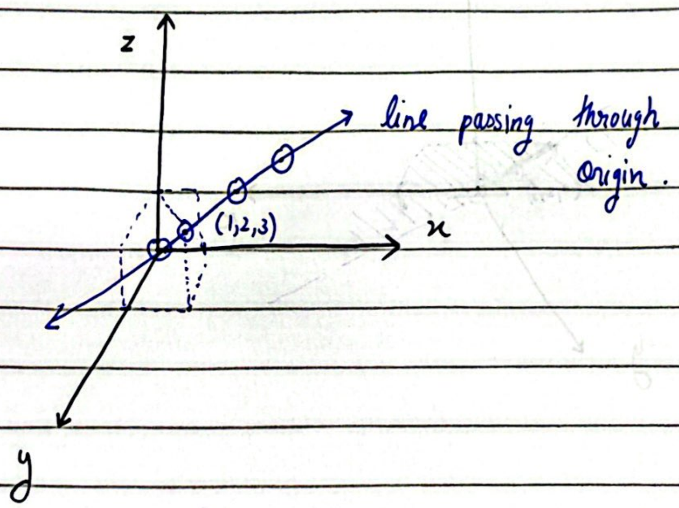
Qs1)

a) $$\begin{bmatrix}1\\2\\3\end{bmatrix}x + \begin{bmatrix}3\\6\\9\end{bmatrix}y \Rightarrow \begin{bmatrix}x+3y\\2x+6y\\3x+9y\end{bmatrix} \Rightarrow \underline{(x+3y)}\begin{bmatrix}1\\2\\3\end{bmatrix}$$

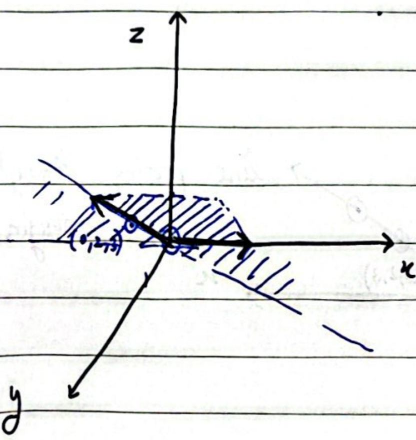


So, it is a line in R^3.

b) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} y = \begin{bmatrix} x \\ 2y \\ 3y \end{bmatrix}$

it is combination of 2 vectors.

We can take different values of x, y.



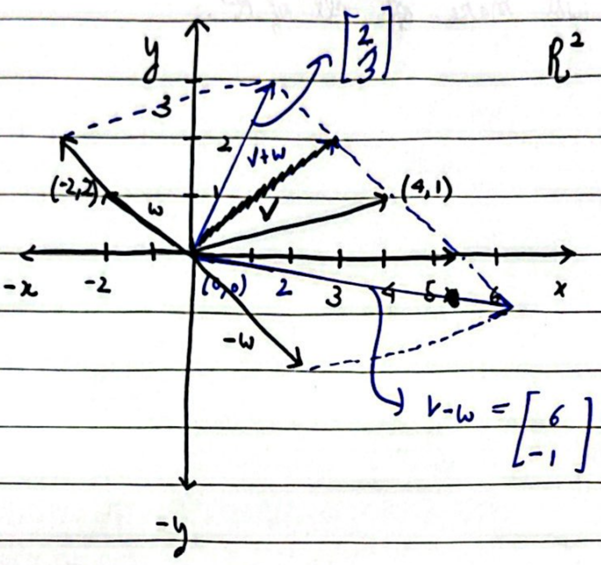
So, it will be a plane in R^3.

c) $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} y + \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} z \Rightarrow$ combination of 3 Linearly independent vector.

This will make all of R^3.

Qs2) $v = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ $w = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

Ans) $v+w = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $v-w = \begin{bmatrix} 6 \\ -1 \end{bmatrix}$



Q3)

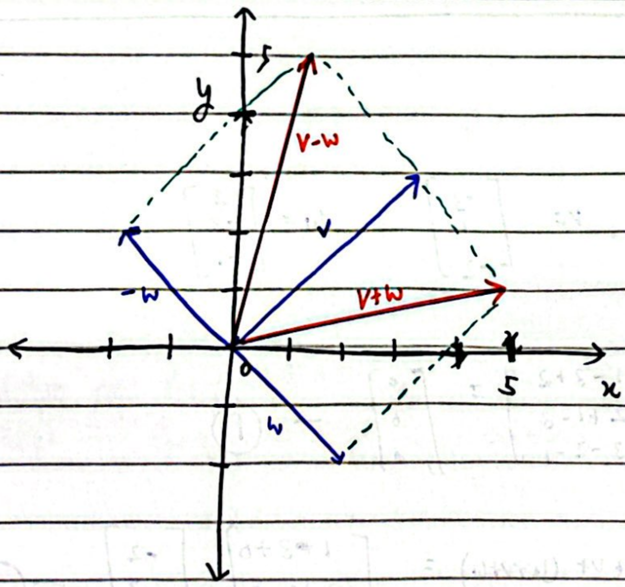
Ans)

$$v + w = \begin{bmatrix} 5 \\ 1 \end{bmatrix} \quad — \quad (1)$$

$$v - w = \begin{bmatrix} 1 \\ 5 \end{bmatrix} \quad — \quad (2)$$

$$(1) + (2) \Rightarrow \quad 2v = \begin{bmatrix} 6 \\ 6 \end{bmatrix} \Rightarrow v = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$\therefore \quad w = \begin{bmatrix} 5 \\ 1 \end{bmatrix} - \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$



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Qn4) $v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $w = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Ans) $3v + w = 3\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$

$cv + dw = c\begin{bmatrix} 2 \\ 1 \end{bmatrix} + d\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 2c + d \\ c + 2d \end{bmatrix}$

Qn5)

Ans) $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $v = \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}$ $w = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$

$u + v + w = \begin{bmatrix} 1 - 3 + 2 \\ 2 + 1 - 3 \\ 3 - 2 - 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ — 1

$2u + 2v + w = u + v + (u + v + w) = \begin{bmatrix} 1 - 3 + 0 \\ 2 + 1 + 0 \\ 3 - 2 + 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$ — 2

These lie in a plane because ÷

$w = cu + dv$ and by 1 $w = -u - v$.

$c = -1$

$d = -1$

Q6)

Ans)

$$v = (1, -2, 1)$$
$$w = (0, 1, -1)$$

$$\alpha v + \beta w = \alpha \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \alpha \\ -2\alpha + \beta \\ \alpha - \beta \end{bmatrix}$$

add to components $\Rightarrow x_1 + x_2 + x_3$

$\Rightarrow \cancel{\alpha} + (-2\alpha + \beta) + (\alpha - \beta)$

$\Rightarrow 0$

(zero)

- $$cv + dw = \begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix}$$

$$c \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + d \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix} \Rightarrow \left.\begin{aligned} c &= 3 \\ -2c + d &= 3 \\ c - d &= -6 \end{aligned}\right\} \quad c = 3,\ d = 9$$

- $$cv + dw = \begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} c \\ -2c + d \\ c - d \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix} \Rightarrow \left.\begin{aligned} c &= 3 \\ d &= 9 \\ c - d &= -6 \end{aligned}\right\} \quad \therefore \text{ not possible as } 3^{rd} \text{ eq contradicts.}$$

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Q 7) $u = \begin{bmatrix} -0.6 \\ 0.8 \end{bmatrix}$ $v = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ $w = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Ans) $u \cdot v = -0.6 \times 4 + 3 \times 0.8$

$= -2.4 + 2.4$

$= \underline{0}$

$u \cdot w = -0.6 + 1.6$

$= \underline{1.0}$

$u \cdot (v + w) = u \cdot v + u \cdot w = \underline{1.0}$

$w \cdot v = 4 + 6 = \underline{10} = v \cdot w$

Q 8) Lengths

Ans) $\|u\| = \sqrt{(-0.6)^2 + (0.8)^2} = \sqrt{1} = \underline{1}$

$\|v\| = \sqrt{4^2 + 3^2} = \sqrt{25} = \underline{5}$

$\|w\| = \sqrt{1^2 + 2^2} = \underline{\sqrt{5}}$

<u>Schwartz inequality</u> - $|u \cdot v| \le \|u\| \|v\|$ & $|v \cdot w| \le \|v\| \|w\|$

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$

0 1 5 10 5 $\sqrt{5}$

$\therefore$ $0 \le 5$ & $10 \le 5\sqrt{5}$

$\downarrow$

$11.180\ldots$

So, the inequality holds true.

Q9)

Ans) a) $v = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$ $w = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\|v\| = \sqrt{\frac{2}{3}}$

$\|w\| = 1$

$v \cdot w = \frac{1}{\sqrt{3}}$

$\cos\theta = \frac{v \cdot w}{\|v\|\|w\|}$

$= \frac{1/\sqrt{3}}{\sqrt{2}/\sqrt{3}}$

$= \frac{1}{\sqrt{2}}$

So, ~~$\theta = 60°$ or $\frac{\pi}{3}$ Radians~~. $\theta = 45°$ or $\frac{\pi}{4}$ Radian.

b) $v = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$ $w = \begin{bmatrix} -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$

$\|v\| = \sqrt{2/3}$

$\|w\| = \sqrt{2/3}$

$v \cdot w = 0$

$\cos\theta = \frac{v \cdot w}{\|v\| \|w\|}$

$= 0$

$\therefore \theta = 90°$ or $\frac{\pi}{2}$

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c) $V = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$ $\quad w = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$

$$\cos\theta = \frac{V \cdot w}{\|v\| \, \|w\|}$$

$$= 0$$

$\|V\| = \sqrt{2^2 + 2^2 + 1^2}$
$= 3$

$\|w\| = \sqrt{2^2 + 1^2 + 2^2}$
$= 3$

$V \cdot w = 4 - 2 - 2 = 0$

$\theta = 90^\circ$ or $\frac{\pi}{2}$

d) $V = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ $\quad w = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

$$\cos\theta = \frac{V \cdot w}{\|v\| \, \|w\|}$$

$$= \frac{-5}{\sqrt{10}\sqrt{5}} = \frac{-1}{\sqrt{2}}$$

$\|v\| = \sqrt{3^2 + 1^2} = \sqrt{10}$
$\|w\| = \sqrt{1^2 + 2^2} = \sqrt{5}$

$V \cdot w = -3 - 2 = -5$

$\theta = 135^\circ$ or $\frac{3\pi}{4}$ Radians.

Q 10)

Ans) $v = (1, 1, \ldots\ldots, 1)$

in 9 dimension.

length $\|v\| = \sqrt{\underbrace{1^2+1^2+1^2 \ldots\ldots +1^2}_{9 \text{ times}}}$

$= \sqrt{9}$

$= 3$

$\underset{\text{(unit vector) in same dir}}{u} = \dfrac{v}{\|v\|} = \left(\frac{1}{3}, \frac{1}{3}, \ldots\ldots, \frac{1}{3}\right)$ in 9D.

→ w (unit vector ⊥ to v)

$\therefore\ w \cdot v = 0$

$\Rightarrow$ take $w = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0, \ldots\ldots 0\right)$ in 9D

$\underline{w \cdot v = 0}$

Q 11)
Ans)

$$\cancel{Ax = \begin{bmatrix} 2 & 1 & 2 \\ 4 & 2 & 4 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 2+1+2 \\ 8+4+8 \\ 0+5+0 \end{bmatrix} =}$$

$$Ax = \begin{bmatrix} 2 & 1 & 2 \\ 4 & 2 & 4 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 2+2+10 \\ 4+4+20 \\ 0+2+0 \end{bmatrix} = \begin{bmatrix} 14 \\ 28 \\ 2 \end{bmatrix}$$

$$By = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 4 \\ 10 \end{bmatrix} = \begin{bmatrix} 4+0+0 \\ 4+4+0 \\ 4+4+10 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 18 \end{bmatrix}$$

$$Iz = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

Q. 12)

Ans)

$$Ax = 1 \cdot \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix} + 2 \cdot \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + 5 \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix} + \begin{bmatrix} 10 \\ 20 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 14 \\ 28 \\ 0 \end{bmatrix}$$

$$By = 4 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 4 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + 10 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 4+4 \\ 4+4+10 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 18 \end{bmatrix}$$

$$Iz = z_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + z_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + z_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

Q13)

Ans) "$Ax=b$ has a solⁿ vector x if the vector b is in the column space of A"

$A =$ matrix

$x:$ unknown

$Ax=b$ iff b is in column space of A.

- Column space consists of all vector b that can be expressed as Ax. It represents "span" of all columns of A.

$\therefore$ if b is in column space, then x exist.
if b is not in column space, then no solⁿ.

- $Ax=b$

i) $$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$x_1 + 3x_2 = 4$ — (1)
$2x_1 + 4x_2 = 6$ — (2)

(2) $-2\times$(1) $\Rightarrow -2x_2 = -2 \Rightarrow x_2 = 1$
$\therefore x_1 = 1$

(1,1) solⁿ

(ii) $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$

$x_1 + 3x_2 = -2$ — (1)
$2x_2 + 4x_2 = -2$ — (2)

(2) $- 2^*$ (1)

$\Rightarrow -2x_2 = 2$
$\Rightarrow x_2 = -1$

$\therefore x_1 = 1$

$(1, -1)$ sol^n

(iii) $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$x_1 + 3x_2 = 1$ — (1)
$2x_1 + 4x_2 = 1$ — (2)

(2) $- 2^*$ (1)

$\Rightarrow -2x_2 = -1 \Rightarrow x_2 = +1/2$
$\Rightarrow x_1 = -1/2$

$\left(\frac{-1}{2}, \frac{1}{2}\right)$ sol^n

Qs 14)

Ans)

(i) $Ax = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} z \\ y \\ x \end{bmatrix}$

ii) $Ax = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 3 \\ 3 & 3 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2+1-3 \\ 1+2-3 \\ 3+3-6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

iii) $Ax = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+1 \\ 1+2 \\ 3+3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix}$

Qs 15)

Ans) a) $\underline{n}$ components , $\underline{m}$ components

b) $\underline{n}$ dim space , $\underline{m}$ dim space

Q6 k) $2x + 3y + z + 5t = 8$

Ans)

$A = \begin{bmatrix} 2 & 3 & 1 & 5 \end{bmatrix}$, → has 1 row, 4 columns

$x = \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}$

$$\underset{A}{\begin{bmatrix} 2 & 3 & 1 & 5 \end{bmatrix}} \underset{x}{\begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}} = \begin{bmatrix} 8 \end{bmatrix}$$

x → 4 rows

$[8]$ → b → (b vector) → 1 component

A → has 1 row

$b = [8]$

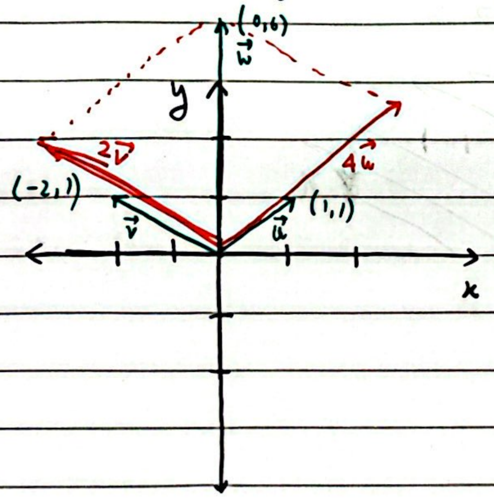
(Qs 17)

Ans) $x - 2y = 0$, $x + y = 6$ } $x = 2y$, $y = 2, x = 4$

<u>Column picture :</u> $Ax = b$

$$\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$$

$$x \underbrace{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}_{u} + y \underbrace{\begin{pmatrix} -2 \\ 1 \end{pmatrix}}_{v} = \underbrace{\begin{pmatrix} 0 \\ 6 \end{pmatrix}}_{w}$$



$\therefore 4\vec{u} + 2\vec{v} = \vec{w}$

Row picture:

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$$

$x = 2y$
$x + y = 6$