13/9/24

Homework 2

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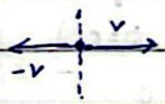
MS in Applied Mathematics

Qs1)

a) v, w - unit vector

$v \cdot -v = |v||v| \cos 180^\circ = -1$



b) $(v+w)\cdot(v-w) = v\cdot v + \cancel{w\cdot v} - \cancel{v\cdot w} - w\cdot w$

$= 1 + \cancel{0} + \cancel{0} - 1$

$= 0$

c) $(v-2w)\cdot(v+2w) = v\cdot v - \cancel{2w\cdot v} + \cancel{2v\cdot w} - 4w\cdot w$

$= 1 - 4$

$= -3$

Q2)
Ans)

let P_1

$$\begin{pmatrix} x & y & z \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} y & z & x \end{pmatrix}$$

$$\begin{pmatrix} ax+dy+gz & bx+ey+zh & cx+fy+zi \end{pmatrix} = \begin{pmatrix} y & z & x \end{pmatrix}$$

$\therefore$
$$ax+dy+gz = y \quad [a=0,\ d=1,\ g=0]$$
$$bx+ey+zh = z \quad [b=0,\ e=0,\ h=1]$$
$$cx+fy+zi = x \quad [c=1,\ f=0,\ i=0]$$

$$\therefore P_1 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

if $\rightarrow$

$$\overset{P_2}{\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}} \overset{X^T}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = \begin{pmatrix} y \\ z \\ x \end{pmatrix}$$

- Then $P_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$$\begin{array}{cc} & Q_1 \\ \begin{pmatrix} y & z & x \end{pmatrix} & \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \end{array} = \begin{pmatrix} x & y & z \end{pmatrix}$$

$$\begin{aligned} ay + dz + gx &= x \qquad & a=0, d=0, g=1 \\ by + ez + hx &= y \qquad & b=1, e=0, h=0 \\ cy + fz + ix &= z \qquad & c=0, f=1, i=0 \end{aligned}$$

$$\therefore Q_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

→ if

$$\begin{array}{ccc} Q_2 & X^T & \\ \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} & \begin{pmatrix} y \\ z \\ x \end{pmatrix} & = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \end{array}$$

Then $Q_2 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

Q3)
Ans)

- $E\begin{bmatrix}3\\5\end{bmatrix} = \begin{bmatrix}3\\2\end{bmatrix}$

take $E = \begin{bmatrix}a & b\\c & d\end{bmatrix}$

$$\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}3\\5\end{bmatrix} = \begin{bmatrix}3\\2\end{bmatrix}$$

$$\left.\begin{aligned}3a+5b &= 3\\3c+5d &= 2\end{aligned}\right\}$$ as it is ~~given~~ specific value, can't find a, b, c, d.

* $E\begin{bmatrix}v_1\\v_2\end{bmatrix} = \begin{bmatrix}v_1\\v_2-v_1\end{bmatrix}$

let E =

$$\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}v_1\\v_2\end{bmatrix} = \begin{bmatrix}v_1\\v_2-v_1\end{bmatrix}$$

$a v_1 + b v_2 = v_1$ — (1)
$c v_1 + d v_2 = v_2 - v_1$ — (1)

By comparing, $a = 1,\ b = 0$
$c = -1,\ d = 1$

$$\therefore E = \begin{bmatrix}1 & 0\\-1 & 1\end{bmatrix}.$$

classmate

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- $E\begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 7 \end{bmatrix}$

So, $E\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 - v_1 \\ v_3 \end{bmatrix}$

Let $E = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 - v_1 \\ v_3 \end{bmatrix}$

$v_1 a + v_2 b + v_3 c = v_1$

$v_1 d + v_2 e + v_3 f = v_2 - v_1$

$v_1 g + v_2 h + v_3 i = v_3$

By comparing,

$\therefore\ E = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Qo 4)

Ans)

$2x - 4y = 6$ — ①

$-x + 5y = 0$ — ②

Subtract $-\frac{1}{2}$① from ②.

$\therefore$ ② $-\left(-\frac{1}{2}①\right)$

$\Rightarrow 3y = 3$

$\Rightarrow y = 1$

$\therefore x = 5y = 5$

$(5, 1)$ - solⁿ

if $2x - 4y = -6$ — ①

$-x + 5y = 0$ — ②

② $+ \frac{1}{2}$①

$\Rightarrow 3y = -3$

$\Rightarrow y = -1$

$x = -5$

$(-5, -1)$ solⁿ

Qs5)
Ans)

$ax + by = f \quad - (1)$

$cx + dy = g \quad - (2)$

$$\boxed{l = \frac{c}{a}}$$

$(2) - \underbrace{\frac{c}{a}}_{l}(1)$

[second pivot = $d - \frac{cb}{a}$]

$\Rightarrow \quad dy - \frac{c}{a}by = g - \frac{c}{a}f$

$\Rightarrow \quad (ad - bc)y = ag - cf$

$$\Rightarrow \left[y = \frac{ag - cf}{ad - bc} \right]$$

When $ad = bc$, second pivot = 0 .

Q6)

Ans)

$$ax + 3y = -3 \quad -(1)$$
$$4x + 6y = 6 \quad -(2)$$

$$A = \left[\begin{array}{cc|c} a & 3 & -3 \\ 4 & 6 & 6 \end{array}\right]$$

$$R_2 \rightarrow R_2 - \frac{4}{a} R_1$$

$$A = \left[\begin{array}{cc|c} a & 3 & -3 \\ 0 & 6 - \frac{12}{a} & 6 + \frac{12}{a} \end{array}\right]$$

Temporarily

- ~~Permanent~~ breakdown : If coefficient of x becomes 0, $a = 0$ so no division possible.

$\therefore$ $a = 0 \Rightarrow$ $3y = -3 \Rightarrow y = -1$ $(3, -1)$

$\therefore$ $4x - 6 = 6 \Rightarrow x = 3$

Permanent

- ~~Temp~~ breakdown : if $\underline{a = 2}$

$$\left.\begin{array}{l} 2x + 3y = -3 \\ 4x + 6y = 6 \end{array}\right\} \text{no sol}^n \text{ as Parallel lines}$$

* **Temp breakdown** :

$a = 0$

$$\left[\begin{array}{cc|c} 0 & 3 & -3 \\ 4 & 6 & 6 \end{array}\right]$$

— there is no pivot.

Row exchange → $R_2 \leftrightarrow R_1$

$$\left[\begin{array}{cc|c} 4 & 6 & 6 \\ 0 & 3 & -3 \end{array}\right]$$

$4x + 6y = 6$

$0x + 3y = -3$

$$\therefore \left.\begin{array}{l} y = -1 \\ x = 3 \end{array}\right\} \text{ans.}$$

Q7)

Ans)

LE

$ax_1 + by_1 + cz_1 = d$

a) $sol^n\ 1 = (x, y, z)$

$sol^n\ 2 = (X, Y, Z)$

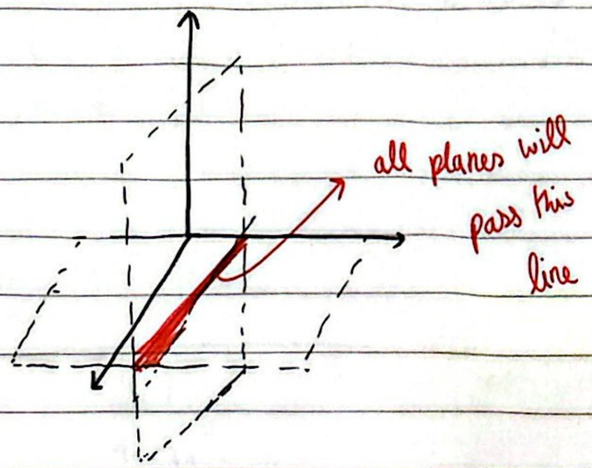
Another $sol^n = \alpha(x, y, z) + \beta(X, Y, Z)$

Where $(\alpha + \beta) = 1$

So take any value, $\alpha = \frac{1}{2}, \beta = \frac{1}{2}$

$\therefore sol^n = \left(\frac{x+X}{2}, \frac{y+Y}{2}, \frac{z+Z}{2}\right)$

b) 25 planes meet at a point.



Qs8)

Ans)

$$2x-3y=3$$
$$4x-5y+z=7$$
$$2x-y-3z=5$$

Augmented matrix (A) : $\left[\begin{array}{ccc|c} 2 & -3 & 0 & 3 \\ 4 & -5 & 1 & 7 \\ 2 & -1 & -3 & 5 \end{array}\right]$

1) $R_2 \to R_2 - 2R_1$
2) $R_3 \to R_3 - R_1$

(Row operation)

i) Subtract $2 \times R_1$ from R_2

ii) Subtract $1 \times R_1$ from R_3

$$A = \left[\begin{array}{ccc|c} 2 & -3 & 0 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & -3 & 2 \end{array}\right]$$

3) $R_3 \to R_3 - 2R_2$

iii) subtract $2 \times R_2$ from R_3

$$A = \left[\begin{array}{ccc|c} 2 & -3 & 0 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -5 & 0 \end{array}\right]$$

pivots

Back substitution:

$-5z = 0 \Rightarrow z = 0$

$y + z = 1 \Rightarrow y = 1$

$2x - 3y = 3 \Rightarrow x = 3$

$\therefore$ Sol$^n \Rightarrow$ $\underline{(3, 1, 0)}$

Q.9)

Ans) • $A \to U$ (without row exchange ~~elimination~~)

∴ row of U -> linear combination of rows of A.
(rows 1 to j) as it is elimination

• Yes if $Ax = 0 \Rightarrow Ux = 0$

as right hand side of Augmented matrix not change as elimination operation on 0 is $\underline{0}$.

• No if $Ax = b \not\Rightarrow Ux = b$

• b can change while elimination operation.

• if A is a lower triangular matrix,

$$A = \begin{bmatrix} a & 0 & \cdot & \cdot \\ a_1 & b & \cdot & \cdot \\ a_2 & \cdot & c & \cdot \\ \vdots & \cdot & \cdot & \ddots \end{bmatrix} \quad \text{then } U = \begin{bmatrix} a_k & & & \\ & b_k & & \\ & & c_k & \\ & & & \ddots \end{bmatrix}$$

↓
become diagonal matrix.

Q10)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 7 & 8 & 9 \\ 1 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix}$$

Ans)

$$AB = \begin{bmatrix} 18 & 20 & 28 \\ 13 & 16 & 22 \\ 41 & 40 & 57 \end{bmatrix}$$

$$BA = \begin{bmatrix} 7 & 8 & 9 \\ 1 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 52 & 76 & 53 \\ 11 & 14 & 3 \\ \cancel{18}\,28 & 40 & 25 \end{bmatrix}$$

No, they are not equal.

Q11)

Ans)

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ 2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 3 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Use columns times row methods.

Let $A = \begin{bmatrix} C_1 & C_2 \end{bmatrix}$ $\qquad B = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$

$$AB = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

$$= C_1 \times R_1 + C_2 \times R_2$$

$$= \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \times \begin{bmatrix} 3 & 3 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 0 \\ 6 & 6 & 0 \\ 6 & 6 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 4 & 8 & 4 \\ 1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 0 \\ 10 & 14 & 4 \\ 7 & 8 & 1 \end{bmatrix}$$

Q12) Block Multiplication.

Ans) $A = \begin{bmatrix} 3 & 3 & 0 \\ 1 & 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ 2 & 1 \end{bmatrix}$

$$\therefore AB = \left[\begin{bmatrix} 3 & 3 \\ 1 & 2 \end{bmatrix} \middle| \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right] \left[\begin{array}{c} \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix} \\ \hline \begin{bmatrix} 2 & 1 \end{bmatrix} \end{array}\right]$$

$$= \begin{bmatrix} 3 & 3 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 12 \\ 5 & 8 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 12 \\ 7 & 9 \end{bmatrix}$$

$\downarrow$

Ans