classmate

Date

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* Homework 1

Q1)

Ans) $\det A = 1/2$ $A \rightarrow 4\times4$ matrix

$\det(2A) = 2^4 \cdot \det(A) = 16 \times \frac{1}{2} = 8$

$\det(-A) = (-1)^4 \cdot \det(A) = 1/2$

$\det(A^2) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\det(A^{-1}) = \frac{1}{\det(A)} = 2$

Qs 2)
Ans)

$$Q_1 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\det(Q_1) = \cos^2\theta + \sin^2\theta$$
$$= \underline{1}$$

$$Q_2 = \begin{bmatrix} 1-2\cos^2\theta & -2\cos\theta\sin\theta \\ -2\cos\theta\sin\theta & 1-2\sin^2\theta \end{bmatrix}$$

$$\det(Q_2) = (1-2\cos^2\theta)(1-2\sin^2\theta) - 4\cos^2\theta\sin^2\theta$$
$$= 1 - 2\cos^2\theta - 2\sin^2\theta + \cancel{4\cos^2\theta\sin^2\theta} - \cancel{4\cos^2\theta\sin^2\theta}$$
$$= 1-2$$
$$= \underline{-1}$$

Q3)
Ans)

$$M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\det(M) = aei - ahf + bgf - bdi + cdh - ceg$$

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\det(A) = 1\times1\times1 - 1\times1\times1 + 1\times(-1)\times1$$
$$= \underline{-1}$$

$$B = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 2 & 8 \\ 6 & 3 & 12 \end{bmatrix}$$

$$\det(B) = 2[24-24] - 1[48-48] + 4[12-12]$$
$$= \underline{0}$$

$$c = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\det(c) = 1\left[45-48\right] - 2\left[36-42\right] + 3\left[32-35\right]$$

$$= -3 - 2\times(-6) + 3\times 3$$

$$= -3 + 12 - 9$$

$$= 0$$

Q5.4)
A)

$$AC^T = (\det A)\, I$$

Take det both sides –

$$\det(AC^T) = \det((\det A)\, I)$$

$$= (\det A)^n$$

$\rightarrow$ $\det(A)\det(C^T) = (\det(A))^n$ | $\det(A) \neq 0$

$$\det(C^T) = (\det(A))^{n-1}$$

$\downarrow$

$$\det(C) = (\det(A))^{n-1}$$

$\rightarrow$ if $\det(A) = 0$

$$AC^T = 0$$

$$\det(A)\det(C^T) = 0$$

$\det(C^T)$ can be anything

Qs 5)

Ans)

$$A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix}$$

$$C_{ij} = (-1)^{i+j} \det(M_{ij})$$

$C_{11} = 6$	$C_{21} = +3$	$C_{31} = -6$
$C_{12} = -3$	$C_{22} = 1$	$C_{32} = 2$
$C_{13} = 0$	$C_{23} = -1$	$C_{33} = 1$

$$C = \begin{bmatrix} 6 & -3 & \\ +3 & 1 & -1 \\ -6 & 2 & 1 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 6 & 3 & -6 \\ -3 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A C^T = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$\det(A) = 3$

- if we change 4 to 100, as cofactor of $(1,3) = 0$

$C_{13} = 0$

$\therefore$ it will not change the value of determinant.

$$\det(A) = \sum_j x_{ij} C_{ij}$$

Q6)
A6)

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\det(A) = ad - bc$$

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\det(A^{-1}) = \det\left[\frac{1}{\det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\right]$$

$$= \frac{1}{ad-bc}(ad-bc) \quad \times \quad \text{Wrong.}$$

$\frac{1}{(\det A)}$ scalar multiple so should have power 2.

$\therefore \rightarrow \det(A^{-1}) = \frac{1}{(\det A)^2}(ad-bc)$

$$= \frac{1}{\det A} = \frac{1}{ad-bc}$$

Q07)

a) $2x_1 + 5x_2 = 1$

$x_1 + 4x_2 = 2$

$|A| = \begin{vmatrix} 2 & 5 \\ 1 & 4 \end{vmatrix}$ | $|B_1| = \begin{vmatrix} 1 & 5 \\ 2 & 4 \end{vmatrix}$ | $|B_2| = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$

$\det A = 3$ | $\det B_1 = -6$ | $\det B_2 = +3$

$x_1 = \dfrac{-6}{3} = -2$ | $x_2 = \dfrac{3}{3} = 1$

$x_1 = -2$

$x_2 = 1$

b)

$$2x_1 + x_2 + 0.x_3 = 1$$
$$x_1 + 2x_2 + x_3 = 0$$
$$x_2 + 2x_3 = 0$$

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\det A = 2[3] - 1[2]$$
$$= 4$$

$$B_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\det B_1 = 3$$

$$B_2 = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\det B_2 = -2$$

$$B_3 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$\det B_3 = 1$

$x_1 = \frac{3}{4}$ | $x_2 = \frac{-2}{4} = \frac{-1}{2}$ | $x_3 = \frac{1}{4}$

Sol^n

Q8)

Ans)

$$E_n = \begin{bmatrix} 1 & 1 & & & \\ 1 & 1 & 1 & & \\ & 1 & 1 & 1 & \\ & & \ddots & \ddots & \ddots \\ & & & & \end{bmatrix}$$

Final row → $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & \dots \end{bmatrix}$

$E_n = 1 \det(E_{n-1}) - 1 \det(E_{n-2})$

$\therefore \boxed{E_n = E_{n-1} - E_{n-2}}$

$E_1 = 1$
$E_2 = 0$
$E_3 = -1$
$E_4 = -1$
$E_5 = 0$
$E_6 = 1$
$E_7 = 1$
$E_8 = 0$

repeats after 6 terms.

$100 \% 6 = 4$

$E_{100} = E_4 = \underline{-1}$

Qo 1)
Ans)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\downarrow R_3 \to R_3 - R_2$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = U$$

$\det(A) = \det(U) = 1 \times 1 \times 1 = 1$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 3 & 3 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -3 \\ 0 & -3 & -6 \end{bmatrix}$$

$$\downarrow R_3 \to R_3 - \frac{3}{2} R_2$$

$$U \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -3 \\ 0 & 0 & -3/2 \end{bmatrix}$$

$-6 + \frac{9}{2}$

$\frac{-3}{2}$

$\det(A) = \det(U) = 1 \times -\frac{3}{2} \times -2 = 3$

Q10)
Ans)

$$A = \begin{bmatrix} 101 & 201 & 301 \\ 102 & 202 & 302 \\ 103 & 203 & 303 \end{bmatrix}$$

$$\Big\downarrow \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\begin{bmatrix} 101 & 201 & 301 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$$

$$\Big\downarrow R_3 \to R_3 - 2R_2$$

$$\begin{bmatrix} 101 & 201 & 301 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ $\det(A) = 0$

$$A = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & a & a^2 \\ 0 & b-a & (b^2-a^2) \\ 0 & c-a & (c^2-a^2) \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} = (b-a)(c-a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}$$

$$= (b-a)(c-a)\left[c+\cancel{a}-b-\cancel{a}\right]$$

$$= \underline{(b-a)(c-a)(c-b)}$$

Q11)

Ans) $M = \begin{bmatrix} A & B \\ 0 & D \end{bmatrix}$

a) $|M| = |A||D|$

works as M is an upper Δ matrix.

$$\begin{vmatrix} A & B \\ 0 & D \end{vmatrix} = \underbrace{\begin{vmatrix} A & 0 \\ 0 & D \end{vmatrix}}_{} \underbrace{\begin{vmatrix} I & A^{-1}B \\ 0 & I \end{vmatrix}}_{1}$$

$$\underbrace{\begin{vmatrix} A & 0 \\ 0 & I \end{vmatrix}}_{|A|} \underbrace{\begin{vmatrix} I & 0 \\ 0 & D \end{vmatrix}}_{|D|}$$

$= |A| \, |D|$

So $|B|$ is not in eq^n.

b) $\begin{vmatrix} A & B \\ C & D \end{vmatrix} \neq |A||D| - |C||B|$

Ex $A = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ $B = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ $D = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ $C = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}$

$|M| = \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{vmatrix} = 0$ (2^{nd}, 4^{th} row same)

$|A| = 1$

$|B| = 1$

$|D| = 1$

$|C| = 0$

$|A||D| - |C||B| = 1$

$\therefore$ $0 \neq 1$

c) $\begin{vmatrix} A & B \\ C & D \end{vmatrix} \neq \det(AD - CB)$

$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = \det(AD - BC)$

$B = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$ $C = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}$ $A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$BC = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$ $CB = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ $AD = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$

$M = \begin{vmatrix} 1 & 2 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{vmatrix} = \det(AD - BC)$

$= \begin{vmatrix} \cancel{0} & \cancel{-2} \\ \cancel{-1} & \cancel{-1} \end{vmatrix} \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}$

$= 0$

$\det(AD - CB) =$

$= \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} = 1$

$\therefore \quad |M| \neq \det(AD - CB)$

Q12)

Ans) Area = det $[v_1 \ v_2]$

- $v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$$\begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix} = 6-2 = 4$$

↳ Area

- $v_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$$\begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} = 6-2 = 4$$

↳ Area

Both have the same area.