Homework 3

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Qs 1)

a)

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b)

$$E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 7 & 1 \end{bmatrix}$$

c)

$$P = \begin{array}{cl} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} & \left.\begin{array}{l} \text{row 2} \\ \text{row 3} \\ \text{row 1} \end{array}\right\} \text{this order.} \end{array}$$

Q.2)
Ans)

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{pmatrix}$$

(Corresponding matrix)

$$E_{21} = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad E_{31} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

$$E_{32} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

→ Take $A = \begin{pmatrix} 1 & 1 & 0 \\ 4 & 6 & 0 \\ -2 & 2 & 1 \end{pmatrix}$

$R_2 \to R_2 - 4R_1$

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ -2 & 2 & 1 \end{pmatrix}$$

So (2,1) element got zero.

$$\therefore E_{21} = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Makes $A_{2\times1} = 0$ when multiply

$R_3 \to R_3 + 2R_1$

$$A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 4 & 0 \end{pmatrix} \qquad E_{31} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

$R_3 \to R_3 - 2R_2$

$$A_3 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad E_{32} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

Therefore those elimination matrix.

$$\to \quad E_{21}A = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ -2 & 2 & 0 \end{pmatrix}$$

$$\to \quad E_{31}E_{21}A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ -2 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 4 & 0 \end{pmatrix}$$

$$\to \quad E_{32}E_{31}E_{21}A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 4 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -2 \end{pmatrix}$$

U

$$E = E_{32}E_{31}E_{21} = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 10 & -2 & 1 \end{pmatrix}$$

$$L = E^{-1} = E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}$$

Q3)
Ans)

a) add 7 times row 1 to row 3.

b) $E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -7 & 0 & 1 \end{bmatrix}$

$E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 7 & 0 & 1 \end{bmatrix}$

c) $E E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -7 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 7 & 0 & 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{I}$

Qs 4)
Ans)
a)

$$M = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \xleftarrow[R_3 \to R_1 + R_3]{R_1 \to R_1 + R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(Ans)

b)

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 \to R_3 + R_1} \underset{A_1}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}} \xrightarrow{R_1 \to R_1 + R_3} \underset{A_2}{\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}}$$

(answer)

Qn5)
Ans)

$Ax = b$ | $Ax^* = b^*$ $\rightarrow$ solve both simultaneously

Use an double augmented matrix to solve.

$$[A \mid b \mid b^*]$$

$$\rightarrow \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Augmented matrix : $\left[\begin{array}{cc|c:c} 1 & 4 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right]$

$R_2 \rightarrow R_2 - 2R_1$

$$\left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{array}\right]$$

$\therefore$ $x + 4y = 1$
$-y = -2$

$\therefore$ $y = 2$
$x = -7$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -7 \\ 2 \end{pmatrix}$$

$u + 4v = 0$
$-v = 1$

$v = -1$
$u = 4$

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

Q6) matrix

An) $A \to 3\times5$

$B \to 5\times3$

$C \to 5\times1$

$D \to 3\times1$

1) $BA = B_{5\times\underline{3}}\ A_{\underline{3}\times5}$ (Yes)

all 3

$$= \begin{pmatrix} 1&1&1 \\ 1&1&1 \\ 1&1&1 \\ 1&1&1 \\ 1&1&1 \end{pmatrix} \begin{pmatrix} 1&1&1&1&1 \\ 1&1&1&1&1 \\ 1&1&1&1&1 \end{pmatrix} = \begin{pmatrix} 3 & \cdots & 3 \\ \vdots & & \vdots \\ 3 & \cdots & 3 \end{pmatrix}_{5\times5}$$

2) $AB = A_{3\times\underline{5}}\ B_{\underline{5}\times3}$ (Yes allowed)

$$= \begin{pmatrix} 5&5&5 \\ 5&5&5 \\ 5&5&5 \end{pmatrix}_{3\times3}$$

3) $ABD = (AB)_{3\times\underline{3}} \times D_{\underline{3}\times1}$ (Yes

$$= \begin{pmatrix} 5&5&5 \\ 5&5&5 \\ 5&5&5 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 15 \\ 15 \\ 15 \end{pmatrix}_{3\times1}$$

4) $DC = D_{3\times 1} \; C_{5\times 1}$ as D has 1 col^n
C has 5 row.

(Not allowed)

5) $A(B+C)$

$B_{5\times 3} + C_{5\times 1}$

not allowed need same dim for addition.

Q 7)

Ans) a) if columns of 1 and 3 of B are same, so are col 1 & 3 of AB.

TRUE

$$B = \begin{bmatrix} b_1 & - & b_1 \\ b_2 & - & b_2 \\ b_3 & - & b_3 \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{bmatrix}$$

same

$$AB = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ & & \\ & & \end{bmatrix} \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{bmatrix} b_1 & - & b_1 \\ b_2 & - & b_2 \\ b_3 & - & b_3 \end{bmatrix} \end{array} = \begin{bmatrix} \alpha_1 b_1 + \alpha_2 b_2 + \alpha_3 b_3 & - & \alpha_1 b_1 + \alpha_2 b_2 + \alpha_3 b_3 \\ \vdots & - & \vdots \\ \vdots & - & \vdots \end{bmatrix}$$

by multiplying we get the same column.

b) if rows of 1 & 3 of B are same.
rows of 1 & 3 of AB are same.

False

Eg:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$AB = \begin{pmatrix} 4 & 0 & 0 \\ 10 & 0 & 0 \\ 16 & 0 & 0 \end{pmatrix}$$

$\therefore$ Rows are different.

Hence false

Q08)

Ans) Yes B is invertible .

As $B = P.A$

↓

permutation matrix

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B^{-1} = (PA)^{-1}$$
$$= A^{-1} P^{-1}$$
$$= A^{-1} P$$

| $P^{-1} = P$
(permutation matrix)

So to get B^{-1} , exchange first 2 rows of A^{-1}

Qs 9)

Ans)

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

a) $A = \begin{pmatrix} 0 & 3 \\ 4 & 0 \end{pmatrix}$ $|A| = -12$ $A^{-1} = \frac{1}{-12} \begin{pmatrix} 0 & -3 \\ -4 & 0 \end{pmatrix}$

$$\therefore A^{-1} = \begin{pmatrix} 0 & 1/4 \\ 1/3 & 0 \end{pmatrix}$$

b) $B = \begin{pmatrix} 2 & 0 \\ 4 & 2 \end{pmatrix}$ $|B| = 4$

$$B^{-1} = \frac{1}{4} \begin{pmatrix} 2 & 0 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 \\ -1 & 1/2 \end{pmatrix}$$

c) $C = \begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix}$ $|C| = 1$

$$C^{-1} = \begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix}$$

2x2

Q.10)
Ans)

$$A = \begin{pmatrix} \text{row } 1 \\ \text{row } 2 \\ \text{row } 3 \\ \vdots \end{pmatrix}$$

row 3 = row 1 + row 2

We find $|A| =$ $\begin{vmatrix} \text{row } 1 \\ \vdots \end{vmatrix}$

Add row 3 = row 3 − row 2 − row 1

$$|A| = \begin{vmatrix} 1 \\ 2 \\ 0 \ 0 \ 0 \ 0 \ 0 \ \dots \\ \vdots \end{vmatrix} = 0$$

so $|A| = 0$, $\therefore$ A is not invertible.

a) $Ax = (0,0,1)$

$$\begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Augmented matrix $\Rightarrow$ $\left(\begin{array}{c|c} r_1 & 0 \\ r_2 & 0 \\ r_3 & 1 \end{array}\right)$

$r_3 = r_3 - r_1 - r_2$

Aug $\Rightarrow$ $\left(\begin{array}{c|c} r_1 & 0 \\ r_2 & 0 \\ 0 & 1 \end{array}\right)$

$\therefore \; 0x + 0y + 0z = 1$ (not possible so no soln) b)

$Ax = b$

$$b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$b_3 = b_2 + b_1$ to have a solⁿ.

c) In elimination process, Row 3 becomes 0 (row of zero).

Qs 11)

Ans) a) $[A|I] \rightarrow$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right]$$

$$\Big\downarrow R_2 \rightarrow R_2 - 2R_1$$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$$\Big\downarrow R_1 \rightarrow R_1 - 3R_2$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 7 & -3 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$$\quad I \qquad\qquad A^{-1}$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

b) $[A|I] \longrightarrow \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 3 & 9 & 0 & 1 \end{array}\right]$

$$\Big\downarrow R_2 \to R_2 - 3R_1$$

$$\left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & -3 & -3 & 1 \end{array}\right]$$

$$\Big\downarrow R_2 \to \frac{R_2}{-3}$$

$$\left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & 1 & 1 & -1/3 \end{array}\right]$$

$$\Big\downarrow R_1 \to R_1 - 4R_2$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -3 & 4/3 \\ 0 & 1 & 1 & -1/3 \end{array}\right]$$

$$\quad I \qquad\qquad A^{-1}$$

$$\therefore A^{-1} = \begin{bmatrix} -3 & 4/3 \\ 1 & -1/3 \end{bmatrix}$$

Qs 12)

Ans) a) $A = 4 \times 4$ has rows of zeros. **TRUE**

Then $|A| = 0$ (no pivot)
det of A

$\therefore$ it can't have an inverse. $A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$

b) Every matrix with 1 down the main diagonal is invertible

FALSE

Eg: $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ $|A| = 0$

c) If A is invertible, A^{-1} & A^2 are invertible **TRUE**

$(A^{-1})^{-1} = A$
$(A^2)^{-1} = (AA)^{-1} = A^{-1}A^{-1}$

- inverse of inverse also invertible.
- multiplication of 2 invertible matrix is invertible.