b) $Ax = b$

0/4

$$\underbrace{\begin{bmatrix} \cdot & - & - & -\cdot \\ \cdot & \cdot 1 \cdot & \cdot\cdot & \cdot\cdot \\ \cdot & \cdot\cdot\cdot & 1 & - - \\ - & \cdot\cdot & \cdot & \cdot\cdot \end{bmatrix}_{2n \times n}}_{A} \underbrace{\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}}_{b}$$

$$\left.\begin{array}{l} r = 2 \text{ (rank)} \\ 2n \text{ columns} \\ n \text{ rows.} \end{array}\right\} \quad \text{As } n \geq 2,$$

→ no. of columns $> r$ | No. of rows $\geq r$

can be written as $A = \begin{bmatrix} I & F \\ 0 & 0 \end{bmatrix}$ it will have either no sol^n or ∞ sol^n.

→ For sol^n to always exist.

$k = \max(i, j)$

and all elements of b from b_k to b_n should be 0.

OR

if $n = 2$, there will be no rows of 0, so sol^n always exist. $A = [I \ F]$

0/8

c) $Ax = u+v$

$A = [uu^T \;\; vv^T]$, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$Ax = uu^T x_1 + vv^T x_2 = u+v$ $\qquad u^T = [0 \ldots 1 \ldots 0]$

Solution alway exist as $u^T x_1 = I$

$v^T x_2 = I$

We can find a lot of values of x_1 & x_2

$x_1 = [\alpha_1 \; \alpha_2 \ldots \underset{i^{th}}{1} \ldots \alpha_n]^T$
$x_2 = [\beta_1 \; \beta_2 \ldots \underset{j^{th}}{1} \ldots \beta_n]^T$ } these both will give I.

$\therefore$ Sol^n exist but not unique.

d) $Ax = u+v$

$A = [uu^T \;\; vv^T]$ $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$uu^T x_1 = u$, $vv^T x_2 = v$ $\Rightarrow$ $u^T x_1 = I$, $v^T x_2 = I$

Take 1) $x_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \rightarrow i^{th}$, $x_2 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 1 \\ 0 \end{bmatrix} \rightarrow j^{th}$

$x_1 = u$ $\qquad$ $x_2 = v$

Sol^n 1

2) $x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

~~x_1 x_2~~

Sol^n 2.