Other Method for Inverse L.T

① Multiplication by S :—

If $L^{-1}\bar{f}(s) = f(t)$ then

$$\boxed{L^{-1} s\bar{f}(s) = \frac{d}{dt} f(t)}$$

② Division by S :—

If $L^{-1}\bar{f}(s) = f(t)$ then

$$\boxed{L^{-1}\left[\frac{\bar{f}(s)}{s}\right] = \int_0^t f(t)\,dt}$$

Ques Find Inverse Laplace Tr. of given fn :—

ⓐ $L^{-1}\left[\frac{1}{s^2(s^2+a^2)}\right]$

Soln → By using division by S :—

$$\bar{f}(s) = \frac{1}{s^2+a^2}$$

then, $L^{-1}\bar{f}(s) = \frac{1}{a}\sin at$

Now, $L^{-1}\frac{1}{s^2(s^2+a^2)} = \int_0^t\int_0^t \frac{1}{a}\sin at\,dt\,dt$

$$= \int_0^t \frac{1}{a}\left\{-\frac{\cos at}{a}\right\}_0^t dt$$

$$= \frac{-1}{a^2}\int_0^t \{\cos at - 1\}\,dt$$

$$= \frac{-1}{a^2}\left[\frac{\sin at}{a} - t\right]_0^t$$

$$= \frac{-1}{a^2}\left[\frac{\sin at}{a} - t\right]$$

$$= \frac{t}{a^2} - \frac{\sin at}{a^3} \quad \text{Ans}$$

(b) $L^{-1} \frac{s}{(s+a)^2}$

Sol^n By Multiplication by s ? —

$\bar{f}(s) = \frac{1}{(s+a)^2}$

$L^{-1}[\bar{f}(s)] = e^{-at} t$

Now, $L^{-1}\left(\frac{s}{(s+a)^2}\right) = \frac{d}{dt}\left[\underset{I}{e^{-at}} \underset{II}{t}\right]$

$= e^{-at} \cdot 1 + ta(-e^{-at})$

$= e^{-at} - te^{-at}a$

$= e^{-at}a(1-t)$ Ans.

* * Such type of Questions are solved by multiplication by t method * *

(c) $L^{-1} \log\left(\frac{1+s}{s}\right)$

$\Rightarrow$ since, $L^{-1} \log\left(\frac{1+s}{s}\right) = f(t)$

By multiplication by t method

we know that

$L[t f(t)] = -\frac{d}{ds} \bar{f}(s)$

$\xrightarrow{or}$ $t f(t) = L^{-1}\left[-\frac{d}{ds}\bar{f}(s)\right]$

$t f(t) = -L^{-1}\left[\frac{d}{ds} \log\left(\frac{1+s}{s}\right)\right]$

$= -L^{-1}\left[\frac{d}{ds}\log(1+s) - \frac{d}{ds}\log s\right]$

$= -L^{-1}\left[\frac{1}{1+s} - \frac{1}{s}\right]$

$= -\left[e^{-t} - 1\right] = 1 - e^{-t}$

So, $f(t) = \frac{1-e^{-t}}{t}$ Ans.

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Que $\Rightarrow$ $L^{-1} \log \left\{ \frac{s+a}{s+b} \right\}$

Soln $\Rightarrow$ using multiplication by t ;

$$L[t f(t)] = -\frac{d}{ds} \bar{f}(s)$$

$$t f(t) = L^{-1}\left[-\frac{d}{ds} \bar{f}(s)\right]$$

$$t f(t) = -L^{-1}\left[\frac{d}{ds} \log\left(\frac{s+a}{s+b}\right)\right]$$

$$= -L^{-1}\left\{\frac{d}{ds} \log(s+a) - \frac{d}{ds}\log(s+b)\right\}$$

$$= -L^{-1}\left[\frac{1}{s+a} - \frac{1}{s+b}\right]$$

$$= -\left(e^{-at} - e^{-bt}\right)$$

So, $f(t) = \dfrac{e^{-bt} - e^{-at}}{t}$

Que $\Rightarrow$ $L^{-1} \log\left[\frac{(s+1)}{(s+2)(s+3)}\right]$

Soln $\Rightarrow$ $t f(t) = L^{-1} -\frac{d}{ds} \bar{f}(s)$

$$= L^{-1} \frac{d}{ds}\left[\log (s+1) - \{\log(s+2) + \log(s+3)\}\right]$$

$$= L^{-1}\left[\frac{1}{s+1} - \left\{\frac{1}{s+2} + \frac{1}{s+3}\right\}\right]$$

$$= -\left\{e^{-t} - e^{-2t} - e^{-3t}\right\}$$

So $f(t) = \dfrac{-e^{-t} + e^{-2t} + e^{-3t}}{t}$

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Que $L^{-1} \log\left(1-\frac{a^2}{s^2}\right)$

Sol^n $L^{-1} \log\left\{\frac{s^2-a^2}{s^2}\right\}$

$$tf(t) = L^{-1}\left(-\frac{d}{ds}\bar{f}(s)\right)$$

$$tf(t) = L^{-1}\left[-\frac{d}{ds}\log\left(\frac{s^2-a^2}{s^2}\right)\right]$$

$$= -L^{-1}\left[\frac{d}{ds}\log(s^2-a^2) - \frac{d}{ds}\log s^2\right]$$

$$= -L^{-1}\left[\frac{2s}{s^2-a^2} - \frac{2s}{s^2}\right]$$

$$tf(t) = -2\cosh at + 2$$

$$f(t) = \frac{2}{t}(1-\cosh at) \quad \text{Ans}$$

Que $L^{-1} \tan^{-1}\frac{2}{s}$

Sol^n $tf(t) = L^{-1} \frac{-d}{ds}\bar{f}(s)$

$$= -L^{-1}\frac{d}{ds}\tan^{-1}2/s$$

$$= -L^{-1}\left\{\frac{1}{1+(2/s)^2} \times \frac{-2}{s^2}\right\}$$

$$= +L^{-1}\left\{\frac{2}{s^2+4}\right\}$$

$$tf(t) = \sin 2t$$

So, $f(t) = \frac{\sin 2t}{t}$ Ans

Imp

Que $L^{-1}\, s \log \frac{s-1}{s+1}$

Sol^n ⟹ I^st we have to solve $L^{-1}\left(\log \frac{s-1}{s+1}\right)$ then apply multiplication by s :—

let, $\bar{f}(s) = \log \frac{s-1}{s+1}$

$$t f(t) = L^{-1} \frac{-d}{ds} \bar{f}(s)$$

$$= L^{-1} \frac{-d}{ds}\left(\log \frac{s-1}{s+1}\right)$$

$$= -L^{-1}\left[\frac{d}{ds} \log(s-1) - \frac{d}{ds} \log(s+1)\right]$$

$$= -L^{-1}\left[\frac{1}{s-1} - \frac{1}{s+1}\right]$$

$$= -\{e^{t} - e^{-t}\}$$

$$= e^{-t} - e^{t}$$

So, $f(t) = \dfrac{e^{-t} - e^{t}}{t}$

Now, multiplication by s

$$L^{-1}\left(s \log \frac{s-1}{s+1}\right) = \frac{d}{dt}\left[\frac{e^{-t} - e^{t}}{t}\right]$$

$$= \frac{t\{-e^{-t} - e^{t}\} - (e^{-t} - e^{t}) \cdot 1}{t^2}$$

$$= \frac{-t\{e^{t} + e^{-t}\} + \{e^{t} - e^{-t}\}}{t^2}$$

$$= \frac{-2t\left\{\frac{e^{t} + e^{-t}}{2}\right\} + 2\left\{\frac{e^{t} - e^{-t}}{2}\right\}}{t^2}$$