5. (*a*) The mean and variance of binomial distribution are 4 and $\frac{4}{3}$ respectively. Find $P(X \geq 1)$. [2]

(*b*) Fit a Poisson distribution to the following data given the number of yeast cells per square for 400 squares: [7]

No. of cells/sq	0	1	2	3	4	5	6	7	8	9	10
No. of squares	103	143	98	42	8	4	2	0	0	0	0

(*c*) In an examination taken among 500 candidates, the average and standard deviation of marks obtained are 40% and 10%. Find approximately the following :

(*i*) How many will pass, if 50% is fixed as a minimum?

(*ii*) What should be the minimum if 350 candidates are to pass?

(*iii*) How many have scored marks above 60%? [7]

(*d*) In a precision bombing attack, there is a 50% chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give a 99% chance or better of completely destroying the target? [7]

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Roll No. ....

322351(14)

BE (3rd Semester)

Examination, April-May, 2017

[New Scheme]

Mathematics-III

Time Allowed : 3 hours *Maximum Marks* : 80

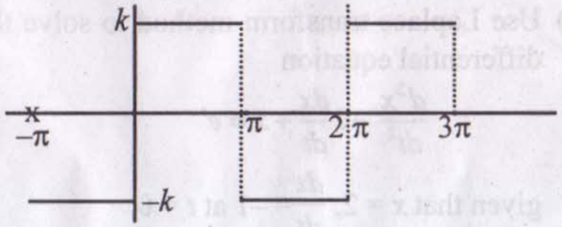
Minimum Pass Marks : 28

Note : (*i*) Part (*a*) of each question is compulsory. Attempt any **two** parts from (*b*), (*c*) and (*d*).

(*ii*) The figures in the right-hand margin indicate marks.

1. (*a*) If $f(x) = x$ in the interval $-\pi < x < \pi$, what is the value of $\frac{a_0}{2}$? [2]

(*b*) Develop the Fourier series for the function defined by the following figure : [7]



(*c*) Obtain the constant term and the coefficients of sine and cosine terms up to second harmonics in the Fourier expansion of y as given in the following table : [7]

x :	0	1	2	3	4	5
y :	9	18	24	28	26	20

(*d*) Prove that

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}, \quad -\pi < x < \pi$$

Hence show that :

$$\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$$ [7]

2. (*a*) If $L(f(t)) = \dfrac{1}{s(s^2+1)}$, find $L\{f(2t)\}$. [2]

(*b*) Find the inverse transforms of the following functions : [7]

(*i*) $f(s) = \dfrac{5s+3}{(s-1)(s^2+2s+5)}$

(*ii*) $f(s) = \log\left[\dfrac{s^2+1}{s(s+1)}\right]$

(*c*) Using convolution theorem, solve

$$y(t) = t + \int_0^t y(\tau)\sin(t-\tau)\, d\tau$$ [7]

(*d*) Use Laplace transform method to solve the differential equation

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$$

given that $x = 2, \dfrac{dx}{dt} = -1$ at $t = 0$. [7]

3. (*a*) State Cauchy-Riemann equation. [2]

(*b*) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at origin even though CR equations are satisfied thereat. [7]

(*c*) Prove that $\displaystyle\int_0^{\pi} \frac{d\theta}{17-8\cos\theta} = \frac{\pi}{15}$. [7]

(*d*) If $f(z) = \begin{cases} \dfrac{\sin z}{(z-\pi)^2}, & \text{if } z \neq \pi \\ \infty, & \text{if } z = \pi \end{cases}$

then prove that $\int_C f(z)\, dz = -2\pi i$, where $|z| < \pi$. [7]

4. (*a*) Form partial differential equation by eliminating arbitrary function $z = f\left(\frac{y}{x}\right)$. [2]

(*b*) Find the surface satisfying $t = 6x^3y$, where $t = \dfrac{\partial^2 z}{\partial y^2}$ containing two lines $y = 0,\ z = 0$ and $y = 2, z = 2$. [7]

(*c*) Solve partial differential equation

$$r - 2s + t = \sin(2x + 3y)$$ [7]

(*d*) Using method of separation of variable, solve partial differential equation

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$$

where $u(x, 0) = 6e^{-3x}$. [7]