Finite Differences

argument → x : x_1 x_2 x_3 - - - - -

entry → $f(x)$: $f(x_1)$ $f(x_2)$ $f(x_3)$ - - - - -

let, $y = f(x)$

finite difference

a	$a+h$	$a+2h$	$a+3h$	- - - - - -
$f(a)$	$f(a+h)$	$f(a+2h)$	$f(a+3h)$	- - - - - -

* Forward Differences *

$$\Delta f(a) = f(a+h) - f(a)$$

$$\Delta^2 f(a) = \Delta f(a+h) - \Delta f(a)$$

$$\Delta^3 f(a) = \Delta^2 f(a+h) - \Delta^2 f(a)$$

operator = Δ

Que Given, $f(0) = 7$, $f(5) = 11$, $f(10) = 14$, $f(15) = 18$, $f(20) = 24$, $f(25) = 32$. Construct forward difference table and find $\Delta^4 f(0)$ and $\Delta^5 f(0)$.

Soln

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
0	$7 = y_0$	Δy_0 $11-7=4$				
5	$11 = y_1$	Δy_1 $14-11=3$	-1			
10	$14 = y_2$	Δy_2 $18-14=4$	1	2	-1	
15	$18 = y_3$	Δy_3 $24-18=6$	2	1		0
20	$24 = y_4$	Δy_4 $32-24=8$	2	0	-1	
25	$32 = y_5$					

From the table it is clear that

$\Delta^4 f(0) = -1$, $\Delta^5 f(0) = 0$ Ans

Backward Difference

$\nabla f(a+h) = f(a+h) - f(a)$

$\nabla f(a+2h) = f(a+2h) - f(a+h)$

$\nabla f(a+3h) = f(a+3h) - f(a+2h)$

Same as prev. Que

Ex:-

x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	
0	$7 = y_0$	$\nabla y_1 = 11-7 = 4$	$\nabla^2 y_1 = 3-4 = -1$	$\nabla^3 y_1 = 1+1 = 2$	$\nabla^4 y_1 = 1-2 = -1$
5	$11 = y_1$	$\nabla y_2 = 14-11 = 3$	$\nabla^2 y_2 = 4-3 = 1$	$\nabla^3 y_2 = 2-1 = 1$	$\nabla^4 y_2 = 0-1 = -1$
10	$14 = y_2$	$\nabla y_3 = 18-14 = 4$	$\nabla^2 y_3 = 6-4 = 2$	$\nabla^3 y_3 = 2-2 = 0$	
15	$18 = y_3$	$\nabla y_4 = 24-18 = 6$	$\nabla^2 y_4 = 8-6 = 2$		
20	$24 = y_4$	$\nabla y_5 = 32-24 = 8$			
25	$32 = y_5$				

Central Difference

$y_1 - y_0 = \delta y_{1/2}$, $y_3 - y_2 = \delta y_{5/2}$

$y_2 - y_1 = \delta y_{3/2}$, — " — — —

Basic

x	y	1st difference	2nd difference	3rd difference	4th difference
x	y_0				
$x+h$	y_1	$\delta y_{1/2}$	$\delta^2 y_{\frac{1}{2}+\frac{3}{2}}$ or $\delta^2 y_1$		
$x+2h$	y_2	$\delta y_{3/2}$	$\delta^2 y_2$	$\delta^3 y_{3/2}$	$\delta^4 y_2$
$x+3h$	y_3	$\delta y_{5/2}$	$\delta^2 y_3$	$\delta^2 y_{5/2}$	
$x+4h$	y_4	$\delta y_{7/2}$			

mean $= \frac{0+1}{2} = \frac{1}{2}$

$\frac{1+2}{2} = \frac{3}{2}$

Forward Difference

variety

- ✓ Evaluate
- missing term

① Evaluate, $\Delta \tan^{-1}x$.

Soln ⟹ $\Delta \tan^{-1}x = \tan^{-1}(x+h) - \tan^{-1}x$

Formula :- $\tan^{-1}A - \tan^{-1}B = \tan^{-1}\left\{\frac{A-B}{1+AB}\right\}$

$$= \tan^{-1}\left[\frac{x+h-x}{1+(x+h)x}\right]$$

$$= \tan^{-1}\left(\frac{h}{1+x^2+hx}\right) \quad \text{Ans}$$

② $\Delta^2 ab^x$

Soln ⟹ $\Delta\left[\Delta ab^x\right]$

$\Delta\left[ab^{x+1} - ab^x\right]$

$\Rightarrow a\,\Delta\left[b^x \cdot b - b^x\right]$

$\Rightarrow a(b-1)\,\Delta\{b^x\}$

again,

$\Rightarrow a(b-1)\left[b^{x+1} - b^x\right]$

$\Rightarrow a(b-1)\,b^x(b-1)$

$\Rightarrow a(b-1)^2 b^x$ Ans

* [In algebraic term we always put $h=1$]

③ $\Delta^n e^x$

Soln ⟹ $\Delta e^x = e^{x+1} - e^x$ (Put $h=1$)

$= e^x(e-1)$

$\Delta^2 e^x = \Delta\left[e^x(e-1)\right]$

$= (e-1)\left[e^{x+1} - e^x\right]$

$= (e-1)\left[e^x(e-1)\right] = (e-1)^2 e^x$

again, $\Delta^3 e^x = \Delta\left[(e-1)^2 e^x\right]$

$$= (e-1)^2\left[e^{x+1} - e^x\right]$$

$$= (e-1)^2 (e-1) e^x$$

$$= (e-1)^3 e^x$$

By continuing this process;

$$\Delta^n e^x = (e-1)^n e^x \quad \text{Ans}$$

(4) $\Delta^2\left(\dfrac{5x+12}{x^2+5x+6}\right)$

Sol^n $\Rightarrow$ 1st we will solve $\dfrac{5x+12}{x^2+5x+16}$ by using partial fraction;

$$\frac{5x+12}{x^2+5x+6} = \frac{5x+12}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3} \quad \text{—— (I)}$$

$$\Rightarrow \frac{5x+12}{\cancel{(x+2)(x+3)}} = \frac{(x+3)A + (x+2)B}{\cancel{(x+2)(x+3)}}$$

$$5x+12 = (x+3)A + (x+2)B \quad \text{—— (II)}$$

Put, $x=-3$ in eq^n (II);

$$-15+12 = (x+2)B$$

$$-3 = -B \Rightarrow \boxed{B=3}$$

again,

$x=-2$ in eq^n (II);

$$-10+12 = (-2+3)A$$

$$\boxed{2 = A}$$

By ① $\rightarrow$

$$= \frac{2}{x+2} + \frac{3}{x+3}$$

$$\Delta^2\left[\frac{2}{x+2} + \frac{3}{x+3}\right]$$

$$= \Delta\left[2\,\Delta\left(\frac{1}{x+2}\right) + 3\,\Delta\left(\frac{1}{x+3}\right)\right]$$

$$= \Delta\left[2\left\{\frac{1}{x+3} - \frac{1}{x+2}\right\} + 3\left\{\frac{1}{x+4} - \frac{1}{x+3}\right\}\right]$$

$$= \Delta\left[2\left\{\frac{x+2-(x+3)}{(x+3)(x+2)}\right\} + 3\left\{\frac{x+3-(x+4)}{(x+4)(x+3)}\right\}\right]$$

$$= \Delta\left[\frac{-2}{(x+3)(x+2)} - \frac{3}{(x+4)(x+3)}\right]$$

$$= -2\left[\frac{1}{(x+4)(x+3)} - \frac{1}{(x+3)(x+2)}\right] - 3\left[\frac{1}{(x+5)(x+4)} - \frac{1}{(x+4)(x+3)}\right]$$

$$\Rightarrow \frac{4}{(x+2)(x+3)(x+4)} + \frac{6}{(x+3)(x+4)(x+5)}$$

$$\Rightarrow \frac{2(5x+16)}{(x+2)(x+3)(x+4)(x+5)}$$ Ans

Missing term

Q①

Yr:	1917	1918	1919	1920	1921
Export:	443	384	–	397	467

Find the missing value.

Solⁿ Taking $y_r - 1917 = x$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	443				
		-59			
1	384		$y_2 - 325$		
		$y_2 - 384$		$1106 - 3y_2$	
2	y_2 (let)		$781 - 2y_2$		$6y_2 - 2214$
		$397 - y_2$		$3y_2 - 1108$	
3	397		$y_2 - 327$		
		70			
4	467				

Put $\Delta^4 y = 0$ (This is near about 0)

$\Rightarrow 6y_2 - 2214 = 0 \Rightarrow \boxed{y_2 = 369}$ Ans

Que② Find the missing values in the following

x:	0	5	10	15	20	25
y:	6	10	–	17	–	31

Solⁿ Form a Forward difference Table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	6				
		4	y_2-14		
5	10			$41-3y_2$	
		y_2-10			y_4-6y_2-102
10	y_2		$27-y_2$		
		$17-y_2$		y_4+3y_2-61	
15	17		y_4+y_2-34		$143-4y_4-4y_2$
		y_4-17		$82-3y_4-y_2$	
20	y_4		$48-2y_4$		
		$31-y_4$			
25	31				

put, $\Delta^4 y = 0$ (This is near about 0)

$y_4 - 6y_2 - 102 = 0 \Rightarrow y_4 - 6y_2 = 102$ — ①

$143 - 4y_4 - 4y_2 = 0 \Rightarrow 4y_4 + 4y_2 = 143$ — ②

By ① & ② :—

$$\boxed{\begin{aligned} y_4 &= 13.25 \\ y_2 &= 22.5 \end{aligned}}$$ Ans.

Homework

Que ⟹ find missing term :—

x:	2.0	2.1	2.2	2.3	2.4	2.5	2.6
y:	0.135	—	0.111	0.100	—	0.082	0.074

Ans:— $y_1 = 0.123$, $y_4 = 0.0904$ } Ans

Newton's Interpolation Formula

Newton's Interpolation Forward Formula :—

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

where $p = \frac{x - x_0}{h}$, h = difference of given data

Q① The Table gives the distances in nautical miles of the visible horizon for the given heights in feet above the earth's surface

x: height :	100	150	200	250	300	350	400
y: distance :	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the values of y when ① $x = 218$ ft ② 410 ft.

Sol^n

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$x_0 = 100$	$10.63 = y_0$	$\Delta y = y_1 - y_0 = 2.40$			
150	$13.03 = y_1$	2.01	-0.39	0.15	
$x_0 = 200$	$15.04 = y_0$	$1.77 = \Delta y_0$	-0.24		-0.07
250	16.81	1.61	$-0.16\ \Delta^2 y_0$	0.08	
300	18.42	1.48	-0.13	$0.03 = \Delta^3 y_0$	-0.05
350	19.90	1.37	-0.11	0.02	$-0.01 = \Delta^4 y_0$
$x_n = 400$	$21.27 = y_n$	$= \nabla y_n$	$\nabla^2 y_n$	$\nabla^3 y_n$	

(i) we have to find value of y at $x = 218$ which is very close to 200,

So, $x_0 = 200$, $h = 50$

So, $p = \frac{x - x_0}{h} = \frac{218 - 200}{50} = 0.36$

Newton's Interpolation Backward Formula →

$$y_p = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \cdots$$

Using Newton's forward formula;

$$y_{1218} = y_0 + p\Delta y_0 + \frac{p(p-1)}{2\cdot 1}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3\cdot 2\cdot 1}\Delta^3 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4\cdot 3\cdot 2\cdot 1}\Delta^4 y_0$$

$$= 15.04 + 0.36(1.77) + \frac{0.36(-0.64)}{2}(-0.16) +$$

$$\frac{0.36(-0.64)(-1.64)}{6}(0.03) + \cdots$$

$$= 15.04 + 0.637 + 0.018 + 0.001 + \cdots$$

$$= 15.696, \text{ i.e. } 15.7 \text{ (approx)}$$ Ans.

(ii) $x = 410$ is near the end of the table i.e 400

$x_0 = 400$, $p = \frac{x - x_0}{h} = \frac{410-400}{50} = 0.2$

$x_0 = 400$ is last data of table, so we will use Backward interpolation Formula :→

$$y_{410} = y_{400} + 0.2(1.37) + \frac{0.2(1.2)}{2}(-0.11) + \cdots$$

$$= 21.27 + 0.2(1.37) + \frac{0.2(1.2)}{2}(-0.11) + \cdots$$

$$= 21.53$$ Ans.

Que ⟹ From the following table, estimate the number of students who obtained marks b/w 40 & 45

marks:	30-40	40-50	50-60	60-70	70-80
No. of students:	31	42	51	35	31

Sol^n ⟹

marks less than (x)	No. of Students (y)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$x_0 = 40$	$y_0 = 31$	42 (Δy_0)			
50	73		9 ($\Delta^2 y_0$)		
		51		-25 ($\Delta^3 y_0$)	
60	124		-16		37 ($\Delta^4 y_0$)
		35		12	
70	159		-4		
		31			
80	190				

we have to find no. of students who scored marks b/w 40 & 45.

Given that marks less than 40 = no. of student 31

Now we will find no. of students who scored less than 45 marks.

i.e. $x = 45$ which is near value of y_0

So, $x_0 = 40$ $\Rightarrow p = \frac{x - x_0}{h} = \frac{45-40}{10}$

$$\boxed{p = 0.5}$$

By using Newton's Forward interpolation Formula;

$$y_{45} = y_{40} + p\Delta y_{40} + \frac{p(p-1)}{2!}\Delta^2 y_{40} + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_{40} + \cdots$$

$$= 31 + 0.5 \times 42 + \frac{0.5(-0.5)}{2} \times 9 + \frac{0.5(0.5)(-0.5)}{6} \times (-25) + \cdots$$

$$= 47.87$$

The no. of students with marks less than 45 is 47.87

But the no. of students with marks less than 40 is 31

Hence the number of students getting marks b/w 40 & 45 $= 48 - 31 = 17$ Ans

Que ⟹ Find the cubic Polynomial which takes the following values:-

x : 0	1	2	3	Hence evaluate $f(4)$
$f(x)$: 1	2	1	10	

Soln ⟹

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
$x_0 = 0$	$1 = y_0$			
		$\Delta y_0 = 1$		
1	2		-2	
		-1		12
2	1		10	
		9		
3	10			

We take, $x_0 = 0$, $p = \frac{x - x_0}{h} = \frac{x - 0}{1} = x$

$$f(x) = f(0) + \frac{x}{1}\Delta f(0) + \frac{x(x-1)}{2!}\Delta^2 f(0) + \frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$$

$$= 1 + x(1) + \frac{x(x-1)}{2}(-2) + \frac{x(x-1)(x-2)}{3!}(12)$$

$= 2x^3 - 7x^2 + 6x + 1$ This is cubic eqn

Now, $f(4) = 2(4)^3 - 7(4)^2 + 6(4) + 1$

$$\boxed{f(4) = 41}$$ Ans

Central Difference Formula

x	y	1st Difference	2nd Difference	3rd Difference
x_0-2h	y_{-2}			
		Δy_{-2}		
x_0-h	y_{-1}		$\Delta^2 y_{-2}$	
		Δy_{-1}		$\Delta^3 y_{-2}$
x_0	y_0		$\Delta^2 y_{-1}$	
		Δy_0		$\Delta^3 y_{-1}$
x_0+h	y_1		$\Delta^2 y_0$	
		Δy_1		
x_0+2h	y_2			

(I) Gauss's Forward Interpolation Formula →

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_{-1} + \frac{(p+1)p(p-1)}{3!}\Delta^3 y_{-1} + \frac{(p+1)p(p-1)(p-2)}{4!}\Delta^4 y_{-2} + \cdots$$

here, $p = \frac{x-x_0}{h}$, * If $0<p<1$ then we get accurate value for required result.

Que ① Find y, at $x=30$ where.

x :	21	25	29	33	37
y :	18.478	17.8144	17.10	16.343	15.5154

Sol^n

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
21	18.478 $= y_{-2}$	$\Delta y_{-2} = -0.6564$	$\Delta^2 y_{-2} = -0.051$	$\Delta^3 y_{-2} = -0.0074$	$\Delta^4 y_{-2} = -0.0022$
25	17.8144 $= y_{-1}$	$\Delta y_{-1} = -0.7074$	$\Delta^2 y_{-1} = -0.0564$	$\Delta^3 y_{-1} = 0.0076$	
$x_0 = 29$	17.10 $= y_0$	$\Delta y_0 = -0.7638$	$\Delta^2 y_0 = -0.0640$		
33	16.343 $= y_1$ ~~17.10~~	$\Delta y_1 = -0.8272$			
37	15.5154 $= y_2$				

$x = 30$ (Given), $x_0 = 29$, $h = 4$

So, $p = \frac{x - x_0}{h} = \frac{30-29}{4} = \frac{1}{4} = 0.25$

$$y_{30} = 17.10 + (0.25)(-0.7638) + \frac{(0.25)(0.25-1)}{2}(-0.0564) + \frac{(0.25+1)(0.25)(0.25-1)}{3!}(0.0076) + \frac{(0.25+1)(0.25)(0.25-1)(0.25-2)}{4!} \times (-0.0022)$$

By calculating, we get

$y_{30} = 16.9216$ (approx)

Que. Using Gauss Forward interpolation formula to obtain $y(32)$. Given that $y(25) = 0.2707$, $y(35) = 0.3386$, $y(30) = 0.3027$ and $y(40) = 0.3794$.

Soln.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
25	$0.2707 = y_{-1}$	$\Delta y_{-1} =$		
$x_0 = 30$	$0.3027 = y_0$	$\Delta y_0 =$	$\Delta^2 y_{-1} =$	$\Delta^3 y_{-1} =$
35	$0.3386 = y_1$	$\Delta y_1 =$	$\Delta^2 y_0 =$	
40	$0.3794 = y_2$			

let, $x_0 = 30$, $x = 32$ & $h = 5$

$p = \frac{x - x_0}{h} = \frac{32-30}{5} = \frac{2}{5} = 0.4$

Now, calculate your own

Ans:- 0.3165

(II) Gauss's Backward Interpolation Formula

Formula

$$y_p = y_0 + p\Delta y_{-1} + \frac{(p+1)p}{2!}\Delta^2 y_{-1} + \frac{(p+1)p(p-1)}{3!}\Delta^3 y_{-2} + \frac{(p+2)(p+1)p(p-1)}{4!}\Delta^4 y_{-2} + \cdots$$

Que. Use Gauss's Backward Interpolation Formula, the Population of a town for the year 1974, given that

year :	1939	1949	1959	1969	1979	1989
population :	12	15	20	27	39	52

Sol^n ⟹

x	p	y_p	Δy_p	$\Delta^2 y_p$	$\Delta^3 y_p$	$\Delta^4 y_p$	$\Delta^5 y_p$
1939	−3	12 $= y_{-3}$	$\Delta y_{-3} = 3$				
				$\Delta^2 y_{-3} = 2$			
1949	−2	15 $= y_{-2}$	$\Delta y_{-2} = 5$		$\Delta^3 y_{-3} = 0$		
				$\Delta^2 y_{-2} = 2$		$\Delta^4 y_{-3} = 3$	
1959	−1	20 $= y_{-1}$	$\Delta y_{-1} = 7$		$\Delta^3 y_{-2} = 3$		$\Delta^5 y_{-3} = -10$
				$\Delta^2 y_{-1} = 5$		$\Delta^4 y_{-2} = -7$	
1969	0	27 $= y_0$	$\Delta y_0 = 12$		$\Delta^2 y_{-1} = 4$		
				$\Delta^2 y_0 = 1$			
1979	1	39 $= y_1$	$\Delta y_1 = 13$				
1989	2	52 $= y_2$					

$p = \frac{x - x_0}{h} = \frac{1974 - 1969}{10}$

$\boxed{p = 0.5}$

Gauss's Backward Interpolation Formula :—

$$y_{0.5} = 27 + (0.5)7 + \frac{(1.5)(0.5)}{2}(5) + \frac{(1.5)(.5)(-0.5)}{6}(3) + \frac{(2.5)(1.5)(-0.5)^2}{24}(-7) + \frac{(2.5)(1.5)(0.5)(-0.5)(-1.5)}{120}(-10)$$

$= 32.345$ (approx) Ans.

Newton's Interpolation Formula

Newton's Interpolation Forward Formula :—

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

where $p = \frac{x - x_0}{h}$, h = difference of given data

Q① The Table gives the distances in nautical miles of the visible horizon for the given heights in feet above the earth's surface

x: height :	100	150	200	250	300	350	400
y: distance :	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the values of y when ① $x = 218$ ft ② 410 ft.

Sol^n x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$x_0 = 100$	$10.63 = y_0$	$\Delta y = y_1 - y_0 = 2.40$			
150	$13.03 = y_1$	2.01	−0.39	0.15	
$x_0 = 200$	$15.04 = y_0$	$1.77 = \Delta y_0$	−0.24	0.08	−0.07
250	16.81	1.61	$-0.16\ \Delta^2 y_0$	$0.03 = \Delta^3 y_0$	−0.05
300	18.42	1.48	−0.13	0.02	$-0.01 = \Delta^4 y_0$
350	19.90	1.37	$-0.11\ \nabla^2 y_n$	$\nabla^3 y_n$	
$x_n = 400$	$21.27 = y_n$	$= \nabla y_n$			

(i) we have to find value of y at $x = 218$ which is very close to 200,

So, $x_0 = 200$, $h = 50$

So, $p = \frac{x - x_0}{h} = \frac{218 - 200}{50} = 0.36$

Newton's Interpolation Backward formula →

$$y_p = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \cdots$$

Using Newton's forward formula;

$$y_{1218} = y_0 + p\Delta y_0 + \frac{p(p-1)}{2\cdot 1}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3\cdot 2\cdot 1}\Delta^3 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4\cdot 3\cdot 2\cdot 1}\Delta^4 y_0$$

$$= 15.04 + 0.36(1.77) + \frac{0.36(-0.64)}{2}(-0.16) +$$

$$\frac{0.36(-0.64)(-1.64)}{6}(0.03) + \cdots$$

$$= 15.04 + 0.637 + 0.018 + 0.001 + \cdots$$

$$= 15.696, \text{ i.e. } 15.7 \text{ (approx)}$$ Ans.

(ii) $x = 410$ is near the end of the table i.e 400

$x_0 = 400$, $p = \frac{x - x_0}{h} = \frac{410-400}{50} = 0.2$

$x_0 = 400$ is last data of table, so we will use Backward interpolation Formula :→

$$y_{410} = y_{400} + 0.2(1.37) + \frac{0.2(1.2)}{2}(-0.11) + \cdots$$

$$= 21.27 + 0.2(1.37) + \frac{0.2(1.2)}{2}(-0.11) + \cdots$$

$$= 21.53$$ Ans.

Que ⟹ From the following table, estimate the number of students who obtained marks b/w 40 & 45

Marks:	30-40	40-50	50-60	60-70	70-80
No. of students:	31	42	51	35	31

Sol^n ⟹

marks less than (x)	No. of Students (y)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$x_0 = 40$	$y_0 = 31$	42 (Δy_0)	9 ($\Delta^2 y_0$)	−25 ($\Delta^3 y_0$)	37 ($\Delta^4 y_0$)
50	73	51	−16	12	
60	124	35	−4		
70	159	31			
80	190				

we have to find no. of students who scored marks b/w 40 & 45.

Given that marks less than 40 = no. of Student 31

Now we will find no. of students who scored less than 45 marks.

i.e. $x = 45$ which is near value of y_0

So, $x_0 = 40$ $\Rightarrow p = \frac{x - x_0}{h} = \frac{45-40}{10}$

$$\boxed{p = 0.5}$$

By using Newton's Forward interpolation Formula;

$$y_{45} = y_{40} + p\Delta y_{40} + \frac{p(p-1)}{2!}\Delta^2 y_{40} + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_{40} + \cdots$$

$$= 31 + 0.5 \times 42 + \frac{0.5(-0.5)}{2} \times 9 + \frac{0.5(0.5)(-1.5)}{6} \times (-25) + \cdots$$

$$= 47.87$$

The no. of students with marks less than 45 is 47.87

But the no. of students with marks less than 40 is 31

Hence the number of students getting marks b/w 40 & 45 = 48 − 31 = (17) Ans

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Que ⇒ Find the cubic Polynomial which takes the following values :-

				Hence evaluate
x : 0	1	2	3	$f(4)$
$f(x)$: 1	2	1	10	

Soln ⇒

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
$x_0 = 0$	$1 = y_0$	$\Delta y_0 = 1$		
1	2	−1	−2	12
2	1	9	10	
3	10			

We take, $x_0 = 0$, $p = \frac{x - x_0}{h} = \frac{x - 0}{1} = x$

$$f(x) = f(0) + \frac{x}{1}\Delta f(0) + \frac{x(x-1)}{2!}\Delta^2 f(0) + \frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$$

$$= 1 + x(1) + \frac{x(x-1)}{2}(-2) + \frac{x(x-1)(x-2)}{3!}(12)$$

$$= 2x^3 - 7x^2 + 6x + 1$$ This is cubic eqn

Now, $f(4) = 2(4)^3 - 7(4)^2 + 6(4) + 1$

$$\boxed{f(4) = -41}$$ Ans

Central Difference Formula

x	y	1st Diff.	2nd Difference	3rd Difference
x_0-2h	y_{-2}			
		Δy_{-2}		
x_0-h	y_{-1}		$\Delta^2 y_{-2}$	
		Δy_{-1}		$\Delta^3 y_{-2}$
x_0	y_0		$\Delta^2 y_{-1}$	
		Δy_0		$\Delta^3 y_{-1}$
x_0+h	y_1		$\Delta^2 y_0$	
		Δy_1		
x_0+2h	y_2			

(I) Gauss's Forward Interpolation Formula →

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_{-1} + \frac{(p+1)p(p-1)}{3!}\Delta^3 y_{-1} + \frac{(p+1)p(p-1)(p-2)}{4!}\Delta^4 y_{-2} + \cdots$$

here, $p = \frac{x-x_0}{h}$, If $0<p<1$ then we get accurate value for required result.

Que ① Find y, at $x=30$ where.

x:	21	25	29	33	37
y:	18.478	17.8144	17.10	16.343	15.5154

Sol^n

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
21	$18.478 = y_{-2}$	$\Delta y_{-2} = -0.6564$			
25	$17.8144 = y_{-1}$	$\Delta y_{-1} = -0.7074$	$\Delta^2 y_{-2} = -0.051$	$\Delta^3 y_{-2} = -0.0074$	
$x_0 = 29$	$17.10 = y_0$	$\Delta y_0 = -0.7638$	$\Delta^2 y_{-1} = -0.0564$	$\Delta^3 y_{-1} = 0.0076$	$\Delta^4 y_{-2} = -0.0022$
33	$16.343 = y_1$ ~~17.10~~	$\Delta y_1 = -0.8272$	$\Delta^2 y_0 = -0.0640$		
37	$15.5154 = y_2$				

$x = 30$ (Given), $x_0 = 29$, $h = 4$

So, $p = \frac{x - x_0}{h} = \frac{30-29}{4} = \frac{1}{4} = 0.25$

$$y_{30} = 17.10 + (0.25)(-0.7638) + \frac{(0.25)(0.25-1)}{2}(-0.0564) + \frac{(0.25+1)(0.25)(0.25-1)}{3!}(0.0076) + \frac{(0.25+1)(0.25)(0.25-1)(0.25-2)}{4!} \times (-0.0022)$$

By calculating, we get

$y_{30} = 16.9216$ (approx)

Que. Using Gauss Forward interpolation formula to obtain $y(32)$. Given that $y(25) = 0.2707$, $y(35) = 0.3386$, $y(30) = 0.3027$ and $y(40) = 0.3794$.

Soln.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
25	$0.2707 = y_{-1}$	$\Delta y_{-1} =$		
$x_0 = 30$	$0.3027 = y_0$	$\Delta y_0 =$	$\Delta^2 y_{-1} =$	$\Delta^3 y_{-1} =$
35	$0.3386 = y_1$	$\Delta y_1 =$	$\Delta^2 y_0 =$	
40	$0.3794 = y_2$			

let, $x_0 = 30$, $x = 32$ & $h = 5$

$p = \frac{x - x_0}{h} = \frac{32-30}{5} = \frac{2}{5} = 0.4$

Now, calculate your own :- - - -

Ans:- 0.3165

(II) Gauss's Backward Interpolation Formula

Formula

$$y_p = y_0 + p\Delta y_{-1} + \frac{(p+1)p}{2!}\Delta^2 y_{-1} + \frac{(p+1)p(p-1)}{3!}\Delta^3 y_{-2} + \frac{(p+2)(p+1)p(p-1)}{4!}\Delta^4 y_{-2} + \cdots$$

Que. Use Gauss's Backward Interpolation Formula, the population of a town for the year 1974, given that

year :	1939	1949	1959	1969	1979	1989
population :	12	15	20	27	39	52

Sol^n ⟹

x	p	y_p	Δy_p	$\Delta^2 y_p$	$\Delta^3 y_p$	$\Delta^4 y_p$	$\Delta^5 y_p$
1939	−3	$12 = y_{-3}$	$\Delta y_{-3} = 3$				
				$\Delta^2 y_{-3} = 2$			
1949	−2	$15 = y_{-2}$	$\Delta y_{-2} = 5$		$\Delta^3 y_{-3} = 0$		
				$\Delta^2 y_{-2} = 2$		$\Delta^4 y_{-3} = 3$	
1959	−1	$20 = y_{-1}$	$\Delta y_{-1} = 7$		$\Delta^3 y_{-2} = 3$		$\Delta^5 y_{-3} = -10$
				$\Delta^2 y_{-1} = 5$		$\Delta^4 y_{-2} = 7$	
1969	0	$27 = y_0$	$\Delta y_0 = 12$		$\Delta^2 y_{-1} = 4$		
				$\Delta^2 y_0 = 1$			
1979	1	$39 = y_1$	$\Delta y_1 = 13$				
1989	2	$52 = y_2$					

$$p = \frac{x - x_0}{h} = \frac{1974 - 1969}{10}$$

$$\boxed{p = 0.5}$$

Gauss's Backward Interpolation Formula :—

$$y_{0.5} = 27 + (0.5)7 + \frac{(1.5)(0.5)}{2}(5) + \frac{(1.5)(.5)(-0.5)}{6}(3) + \frac{(2.5)(1.5)(-0.5)}{24}(-7) + \frac{(2.5)(1.5)(0.5)(-0.5)(-1.5)}{120}(-10)$$

$$= 32.345 \quad \text{(approx)}$$

Ans.

(III) Stirling Formula

$$y_p = y_0 + p\left\{\frac{\Delta y_0 + \Delta y_{-1}}{2}\right\} + \frac{p^2}{2!}\Delta^2 y_{-1} + \frac{p(p^2-1)}{3!}\times\left\{\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right\} + \frac{p^2(p^2-1)}{4!}\Delta^4 y_{-2} + \cdots\infty$$

here $\boxed{p = \frac{x - x_0}{h}}$

Que Use stirling formula to find y for $x = 35$ from the following table:

x :	20	30	40	50
y :	512	439	346	243

Soln

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
20	$512 = y_{-2}$	$\Delta y_{-2} = -73$	$\Delta^2 y_{-2} = -20$	$\Delta^3 y_{-2} = 10$
30	$439 = y_{-1}$	$\Delta y_{-1} = -93$	$\Delta^2 y_{-1} = -10$	
$x_0 = 40$	$346 = y_0$	$\Delta^2 y_0 = -103$		
50	$243 = y_1$			

$x = 35$ (A/c to Que), $x_0 = 40$, $h = 10$

then $p = \frac{x - x_0}{h} = \frac{35-40}{10} = -0.5$

By stirling Formula:-

$$y_{35} = 346 + (-0.5)\left\{\frac{-103-93}{2}\right\} + \frac{(-0.5)^2}{2}(-10) + \frac{(-0.5)\{(-0.5)^2-1\}}{3!}\times\left\{\frac{-10+10}{2}\right\}$$

$$\boxed{y_{35} = 393.75}$$

H.W.

Que:- Employ stirling Formula to compute $y_{12.2}$ from the data

x =	10	11	12	13	14
$10^5 u_x$ =	23967	28060	31788	35209	38368

Use → ($y_x = 1 + \log_{10}\sin x$)

Note $y = 10^5 u_x$; so 1st divide $10^5 u_x$ by 10^5 then it become u_x or y.

Bessel's Formula

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\cdot\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2} + \frac{(p-1/2)\,p(p-1)}{3!}\,\Delta^3 y_{-1} + \frac{(p+1)\,p(p-1)(p-2)}{4!}\,\frac{\Delta^4 y_{-2} + \Delta^4 y_{-1}}{2} + \cdots \infty$$

$p = \frac{x - x_0}{h}$, If $\frac{1}{4} < p < \frac{3}{4}$ then result is accurate

Que Apply Bessel's formula to obtain y_{25}, Given $y_{20} = 2854$, $y_{24} = 3162$, $y_{28} = 3544$, $y_{32} = 3992$.

Sol^n

x	p	y	Δy	$\Delta^2 y$	$\Delta^3 y$
20	−1	2854 $= y_{-1}$			
			$\Delta y_{-1} = 308$		
24	0	3162 $= y_0$		$\Delta^2 y_{-1} = 74$	$\Delta^3 y_{-1} = -8$
			$\Delta y_0 = 382$		
28	1	3544 $= y_1$		$\Delta^2 y_0 = 66$	
			$\Delta y_1 = 448$		
32	2	3992 $= y_2$			

$x = 25$, $x_0 = 24$, $h = 4$

$$p = \frac{x - x_0}{h} = \frac{25-24}{4} = \frac{1}{4}$$

By Bessel's Formula;

$$y_{25} = 3162 + \frac{1}{4}(382) + \frac{\frac{1}{4}\left(\frac{1}{4}-1\right)}{2}\left\{\frac{74+66}{2}\right\} + \frac{\left(\frac{1}{4}-\frac{1}{2}\right)\frac{1}{4}\left(\frac{1}{4}-1\right)}{3!} \times (-8)$$

$$= 3250.875 \quad \text{(approx)}$$

Interpolation with Unequal Intervals

(I) Langranges' formula for unequal Interval

(II) Newton's divided difference formula

(I) Langrange's Interpolation Formula :—

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)\cdots}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\cdots} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdots y_1$$

Yaha x_0 Nahi ayega

Yaha sare terms x_0 se subtract kerna hai

x_1 se sare terms subtract

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)\cdots}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\cdots} y_2 + \cdots$$

x_2

x_2 se subtraction

Pehle ko terms formula

Que (1) Given the values :

x :	5	7	11	13	17
$f(x)$:	150	392	1452	2366	5202

evaluate $f(9)$, using Langrange's formula

Solⁿ ⟹ we have to find $f(9)$ by Langrange's formula.

Given, $x_0 = 5$, $x_1 = 7$, $x_2 = 11$, $x_3 = 13$, $x_4 = 17$

$y_0 = 150$, $y_1 = 392$, $y_2 = 1452$, $y_3 = 2366$, $y_4 = 5202$

A/c to the que. $[x = 9]$ ∴ by Langrange's formula

$$f(9) = \frac{(9-7)(9-11)(9-13)(9-17)}{(5-7)(5-11)(5-13)(5-17)} \times (150) + \frac{(9-5)(9-11)(9-13)(9-17)}{(7-5)(7-11)(7-13)(7-17)} \times (392)$$

$$+ \frac{(9-5)(9-7)(9-13)(9-17)}{(11-5)(11-7)(11-13)(11-17)} \times (1452) + \frac{(9-5)(9-7)(9-11)(9-17)}{(13-5)(13-7)(13-11)(13-17)} \times (2366)$$

$$+ \frac{(9-5)(9-7)(9-11)(9-13)}{(17-5)(17-7)(17-11)(17-13)} \times (5202)$$

$$= -\frac{50}{3} + \frac{3136}{15} + \frac{3872}{3} - \frac{2366}{3} + \frac{578}{5}$$

$$= 810$$

Central Difference Formula

x	y	1st Difference	2nd Difference	3rd Difference
x_0-2h	y_{-2}			
		Δy_{-2}		
x_0-h	y_{-1}		$\Delta^2 y_{-2}$	
		Δy_{-1}		$\Delta^3 y_{-2}$
x_0	y_0		$\Delta^2 y_{-1}$	
		Δy_0		$\Delta^3 y_{-1}$
x_0+h	y_1		$\Delta^2 y_0$	
		Δy_1		
x_0+2h	y_2			

(I) Gauss's Forward Interpolation Formula →

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_{-1} + \frac{(p+1)p(p-1)}{3!}\Delta^3 y_{-1} + \frac{(p+1)p(p-1)(p-2)}{4!}\Delta^4 y_{-2} + \cdots$$

here, $p = \frac{x-x_0}{h}$, * If $0<p<1$ then we get calculate value for required result.

Que ① Find y, at $x=30$ where.

x:	21	25	29	33	37
y:	18.478	17.8144	17.10	16.343	15.5154

Soln

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
21	18.478 $=y_{-2}$	$\Delta y_{-2} = -0.6564$			
25	17.8144 $=y_{-1}$	$\Delta y_{-1} = -0.7074$	$\Delta^2 y_{-2} = -0.051$	$\Delta^3 y_{-2} = +0.0074$	
$x_0 = 29$	17.10 $=y_0$	$\Delta y_0 = -0.7638$	$\Delta^2 y_{-1} = -0.0564$	$\Delta^3 y_{-1} = 0.0076$	$\Delta^4 y_{-2} = -0.0022$
33	16.343 $=y_1$ ~~17.10~~	$\Delta y_1 = -0.8272$	$\Delta^2 y_0 = -0.0640$		
37	15.5154 $=y_2$				

$x = 30$ (Given), $x_0 = 29$, $h = 4$

So, $p = \frac{x - x_0}{h} = \frac{30-29}{4} = \frac{1}{4} = 0.25$

$$y_{30} = 17.10 + (0.25)(-0.7638) + \frac{(0.25)(0.25-1)}{2}(-0.0564)$$
$$+ \frac{(0.25+1)(0.25)(0.25-1)}{3!}(0.0076) + \frac{(0.25+1)(0.25)\times(0.25-1)(0.25-2)}{4!} \times (-0.0022)$$

By calculating, we get

$y_{30} = 16.9216$ (approx)

Que Using Gauss Forward interpolation formula to obtain $y(32)$. Given that $y(25) = 0.2707$, $y(35) = 0.3386$, $y(30) = 0.3027$ and $y(40) = 0.3794$.

Solⁿ

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
25	$0.2707 = y_{-1}$	$\Delta y_{-1} =$		
$x_0 = 30$	$0.3027 = y_0$	$\Delta y_0 =$	$\Delta^2 y_{-1} =$	$\Delta^3 y_{-1} =$
35	$0.3386 = y_1$	$\Delta y_1 =$	$\Delta^2 y_0 =$	
40	$0.3794 = y_2$			

let, $x_0 = 30$, $x = 32$ & $h = 5$

$p = \frac{x - x_0}{h} = \frac{32-30}{5} = \frac{2}{5} = 0.4$

Now, calculate your own

Ans:- 0.3165

(II) Gauss's Backward Interpolation Formula

Formula

$$y_p = y_0 + p\Delta y_{-1} + \frac{(p+1)p}{2!}\Delta^2 y_{-1} + \frac{(p+1)p(p-1)}{3!}\Delta^3 y_{-2} + \frac{(p+2)(p+1)p(p-1)}{4!}\Delta^4 y_{-2} + \cdots$$

Que. Use Gauss's Backward Interpolation Formula, the Population of a town for the year 1974, given that

Year :	1939	1949	1959	1969	1979	1989
Population :	12	15	20	27	39	52

Sol^n ⇒

x	p	y_p	Δy_p	$\Delta^2 y_p$	$\Delta^3 y_p$	$\Delta^4 y_p$	$\Delta^5 y_p$
1939	−3	12 $= y_{-3}$					
			$\Delta y_{-3} = 3$				
1949	−2	15 $= y_{-2}$		$\Delta^2 y_{-3} = 2$			
			$\Delta y_{-2} = 5$		$\Delta^3 y_{-3} = 0$		
1959	−1	20 $= y_{-1}$		$\Delta^2 y_{-2} = 2$		$\Delta^4 y_{-3} = 3$	
			$\Delta y_{-1} = 7$		$\Delta^3 y_{-2} = 3$		$\Delta^5 y_{-3} = -10$
1969	0	27 $= y_0$		$\Delta^2 y_{-1} = 5$		$\Delta^4 y_{-2} = 7$	
			$\Delta y_0 = 12$		$\Delta^3 y_{-1} = 4$		
1979	1	39 $= y_1$		$\Delta^2 y_0 = 1$			
			$\Delta y_1 = 13$				
1989	2	52 $= y_2$					

$p = \frac{x - x_0}{h} = \frac{1974 - 1969}{10}$

$\boxed{p = 0.5}$

Gauss's Backward Interpolation Formula :—

$$y_{0.5} = 27 + (0.5)7 + \frac{(1.5)(0.5)}{2}(5) + \frac{(1.5)(.5)(-0.5)}{6}(3) + \frac{(2.5)(1.5)(-0.5)^2}{24}(-7) + \frac{(2.5)(1.5)(0.5)(-0.5)(-1.5)}{120}(-10)$$

$= 32.345$ (approx)

(III) Stirling Formula

$$y_p = y_0 + p\left\{\frac{\Delta y_0 + \Delta y_{-1}}{2}\right\} + \frac{p^2}{2!}\Delta^2 y_{-1} + \frac{p(p^2-1)}{3!}\times\left\{\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right\} + \frac{p^2(p^2-1)}{4!}\Delta^4 y_{-2} + \cdots\cdots\infty$$

here $\boxed{p = \frac{x - x_0}{h}}$

Que Use stirling formula to find y for $x = 35$ from the following table:

x :	20	30	40	50
y :	512	439	346	243

Sol^n

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
20	$512 = y_{-2}$	$\Delta y_{-2} = -73$	$\Delta^2 y_{-2} = -20$	
30	$439 = y_{-1}$	$\Delta y_{-1} = -93$		$\Delta^3 y_{-2} = 10$
$x_0 = 40$	$346 = y_0$		$\Delta^2 y_{-1} = -10$	
		$\Delta y_0 = -103$		
50	$243 = y_1$			

$x = 35$ (A/c to Que), $x_0 = 40$, $h = 10$

then $p = \frac{x - x_0}{h} = \frac{35-40}{10} = -0.5$

By stirling Formula :-

$$y_{35} = 346 + (-0.5)\left\{\frac{-103-93}{2}\right\} + \frac{(-0.5)^2}{2}(-10) + \frac{(-0.5)\{(-0.5)^2-1\}}{3!}\times\left\{\frac{-10+10}{2}\right\}$$

$$\boxed{y_{35} = 393.75}$$

Que:- Employ stirling Formula to compute $y_{12.2}$ from the data

x =	10	11	12	13	14
$10^5 u_x$ =	23967	28060	31788	35209	38368

Use → ($y_x = 1 + \log_{10} \sin x$)

Note $y = 10^5 u_x$; so 1st divide $10^5 u_x$ by 10^5 then it become u_x or y.

Bessel's Formula

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\cdot\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2} + \frac{(p-1/2)\,p(p-1)}{3!}\,\Delta^3 y_{-1} + \frac{(p+1)\,p(p-1)(p-2)}{4!}\,\frac{\Delta^4 y_{-2} + \Delta^4 y_{-1}}{2} + \cdots \infty$$

$p = \frac{x - x_0}{h}$, If $\frac{1}{4} < p < \frac{3}{4}$ then result is accurate

Que ⟹ Apply Bessel's Formula to obtain y_{25}, Given $y_{20} = 2854$, $y_{24} = 3162$, $y_{28} = 3544$, $y_{32} = 3992$.

Sol^n ⟹

x	p	y	Δy	$\Delta^2 y$	$\Delta^3 y$
20	−1	2854 $= y_{-1}$			
			$\Delta y_{-1} = 308$		
24	0	3162 $= y_0$		$\Delta^2 y_{-1} = 74$	
			$\Delta y_0 = 382$		$\Delta^3 y_{-1} = -8$
28	1	3544 $= y_1$		$\Delta^2 y_0 = 66$	
			$\Delta y_1 = 448$		
32	2	3992 $= y_2$			

$x = 25$, $x_0 = 24$, $h = 4$

$$p = \frac{x - x_0}{h} = \frac{25-24}{4} = \frac{1}{4}$$

By Bessel's Formula ;

$$y_{25} = 3162 + \frac{1}{4}(382) + \frac{\frac{1}{4}\left(\frac{1}{4}-1\right)}{2}\left\{\frac{74+66}{2}\right\} + \frac{\left(\frac{1}{4}-\frac{1}{2}\right)\frac{1}{4}\left(\frac{1}{4}-1\right)}{3!} \times (-8)$$

$$= 3250.875 \quad \text{(approx)}$$

Interpolation with Unequal Intervals

(I) Langranges' Formula for unequal Interval

(II) Newton's divided difference formula

(I) Langrange's Interpolation Formula :—

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)\cdots}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\cdots} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdots y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)\cdots}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\cdots} y_2 + \cdots$$

Yaha x_0 Nahi ayega

Yaha sare terms x_0 se subtract Karna hai

x_1 se sare terms subtract

x_2 se subtraction

many to many formula

Que (1) Given the values :

x :	5	7	11	13	17
$f(x)$:	150	392	1452	2366	5202

evaluate $f(9)$, using Langrange's formula

Solⁿ ⟹ we have to find $f(9)$ by Langrange's formula.

Given, $x_0 = 5$, $x_1 = 7$, $x_2 = 11$, $x_3 = 13$, $x_4 = 17$

$y_0 = 150$, $y_1 = 392$, $y_2 = 1452$, $y_3 = 2366$, $y_4 = 5202$

A/c to the que. $[x = 9]$, by Langrange's formula

$$f(9) = \frac{(9-7)(9-11)(9-13)(9-17)}{(5-7)(5-11)(5-13)(5-17)} \times (150) + \frac{(9-5)(9-11)(9-13)(9-17)}{(7-5)(7-11)(7-13)(7-17)} \times (392)$$

$$+ \frac{(9-5)(9-7)(9-13)(9-17)}{(11-5)(11-7)(11-13)(11-17)} \times (1452) + \frac{(9-5)(9-7)(9-11)(9-17)}{(13-5)(13-7)(13-11)(13-17)} \times (2366)$$

$$+ \frac{(9-5)(9-7)(9-11)(9-13)}{(17-5)(17-7)(17-11)(17-13)} \times (5202) =$$

$$= -\frac{50}{3} + \frac{3136}{15} + \frac{3872}{3} - \frac{2366}{3} + \frac{578}{5}$$

$$= 810$$

Newton's Divided Difference Formula

let, $x : x_0 \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad \ldots \quad x_n$

$y : y_0 \quad y_1 \quad y_2 \quad y_3 \quad y_4 \quad \ldots \quad y_n$

Divided difference formula is denoted by "Δ" or $T(x_0, x_1, x_2, \ldots x_n)$ and

$\Delta y_0 = \dfrac{y_1 - y_0}{x_1 - x_0}$	$\Delta^2 y_0 = \dfrac{\Delta y_1 - \Delta y_0}{x_2 - x_0}$	$\Delta^3 y_0 = \dfrac{\Delta^2 y_1 - \Delta^2 y_0}{x_3 - x_0}$
$\Delta y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}$	$\Delta^2 y_1 = \dfrac{\Delta y_2 - \Delta y_1}{x_3 - x_1}$	$\Delta^3 y_1 = \dfrac{\Delta^2 y_2 - \Delta^2 y_1}{x_4 - x_1}$
$\Delta y_2 = \dfrac{y_3 - y_2}{x_3 - x_2}$	$\Delta^2 y_2 = \dfrac{\Delta y_3 - \Delta y_2}{x_4 - x_2}$	

then,

$$y = f(x) = y_0 + (x - x_0)\,\Delta y_0 + (x - x_0)(x - x_1)\,\Delta^2 y_0 + (x - x_0)(x - x_1)(x - x_2)\,\Delta^3 y_0 + \ldots$$

Formation of Table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	y_0	$\Delta y_0 = \dfrac{y_1 - y_0}{x_1 - x_0}$	$\Delta^2 y_0 = \dfrac{\Delta y_1 - \Delta y_0}{x_2 - x_0}$	$\Delta^3 y_0 = \dfrac{\Delta^2 y_1 - \Delta^2 y_0}{x_3 - x_0}$
x_1	y_1	$\Delta y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}$	$\Delta^2 y_1 = \dfrac{\Delta y_2 - \Delta y_1}{x_3 - x_1}$	
x_2	y_2	$\Delta y_2 = \dfrac{y_3 - y_2}{x_3 - x_2}$		
x_3	y_3			

(How)

Que → Use Newton's Divided difference and evaluate $f(9)$,

x:	5	7	11	13	17
y:	150	392	1452	2366	5202

Que → Use divided difference formula and evaluate $f(4)$ &

$f'(4)$.

x:	0	2	3	6
$f(x)$ =	648	704	729	792

Soln →

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
$0 = x_0$	$648 = y_0$	$\Delta y_0 = \frac{704-648}{2-0} = 28$	$\Delta^2 y_0 = \frac{25-28}{3-0} = -1$	$\Delta^3 y_0 = \frac{-1+1}{6-0} = 0$
$2 = x_1$	$704 = y_1$	$\Delta y_1 = \frac{729-704}{3-2} = 25$	$\Delta^2 y_1 = \frac{21-25}{6-2} = -1$	
$3 = x_2$	$729 = y_2$	$\Delta y_2 = \frac{792-729}{6-3} = 21$		
$6 = x_3$	$792 = y_3$			

First we have to find $f(x) = ?$ (A/c to the Que) by Newton's divided difference formula;

$$f(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0$$

Now, $= 648 + (x-0)28 + (x-0)(x-2)(-1)$ —— (1)

$f(4) = 648 + 4\times 28 + 4\times 2\times -1$

$f(4) = 752$

Now, $f'(x) = ?$

By eqn (1) → $f(x) = 648 + 28x - \{x^2 - 2x\}$

$f'(x) = 28 - 2x + 2 = 30 - 2x$

So $f'(4) = 30 - 2\times 4 = 30 - 8 = 22$

Ans

Operators

① **Shift operator "E" :—**

Let $y = f(x)$ be a given function and h is any fixed value.

let $x = x_0 \quad x_1 \quad x_2 \quad x_3 \cdots x_n$

Such that $y = y_0 \; y_1 \; y_2 \; y_3 \cdots y_n$

where $y_1 = x_0 + h$, $y_2 = x_1 + h$ ----

Then shift operator "E" is defined by $E(f(x)) = f(x+h)$ (increament of h in x)

$$E^2 f(x) = f(x+2h)$$

$$E^3 f(x) = f(x+3h)$$

Inverse shift operator E^{-1} is defined by,

$$E^{-1} f(x) = f(x-h)$$

$$E^{-2} f(x) = f(x-2h)$$

$E y_0 = y_1$	$E^{-1} y_1 = y_0$
$E^2 y_0 = y_2$	$E^{-2} y_2 = y_0$
$E^3 y_0 = y_3$	$E^{-3} y_3 = y_0$

Similarly, $E^{1/2} f(x) = f(x + \frac{h}{2})$

② **Forward difference operator :— "Δ".**

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta^2 f(x) = \Delta[f(x+h) - f(x)]$$

$$= [f(x+2h) - f(x+h)] - [f(x+h) - f(x)]$$

$$= f(x+2h) - 2f(x+h) + f(x)$$

Similarly, $\Delta y_0 = y_1 - y_0$

$\Delta y_1 = y_2 - y_1$

$\vdots \qquad \vdots$

③ Backward difference operator :- " ∇ "

$$\nabla f(x) = f(x) - f(x-h)$$

$$\nabla^2 f(x) = \nabla\left[f(x) - f(x-h)\right]$$

$$= \left[f(x) - f(x-h)\right] - \left[f(x-h) - f(x-2h)\right]$$

$$= f(x) - 2f(x-h) + f(x-2h)$$

Similarly, $\nabla y_1 = y_1 - y_0$

$\nabla y_2 = y_2 - y_1$

$\nabla y_3 = y_3 - y_2$

$\vdots \qquad \vdots$

④ Central difference operator :-

symbol " δ "

$$f(x) = f\left(x + \frac{h}{2}\right) - f\left(x - \frac{h}{2}\right)$$

⑤ Averaging operator :- " μ " "mu"

$$\mu\left[f(x)\right] = \frac{1}{2}\left[f\left(x + \frac{h}{2}\right) - f\left(x - \frac{h}{2}\right)\right]$$

Relation B/w Difference Operators

Prove that,

① $\Delta = E^{-1}$

② $\nabla = 1 - E^{-1}$

③ $\delta = E^{1/2} - E^{-1/2}$

④ $\mu = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]$

⑤ $\left[E^{1/2} + E^{-1/2}\right](1+\Delta)^{1/2} = 2+\Delta$

⑥ $E = e^{hD}$, where, $D = \frac{d}{dx}$

⑥ $\Delta = E\nabla = \delta E^{1/2}$

Proof ①

$$\Delta f(x) = f(x+h) - f(x)$$
$$= Ef(x) - f(x)$$

{Here, $E(x) = f(x+h)$}

$$\Delta f(x) = f(x)\{E-1\}$$

So, $\boxed{\Delta = E-1}$

②

$$\nabla f(x) = f(x) - f(x-h)$$
$$= f(x) - E^{-1}f(x)$$
$$= f(x)\{1 - E^{-1}\}$$

So, $\boxed{\nabla = 1 - E^{-1}}$

③

$$\delta f(x) = f\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)$$
$$= E^{1/2} f(x) - E^{-1/2} f(x) \quad \text{—— ①}$$
$$= \left[E^{1/2} - E^{-1/2}\right] f(x)$$

So, $\boxed{\delta = E^{1/2} - E^{-1/2}}$

④

$$\mu[f(x)] = \frac{1}{2}\left[f\left(x+\frac{h}{2}\right) + f\left(x-\frac{h}{2}\right)\right]$$

Average operator

$$= \frac{1}{2}\left[E^{1/2} f(x) + E^{-1/2} f(x)\right] \quad \{\text{By ①}\}$$

$$\mu f(x) = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right] f(x)$$

So, $$\boxed{\mu = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]}$$

(5) $E = e^{hD}$

By Definition of shifting op. (expand the f^n By Taylor's thm)

$$E f(x) = f(x+h)$$

$$= f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) \cdots \infty$$

$$E\{f(x)\} = f(x) + hDf(x) + \frac{h^2}{2!} D^2 f(x) + \frac{h^3}{3!} D^3 f(x) + \cdots \infty$$

$$= f(x)\left[1 + hD + \frac{h^2}{2!} D^2 + \frac{h^3}{3!} D^3 + \cdots \infty\right]$$

So, $$E = \underbrace{1 + hD + \frac{h^2 D^2}{2!} + \frac{h^3 D^3}{3!} + \cdots \infty}_{\text{This is expansion of } e^{hD}}$$

$$\left[* \text{ Formula } e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \infty\right]$$

So, $$\boxed{E = e^{hD}}$$

(6) $\Delta = E\nabla = \delta E^{1/2}$

~~$\Delta f(x) = f(x+h) - f(x)$~~

$$E\nabla f(x) = E\left[f(x) - f(x-h)\right]$$

$$= E f(x) - E f(x-h)$$

$$= f(x+h) - f(x)$$

$$= \Delta f(x)$$

So, $$\boxed{E\nabla = \Delta}$$

$$\nabla E f(x) = \nabla \{ f(x+h) \}$$
$$= f(x+h) - f(x)$$
$$= \Delta f(x)$$

So, $\boxed{\nabla E = \Delta}$

Now, $\delta E^{1/2} f(x) = \delta f(x + \frac{h}{2})$

$$= f(x + \frac{h}{2} + \frac{h}{2}) - f(x + \frac{h}{2} - \frac{h}{2})$$
$$= f(x+h) - f(x)$$
$$= \Delta f(x)$$

So, $\boxed{\delta E^{1/2} = \Delta}$

* All the Best *