Question Bank

Semester: B. Tech- 3rd sem	Branch: Common to all branches
Subject: Mathematics-III	Course code – All branches

Unit-I
LAPLACE TRANSFORM

2-marks.

Q. 1. Write the condition for existence of Laplace transform.
Q. 2. Define unit impulse function.
Q. 3. Find $L^{-1}\left\{\frac{1}{(s+3)^5}\right\}$
Q. 4. Find $L\{4^t\}$
Q. 5. If $f(t)$ is a periodic function with period T, then $L\{f(t)\}$

4-marks.

Q. 1. Evaluate $L\left\{e^{-t}\int_0^t \frac{\sin t}{t}\,dt\right\}$.
Q. 2. Find $L^{-1}\left\{\frac{1}{s(s^2+1)}\right\}$.
Q. 3. Express the following function in terms of unit step function and find its Laplace transform:

$$f(t)=\begin{cases}0, & 0<t<1\\ t-1, & 1<t<2\\ 1, & t>2\end{cases}$$

Q. 4. If $L\{f(t)\}=\bar{f}(s)$. Then prove that $L\{e^{at}f(t)\}=\bar{f}(s-a)$.
Q. 5. Find the inverse transform of $\frac{4s+5}{(s-1)^2(s+2)}$

8-marks.

Q. 1. Find the Laplace transform of
i. $(e^{-t}\sin t)t$
ii. $\left(\sqrt{t}-\frac{1}{\sqrt{t}}\right)^3$

Q. 2. Find the Laplace transform of
i. $\frac{\cos at-\cos bt}{t}$.
ii. $cos^3 2t$

Q. 3. Find the Laplace transform of $\frac{1-\cos t}{t^2}$.
Q. 4. Find the inverse Laplace transform of
i. $\frac{s^2+6}{(s^2+1)(s^2+4)}$

ii. $\frac{s}{(s^2+1)(s^2+4)}$ using Convolution theorem.

Q. 5. Find the inverse Laplace transform of

i. $\frac{s^2+s-2}{s(s+3)(s-2)}$

ii. $\tan^{-1}\frac{2}{s^2}$.

Q. 6. Apply convolution theorem to prove that $L^{-1}\left\{\frac{8}{(s^2+1)^3}\right\} = (3-t^2)\sin t - 3t\cos t$.

Q. 7. Use the method of partial fraction to find the inverse transform of $\frac{s}{s^4+s^2+1}$

Q. 8. (a)Find the Laplace transform of $\frac{1-\cos 2t}{t}$

(b) Evaluate the following: $\int_0^\infty te^{-3t}\sin t\, dt$.

Q. 9. (a)Find the Laplace transform of $\sin 2t \sin 3t + \cos^2 t$.

(b) Show that $\int_0^\infty te^{-2t}\cos t\, dt = \frac{3}{25}$

Q. 10. Solve the differential equation by transform method $\frac{d^2x}{dt^2} + 9x = \cos 2t, when\ x(0) = 1,\ x\left(\frac{\pi}{2}\right) = -1$.

Q. 11. Solve the differential equation by transform method $ty'' + 2y' + ty = \sin t, when\ y(0) = 1$.

Q. 12. Solve the differential equation by transform method $ty'' + (1-2t)y' - 2y = 0, when\ y(0) = 1\ and\ y'(0) = 2$.

Q. 13. Solve the differential equation by transform method $y'' - 3y' + 2y = 4t - e^{3t}, when\ y(0) = 1\ and\ y'(0) = -1$.

Q. 14. Solve $(D^2 + m^2)x = a\cos nt, t > 0, when\ x = x_0\ and\ Dx = x_1, when\ t = 0,\ m \neq n$.

Q. 15. Solve $(D^3 - 3D^2 + 3D - 1)y = t^2e^t,\ when\ y(0) = 1, Dy(0) = 0\ and\ D^2y(0) = -2$.

Unit-II
PARTIAL DIFFERENTIAL EQUATIONS

2 marks Questions

Q. 1 Form the partial differential equation if $z = e^{my}\,\phi(x - y)$.

Q. 2 Form the partial differential equation if $z = y^2 + 2f\left[\frac{1}{x} + \log y\right]$

Q. 3 Write Lagrange's linear equation.

Q. 4 Solve $x\,p + y\,q = 3z$.

Q. 5 Solve $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 0$

4 marks Questions

Q. 1 Solve $\frac{\partial^3 z}{\partial x^2 \partial y} + 18xy^2 + \sin(2x - y) = 0$

Q. 2 Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x + 3y)$

Q. 3 Form the partial differential equation from $z = f(x^2 + y^2, z - xy)$

Q. 4 Solve $p - q = \log(x + y)$

Q. 5 Solve $p\tan x + q\tan y = \tan z$

8- marks Questions

Q. 1 Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$

Q. 2 Solve $p\,x\,(z - 2y^2) = (z - q\,y)\,(z - y^2 - 2x^3)$

Q. 3 Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$

Q. 4 Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$

Q. 5 Solve $x(y^2 - z^2)\,p + y\,(z^2 - x^2)\,q = z\,(x^2 - y^2)$

Q. 6 Solve $(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} = ly - mx$

Q. 7 Solve $4\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 16\log(x + 2y)$

Q. 8 Solve $(D^2 - DD' - 2D'^2)z = (y - 1)e^x$

Q. 9 Solve $(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x + 2y)$

Q. 10 Solve $(D^2 + DD' - 6D'^2)z = \cos(2x + y)$

Q. 11 Solve $(D^2 + 3DD' + 2D'^2)z = 24xy$

Q. 12 Solve $\frac{\partial^2 z}{\partial x\, \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$ when $x = 0$ and $z = 0$ when y is an odd multiple of $\frac{\pi}{2}$.

Q. 13 Solve by method of separation of variables

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}\ \text{,given } u(0, y) = 8e^{-3y}$$

Q. 14 Solve by method of separation of variable; $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$

given $u = 3e^{-y} - e^{-5y}$ when $x = 0$.

Q. 15 Solve by method of separation of variable $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6\,e^{-3x}$.

UNIT-III
RANDOM VARIABLES

2 marks Questions
Q. 1 Define probability density function.
Q. 2 Define moment generating function of discrete and continuous probability distribution.
Q. 3 Define expectation and variance.
Q. 4 Define random variable and random experiment.
Q. 5 Write applications of binomial distribution.

4 marks Questions
Q. 1 If a random variable has a Poisson distribution such that P(1) = P(2), find mean of the distribution and P(4).

Q. 2 A variate X has a probability distribution

x :	-3	6	9
$P(X = x)$:	1/6	½	1/3

Find E(X) and E(X^2). Hence evaluate E(2x +1) 2

Q. 3 Is the function $f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$ a density function?

Q. 4 The mean and variance of binomial distribution are 4 and 4/3 respectively. Find $P(X \geq 1)$.

Q. 5 In 256 sets of 12 tosses of a coin, in how many cases one can expect 8 heads and 4 tails.

8 marks Questions
Q. 1 The probability density function of a variate X is

X :	0	1	2	3	4	5	6
P(X) :	k	3k	5k	7k	9k	11k	13k

(i) Find P(X < 4), P(X ≥ 5), P(3 < X ≤ 6).
(ii) What will be the minimum value of k so that P(X ≤ 2) >0. 3.

Q. 2 The probability density $p(x)$ of a continuous random variable is given by $p(x) = y_0 e^{-|x|}$, $-\infty < x < \infty$ Prove that y_0 = ½. Find the mean and variance of the distribution.

Q. 3 If x is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} kx & (0 \leq x \leq 2) \\ 2k & (2 \leq x < 4) \\ -kx + 6k & (4 \leq x < 6) \end{cases}$$

Find k and mean value of x.

Q. 4 Find the moment generating function of the exponential distribution
$f(x) = \frac{1}{c} e^{-x/c}, 0 \leq x \leq \infty, c > 0$
Hence find its mean and S.D.

Q. 5 A bag contains 5 black, 6 white and 7 red balls. Four balls are drawn at random from it. If x denotes the number of white balls, then find E(x).

Q. 6 The probability that a pen manufactured by a company will be defective is 1/10. If 12 such pens are manufactured find the probability that

a) Exactly 2 will be defective.
(b) Atleast two will be defective.
(c) None will be defective.

Q. 7 Fit a binomial distribution for the following data and compare the theoretical frequencies with the actual ones.

x	0	1	2	3	4	5
f	2	14	20	34	22	8

Q. 8 Out of 800 families with 5 children each, how many would you expect to have
(a) 3 boys (b) 5 girls (c) Either 2 or 3 boy?
Assume equal probabilities for boys and girls.

Q. 9 In a certain factory turning out razor blades there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 10,000 packets.

Q. 10 Fit a Poisson distribution to the set of observation:

X	0	1	2	3	4
F	122	60	15	2	1

Q. 11 A car hire firm has 2 cars which it hires out day by day. The number of demands for a car on each day is distribution as a Poisson distribution with mean 1.5. Calculate the probability of day
(i) on which there is no demand
(ii) on which demand is refused. ($e^{-1.5} = 0.2231$).

Q. 12 In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D of the distribution.

Q. 13 Fit a normal curve to the following distribution.

X	2	4	6	8	10
F	1	4	6	4	1

Q. 14 In a precision bombing attack there is a 50% chance that any bomb will strike the target. Two direct hits are needed to destroy the target completely. How many bombs must be dropped to give a 99% chance or better of completely destroying the target?

Q. 15 In a test on 2000 electric bulbs, it was found that the life of particular make, was normally distributed with an average life of 2040 hours and S.D of 60 hours. Estimate the number of bulbs likely to burn for
(a) More than 2150 hours.
(b) Less than 1950 hours and
(c) More than 1920 hours and less than 2160 hours.

Unit IV

INTERPOLATION WITH EQUAL AND UNEQUAL INTERVALS

2 Marks Questions

Q. 1 Explain forward and backward difference.

Q. 2 Which of the following is correct (i) $\Delta\ x^n = n \cdot x^{n-1}$ (ii) $\Delta\ [x]^n = n \cdot [x]^{n-1}$ (iii) $\Delta^n\ e^x = e^x$

(iv) $\Delta\ \cos x = -\sin x$

Q. 3 Prove that $\Delta = E - 1$.

Q. 4 Write relation between E and D

Q. 5 Write formula for Newton's forward and backward interpolation.

4 Marks Questions

Q. 1 Find the missing values in the following:

x:	0	5	10	15	20	25
$f(x)$:	6	10	----	17	----	31

Q. 2 Fit a polynomial of degree three which takes the following values:

x:	3	4	5	6
$f(x)$:	6	24	60	120

Q. 3 From the following table estimate the number of students who obtained marks between 40 and 45

Marks x:	30-40	40-50	50-60	60-70	70-80
No. of Candidate $f(x)$:	31	42	51	35	31

Q. 4 Find the value of $\log\ 337 \cdot 5$ by Stirling formula.

x:	310	320	330	340	350	360
$\log x$:	2.4914	2.5052	2.5185	2.5315	2.5441	2.5563

Q. 5 Using Lagrange's formula, evaluate $f(9)$, given

x:	5	7	11	13	17

$f(x)$:	150	392	1452	2366	5202

8 Marks Questions

Q. 1 Given $\sin 45° = 0.7071$, $\sin 50° = 0.7660$ $\sin 55° = 0.8192$ $\sin 60° = 0.8660$, find $\sin 52°$, using Newton's forward interpolation

Q. 2 Given $\tan 0° = 0.0$, $\tan 5° = 0.8755$ $\tan 10° = 0.1763$ $\tan 15° = 0.2679$, $\tan 20° = 0.3640$, $\tan 25° = 0.4663$, $\tan 30° = 0.5774$, find Using Stirling's formula, show that $\tan 16° = 0.2867$.

Q. 3 The population of a town is as follows:

Year	x:	1941	1951	1961	1971	1981	1991
Population	$f(x)$:	20	24	29	36	46	51

Estimate the population increase during the period 1946 to 1976. Do calculation for 4 decimal places.

Q. 4 Estimate the sale for 1966 correct up to 4 decimal places using the following table:

Year	x:	1941	1951	1961	1971	1981	1991
Population	$f(x)$:	20	24	29	36	46	51

Q. 5 Given the following table, find $f(35)$ correct upto 2 places, by using Stirling's & Bessel's formula.

x:	20	30	40	50
$f(x)$:	512	439	346	243

Q. 6 Using Stirling's formula, find $f(1.22)$

x:	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
$f(x)$:	0.84147	0.89121	0.93204	0.96356	0.98540	0.99749	0.99957	0.99385	0.97385

Q. 7 Find $f(25)$ correct upto 2 places by using Bessel's/(Stirling) formula given

x:	20	24	28	32
$f(x)$:	2854	3162	3544	3992

Q. 8 Find the value of $f(21)$ and $f(28)$ correct upto 4 places of decimal from the following table

x:	20	23	26	29
$f(x)$:	0.3420	0.3907	0.4384	0.4848

Q. 9 Using Lagrange's formula to fit a polynomial and find $f(1)$ to the data

x:	-1	0	2	3
$f(x)$:	-8	3	1	12

Q. 10 Using Lagrange's formula, express $\frac{3x^2+x+1}{(x-1)(x-2)(x-3)}$ as a sum of partial fraction.

Q. 11 Find the cubic polynomial by Lagrange's formula which takes the following values, then find $f(3)$

x:	0	1	2	5
$f(x)$:	2	3	12	147

Q. 12 Using Lagranges interpolation formula find $f(10)$

x:	5	6	9	11
$f(x)$:	12	13	14	16

Q. 13 Using Newton's divided difference formula, evaluate $f(9)$ & $f(15)$, given

x:	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

Q. 14 Find the value of log 656 by Newton's divide difference formula.

x:	654	658	659	661
$\log x$:	2.8156	2.8182	2.8189	2.8202

Q. 15 Determine $f(x)$ as a polynomial in x for the following data

x:	-4	-1	0	2	5
$f(x)$:	1245	33	5	9	1335

Unit V

NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

2-marks.

Q. 1. Adams-Bashforth predictor formula for solving $y' = f(x, y)$ given $y_0 = y(x_0)$ is…………………

Q. 2. Write the formula for 4^{th} order Runge-Kutta method.

Q. 3. What is the disadvantage of Picard's method?

Q. 4. Write the name of two self-starting methods to solve $y' = f(x, y)$ given $y_0 = y(x_0)$.

Q. 5. Write the name of two multi-steps methods available for solving ordinary differential equations.

4-marks.

Q. 1. Taylor's series solution of $y' - xy = 0, \quad y(0) = 1$ upto x^4 is ……………………

Q. 2. Using Euler's method solve $\frac{dy}{dx} = \frac{y-2x}{y}$, $y(0) = 1$ to find, $y(0.1) =$……………

Q. 3. If $y' = x - y, \quad y(0) = 1$, then by Picard's method the value of $y^{(1)}(1)$

Q. 4. Using Runge-Kutta method of fourth order find the value of $y(0.1)$ for

$$y' = x - 2y, \quad y(0) = 1, \text{ taking } h = 0.1.$$

Q. 5. Using modified Euler's method find the value of $y(0.05)$ for

$$\frac{dy}{dx} = x + y, \quad y(0) = 1.$$

8-marks.

Q. 1. Find the value of y for $x = 0.1$ by Picard's method, given that $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$.

Q. 2. Use Picard's method to approximate the value of y for $x = 0.1$, given that $\frac{dy}{dx} = 3x + y^2$ and $y = 1$ for $x = 0$.

Q. 3. Solve $y' = x + y, \quad y(0) = 1$ by Taylor's series method. Hence find the value of y at $x = 0.1$ and $x = 0.2$.

Q. 4. Employ Taylor's method to obtain approximate value of y at $x = 0.2$ for the differential equation $\frac{dy}{dx} = 2y + 3e^x, y(0) = 0$. Compare the numerical solution obtained with the exact solution.

Q. 5. Using Euler's method solve the differential equation $y' = x + y, \quad y(0) = 1$, taking step length $h = 0.2$ (*carry out six steps*).

Q. 6. Apply Euler's method to solve for y at $x = 0.6$ for $\frac{dy}{dx} = 1 - 2xy, y(0) = 0$ take $h = 0.2$.

Q. 7. Using modified Euler's method find the solution of the equation $\frac{dy}{dx} = x + |\sqrt{y}|$, with initial conditions $y(0) = 1$ for the range $0 \le x \le 0.6$ in steps of 0.2.

Q. 8. Solve the following differential equation by modified Euler's method $\frac{dy}{dx} = \log(x + y), y(0) = 2$ at $x = 1.2$ and $x = 1.4$ with $h = 0.2$.

Q. 9. Using Runge-Kutta method of fourth order solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$, $y(0) = 1$ at $x =$ 0.2 and $x = 0.4$.

Q. 10. Apply Runge-Kutta method of fourth order to approximate the value of y for at $x =$ 0.2 in steps of 0.1 if $\frac{dy}{dx} = x + y^2$, given that $y = 1$ when $x = 0$.

Q. 11. Using Milne's method find $y(4.4)$ given $5xy' + y^2 - 2 = 0$, and $y(4) = 1, y(4.1) = 1.0049, y(4.2) = 1.0097, y(4.3) = 1.0143$.

Q. 12. Given $2\frac{dy}{dx} = (1 + x^2)y^2$ and $y(0) = 1, y(0.1) = 1.06, y(0.2) = 1.12, y(0.3) = 1.21$. Evaluate $y(0.4)$ by Milne's predictor -Corrector method.

Q. 13. Using Adams-Bashforth method obtain the solution of $\frac{dy}{dx} = x - y^2$ at $x = 0.8$ given the values

x:	0	0.2	0.4	0.6
y:	0	0.0200	0.0795	0.1762

Q. 14. Given $y' = x^2 - y, y(0) = 1$ and starting values are and $y(0.1) = 0.90516, y(0.2) = 0.82127, y(0.3) = 0.74918$. Evaluate $y(0.4)$ by Adams-Bashforth method.

Q. 15. Use Adams-Bashforth method to find $y(0.4)$ given that $2y' = xy$, and $y(0) = 1, y(0.1) = 1.01, y(0.2) = 1.0097, y(0.3) = 1.023$.