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B. E. (Third Semester) Examination,
Nov.-Dec. 2019

(Old Scheme)

(AEI, EI, Et & T, Mechatronics Engg. Branch)

MATHEMATICS-III

Time Allowed : Three hours

Maximum Marks : 80

Minimum Pass Marks : 28

***Note** : Attempt all questions. Part (a) of each question is compulsory and carries 2 marks each. Attempt any **two** parts from (b), (c) and (d) and carries 7 marks each.*

(a) Write Dirichelet's conditions for Fourier expansion of a function. 2

(b) An alternating current after passing through rectifier has the form

$$i = \begin{cases} I_0 \sin x & \text{for } 0 \le x < \pi \\ 0 & \text{for } \pi < x < 2\pi \end{cases}$$

where I_0 is the maximum current and the period is 2π. Express i as a Fourier series. 7

(c) Obtain Fourier series for the function $f(x)$ given by

$$f(x) = \begin{cases} 1+\dfrac{2x}{\pi} & -\pi < x < 0 \\ 1-\dfrac{2x}{\pi} & 0 < x < \pi \end{cases}$$

Deduce that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}\ldots = \frac{\pi^2}{8}$$ 7

(d) Obtain a half range cosine series

$$f(x) = \begin{cases} Kx & 0 \le x < l/2 \\ K(l-x) & l/2 < x < l \end{cases}$$

Deduce the sum of the series

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots$$ 7

2. (a) State Convolution theorem for Fourier transform. 2

(b) Prove that : 7

$$\int_0^\infty \frac{\sin \pi\alpha \cdot \sin \alpha\theta}{1-\alpha^2} d\alpha = \begin{cases} \pi/2 \sin \alpha & , \quad 0 \le \theta < \pi \\ 0 & \theta > \pi \end{cases}$$

(c) Find the Fourier sine transform of $e^{-|x|}$. Hence show that 7

$$\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi e^{-m}}{2} \qquad m \ge 0$$

(d) Find the Fourier transform of

$$f(x) = \begin{cases} 1-x^2 & |x| \le 1 \\ 0 & |x| \ge 1 \end{cases}$$

Hence evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3}\cos x/2\,dx$. 7

3. (a) Write convolution theorem for inverse Laplace transform. 2

(b) Find the Laplace transform of : 7

(i) $\frac{e^{-at} - e^{-bt}}{t}$

(ii) $\frac{1-\cos t}{t^2}$

(c) (i) Apply convolution theorem to evaluate :

$$L^{-1}\frac{s}{\left(s^2+a^2\right)^2}$$

(ii) Find the inverse of the transform :

$$\log\left(\frac{s(s+1)}{s^2+4}\right)$$ 7

(d) Solve the equation by the transform method

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^t \sin t$$

where $y(0) = 0$ and $y'(0) = 1$. 7

4. (a) State Cauchy's Residue theorem. 2

(b) If $f(z)$ is a regular function of z, prove that :

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left|f(z)\right|^2 = 4\left|f'(z)\right|^2$$ 7

(c) Evaluate :

(i) $\int_C \tan z\,dz$ where C is the circle $|z| = z$.

(ii) $\int_C z^2 e^{1/2} dz$ where $C\ |z| = 1$. 7

(d) Apply the calculus of residues to prove that

$$\int_0^{2\pi} \frac{d\theta}{1 - 2p\sin\theta + p^2} = \frac{2\pi}{1-p^2} \quad (0 < p < 1)$$ 7

5. (a) Write the Karl Pearson's coefficient of correlation. 2

(b) Find the coefficient of correlation for the folllowing table : 7

x :	10	14	18	22	26	30
y :	18	12	24	6	30	36

(c) Obtain the rank of correlation coefficient for the following data : 7

x	y
68	62
64	58
75	68
50	45
64	81
80	60
75	68
40	48
55	50
64	70

(d) In a partially destroyed laboratory record of an analysis of correlation data, the following result only are eligible :

Variance of $x = 0$

Regressive equations : $8x - 10y + 66 = 0$

$40x - 18y - 214 = 0$

What were (i) the means values of x and y, (ii) the standard deviation of y, (iii) the coefficient of correlation between x and y. 7