Assignment - I
Laplace transform

Q.1(a) Define Laplace transform. (b) Find $L\left[\frac{\cos\sqrt{t}}{\sqrt{t}}\right]$

Q.2. Find the laplace transform of the function.

(i) $f(t) = |t-1| + |t+1|$: $t \geq 0$.

(ii) $f(t) = \begin{cases} t, & 0<t<\pi \\ \pi - t; & \pi < t < 2\pi \end{cases}$ where $f(t) \to$ periodic function

Q.3. Find the laplace transform of

(i) $t e^{-t} \sin 3t$. (ii) $\frac{1-\cos t}{t^2}$ (iii) $t\int_0^t \frac{e^{-t}\sin t}{t}\,dt$.

Q.4. Evaluate (i) $\int_0^\infty t e^{-2t} \sin t\, dt$ (ii) $\int_0^\infty \frac{e^{-t}\sin^2 t}{t}\, dt$

Q.5 Find the inverse transforms of

(i) $\frac{s}{s^4+4a^4}$ (ii) $\frac{s}{s^4+s^2+1}$ (iii) $\frac{1}{s^3-a^3}$

Q.6 Find the inverse laplace transforms of the following

(i) $s\log\frac{s-1}{s+1}$ (ii) $\tan^{-1}\frac{2}{s^2}$

Q.7 Apply convolution theorem to evaluate

(i) $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$ (ii) $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$ (iii) $L^{-1}\left[\frac{1}{s}\cos\frac{1}{s}\right]$

Q.8 Solve $\frac{d^2x}{dt^2} + 9x = \cos 2t$; if $x(0)=1$, $x(\pi/2)=-1$.

Q.9. Solve $ty'' + 2y' + ty = \cos t$ given that $y(0)=1$

Q.10. Solve $ty'' + (1-2t)y' - 2y = 0$, when $y_0 = 1$; $y'_0 = 2$

Q.11 Solve $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t$, $y(0)=0$, $y'(0)=1$

Q.12. Solve $y'' + y = t$; $y(0)=1$; $y'(0)=0$.

Q.13. Define Unit step function. Using unit step function, find the laplace transform of $f(t) \begin{cases} \sin t; & 0 \le t < \pi \\ \sin 2t; & \pi \le t \le 2\pi \\ \sin 3t; & t \geq 2\pi \end{cases}$

Q.14. Evaluate $L^{-1}\left[\frac{e^{-s} - 3e^{-3s}}{s^2}\right]$

Q.15. Find the inverse laplace transform of : $\frac{se^{-s/2} + \pi e^{-s}}{s^2+\pi^2}$

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Fourier Series

Q.1. Obtain a Fourier series to represent e^x from $-\pi$ to π. Hence derive series for $\pi/\sinh\pi$

Q.2. Find the Fourier series expansion for $f(x)$, if

$$f(x) = -\pi, \quad -\pi < x < 0$$
$$= x \; ; \quad 0 < x < \pi$$

Deduce that $1/1^2 + 1/3^2 + \frac{1}{5^2} \cdots = \pi^2/8$

Q3. If $f(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ \sin x, & 0 \le x \le \pi \end{cases}$

P.T. $f(x) = \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos 2nx}{4n^2-1}$

Hence show that $\frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} \cdots = \frac{1}{4}(\pi - 2)$

Q.4. Obtain Fourier series for the function

$$f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} \cdots = \pi^2/8$

Q.5. If $f(x) = |\cos x|$, expand $f(x)$ as a Fourier series in the interval $(-\pi, \pi)$.

Q.6. Given $f(x) = \begin{cases} -x+1, & \text{for } -\pi \le x \le 0 \\ x+1, & \text{for } 0 \le x \le \pi \end{cases}$

Is the function even or odd? Find the Fourier series for $f(x)$ and deduce the value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} \cdots$

Q7. Obtain the Fourier expansion of $x\sin x$ as a cosine series in $(0, \pi)$. Hence show that $\frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} \cdots = \frac{\pi - 2}{4}$

Q.8 Expand $f(x) = \frac{1}{4} - x$, if $0 < x < 1/2$

$= x - 3/4$, if $1/2 < x < 1$

as the Fourier series of Sine terms.

Q9. Find the Fourier series to represent the function $|\sin x|$, $-\pi < x < \pi$.

Q.10. obtain a half range cosine series for

$$f(x) = \begin{cases} Kx \ ; & 0 \le x \le l/2 \\ K(l-x), & l/2 \le x \le l. \end{cases}$$

Deduce the sum of the series $1/1^2 + 1/3^2 + 1/5^2 \cdots \infty$

Q.11. The Following table gives the variations of periodic current over a period

t(sec)	0	T/6	T/3	T/2	2T/3	5T/6	T
A (amp)	1.98	1.30	1.05	1.30	−0.88	−0.25	1.98

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic.

Q12. Compute the first two harmonic of the Fourier series of f(x) given in the following table

x:	0	π/3	2π/3	3π/3	4π/3	5π/3	6π/3
y.	1	1.4	1.9	1.7	1.5	1.2	1.0

Q13 obtain the constant term and the coefficients of the first sine and cosine terms in the fourier expansion of y as given

x	0	1	2	3	4	5
y	9	18	24	28	26	20

Q.14. The turning moment T on the crankshaft of a steam engine for the crank angle θ degrees is given as follows

θ	0	15	30	45	60	75	90	105	120	135
T	0	2.7	5.2	7.0	8.1	8.3	7.9	6.8	5.5	4.1

θ	150	165	180
T	2.6	1.2	0

Expand T in a series of sines upto the fourth harmonics.

Partial Differential Equations

Q.1. Form the Partial differential equations from

(i) $z = a\log\left\{\frac{b(y-1)}{1-x}\right\}$ (ii) $z = f\left(\frac{xy}{z}\right)$

(iii) $F(x+y+z,\ x^2+y^2+z^2) = 0$ (iv) $z = f(x+at) + g(x-at)$

Q.2 Solve (i) $(mz-ny)\frac{\partial z}{\partial x} + (nx-lz)\frac{\partial z}{\partial y} = ly - mx$

(ii) $(x^2-y^2-z^2)p + 2xyq = 2xz$

(iii) $(z^2-2yz-y^2)p + (xy+zx)q = xy - zx$

(iv) $px(z-2y^2) = (z-qy)(z-y^2-2x^3)$

Q3. Solve (i) $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2y$

(ii) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$

(iii) $4\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 16\log(x+2y)$

(iv) $(D^2 - DD' - 2D'^2)z = (y-1)e^x$

(v) $(D^2 + 2DD' + D'^2)z = 2\cos y - x\sin y$

(vi) $(D^2 - DD' + D' - 1)z = \cos(x+2y)$

Q.4. Using the method of separation of variables,

(i) Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x,0) = 6e^{-3x}$

(ii) $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, given that $u(0,y) = 8e^{-3y}$

(iii) $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given $u = 3e^{-y} - e^{-5y}$ when $x = 0$

(iv) $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$, $u(x,0) = 4e^{-x}$

Q.5. A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = a\sin\left(\frac{\pi x}{l}\right)$ from which it is released at time $t=0$. Show that the displacement of any point at a distance x from one end at time t is given by

$$y(x,t) = a\sin\left(\frac{\pi x}{l}\right)\cos(\pi ct/l)$$

Q6. A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y = y_0\sin^3(\pi x/l)$. If it is released from rest from this position, find the displacement $y(x,t)$.

1. If a random variable has a Poisson distribution such that $P(1) = P(2)$ find (i) mean of the distribution (ii) $P(4)$

2. X is a Poisson variable and it is found that the probability that $X=2$ is two thirds of the probability that $X=1$. Find the probability that $X=0$ and the probability that $X=3$. what is the probability that X exceeds 3!

3. A certain screw making machine produces an average of 2 defective screws out of 100, and packs them in boxes of 500. Find the probability that a box contains 15 defective screws?

4. A car-hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a poisson distribution with mean 1.5. Calculate the proportion of days (i) on which their is no demand (ii) on which demand is refused.

5. In a certain factory ~~at~~ turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10, use Poisson distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 10,000 packets.