Unit-4

Random Variable

* **Sample Space** → A set S that consists of all possible outcomes of a random experiments are ~~is~~ called "Sample Space".

Example → If we have a coin then Sample Space $S = \{H, T\}$.

* **Random Variable** → A real valued function defined on a sample space is called "Random Variable". It is called "simply variant". It is denoted by x or y.

Example: → Suppose that a coin is tossed twice so that sample space,

Sample Space - $S = \{HH, TT, HT, TH\}$

~~Assign a real Valued~~ no. $= \{2, 0, 1, 1\}$

{If favorable outcome is H}

So, Random Variable $X = \{0, 1, 2\}$

(1)

Random Variable

Discreate R.V.

(I) Probability Distribution

x	x_1	x_2	x_3	x_4
$P(x)$	p_1	p_2	p_3	p_4

(II) Distribution Function

$$F(x) = \begin{cases} 0 & , -\infty \le x \le 0 \\ 0 + f_1(x) & , 0 \le x \le 1 \\ 0 + f_1(x) + f_2(x) & , 1 \le x \le 2 \\ 0 + f_1(x) + f_2(x) + f_3(x) & , 2 \le x \le 3 \\ 0 + f_1(x) + f_2(x) + f_3(x) + f_4(x) = 1 & , 3 \le x \le \infty \end{cases}$$

(III) Density Function

(a) $f(x) \geq 0$

(b) $\sum_{\forall i} f(x_i) = 1$

Continuous R.V

(I) Not possible to show in tabular form.

Imp

(II) Distribution function (or) "Cummulative Dist. fun^n"

$$\boxed{P(x) = \int_{-\infty}^{x} f(u)\,du}$$

(III) Density Function

(a) $f(x) \geq 0$

(b) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(IV) If we want to find probability b/w 2 values in continuous R.V ($-\infty$ to ∞) then

$$\boxed{P(a \le x \le b) = \int_a^b f(x)\,dx}$$

Discrete R.V.

(2)

Q. A coin tossed 3 times. If X is R.V. giving the no. of Heads which arise, then construct a table showing the prob. distr. funⁿ, also find Density function

Solⁿ

Sample space → $2^n = 2^3 = 8$

$S = \{HHH, HTT, THT, TTH, HHT, HTH, THH, TTT\}$

Total 8 combinations.

$X = R.V. = \{0, 1, 2, 3\}$

then

$P(X=0) = 1/8$

$P(X=1) = 3/8$

$P(X=2) = 3/8$

$P(X=3) = 1/8$

Probability Distribution →

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

Distribution funⁿ →

$$F(x) = \begin{cases} 0 & , -\infty \le x \le 0 \\ 0 + 1/8 & , 0 \le x \le 1 \\ 0 + 1/8 + 3/8 & , 1 \le x \le 2 \\ 0 + 1/8 + 3/8 + 3/8 & , 2 \le x \le 3 \\ 0 + 1/8 + 3/8 + 3/8 + 1/8 = 1 & , 3 \le x < \infty \end{cases}$$

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Density function →

(i) $f(x) \geq 0$

It is obvious that f^n is $f(x) \geq 0$.

(ii) $\sum_{\forall i} f(x_i) = 1$

$\Rightarrow \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1$

Then given f^n is a Density f^n.

Imp

Q. The probability Density f^n of a variate (R.V.) X is

X:	0	1	2	3	4	5	6
P(X):	K	3K	5K	7K	9K	11K	13K

(a) Find $P(X<4)$, $P(X \geq 5)$, $P(3<x\leq 6)$

(b) what will be the minimum value of K so that $P(X \leq 2) > 0.3$

Sol^n

I^{st} we will find $K = ?$

for this,

$$\sum_{i=1}^{6} P(x_i) = K + 3K + 5K + 7K + 9K + 11K + 13K$$

Sum of all probabilities $= 1$

then, $1 = 49K$

then $\boxed{K = \frac{1}{49}}$

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(a) $P(x<4) = K+3K+5K+7K$

$= 16K$

since, $K = \frac{1}{49}$

then $= 16 \times \frac{1}{49} = \frac{16}{49}$

(2) $P(x \geq 5) = 11K + 13K$

$= 24K$

$= 24 \times \frac{1}{49}$

$= \frac{24}{49}$

$P(3 < x \leq 6) = 9K + 11K + 13K$

$= 33K$

$= 33 \times \frac{1}{49}$

$= \frac{33}{49}$

(b) $P(x \leq 2) > 0.3$

$K + 3K + 5K > 0.3$

$9K > 0.3$

then $K > \frac{0.3}{9}$

$K > \frac{1}{30}$

Then the minimum value of K is $\Rightarrow \boxed{K > 1/30}$.

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Continuous R.V

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Imp.

Q.1 Find the constant c such that the fu^n $f(x) = cx^2, \; 0 < x < 3$
$\qquad\qquad 0, \;$ otherwise.

is (a) Density fu^n.
(b) compute $(1 < x < 2)$
(c) cummulative Dist. fu^n,

Sol^n → ("In continuous fu^n Question is always in the form of interval.")

(a) for Density fu^n. Ist we will find $c = ?$
for this,

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\boxed{\cancel{\int_{-\infty}^{\infty} f(x)\,dx}}$$ Break ~~in~~ the fu^n in subintervals

$$\int_{-\infty}^{0} f(x)\,dx + \int_0^3 f(x)\,dx + \int_3^{\infty} f(x)\,dx = 1$$

$$0 + \int_0^3 cx^2\,dx + 0 = 1$$

$$c\left(\frac{x^3}{3}\right)_0^3 = 1$$

$$\frac{c}{3} \times 27 = 1$$

$$\boxed{c = 1/9}$$

since, $c = 1/9$ then it is clear that $f(x) > 0$
So given fu^n is Density fu^n.

(b) $P(1 < x < 2) = \int_1^2 cx^2\, dx$

$$= c\left(\frac{x^3}{3}\right)_1^2$$

$$= \frac{c}{3} \times (8-1)$$

$$= \frac{c}{3} \times 7$$

Put, $\boxed{c = 1/9}$ $= \frac{1}{9 \times 3} \times 7$

$$= \frac{7}{27}$$

(c) Cummulative Distribution →

$$F(x) = \int_{-\infty}^{x} f(u)\, du$$

Since, A/c to the Question

$$f(x) = cx^2, \quad 0 < x < 3$$
$$0, \quad \text{otherwise}$$

then $\boxed{x = 3}$ = upper limit

then, $F(x) = \int_{-\infty}^{x=3} f(u)\, du$

$$= \int_{-\infty}^{3} f(u)\, du$$

we break the fn in sub intervals → $= \int_{-\infty}^{0} + \int_{0}^{3}$

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$$= \int_{-\infty}^{0} 0\, du + \int_{0}^{3} cx^2\, dx$$

$$= c\left(\frac{x^3}{3}\right)_0^3 = \frac{c}{3} \times 27$$

$$= \frac{1}{9 \times 3} \times 27 = 1$$

then $\boxed{F(x) = 1}$

Que Is the function defined as follows a density fu^n.

$$f(x) = e^{-x} \;, \quad x \geq 0$$
$$\quad\quad 0 \;, \quad x < 0$$

(a) If so determine the Probability that the variate has this density will fall in the interval (1, 2)

(b) Also find the cummulative prob. fu^n. F(2).

Sol^n Ist we will prove that Density fu^n,

(i) It is clear that $f(x) \geq 0$

and (ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$$= \int_{-\infty}^{0} + \int_{0}^{\infty}$$

$$+ \int_{0}^{\infty} e^{-x}\, dx = 1$$

$= \left[-e^{-x}\right]_0^{\infty} = 1$

(a) $P(1 \leq x \leq 2) = \int_1^2 e^{-x}\,dx$

$= \left(-e^{-x}\right)_1^2$

$= e^{-2} - e^{-1}$

$= 0.233$ //.

(c) Cummulative Distribution

$fu^n = \int_{-\infty}^{x} f(x)\,dx$

$F(2) = \int_{-\infty}^{2} f(x)\,dx$

$= \int_{-\infty}^{0} 0\,dx + \int_0^2 e^{-x}\,dx$

$= \left(-e^{-x}\right)_0^2$

$= -e^{-2} + 1$

$= 0.865$ //.

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E(X) Mathematical Expectation

Discrete R.V.	Continuous R.V

Discrete R.V.:

x	x_1	x_2	x_3	x_4
$P(x)$	P_1	P_2	P_3	P_4

$$E(x) = x_1 P_1 + x_2 P_2 + x_3 P_3 + x_4 P_4$$

expectation

Continuous R.V: uncountable terms

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

It is also known as "Expectation".

Q① Discrete R.V

Suppose that a game is to be played with a single die assumed fair. In this game a player wins \$ 20, if 2 turns up, \$ 40 if 4 turns up, loses \$ 30 if 6 turns up, while the player neither wins nor losses if any other face turns up. Find the expected sum of money to be won.

Sol^n → Let X be the R.V. giving the amount of money won on any toss. The possible amounts won when the die turns up.

outcomes :	1	2	3	4	5	6
R.V. :	x_1	x_2	x_3	x_4	x_5	x_6
prob :	$f(x_1)$	$f(x_2)$	$f(x_3)$	$f(x_4)$	$f(x_5)$	$f(x_6)$

S	1	2	3	4	5	6
x_i	0	+20	0	+40	0	−30
$f(x_i)$	1/6	1/6	1/6	1/6	1/6	1/6

$$E(x) = 0\times 1/6 + 20\times 1/6 + 0\times 1/6 + 40\times 1/6 + 0\times 1/6 + (-30)\times 1/6$$

$$\boxed{E(x) = 5}$$

Continuous R.V.

Q. → The Density fu^n of a R.V. X is given by

$$f(x) = \begin{cases} \frac{1}{2}x, & 0 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

Find $E(X)$.

Sol^n

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{0} + \int_{0}^{2} + \int_{2}^{\infty}$$

$$= \int_0^2 x \times \frac{x^2}{2} dx = \left(\frac{x^3}{6}\right)_0^2$$

$$= \frac{8}{6} = \frac{4}{3} //$$

V.V Imp

Q The Probability Density function

$$p(x) = y_0 e^{-|x|} , -\infty \text{ to } \infty$$

Then Prove that $\boxed{y_0 = 1/2}$

also find

mean and variance.

Sol^n Mod function is Defined by

$$|x| = \begin{cases} +x, & x > 0 \\ -x, & x < 0 \end{cases}$$

$$\boxed{e^{-|x|} = \begin{cases} e^{-x}, & x > 0 \\ e^{x}, & x < 0 \end{cases}}$$

Given that $p(x)$ is Density fu^n;

$$\Rightarrow \int_{-\infty}^{\infty} p(x)\,dx = 1$$

$$\Rightarrow \int_{-\infty}^{\infty} y_0 e^{-|x|} dx = 1$$

$$\Rightarrow \cancel{\int_{-\infty}^{0} y_0 e^{+(x)} dx + \int_0^{\infty} y_0 e^{-(x)} dx = 1}$$

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since, $|x|$ is an even fu^n and range is $-\infty$ to ∞ then By Definite Integral

$$p(x) = 2\int_0^\infty y_0 e^{-x}\,dx = 1$$

$$\Rightarrow 2y_0\left[-e^{-x}\right]_0^\infty = 1$$

$$\Rightarrow 2y_0[0+1] = 1$$

$$\Rightarrow 2y_0 = 1$$

then $\boxed{y_0 = 1/2}$

Now, we have to find mean (μ) and Variance

By Formula

$$\boxed{\text{mean} = E(x) = \int_{-\infty}^{\infty} x\,P(x)\,dx}$$

$$E(x) = \int_{-\infty}^{0} y_0\, x e^{x}\,dx + \int_0^\infty y_0\, x e^{-x}\,dx$$

$$= y_0\left[xe^{x} - e^{x}\right]_{-\infty}^{0} + y_0\left[x(-e^{-x}) - (e^{-x})\right]_0^\infty$$

$$= y_0[-1+1] = 0$$

$$\boxed{E(x) = 0}$$

Note:- $\int_{-\infty}^{\infty} \underbrace{x}_{\text{odd } fu^n} \underbrace{P(x)}_{\text{even } fu^n} dx = 0 \times e = 0$

$= 0$

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$$\text{Variance} = E(x-\mu)^2 = \int_{-\infty}^{\infty} (x-\mu)^2 \, p(x)\, dx$$

$$= \int_{-\infty}^{0} (x-\mu)^2 y_0 e^{x} dx + \int_{0}^{\infty} (x-\mu)^2 y_0 e^{-x} dx$$

$\boxed{\mu = 0}$

$$= \int_{-\infty}^{0} x^2 y_0 e^{+x} dx + \int_{0}^{\infty} x^2 y_0 e^{-x} dx$$

$$= y_0 \left[\left(x^2 e^{x} - 2x e^{x} + 2e^{x} \right)_{-\infty}^{0} + \left\{ x^2(-e^{-x}) - 2x(e^{-x}) + 2(-e^{-x}) \right\}_{0}^{\infty} \right]$$

$\text{Var.} = 2$

$\boxed{\text{Put } y_0 = 1/2}$

then, Std Deviation

$$\sigma = \sqrt{2}$$

—x—

Q. From an urn containing 3 red, 2 white balls, a man is to draw 2 balls at random without replacement being promised Rs. 20, for each red ball, and Rs. 10 for each white one. Find his expectation.

Sol^n 3 red 2 white = 5 Balls

$S =$	RR	WW	RW
$X \Rightarrow$	40/-	20/-	30/-
$P(x) =$	?	?	?

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$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

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$$P(X=40) = \frac{3C_2}{5C_2} = \frac{\frac{3!}{1!\,2!}}{\frac{5!}{2!\,3!}}$$

$$= \frac{3}{10}$$

This is Permutation / Combination, we use this formula when we have different color balls. But If we have Identical ball then we use prob. $\frac{3 \to \text{red}}{5 \to \text{Total}}$.

$$P(X=20) = \frac{2C_2}{5C_2} = \frac{\frac{2!}{0!\,2!}}{\frac{5!}{3!\,2!}}$$

$$= \frac{1}{10}$$

$$P(X=30) = \frac{3C_1 \cdot 2C_1}{5C_2}$$

$$= \frac{\frac{3!}{2!\cdot 1!} \times \frac{2!}{1!}}{\frac{5!}{3!\cdot 2!}}$$

$$= \frac{3}{5}$$

S	RR	WW	RW
X	40	20	30
P(X)	3/10	1/10	3/5

then expectation

$$E(X) = 40 \times \frac{3}{10} + 20 \times \frac{1}{10} + 30 \times \frac{3}{5}$$

$$= 32$$

Q A continuous Distribution of a R.V. x in the range $(-3,3)$ is defined as

$$f(x) = \frac{1}{16}(3+x)^2, \quad -3 \le x \le -1$$
$$= \frac{1}{16}(6-2x^2), \quad -1 \le x \le 1$$
$$= \frac{1}{16}(3-x^2), \quad 1 \le x \le 3$$

verify that the area under the curve is unity. Show that the mean is 0.

Sol^n Area under the curve:—

$$= \frac{1}{16}\int_{-3}^{-1}(x+3)^2\,dx + \frac{1}{16}\int_{-1}^{1}\cancel{(2-6x)^2}\ 6-2x^2\,dx$$
$$+ \frac{1}{16}\int_{1}^{3}(3-x^2)^2\,dx$$

$$= \frac{1}{16}\times 16 = 1$$

Mean $\mu = \frac{1}{16}\int_{-3}^{-1} x(x+3)^2\,dx$

$$+ \frac{1}{16}\int_{-1}^{1} x(2-6x)^2\,dx$$
$$+ \frac{1}{16}\int_{1}^{3} x(3-1)^2\,dx$$

$= 0$

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Q. X is a random variable giving time (in min) during which a certain electrical equipment is used at max. load in a specified time period if the probability distribution function is given by,

$$f(x) = \frac{x}{(1500)^2}, \quad 0 \le x \le 1500$$

$$= -\frac{(x-3000)}{(1500)^2}, \quad 1500 \le x \le 3000$$

$$= 0, \quad \text{otherwise}$$

Find the expected value of x.

Sol^n ⇒

$$E(x) = \int_0^{1500} x \times \frac{x}{(1500)^2}\,dx - \int_{1500}^{3000} x \times \frac{x-3000}{(1500)^2}\,dx$$

$$= \frac{1}{(1500)^2}\left[\frac{x^3}{3}\right]_0^{1500} - \frac{1}{(1500)^2}\left\{\frac{x^3}{3} - 1500x^2\right]_{1500}^{3000}$$

$$= 500 - (-1{,}000)$$

$$= 1500 \; //$$

— * —

The End

Moment & Moment Generating Function

$$M_x(t) = \sum e^{tx} f(x) \rightarrow \text{Discrete}$$
$$M_x(t) = \int_{-\infty}^{\infty} e^{tx} f(x)\,dx \rightarrow \text{continuous}$$

* $M_x(t) = 1 + t\mu_1' + \mu_2' \frac{t^2}{2!} + \mu_3' \frac{t^3}{3!} + \cdots$

This is M.G.F. in series form.

* Moment Formula $\mu_r' = \frac{d^r}{dt^r}\left[M_x(t)\right]_{\text{at } t=0}$ about the origin

put $r=1$
mean $= \mu = \mu_1' = \frac{d}{dt}\left[M_x(t)\right]$

put $r=2$ Var $= \mu_2' = \frac{d^2}{dt^2}\left[M_x(t)\right]$ at $t=0$

$$\text{S.D.} = \sigma = \sqrt{\mu_2} = \sqrt{\mu_2' - (\mu_1')^2}$$

Que $\rightarrow$ Find the moment generating function of the exponential distribution

$$f(x) = \frac{1}{c} e^{-x/c} \quad , 0 \le x \le \infty$$

Find mean and S.D.

Sol^n $\Rightarrow$

$$M_x(t) = \int_{-\infty}^{\infty} e^{tx} f(x)\,dx$$
$$= \int_{-\infty}^{0} 0\,dx + \int_{0}^{\infty} e^{tx} \cdot \frac{1}{c} e^{-x/c}\,dx$$
$$= \frac{1}{c}\int_{0}^{\infty} e^{(t-1/c)x}\,dx$$
$$= \frac{1}{c}\left[\int_{0}^{\infty} e^{-\left(\frac{1}{c}-t\right)x}\,dx\right]$$

$$\Rightarrow \frac{1}{c}\left[\frac{-e^{-\left(\frac{1}{c}-t\right)x}}{\frac{1}{c}-t}\right]_0^\infty$$

$$= \frac{1}{c}\left[\frac{1}{\frac{1}{c}-t}\right]$$

$$= \frac{\frac{1}{c}}{\frac{1-tc}{c}} = \frac{1}{1-tc}$$

$$= (1-tc)^{-1}$$

By Binomial thm;

$$\therefore M_x(t) = 1 + tc + (tc)^2 + (tc)^3 + \cdots\cdots\infty$$

Now, $\mu_r' = \frac{d^r}{dt^r} M_x(t)$ (at $t=0$)

$r=1 \rightarrow$ $\mu_1' = \frac{d}{dt}\left\{1 + tc + (tc)^2 + (tc)^3 + \cdots\infty\right\}$

Mean

$$= \left[c + 2tc^2 + 3t^2c + \cdots\infty\right] \quad (1)$$

Put $t=0$

$$= c$$

$r=2 \rightarrow$ $\mu_2' = \frac{d^2}{dt^2} M_x(t)$ at $t=0$

Variance

$$= \frac{d}{dt}\left[\frac{d}{dt} M_x(t)\right]$$

By (1)

$$= \frac{d}{dt}\left[c + 2tc^2 + 3t^2c + \cdots\infty\right]$$

$$= \left(2c^2 + 6tc + \cdots\infty\right)$$

Put $t=0$

$$= 2c^2$$

Now, Standard deviation

S.D. σ or $\mu_2 = \sqrt{\mu_2' - (\mu_1')^2}$

$= \sqrt{\text{Var.} - (\text{Mean})^2}$

$= \sqrt{2c^2 - c^2}$

$= \sqrt{c^2}$

S.D $\boxed{\sigma = c}$

Binomial Distribution

** mean = np
variance = npq
S.D. = $\sqrt{npq}$ **

$n \rightarrow$ minimum (Between 1 to 50)

" In some trials the probability will not change from 1 trial to the next such trials are known as "Bernoulli's Trial" or "Binomial distribution".

p = Prob. of Successful event
q = Prob. of faluire event
n = Total trials.

Formula for Binomial Distribution $\longrightarrow$

$$P(X=x) = {}^nC_x \, p^x q^{n-x}$$

Que ① $\rightarrow$ An ordinary six faced die is thrown 4 times what are the prob. of obtaining 4, 3, 2, 1, 0 faces.

Sol^n $\rightarrow$ p = prob. of obtaining any single face $= \frac{1}{6}$

$q = \cancel{\text{pro}} \; (1-p) = \frac{5}{6}$

n = total trial = 4

* $\boxed{{}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}}$ *

$$P(x=4) = {}^4C_4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^0$$
$$= 1 \times \left(\frac{1}{6}\right)^4 =$$

$$P(x=3) = 4C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^{4-3}$$
$$= 4C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^1$$
$$=$$

$$P(x=2) = 4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{4-2}$$
$$= 4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 =$$

$$P(x=1) = {}^4C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^3 =$$

$$P(x=0) = {}^4C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^4 =$$

x x x

Que ⇒ **Imp**

If the chance that 1 of the 10. telephone lines is busy at an instant is 0.2

(a) What is the chance that 5 of the lines are busy.

(b) What is the ~~chance that 5 of the lines~~ most probable number of busy lines, and what is probability of this number.

(c) What is the probability that all the lines are busy.

Soln ⇒

p = prob. of 1 telephone lines is busy
$= 0.2$ (given)

$q = (1-p) = 0.8$

$n = 10$

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(a) $p(x=5) = {}^{10}C_5 \, (0.2)^5 \, (0.8)^{10-5}$

$= {}^{10}C_5 \, (0.2)^5 \, (0.8)^5$

$= 0.026$

(b) Most probable no. of busy lines = Expectation (mean) $= np$

$= 10 \times 0.2 = 2$

probability of most probable no. is

$P(x=2) = {}^{10}C_2 \, (0.2)^2 \, (0.8)^8$

$=$

(c) $P(x=10) = {}^{10}C_{10} \, (0.2)^{10} \, (0.8)^0$

$=$

Ques If the prob. that a new born child is a male is 0.6, find the prob. that in a family of 5 children there are exactly 3 boys.

Sol^n prob. that a new born child is a male $= p = 0.6$

$q = 1 - p = 0.4$

$n = 5$

Then Prob. of exactly 3 boys

$P(x=3) = {}^5C_3 \, (0.6)^3 \, (0.4)^2$

$= 0.3456$

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Que ⟹ If on an average 1 vessel in every 10 is wrecked, find the prob that out of 5 vessels expected to arrive at least 4 will arrive safely?

Sol^n ⟹ Prob. of 1 vessel is wrecked out of 10, i.e nine vessels out of 10 are safe

$$= p = \frac{9}{10}$$

and $q = 1 - p = \frac{1}{10}$

Required prob. is → At least 4 ~~will~~ are safe ~~out~~ of 5

$$= P(x=4) + P(x=5)$$

or

$$= 1 - \{P_0 + P_1 + P_2 + P_3\}$$

$$\Rightarrow P(x=4) + P(x=5)$$

$$\Rightarrow {}^5C_4 \left(\frac{9}{10}\right)^4 \left(\frac{1}{10}\right)^1 + {}^5C_5 \left(\frac{9}{10}\right)^5 \left(\frac{1}{10}\right)^0$$

$=$

Que ⟹ The bomb that a bomb dropped from a plane will strike the target is 1/5 if six bombs are dropped find the prob. that

(a) exactly 2 will strike the target.

(b) At least 2 will strike the target.

Sol^n ⟹ Prob. that a bomb dropped from a plane will strike the target $p = \frac{1}{5}$

$q = 1 - \frac{1}{5} = \frac{4}{5}$

$n = 6$

(a) exactly 2 :- $P(x=2) = {}^6C_2 \cdot \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^4$

$= 0.245$

(b) At least 2 :- $1 - [P_0 + P_1]$

$\Rightarrow 1 - \left[{}^6C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^6 + {}^6C_1 \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^5\right]$

$= 0.3447$

Que ⟹ A Sortie of 20 aeroplanes is sent on an operational flight. The chances that an aeroplane flails to return is 5%. Find the prob that (a) One plane does not return. (b) At the most 5 planes do not return. (c) What is the most probable no. of returns

Sol^n ⟹ prob. of an aeroplane fails to return (do not return = means went successfully)

$p = \frac{5}{100} = \frac{1}{20}$

return $= q = 1 - p = 1 - \frac{1}{20} = \frac{19}{20}$

$n = 20$

(a) $P(x=1) = {}^{20}C_1 \left(\frac{1}{20}\right)^1 \left(\frac{19}{20}\right)^{19} =$

(b) At Most 5 $= 1 - [P_6 + P_7 + P_8 + \cdots + P_{20}]$

or $P_0 + P_1 + P_2 + P_3 + P_4 + P_5$

$=$

(c) most probable number to return = Expectation : mean $= nq$

$= 20 \times \frac{19}{20}$

$= 19$

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Que ⟹ The prob. that an entering Student will graduate is 0.4, determine the prob. that out of 5 Student
(a) None (b) One (c) At least 1
will graduate.

Sol^n ⟹ prob that entering student will graduate, $p = 0.4$
$q = 0.6$

(a) None → $P(x=0) = {}^5C_0 (0.4)^0 (0.6)^5$
$=$

(b) one → $P(x=1) = {}^5C_1 (0.4)^1 (0.6)^4$
$=$

(c) at least one $= 1 - p_0$
$= 1 - \{{}^5C_0 (0.4)^0 (0.6)^5\}$
$= 0.92$

Imp

Que ⟹ out of 800 families with 5 children each how many would you expect to have (a) 3 boys (b) 5 girls (c) either 2 or 3 boys. Assume equal prob. for boys & girls.

Sol^n ⟹ prob of girls and boys $= \frac{1}{2}$
$n = 5$
Total family $= 800$

(a) 3 boys → $P(x=3) = {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$
$= \frac{\lfloor 5}{\lfloor 3 \cdot \lfloor 2} \times \frac{1}{8} \times \frac{1}{4}$
$= \frac{10}{32}$

Since, Total 800 families $= \frac{10}{32} \times 800 = 250$

(b) 5 girls $\Rightarrow$ $p(x=5) = \left[{}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0\right] \times 800$

$= 25$

(c) either 2 or 3 boys $\Rightarrow$

$P(x = 2 \text{ or } 3) =$

$= 800\,[P(x=2) + P(x=3)]$

$= 800\left[{}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 + {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2\right]$

$= 500$

Que $\Rightarrow$ If 10 % of the rivets produced by a machine are defective, find the prob. that out of 5 rivets choose at random

(a) None will be defective.

(b) 1 will be defective.

(c) at least 2 will be defective.

Solⁿ $\Rightarrow$ Prob of the rivets produced are defective $p = \frac{10}{100} = 0.1$

$q = 1 - 0.1 = 0.9$, $n = 5$

(a) none $\rightarrow$ $P(x=0) = {}^5C_0 (0.1)^0 (0.9)^5 = 0.59$

(b) one $\rightarrow$ $P(x=1) = {}^5C_1 (0.1)^1 (0.9)^4 = 0.328$

(c) At least 2 $= 1 - [P_0 + P_1]$

$= 1\left[{}^5C_0 (0.1)^2 (0.9)^5 + {}^5C_1 (0.1)^1 (0.9)^4\right]$

$= 0.08146$

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Que ⇒ Fit a Binomial distribution to these data

x :	0	1	2	3	4	5	6	7	8	9	10
f :	6	20	28	12	8	6	0	0	0	0	0

Solⁿ ⇒ Formula for Best fit of Binomial distribution, $\boxed{N(p+q)^n}$

We have $x \to 0$ to 10 i.e total $11 = n$ term, but we will take $n = 10$ Because in Binomial thm we know that

$$\boxed{\begin{array}{l} \text{Ex:- } (a+b)^2 = \underbrace{a^2+b^2+2ab}_{3\text{ terms}} \\ (1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2 + \cdots \infty \end{array}}$$

$N = \Sigma f_i = 80$

$\text{mean} = \mu = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{20+56+36+32+30}{80}$

$\mu = 2.175$

$np = \mu = 2.175 \Rightarrow 10p = 2.175$

So, $p = 0.2175$

& $q = 1 - p = 0.782$

Best fit $\rightarrow \underset{N}{80}\left[\underset{a}{0.2175} + \underset{b}{0.7825}\right]^{10 \to n}$

By Binomial thm; $(a+b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + \cdots\cdots$

$$\Rightarrow 80\left[{}^{10}C_0 (0.2175)^{10} (0.7825)^{0} + {}^{10}C_1 (0.2175)^{9} (0.7825)^{1} + {}^{10}C_2 (0.2175)^{8} (0.7825)^{2} + {}^{10}C_3 (0.2175)^{7} (0.7825)^{3} + \cdots\cdots\right)$$

After solving, we get,

$$\Rightarrow \underset{\substack{\downarrow \\ 10^{th} \\ \text{value}}}{0.0002} + \underset{\substack{\downarrow \\ 9^{th}}}{0.0007} + \underset{\substack{\downarrow \\ 8^{th}}}{\cdots} \cdots\cdots + 19.13 + \underset{1^{st}\ \text{value}}{5.42}$$

Normal Distribution (Continuous R.V.)

formula $p(X=x) = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} dx$ --- ①

But it is very difficult to evaluate. so we will change the variable. i.e. z. the variable in normal

Normal variable $z = \frac{x-\mu}{\sigma}$

Here, $\begin{cases} \mu=0 \\ \mu=1 \end{cases}$

So, $P(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-z^2/2} dz$

Now It is very easy to Solve,

$p(a<x<b) = \int_a^b f(x)dx$

$p(z_1<z<z_2) = \int_{z_1}^{z_2} f(x)dz$

Here, formula $\boxed{z_1 = \frac{a-\mu}{\sigma}}$, formula $\boxed{z_2 = \frac{b-\mu}{\sigma}}$

and formula $\boxed{F(z) = F(z_2) - F(z_1)}$

Important Property :- 1) $F(-z) = \cancel{F} - F(z)$

2) $F(\infty) = 0.5$

3) $F(0) = 0$

Que For a normally distributed variate with mean 1 and S.D. 3 find, the prob that,

① $3.43 \le x \le 6.19$

② $-1.43 \le x \le 6.19$

Soln Given, $\mu = 1$, $\sigma = 3$

ⓐ $\underset{a}{3.43} \le x \le \underset{b}{6.19}$

Converting in normal variable;

$z_1 \leq z \leq z_2$

$$z_1 = \frac{a-\mu}{\sigma} = \frac{3.43-1}{3} = 0.81$$

$$z_2 = \frac{b-\mu}{\sigma} = \frac{6.19-1}{3} = 1.73$$

$$P(z) = F(z_2) - F(z_1)$$
$$= F(1.73) - F(0.81)$$
$$= 0.4582 - 0.2910$$
$$= 0.1672$$ Ans.

(b)

$$-1.43 \leq x \leq 6.19$$
a, b

$$z_1 = \frac{a-\mu}{\sigma} = \frac{-1.43-1}{3} = -0.81$$

$$z_2 = \frac{b-\mu}{\sigma} = \frac{6.19-1}{3} = 1.73$$

$$P(z) = F(z_2) - F(z_1)$$
$$= F(1.73) - F(-0.81)$$
$$= 0.4582 + 0.2910$$
$$= 0.7492$$ Ans.

~~Soln~~ **Que** If z is normally distributed with mean 0 and variance 1, find

(a) $Pr(z \leq -1.64)$

(b) z_1, if $Pr(z \geq z_1) = 0.84$

Soln

$$Pr(z_1 \leq z \leq -1.64) =$$
$$Pr(-\infty \leq z \leq -1.64) =$$
$$F(z_2) - F(z_1)$$

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$= F(-1.64) - F(-\infty)$

$= F(-1.64) + F(\infty)$

$= -0.4495 + 0.5$

$= 0.0505$

since, $\left\{ F(\infty) = 0.5 \right.$

(b) z_1, if $Pr(z \geq z_1) = 0.84$

$\therefore Pr(z_1 \leq z \leq \infty) = 0.84$

Here prob. is already given in the Que we have to find z_1,

$\cancel{F(z_2) - f(z_1) = 0.84}$

$F(\infty) - F(z_1) = 0.84$

$0.5 - f(z_1) = 0.84$

$F(z_1) = -0.34$

So, $z_1 = -0.99$ [By Anti normal Table]

Que ⇒ A Manufacturer of air-mail envelops knows from experience that the weight of the envelops is 'normally Distributed' with mean 1.95 gm and S.D 0.05 gm. about how many envelops weighting

1) 2 gm. or more

2) 2.05 gm. or more can be expected in a given packet of 100 envelops.

Soln ⇒ Given, Mean $\mu = 1.95$ gm.

$\sigma = 0.05$ gm.

(1) 2 gm. or More :-

$$P(\underset{a}{2} \leq x \leq \underset{b}{\infty}) = ?$$

$$z_1 = \frac{a-\mu}{\sigma} = \frac{2-1.95}{0.05} = 1$$

$$z_2 = \frac{b-\mu}{\sigma} = \infty$$

$$P(z) = F(z_2) - F(z_1)$$
$$= F(\infty) - F(1)$$
$$= 0.5 - 0.3413$$
$$= 0.1587$$

There are 100 envelops, so

$$= 100 \times 0.1587$$
$$= 15.87 \quad \text{or} \quad 16 \text{ (approx)}$$

Solⁿ ②

$$P(\underset{a}{2.05} \le x \le \underset{b}{\infty})$$

$$z_1 = \frac{a-\mu}{\sigma} = \frac{2.05-1.95}{0.05} = 2$$

$$z_2 = \frac{b-\mu}{\sigma} = \infty$$

$$\therefore P(z) = F(z_2) - F(z_1)$$
$$= F(\infty) - F(2)$$
$$= 0.5 - 0.4772$$
$$= 0.0228$$

Total 100 envelopes are there,

so 100×0.0228

$= 2.28 = 2$ (approx)

Que: The mean Hight of 500 students is 151 cm and the S.D 15 cm. assuming that the hight are normally distributed. Find how many students heights lies b/w 120 cm & 155 cm.

Solⁿ

Mean $\mu = 151$ cm.

S.D. $\sigma = 15$ cm.

Firstly,

$$P(\underset{a}{120} \le x \le \underset{b}{155}) = ?$$

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$Z_1 = \frac{a-\mu}{\sigma} = \frac{120-151}{15} = -2.0667$

& $Z_2 = \frac{b-\mu}{\sigma} = \frac{155-151}{15} = 0.2667$

$P(Z) = F(Z_2) - F(Z_1)$

$= F(0.2667) - F(-2.0667)$

$= F(0.2667) + F(2.0667)$

$= 0.1026 + 0.4803$

$= 0.5829$

There are Total 500 students, so 500×0.5829

$= 291.45$

$= 291$ (approx)

Que: The μ and σ of the marks obtained by 1000 students in an exam are resp. 34.4 and 16.5, Assuming the normally distribution. Find the approximate no. of students expected to obtain marks b/w 30 and 60.

Sol: $\mu = 34.4$, $\sigma = 16.5$

we have to find

$P(\underset{a}{30} \le x \le \underset{b}{60}) = ?$

$Z_1 = \frac{30-34.4}{16.5} = -0.2667$

$Z_2 = \frac{60-34.4}{16.5} = 1.5515$

$P(Z) = F(Z_2) - F(Z_1)$

$= F(1.55) - F(-0.26)$

$= 0.4394 + 0.1026$

$= 0.5420$

There are 1000 students $\Rightarrow$ $1000 \times 0.5420 = 542$ Ans.

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Que Assuming that the diameters 1000 brass plugs taken consequentively from a machine normal distribution with mean 0.7515 c.m and S.D. 0.002 cm., how many of the plugs are approved diameter is 0.752 ± 0.004 cm.

Sol^n $\mu = 0.7515$ cm, $\sigma = 0.0020$ cm.

Given, diameter

(0.752 ± 0.004) cm.

Taking (+ve) $0.752 + 0.004 = 0.756 = a$

Taking (−ve) $0.752 - 0.004 = 0.748 = b$

we have, $P(\underset{a}{0.748} \le x \le \underset{b}{0.756}) = ?$

$$Z_1 = \frac{a-\mu}{\sigma} = \frac{0.7480 - 0.7515}{0.002} = -1.75$$

$$Z_2 = \frac{b-\mu}{\sigma} = \frac{0.756 - 0.7515}{0.0020} = 2.25$$

$$P(z) = F(z_2) - F(z_1) = F(2.25) - F(-1.75)$$
$$= 0.9477$$

Total 1000 $\longrightarrow$ $0.9477 \times 1000 = 947.7$ **A.**

Que In a normal Distribution 31 % of the items are under 45 and 8 % are over 64 find the mean and S.D. of the distribution.

Imp

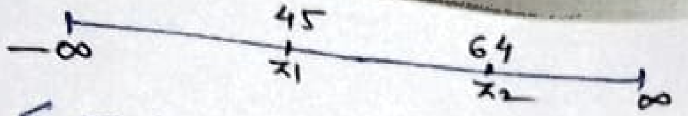
Sol^n $P(-\infty \le x \le 45) = 31\%$

$\Rightarrow P(-\infty \le x \le 45) = 0.31$ —— (1)

$P(64 \le x \le \infty) = 8\%$

$\Rightarrow P(64 \le x \le \infty) = 0.08$ —— (2)

Now we will convert variable in normal variable z such that



$(-\infty \le z \le z_1) = 0.31$ — (3) (According to eqⁿ (1))

& $(z_2 \le z \le \infty) = 0.08$ — (4) (According to eqⁿ (2))

$P(z) = F(z_2) - F(z_1) = 0.31$. [By eqⁿ (3)]

$P(z) = F(z_1) - F(-\infty) = 0.31$

$F(z_1) = 0.31 - 0.5 = -0.19$

So, $\boxed{z_1 = -0.49}$ (by Antinormal)

Now, $\cancel{P(z) = F(z_2) - F(z_1) = 0.08}$ [By eqⁿ (4)]

$P(z) = F(\infty) - F(z_2) = 0.08$

$= 0.5 - F(z_2) = 0.08$

$F(z_2) = 0.5 - 0.08$

$\boxed{z_2 = 1.40}$ (By Antinormal)

Now, $z_1 = \frac{a-\mu}{\sigma} = \frac{45-\mu}{\sigma}$

$-0.49 = \frac{45-\mu}{\sigma} \Rightarrow \mu - 0.40\sigma = 45$ — (5)

& $z_2 = \frac{b-\mu}{\sigma} = \frac{64-\mu}{\sigma}$

$1.40 = \frac{64-\mu}{\sigma} \Rightarrow 1.40\sigma = 64-\mu$

or, $\mu + 1.40\sigma = 64$ — (6)

By eqⁿ (5) & (6) :—

$$\mu - 0.40\sigma = 45$$
$$\mu + 1.40\sigma = 64$$

$$\boxed{\mu = 50 \quad \& \quad \sigma = 10}$$

Que ⟹ In an exam taken by 500 candidates, the average and S.D. 40% and 10% find approx. normal dist.

(a) How many will pass, if 50% is fixed as a min.

(b) what should be the minimum, if 350 candidates are to pass.

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(c) How many have scored marks above 60 %.

Sol^n

$\Longrightarrow$ $\mu = 0.4$ and $\sigma = 0.1$

(a) How many will pass, if 50 % is fixed as a min, $p(50\% \leq x < \infty) = ?$

$$Z_1 = \frac{a-\mu}{\sigma} = \frac{0.5-0.4}{0.1} = 1$$

$$Z_2 = \frac{b-\mu}{\sigma} = \infty$$

$$P(Z) = F(Z_2) - F(Z_1)$$
$$= F(\infty) - F(1)$$
$$= 0.5 - 0.3413 = 0.1587$$

required result is for 500 candidates

$$= 0.1587 \times 500$$
$$= 79$$

(b) What should be min if 350 candidates are to pass ?

let Z_1 be the min value which is required

$$P(Z_1 \leq Z \leq \infty) = F(Z_2) - F(Z_1)$$

Probability $= \frac{350}{500}$

$$\frac{350}{500} = F(\infty) - F(Z_1)$$

$$F(Z_1) = 0.5 - \frac{35}{50} = 0.5 - 0.7$$
$$= -0.2$$

So $Z_1 = -0.52$ (By Antinormal table)

then required result is

$$Z_1 = \frac{a-\mu}{\sigma} \Rightarrow -0.52 = \frac{a-0.4}{0.1}$$

$$a = 0.3480$$

$$\boxed{a = 35\ \%}$$

(c) How many have scored marks above 60 %

$P(60\% \le x \le \infty) = ?$

$$Z_1 = \frac{a-\mu}{\sigma} \Rightarrow \frac{0.6 - 0.4}{0.1} = \frac{0.2}{0.1}$$

$$Z_1 = 2$$

&

$$Z_2 = \frac{b-\mu}{\sigma} = \infty$$

$$P(Z) = F(Z_2) - F(Z_1)$$
$$= F(\infty) - F(2)$$
$$= 0.5 - 0.4772 = 0.0228$$

required result $= 0.0228 \times 500$

$= 11.4$ Ans

Que ⟹ Fit a normal curve for the data

x:	2	4	6	8	10
$f(x)$:	1	4	6	4	1

Sol^n ⟹ mean $\mu = \frac{\Sigma f_i x_i}{\Sigma f_i} = 6$

Formula S.D. $\sigma = \sqrt{\left\{\frac{\Sigma f_i (x_i^2)}{\Sigma f_i}\right\} - \left(\frac{\Sigma f_i x_i}{\Sigma f_i}\right)^2}$

$$= \sqrt{\frac{1(2)^2 + 4(4)^2 + 6(6)^2 + 4(8)^2 + 1(10)^2}{16} - \left(\frac{96}{16}\right)^2}$$

$= 2$

Table

Mid x	(x_1, x_2)	(z_1, z_2) * $z_1 = \frac{a-\mu}{\sigma}$ / $z_2 = \frac{b-\mu}{\sigma}$	Area under the curve * $F(z_2) - F(z_1) = P(z)$	Best fit * $P(x) \times \Sigma f_i$
2 $1 \leftarrow \rightarrow 3$	(1,3) a b	$z_1 = \frac{1-6}{2} = -2.5$ $z_2 = \frac{3-6}{2} = -1.5$	$F(-1.5) - F(-2.5)$ $= -0.4322 + 0.4938$ $= 0.0606$	16×0.0606 $= 0.97$
4 $3 \leftarrow \rightarrow 5$	(3,5)	(-1.5, -0.5)	0.2417	3.9
6 $5 \leftarrow \rightarrow 7$	(5,7)	(-0.5, 0.5)	0.383	6.1
8 $7 \leftarrow \rightarrow 9$	(7,9)	(0.5, 1.5)	0.2417	3.9
10 $9 \leftarrow \rightarrow 11$	(9,11	(1.5, 2.5)	0.606	0.97

Best fit of Given data is $\rightarrow$

x	2	4	6	8	10
f	0.97	3.9	6.1	3.9	0.97

Ans

*

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(>100) Poisson Distribution (Discrete R.V.)

1) The events which are rare.
2) which have a large no. of trials.

$$\boxed{P(X=r) = \frac{m^r e^{-m}}{r!}}$$

m = mean = np
r = prob of Success
n = no. of trials. (large)

Que ⟹ If a random var. has a poisson dist. such that $P(1) = P(2)$ find
① mean ② $P(4)$

Soln ① ⟶ $P(1) = P(2)$

$$\frac{m e^{-m}}{1!} = \frac{m^2 e^{-m}}{2!}$$

$$\boxed{m=2}$$

Soln ② ⟶ $P(4) = \frac{(2)^4 e^{-2}}{4!} = \frac{16 \times e^{-2}}{4 \times 3 \times 2} = \frac{2}{3} e^{-2}$

$= 0.09022$

Que ⟹ x is a poisson dist. and it is found that the prob. that $x=2$ is $2/3$ of the prob. that $x=1$. Find the prob that $x=0$ and the prob. that $x=3$, what is the prob. that x exceeds 3 ?

Soln ⟹

$$P(x=2) = \frac{2}{3} P(x=1)$$

$$\frac{m^2 e^{-m}}{2!} = \frac{2}{3} \frac{m^1 e^{-m}}{1!}$$

$$m = \frac{4}{3}$$

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$$P(x=0) = \frac{\left(\frac{4}{3}\right)^0 e^{-4/3}}{0!} = 0.2636$$

$$P(x=1) = \frac{\left(\frac{4}{3}\right)^1 e^{-4/3}}{1!} = 0.1041$$

exceeds more than 3 → $P(X>3)$
strictly Greater than

$$= 1 - [P_0 + P_1 + P_2 + P_3]$$

$$= 1 - \left[\left(\frac{4}{3}\right)^0 \frac{e^{-4/3}}{0!} + \left(\frac{4}{3}\right)^1 \frac{e^{-4/3}}{1!} + \left(\frac{4}{3}\right)^2 \frac{e^{-4/3}}{2!} \right]$$

$$= 0.1506$$

Que ⟹ A certain screw making machine produces on average of 2 defective screws out of 100, and packs them in boxes of 500. find the prob. that a box contains 15 defective screws.

Solve ⟹ Prob. of defectives $p = \frac{2}{100}$

$p = 0.02$

$n = 500$

mean $= np = m = 0.02 \times 500 = 10$

let r be the prob. of defectives

$$P(x=15) = \frac{m^r e^{-m}}{r!} = \frac{(10)^{15} e^{-10}}{15!}$$

$$= 0.0347$$

Que ⟹ A manufacturer knows that the condensor he makes on the average 1% defectives he packs them in boxes of 100, what is the prob. that a box picked at random will contain 3 or more faulty condensors.

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Sol^n $\Rightarrow$ $p = \frac{1}{100} = 0.01$

$n = 100$

$np = 100 \times 0.01 = 1$

$P(x \geq 3) = 1 - [P_0 + P_1 + P_2]$
at least 3

$= 1 - \left[\frac{(1)^0 e^{-1}}{0!} + \frac{(1)^1 e^{-1}}{1!} + \frac{(1)^2 e^{-1}}{2!}\right]$

$= 0.08$

Imp

Que $\Rightarrow$ A car hire firm has 2 cars which it hires out day by day. The no. of demands for a car on each day is distributed as a poisson dis. with mean 1.5 calculate the proportion of day.

rare cases

(a) on which is no demand.
(b) on which demand is refused.

Sol^n $\rightarrow$ mean $m = 1.5$

(a) prob. on which there is no demand

$P(x=0) = \frac{(1.5)^0 e^{-1.5}}{0!} = 0.2231$

(b) Prob. on which demand is refused

$= 1 - [\text{Prob. of all demands}]$

$= 1 - [\underbrace{P(x=0)}_{\text{0 demand}} + \underbrace{P(x=1)}_{\text{demand of 1 car}} + \underbrace{P(x=2)}_{\text{demand of 2 cars}}]$

$= 0.1913$

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Que ⇒ The incidence of occupational disease in an industry in such that the workmen have a 10% chance of suffering from it what is the prob. that in a grp of 7; 5 or more will suffer from it.

Sol^n ⇒ prob of suffering disease is

$$p = \frac{10}{100} = 0.1$$

$\underrightarrow{n=7}$ mean $np = 7 \times 0.1$.

Mean $m = \frac{7}{10}$

$P(x \geq 5) = P(x=5) + P(x=6) + P(x=7)$

at least 5

$$= \frac{\left(\frac{7}{10}\right)^5 e^{-7/10}}{5!} + \frac{\left(\frac{7}{10}\right)^6 e^{-7/10}}{6!} + \frac{\left(\frac{7}{10}\right)^7 \left(e^{-7/10}\right)}{7!}$$

$$= 0.0008$$

Que ⇒ If the prob. of a bad reaction from a certain infection is 0.001 determine the chance that out of 2000 individuals more than two will get a bad reaction.

Sol^n ⇒ mean $m = np = 2000 \times 0.001 = 2$

Prob. that more than 2 will get a bad reaction $= 1 - (P_0 + P_1)$

$$= 1 - \left[\frac{2^0 \times e^{-2}}{0!} + \frac{2^1 \times e^{-2}}{1!}\right]$$

$=$

V. IMP Q Best fit of poisson dist to the set of observation

x:	0	1	2	3	4
f:	122	60	15	2	1

Sol^n ⟹ Formula for Best fit :-

$$N\left[\frac{m^r e^{-m}}{r!}\right]$$

$N = \Sigma f_i = 200$

$m = \text{mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{60+30+6+4}{200}$

$m = 0.5$

$$\Rightarrow N\left[\frac{m^r e^{-m}}{r!}\right]$$

$$= 200\left[\frac{(0.5)^r e^{-0.5}}{r}\right]$$

Put $r = 0, 1, 2, 3, 4$

$f(x) = 121, 61, 15, 2, 0$

x or r :	0	1	2	3	4
f :	121	61	15	2	0

Normal Distribution (Continuous R.V)

Formula $P(X=x) = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} dx$ --- ①

But it is very difficult to evaluate. So we will change the variable in normal variable. i.e. Z.

Normal variable $Z = \frac{x-\mu}{\sigma}$ Here, $\left\{ \begin{array}{l} \mu=0 \\ \mu=1 \end{array} \right.$

So, $P(Z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-Z^2/2} dZ$

Now It is very easy to solve,

$P(a<x<b) = \int_a^b f(x)\,dx$

$P(Z_1<Z<Z_2) = \int_{Z_1}^{Z_2} f(z)\,dz$

Here, formula $\boxed{Z_1 = \frac{a-\mu}{\sigma}}$, formula $\boxed{Z_2 = \frac{b-\mu}{\sigma}}$

and formula $\boxed{F(Z) = F(Z_2) - F(Z_1)}$

Important Property :-
1) $F(-Z) = -F(Z)$
2) $F(\infty) = 0.5$
3) $F(0) = 0$

Que → For a normally distributed variate with mean 1 and S.D. 3 find, the prob that,

① $3.43 \le x \le 6.19$

② $-1.43 \le x \le 6.19$

Sol → Given, $\mu = 1$, $\sigma = 3$

ⓐ $\underset{a}{3.43} \le x \le \underset{b}{6.19}$

Converting in normal variable,
$z_1 \le z \le z_2$

$z_1 = \frac{a-\mu}{\sigma} = \frac{3.43-1}{3} = 0.81$

$z_2 = \frac{b-\mu}{\sigma} = \frac{6.19-1}{3} = 1.73$

$P(z) = F(z_2) - F(z_1)$
$= F(1.73) - F(0.81)$
$= 0.4582 - 0.2910$
$= 0.1672$ Ans.

(b)
$-1.43 \le x \le 6.19$
$\quad a \qquad\qquad b$

$z_1 = \frac{a-\mu}{\sigma} = \frac{-1.43-1}{3} = -0.81$

$z_2 = \frac{b-\mu}{\sigma} = \frac{6.19-1}{3} = 1.73$

$P(z) = F(z_2) - F(z_1)$
$= F(1.73) - F(-0.81)$
$= 0.4582 + 0.2910$
$= 0.7492$ Ans.

Que If z is normally distributed with mean 0 and variance 1, find
(a) $Pr(z \le -1.64)$

Table VI : Area under the Normal curve

z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993