5. (*a*) If X is a continuous random variable with probability density function given by

$$f(x) = kx \quad , \quad 0 \le x < 2$$
$$= 2x \quad , \quad 2 \le x < 4$$
$$= -kx + 6k \,, \quad 4 \le x < 6$$

find the value of k. [2]

(*b*) A car-hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days on which neither car is used and the proportion of days on which some demand is refused.

(Given $e^{-1.5} = 0.2231$) [7]

(*c*) The probability density $P(x)$ of a continuous random variable is given by

$$P(x) = y_0 e^{-|x|} \; dx \,, \quad -\infty < x < \infty$$

Prove that $y_0 = \frac{1}{2}$, $\mu_1' = 0$, $\sigma = \sqrt{2}$ and mean deviation about mean is 1. [7]

(*d*) Fit Poisson's distribution to the following and calculate theoretical frequencies ($e^{-0.5} = 0.61$) : [7]

Deaths	0	1	2	3	4
Frequency	122	60	15	2	1

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Roll No. ....

322351(14)

BE (3rd Semester)

Examination, April-May, 2018

(New Scheme)

Mathematics–III

Time Allowed : 3 hours *Maximum Marks* : 80

Minimum Pass Marks : 28

Note : (*i*) Part (*a*) of each question is compulsory. Attempt any **two** parts from (*b*), (*c*) and (*d*) of each question.

(*ii*) The figures in the right-hand margin indicate marks.

1. (*a*) Write Fourier series of even and odd functions. [2]

(*b*) Find a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$. Hence show that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots = \frac{\pi^2}{12}$$ [7]

(*c*) Obtain Fourier's series in the interval $(-\pi, \pi)$ for the function $f(x) = x\cos x$. [7]

(*d*) The following table gives the variations of periodic current over a period :

t (sec)	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
A (amp)	1.98	1.30	1.05	1.30	−0.88	−0.25	1.98

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic. [7]

2. (*a*) Find the Laplace transform of the function $F(t) = e^{at}$. [2]

(*b*) Find the Laplace transform of the following : [7]

(*i*) $te^{-4t} \sin 4t$

(*ii*) $\dfrac{1-e^{-t}}{t}$

(*c*) Find the following : [7]

(*i*) $L^{-1}\left\{\dfrac{3s+2}{4s^2+12s+9}\right\}$

(*ii*) $L^{-1}\left\{\dfrac{1}{(s+a)^2}\right\}$

(*d*) Solve by Laplace transform

$$(D^2-3D+2)y = e^{3t},\ y(0)=1,\ y'(0)=0$$ [7]

3. (*a*) Write the necessary conditions for $f(z)$ to be analytic in Cartesian co-ordinates. [2]

(*b*) If $w = \phi + i\psi$ represents the complex potential for an electric field and $\psi = x^2 - y^2 + \dfrac{x}{x^2+y^2}$, determine the function ϕ. [7]

(*c*) Find the Laurent's series expansion of $f(z) = \dfrac{7z-2}{(z+1)\,z(z-2)}$ in the region $1 < z+1 < 3$. [7]

(*d*) Apply calculus of residues to prove that

$$\int_0^{\pi} \frac{a\,d\theta}{1+2a^2-\cos 2\theta} = \frac{\pi}{\sqrt{1+a^2}}$$ [7]

4. (*a*) Find the partial differential equation by eliminating the arbitrary functions from the relation

$$z = f(x-at) + \phi(x+at)$$ [2]

(*b*) Solve $y^2p - xyq = x(z-2y)$. [7]

(*c*) Solve $(D^2 - 2DD' + D'^2)z = 12xy$. [7]

(*d*) Using the method of separation of variables, solve

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$$

given that $u(x, 0) = 6e^{-3x}$. [7]