(*d*) Using the method of separation of variables, solve

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$$

given that $u(x, 0) = 6e^{-3x}$. [7]

5. (*a*) Define the terms random variable and probability density function. [2]

(*b*) A random variable X has the density function

$$\begin{aligned} f(x) &= ax, \quad 0 \le x < 1 \\ &= a, \quad 1 \le x \le 2 \\ &= -ax + 3a, \quad 2 < x \le 3 \\ &= 0, \text{ otherwise} \end{aligned}$$

Determine a and compute $P(X \le 1.5)$. [7]

(*c*) The frequency distribution of a measurable characteristic varying between 0 and 2 is as under :

$$\begin{aligned} f(x) &= x^3 \quad , \quad 0 \le x \le 1 \\ &= (2-x)^3 , \quad 1 \le x \le 2 \end{aligned}$$

Calculate the standard deviation and also the mean of deviation about the mean. [7]

(*d*) Find the probability that at most 5 defective fuses will be found in a box of 200 fuses, if experience shows that 2 percent of such fuses are defective. [7]

CSE

Printed Pages : 4

Roll No. ....

322311(14)

BE (3rd Semester)

Examination, April-May, 2018

(Old Scheme)

Mathematics–III

Time Allowed : 3 hours *Maximum Marks* : 80

Minimum Pass Marks : 28

Note : (*i*) Attempt **all** questions. Part (*a*) of each question is compulsory. Attempt any **two** parts from (*b*), (*c*) and (*d*) of each question.

(*ii*) The figures in the right-hand margin indicate marks.

1. (*a*) What are the conditions of function $f(x)$, that can be developed as a Fourier series? [2]

(*b*) Find a series of sines and cosines of multiples of x which will represent $f(x)$ in the interval $(-\pi, \pi)$ when

$$f(x) = \begin{cases} 0 \quad , & -\pi < x < 0 \\ \frac{1}{4}\pi x , & 0 < x < \pi \end{cases}$$

Hence deduce that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots$$

[7]

(*c*) Find the Fourier series for the function $f(x)$ defined by

$$f(x)=\begin{cases}-1 & , \quad -3<x<0 \\ 0 & , \quad x=0 \\ 1 & , \quad 0<x<3\end{cases}$$ [7]

(*d*) The turning moment T on the crankshaft of a steam engine for the crank angle θ degree is given as :

θ	0	15	30	45	60	75	90	105	120	135	150	165	180
T	0	2.7	5.2	7.0	8.1	8.3	7.9	6.8	5.5	4.1	2.6	1.2	0

Expand T in a series of sine up to the second harmonics. Calculate T for $\theta = 75°$. [7]

2. (*a*) Write the convolution theorem for inverse Laplace transform. [2]

(*b*) Find the Laplace transform of : [7]

(*i*) $f(t)=t\sin^2 t$, (*ii*) $f(t)=t^2e^{-at}$ and

(*iii*) $f(t)=\dfrac{\sinh t}{t}$.

(*c*) Evaluate the following : [7]

(*i*) $L^{-}\left\{\dfrac{1}{s}\right\}$, (*ii*) $L^{-1}\left\{\dfrac{1}{s-a}\right\}$, (*iii*) $L^{-1}\left\{\dfrac{1}{s^2+a^2}\right\}$

(*d*) Using Laplace transform, solve

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=e^{3x},\ y(0)=1,\ \left(\frac{dy}{dx}\right)_{x=0}=0$$ [7]

3. (*a*) Define analytic function. [2]

(*b*) State Cauchy-Riemann equations. If $w = \phi + i\psi$ represents the complex potential for an electric field and $\psi = x^2 - y^2 + \dfrac{x}{x^2+y^2}$, determine the function $\phi\,(x, y)$. [7]

(*c*) Using Cauchy's integral formula, evaluate

$$\int_C \frac{e^{2z}}{(z-1)(z-2)}dz$$

where C is the circle $|z| = 3$. [7]

(*d*) Expand $f(z)=\dfrac{1}{(z+1)(z+3)}$ in a Laurent's and Taylor series valid for the regions

(*i*) $|z|<1$, (*ii*) $1<|z|<3$, (*iii*) $|z|>3$. [7]

4. (*a*) Form a partial differential equation by eliminating the arbitrary function f from the relation

$$z=y^2+2f\left(\frac{1}{x}+\log y\right)$$ [2]

(*b*) Solve : [7]

$$(y^3x-2x^4)p+(2y^4-x^3y)q=9z(x^3-y^3)$$

(*c*) Solve

$$\left(2\frac{\partial^2}{\partial x^2}-5\frac{\partial^2}{\partial y\partial x}+2\frac{\partial^2}{\partial y^2}\right)z=24(y-x)$$ [7]