Variety

Que→ Evaluate, $\int_0^\infty t\, e^{-2t} \cos t \, dt$

Sol^n → ~~Here~~, ~~S=~~

Since, $L\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$ ①

By comparing given integral with ① :

$$s = 2 \quad \text{and} \quad f(t) = t\cos t$$

Now, we have to ~~to~~ find

$$\bar{f}(s) = L\{f(t)\} = L[t\cos t]$$

$$= (-1)^1 \frac{d^1}{ds^1}\left[\frac{s}{s^2+1}\right]$$

$$= -\left[\frac{(s^2+1)\cdot 1 - s(2s)}{(s^2+1)^2}\right]$$

$$= -\left[\frac{s^2+1-2s^2}{(s^2+1)^2}\right]$$

$$= -\left[\frac{-s^2+1}{(s^2+1)^2}\right]$$

put $\xrightarrow{s=2}$ $= -\left[\frac{-(2)^2+1}{(2^2+1)^2}\right]$

$$= -\left[\frac{-4+1}{(5)^2}\right]$$

$$= 3/25$$

Q① → Find L.T. of given Periodic fuⁿ.

$$f(t) = \begin{cases} \sin \omega t, & 0 < t < \pi/\omega \\ 0, & \pi/\omega < t < 2\pi/\omega \end{cases}$$

Solⁿ ⟹ $L\{f(t)\} = \dfrac{\int_0^T e^{-st} f(t)\,dt}{1-e^{-sT}}$

Periodic fuⁿ

Here, T = Total period $= 2\pi/\omega$.

$$= \frac{1}{1-e^{-2\pi s/\omega}} \left[\int_0^{\pi/\omega} e^{-st} \sin \omega t \, dt + \int_{\pi/\omega}^{2\pi/\omega} e^{-st} \cdot 0 \, dt \right]$$

$$= \frac{1}{1-e^{-2\pi s/\omega}} \left[\int_0^{\pi/\omega} e^{-st} \sin \omega t \, dt \right]$$

$$= \frac{1}{1-e^{-2\pi s/\omega}} \left[\frac{e^{-st}}{s^2+\omega^2} \{ -s \sin \omega t - \omega \cos \omega t \} \right]_0^{\pi/\omega}$$

$$= \frac{1}{1-e^{-2\pi s/\omega}} \left[\frac{e^{-s\pi/\omega}}{s^2+\omega^2} \{ -s \sin \omega \times \pi/\omega - \omega \cos \omega \times \pi/\omega \} - \frac{e^{-s \times 0}}{s^2+\omega^2} \{ -s \sin 0\omega - \omega \cos 0\omega \} \right]$$

$$= \frac{1}{1-e^{-2\pi s/\omega}} \left[\frac{e^{-s\pi/\omega}}{s^2+\omega^2} \{ \omega \} - \frac{1}{s^2+\omega^2} \{ -\omega \} \right]$$

$$= \frac{1}{1-e^{-2\pi s/\omega}} \left[\frac{\omega}{s^2+\omega^2} \{ e^{-s\pi/\omega} + 1 \} \right]$$

$$= \frac{\omega \left(1+e^{-s\pi/\omega}\right)}{(s^2+\omega^2)\left(1-e^{-2\pi s/\omega}\right)}$$

Que $L^{-1}\left[\frac{s^2+s-2}{s(s+3)(s-2)}\right]$

Sol^n $\Rightarrow$ By Using Partial fraction :

$$\frac{s^2+s-2}{s(s+3)(s-2)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s-2}$$

$$s^2+s-2 = A(s+3)(s-2) + Bs(s-2) + Cs(s+3)$$

Put $s=0$, $s=-3$, $s=2$

$\underrightarrow{s=0}$ $-2 = -6A \Rightarrow \boxed{A = 1/3}$

$\underrightarrow{s=-3}$ $9-3-2 = -3B\times -5$

$4 = 15B \Rightarrow \boxed{B = \frac{4}{15}}$

$\underrightarrow{s=2}$ $4+\cancel{2}-\cancel{2} = 2C\times 5$

$4 = 10C \Rightarrow C = \frac{\cancel{4}^{2}}{\cancel{10}_{5}} = \frac{2}{5}$

$\boxed{C = \frac{2}{5}}$

$$= \frac{1/3}{s} + \frac{4/15}{s+3} + \frac{2/5}{s-2}$$

$$L^{-1}\left[\frac{1}{3s} + \frac{4}{15(s+3)} + \frac{2}{5(s-2)}\right] =$$

$$\frac{1}{3} + \frac{4}{15}e^{-3t} + \frac{2}{5}e^{2t}$$

* Convolution Theorem *

Convolution means "Extremly complex" or "Twisted" in the complex way.

Imp

Statement → If $L^{-1}\bar{f}(s) = f(t)$ and $L^{-1}\bar{g}(s) = g(t)$

Then

$$\boxed{L^{-1}\{\bar{f}(s)\cdot\bar{g}(s)\} = \int_0^t f(u)\cdot g(t-u)\,du}$$

Application of L.T. – 2nd Form

Que ① $ty'' + 2y' + ty = \sin t$

when $y(0) = 1$.

Sol^n Taking L.T. on both sides;

$\Rightarrow -\frac{d}{ds}[L(y'')] + 2L(y') + t\,L(y) = L(\sin t)$

$\Rightarrow -\frac{d}{ds}[s^2\bar{y} - sy(0) - y'(0)] + 2[s\bar{y} - y(0)]$

$-\frac{d\bar{y}}{ds} = \frac{1}{s^2+1}$ $\left\{\begin{array}{l}\text{Given, } y(0)=1 \\ \& \ y'(0) = A \text{ (let)}\end{array}\right.$

$\Rightarrow -\frac{d}{ds}[\underset{I}{s^2}\underset{II}{\bar{y}} - s - A] + 2[s\bar{y} - y(0)] - \frac{d\bar{y}}{ds} = \frac{1}{s^2+1}$

$\Rightarrow -\left[s^2\frac{d\bar{y}}{ds} + \bar{y}(2s) - 1\right] + 2s\bar{y} - 2 - \frac{d\bar{y}}{ds} = \frac{1}{s^2+1}$

$\Rightarrow \frac{d\bar{y}}{ds}[-s^2 - 1] + \bar{y}[\cancel{-2s} + \cancel{2s}] + 1 - 2 = \frac{1}{s^2+1}$

$\Rightarrow \frac{d\bar{y}}{ds}(-s^2 - 1) - 1 = \frac{1}{s^2+1}$

$\Rightarrow -\frac{d\bar{y}}{ds} = \frac{1}{(s^2+1)^2} + \frac{1}{(s^2+1)}$ —— (1)

** $-\frac{d\bar{y}}{ds} = ?$

classmate

Date ____

Page ____

we know that multiplication by t :—

$L[t f(t)] = -\frac{d}{ds}\bar{f}(s)$

or $\rightarrow$ $t f(t) = L^{-1}\left(\frac{-d\bar{f}(s)}{ds}\right)$

So we will put $-\frac{d\bar{y}}{ds} = t y(t)$ — (2)

By (1) $\rightarrow$ Taking Inverse L.T :—

$L^{-1}\left[\frac{-d\bar{y}}{ds}\right] = L^{-1}\frac{1}{(s^2+1)^2} + \frac{1}{s^2+1}$

by(2) $\rightarrow$ $t y(t) = L^{-1}\frac{1}{(s^2+1)^2} + \frac{1}{s^2+1}$

* Formula $\rightarrow$ $\boxed{L^{-1}\frac{1}{(s^2+a^2)^2} = \frac{1}{2a^3}\left(\sin at - at\cos at\right)}$

So,

$t y(t) = \left\{\frac{1}{2}(\sin t - t\cos t)\right\} + \sin t$

$t y(t) = \frac{3}{2}\sin t - \frac{t\cos t}{2}$

therefore, $y(t) = \frac{1}{2t}\left[3\sin t - t\cos t\right]$

Imp

Que $\Rightarrow$ $ty'' + (1-2t)y' - 2y = 0$

when $y'(0) = 1$, $y(0) = 2$.

Sol^n $\Rightarrow$ $ty'' + (1-2t)y' - 2y = 0$

Taking L.T. on both sides;

$\Rightarrow -\frac{d}{ds} L(y'') + L(y') + 2\frac{d}{ds} L(y') - 2L(y) = 0$

$\Rightarrow -\frac{d}{ds}\left[s^2\bar{y} - s\,y(0) - y'(0)\right] + \left[s\bar{y} - y(0)\right] + 2\frac{d}{ds}\left[s\bar{y} - y(0)\right] - 2\bar{y} = 0$

$\Rightarrow -\frac{d}{ds}\left[\underset{I}{s^2}\underset{II}{\bar{y}} - 2s - 1\right] + (s\bar{y} - 2) + 2\frac{d}{ds}\left[\underset{I}{s}\underset{II}{\bar{y}} - 2\right] - 2\bar{y} = 0$

$\Rightarrow -\left[s^2\frac{d\bar{y}}{ds} + 2s\bar{y} - 2\right] + s\bar{y} - 2 + 2\left[s\frac{d\bar{y}}{ds} + \bar{y}\right] - 2\bar{y} = 0$

$\Rightarrow \frac{d\bar{y}}{ds}\left[-s^2 + 2s\right] + \bar{y}\left[-2s + s + \cancel{2} - \cancel{2}\right] + \cancel{2} - \cancel{2} = 0$

$\Rightarrow (2s - s^2)\frac{d\bar{y}}{ds} - s\bar{y} = 0$

$\Rightarrow \frac{d\bar{y}}{ds} = \frac{s\bar{y}}{2s - s^2}$

$\Rightarrow \frac{d\bar{y}}{ds} = \frac{\bar{y}}{2-s}$ This is form of

* Linear diff eq^n of Ist order

* $\frac{dy}{dx} + Py = Q$

I.F. $\rightarrow$ $e^{\int P\,dx}$

Sol^y $\rightarrow$ $y(\text{I.F}) = \int Q(\text{I.F}) + c$

$\Rightarrow \frac{d\bar{y}}{ds} - \frac{1}{2-s}\bar{y} = 0$

or $\Rightarrow$ $\frac{d\bar{y}}{ds} + \frac{1}{s-2}\bar{y} = 0$

$\Rightarrow$ I.F. $\rightarrow$ $e^{\int \frac{1}{s-2}ds} \Rightarrow e^{\log(s-2)} = (s-2)$

we can also solve this by variable separable method.

Sol^n $\bar{y} \cdot (s-2) = \int 0 (s-2) + c$

or $\bar{y}(s-2) = c$

$\bar{y} = \frac{c}{s-2}$ Taking Inverse L.T.

$\boxed{y = ce^{2t}}$ —— ①

Now we have to find $c = ?$

Given, $y(0) = 2$ putting this value in ①

$2 = ce^0 \Rightarrow \boxed{c = 2}$ Put $c = 2$ in ①

$\boxed{y = 2e^{2t}}$ Ans..

Dirichlet Condition $\longrightarrow$ A function is defined in the interval $(-\pi$ to $\pi)$ can be expressed in the fourier series. If the following conditions are satisfy in the interval $(-\pi$ to $\pi)$

(1) $f(x)$ is peridic, ~~sig~~ single valued and finite.

(2) $f(x)$ ~~is~~ has finite no. of discontinuity.

(3) $f(x)$ has at most finite no. of maxima and minima.

$$* \quad 2\sin A \cdot \cos B = \sin(A+B) + \sin(A-B)$$
$$2\sin A \cdot \sin B = \cos(A-B) - \cos(A+B)$$

classmate
Date
Page

Que (2) $f(x) = x\sin x$, $(0 \text{ to } 2\pi)$
Expand the fu^n by fourier series.

Sol^n $f(x) = x\sin x = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$ —— (1)

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$
$$= \frac{1}{\pi}\int_0^{2\pi} x\sin x\,dx \Rightarrow \frac{1}{\pi}\int_0^{2\pi} \underset{u}{x}\ \underset{v}{\sin x}\,dx$$

Formula $\int uv = uv_1 - u'v_2 + u''v_3 - u'''v_4 \ldots\ldots \infty$

$$\Rightarrow \frac{1}{\pi}\left[x(-\cos x) - (-\sin x)\right]_0^{2\pi}$$
$$= \frac{1}{\pi}\left[-2\pi\right] = -2 \quad \text{—— (2)}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$
$$= \frac{1}{\pi}\int_0^{2\pi} x\sin x\cos nx\,dx$$
$$= \frac{1}{2\pi}\int_0^{2\pi} x(2\sin x\cos nx)\,dx$$
$$= \frac{1}{2\pi}\int_0^{2\pi} x\left[\sin(1+n)x + \sin(1-n)x\right]dx$$
$$= \frac{1}{2\pi}\left[\int_0^{2\pi} \underset{u}{x}\ \underset{v}{\sin(1+n)x}\,dx + \int_0^{2\pi} \underset{u}{x}\ \underset{v}{\sin(1-n)x}\,dx\right]$$
$$= \frac{1}{2\pi}\left[x\left(\frac{-\cos(1+n)x}{1+n}\right) - \left(\frac{-\sin(1+n)x}{(1+n)^2}\right) + x\left(\frac{-\cos(1-n)x}{1-n}\right) - \left(\frac{-\sin(1-n)x}{(1+n)^2}\right)\right]_0^{2\pi}$$
$$= \frac{1}{2\pi}\left[2\pi\left(\frac{-1}{1+n}\right) + 2\pi\left(\frac{-1}{1-n}\right)\right]$$

Formula
$\cos(2\pi+\theta) = \cos\theta$
$\cos(2\pi-\theta) = \cos\theta$
$\sin(2\pi+\theta) = \sin\theta$
$\sin(2\pi-\theta) = -\sin\theta$

classmate

Date ____
Page ____

$$= -\left\{\frac{1}{1+n} + \frac{1}{1-n}\right\}$$

$$= -\left\{\frac{1-n+1+n}{1-n^2}\right\} = -\left\{\frac{2}{1-n^2}\right\}$$

$$= \frac{-2}{1-n^2} \quad , \text{ Here } n \neq 1$$

when $n=1$ $\longrightarrow$ $a_1 = \frac{1}{\pi}\int_0^{2\pi} x \sin x (\cos x)\, dx$

$$a_1 = \frac{1}{2\pi}\int_0^{2\pi} x\,(2\sin x \cos x)\, dx$$

$$a_1 = \frac{1}{2\pi}\int_0^{2\pi} \underset{u}{x}\, \underset{v}{\sin 2x}\, dx$$

$$a_1 = \frac{1}{2\pi}\left[x\left(\frac{-\cos 2x}{2}\right) - \left(\frac{-\sin 2x}{4}\right)\right]_0^{2\pi}$$

$$a_1 = \frac{1}{2\pi}\left[\frac{-2\pi}{2}\right] = -1/2 \quad \text{———} (3)$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x \sin x \sin nx\, dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi} x\,(2\sin x \cdot \sin nx)\, dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi} x\left\{\cos(1-n)x - \cos(1+n)x\right\} dx$$

$$= \frac{1}{2\pi}\left[\int_0^{2\pi} \underset{u}{x}\, \underset{v}{\cos(1-n)x}\, dx - \int_0^{2\pi} \underset{u}{x}\, \underset{v}{\cos(1+n)x}\, dx\right]$$

$$= \frac{1}{2\pi}\left[x\left\{\frac{\sin(1-n)x}{1-n}\right\} - \left\{\frac{-\cos(1-n)x}{(1-n)^2}\right\} - x\left\{\frac{\sin(1+n)x}{1+n}\right\} - \left\{\frac{-\cos(1+n)x}{(1+n)^2}\right\}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[\cancel{\left\{\frac{1}{(1-n)^2} + \frac{1}{(1+n)^2}\right\}} - \cancel{\left\{\frac{1}{(1-n)^2} + \frac{1}{(1+n)^2}\right\}}\right]$$

$$= 0 \quad \text{———} (4)$$

$$b_1 = \frac{1}{\pi}\int_0^{2\pi} x \sin x \cdot \sin x\, dx = \frac{1}{\pi}\int_0^{2\pi} x \sin^2 x\, dx$$

$$= \frac{1}{\pi}\left[\int_0^{2\pi} x\left\{\frac{1-\cos 2x}{2}\right\} dx\right.$$

$$= \frac{1}{2\pi}\left[\int_0^{2\pi} x\, dx - \int_0^{2\pi} \underset{u}{x}\, \underset{v}{\cos 2x}\, dx\right]$$

$$= \frac{1}{2\pi}\left[\frac{x^2}{2} - \left\{x\left(\frac{\sin 2x}{2}\right) - \left(\frac{-\cos 2x}{4}\right)\right\}\right]_0^{2\pi}$$

$$\cancel{= \frac{1}{2\pi}\left[\left(\frac{4\pi^2}{2} - \frac{\sin 2x}{8}\right) - \left(\frac{1}{4}\right)\right]}$$

$$= \frac{1}{2\pi}\left[\frac{4\pi^2}{2} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}}\right]$$

$$= \pi \text{ ——— } (5)$$

Put the values of (2), (3), (4) and (5) in (1):—

$$-\tfrac{1}{2}\cos x$$

$$f(x) = x\sin x = -1 + \sum_{n=2}^{\infty}\left\{\frac{-2}{1-n^2}\cos nx\right\} + \pi\sin x$$

$$= -1 - \frac{1}{2}\cos x + \frac{2}{3}\cos 2x + \frac{2}{10}\sin 3x + \text{---}\infty + \pi\sin x$$

Q ⇒ $f(x) = |x|$, $-\pi$ to π also P.T. $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \infty$

Solⁿ $f(x) = \begin{cases} -x, & -\pi \text{ to } 0 \\ x, & 0 \text{ to } \pi \end{cases}$

$|x|$ is an even fuⁿ. So we have to find a_0 and a_n, $b_n = 0$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} |x|\,dx = \frac{2}{\pi}\int_0^{\pi} x\,dx$$ (By Definite Integral

$$= \frac{2}{\pi}\left(\frac{x^2}{2}\right)_0^{\pi} = \pi$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} |x|\cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x\cos nx\,dx$$ by definite Integral

$$= \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(\frac{-\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right] = \frac{2}{\pi n^2}\{(-1)^n - 1\}$$ $\forall\, n = 1, 2, 3 \cdots$

So, $$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{\pi n^2}\{(-1)^n - 1\}\cos nx$$

$$= \frac{\pi}{2} - \frac{4}{\pi}\cos x - \frac{4}{9\pi}\cos 3x - \frac{4}{25\pi}\cos 5x \cdots \infty$$

$$= \frac{\pi}{2} - \frac{4}{\pi}\left[\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \cdots \infty\right] \quad (1)$$

Put $x = 0$ ⇒ Point of Discontinuity :-

A.M. $$f(x) = \frac{f(x-0) + f(x+0)}{2}$$

$$f(x) = 0$$

By (1) ⇒ $$0 = \frac{\pi}{2} - \frac{4}{\pi}\left[1 + \frac{1}{9} + \frac{1}{25} + \cdots \infty\right]$$

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\cdots \infty$$

Hence Proved

Date
Page

Q $\Rightarrow$ $f(x) = \pi x \quad , 0<x<1$
$\pi(2-x) \quad , 1<x<2$

also show that,

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \infty$$

Sol^n $\Rightarrow$

$$f(x) = \pi x \quad , 0<x<1$$
$$\pi(2-x) \quad , 1<x<2$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$2l$ = Total length
$2l = 2 \Rightarrow \boxed{l=1}$

$$a_0 = \frac{1}{1}\left[\int_0^1 \pi x\,dx + \int_1^2 \pi(2-x)\,dx\right]$$
$$= \pi\left[\left\{\frac{x^2}{2}\right\}_0^1 + \left(2x - \frac{x^2}{2}\right)_1^2\right]$$
$$= \pi\left[\frac{1}{2} + \left\{(4-2) - \left(2-\frac{1}{2}\right)\right\}\right]$$
$$= \pi$$

$$a_n = \frac{1}{1}\left[\int_0^1 \pi x \cos\frac{n\pi x}{1}\,dx + \int_1^2 \pi(2-x)\cos\frac{n\pi x}{1}\,dx\right]$$
$$= \pi\left[\left\{x\left(\frac{\sin n\pi x}{n\pi}\right) - \left(-\frac{\cos n\pi x}{n^2\pi^2}\right)\right\}_0^1 + \left\{(2-x)\left(\frac{\sin n\pi x}{n\pi}\right) - (-1)\left(-\frac{\cos n\pi x}{n^2\pi^2}\right)\right\}_1^2\right]$$
$$= \pi\left[\frac{(-1)^n}{n^2\pi^2} - \frac{1}{n^2\pi^2} - \frac{1}{n^2\pi^2} + \frac{(-1)^n}{n^2\pi^2}\right]$$
$$= \frac{2\pi}{n^2\pi^2}\left\{(-1)^n - 1\right\}$$
$$= \frac{2}{n^2\pi}\left\{(-1)^n - 1\right\} \quad \forall\, n = 1, 2, 3 \cdots$$

Date
Page

$$b_n = \frac{1}{1}\left[\int_0^1 \pi x \sin\frac{n\pi x}{1}\,dx + \int_1^2 \pi(2-x)\sin\frac{n\pi x}{1}\,dx\right]$$

$$b_n = \pi\left[\left\{x\left(\frac{-\cos n\pi x}{n\pi}\right) - \left(\frac{-\sin n\pi x}{n^2\pi^2}\right)\right\}_0^1 + \left\{(2-x)\left(\frac{-\cos n\pi x}{n\pi}\right) - (-1)\left(\frac{-\sin n\pi x}{n^2\pi^2}\right)\right\}_1^2\right]$$

$$= \pi\left[\cancel{\frac{-(-1)^n}{n\pi}} + \cancel{\frac{(-1)^n}{n\pi}}\right]$$

$$= 0$$

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2\{(-1)^n - 1\}}{n^2\pi}\cos n\pi x$$

$$= \frac{\pi}{2} + \frac{-4}{\pi}\cos\pi x - \frac{4}{\pi 3^2}\cos 3\pi x - \frac{4}{\pi 5^2}\cos 5\pi x - - - \infty$$

$$= \frac{\pi}{2} - \frac{4}{\pi}\left[\cos\pi x + \frac{\cos 3\pi x}{3^2} + \frac{\cos 5\pi x}{5^2} + - - - \infty\right] \quad (1)$$

put $x=0$ $\Longrightarrow$ then $f(x) = \pi x$
$= \pi \times 0 = 0$

So put $x=0$ & $f(x)=0$ in eqn (1):

$$0 = \frac{\pi}{2} - \frac{4}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + - - - \infty\right]$$

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + - - - \infty$$

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classmate

Date ______
Page ______

Que ②

x :	0	1	2	3	4	5
y :	4	8	15	7	6	2

Taking $\theta = 60^\circ$ Find 1^{st} 3 terms of cosine series.

$\theta =$	0	60°	120°	180°	240°	300°
$x =$	0	1	2	3	4	5
$y =$	4	8	15	7	6	2

$\boxed{N = 6}$ we will form following tables

$\cos\theta$	$\cos 2\theta$	$\cos 3\theta$	$y\cos\theta$	$y\cos 2\theta$	$y\cos 3\theta$
			Σ	Σ	Σ

$a_0 = 2 \times \frac{\Sigma y}{N}$, $a_1 = 2 \times \frac{\Sigma y \cos\theta}{N}$,

$a_2 = 2 \times \frac{\Sigma y \cos 2\theta}{N}$, $a_3 = 2 \times \frac{\Sigma y \cos 3\theta}{N}$

classmate
Date
Page

Que $\Rightarrow$ t(sec) : 0, T/6, T/3, T/2, 2T/3, 5T/6, T

A (AMP) : 1.98, 1.30, 1.05, 1.30, −0.88, −0.25, 1.98

Find I^{st} Harmonic. Also find Amplitude of I^{st} harmonic

Sol^n $\Rightarrow$ $2c = T \Rightarrow c = T/2$

$$A = \frac{a_0}{2} + a_1 \cos\frac{\pi t}{T/2} + b_1 \sin\frac{\pi t}{T/2}$$

* last term i.e. $t = T$ gives repeated value, so we skip this.

$$A = \frac{a_0}{2} + a_1 \cos\frac{2\pi t}{T} + b_1 \sin\frac{2\pi t}{T}$$

$\boxed{N = 6}$

$$a_0 = 2 \times \frac{\Sigma A}{N}, \quad a_1 = 2 \times \frac{\Sigma A \cos\frac{2\pi t}{T}}{N}$$

$$b_1 = 2 \times \frac{\Sigma A \sin\frac{2\pi t}{T}}{N}$$

Tables $\Rightarrow$

t	A	$\frac{2\pi t}{T}$	$\cos\frac{2\pi t}{T}$	$A\cos\frac{2\pi t}{T}$	$\sin\frac{2\pi t}{T}$	$A\sin\frac{2\pi t}{T}$

Amplitude of I^{st} harmonic $= \sqrt{a_1^2 + b_1^2}$

$= 1.072$

Q $z = y f(x) + x g(y)$ — 1)

$p = \frac{\partial z}{\partial x} = y f'(x) + g(y)$ — 2)

$q = \frac{\partial z}{\partial y} = f(x) + x g'(y)$ — 3)

$r = \frac{\partial^2 z}{\partial x^2} = y f''(x)$ — 4)

$t = \frac{\partial^2 z}{\partial y^2} = x g''(y)$ — 5)

$s = \frac{\partial^2 z}{\partial x \partial y} = f'(x) + g'(y)$ — 6)

classmate

Date

Page

By eqⁿ (6) $\longrightarrow$ $S = f'(x) + g'(y)$

By eqⁿ (2), $f'(x) = \dfrac{p - g(y)}{y}$

& By eqⁿ (3) $\longrightarrow$ $g'(y) = \dfrac{q - f(x)}{x}$

Putting the values of $f'(x)$ & $g'(x)$ in S.

$$S = \frac{p - g(y)}{y} + \frac{q - f(x)}{x}$$

$$S = \frac{xp - xg(y) + qy - yf(x)}{xy}$$

$$Sxy = xp + qy - (xg(y) + yf(x))$$

(By (1))

$$\boxed{Sxy = xp + qy - Z}$$ Ans

classmate
Date
Page

Imp

Que (7) $(mz-ny)\frac{\partial z}{\partial x} + (nx-lz)\frac{\partial z}{\partial y} = ly-mx$

Sol → $(mz-ny)p + (nx-lz)q = ly-mx$

This is Laungrange's LDE;

$$\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx}$$

By Taking multiplier, x, y, z

$$\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx} = \frac{xdx+ydy+zdz}{mxz-nxy+nxy-lyz+lzy-mxz}$$

or → $\frac{xdx+ydy+zdz}{0}$

IV

By Taking new member;
$xdx+ydy+zdz=0$

By Integration, $\boxed{\frac{x^2}{2}+\frac{y^2}{2}+\frac{z^2}{2}=c_1}$

again, By Taking multiplier, l, m, n.

$$\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx} = \frac{ldx+mdy+ndz}{mlz-nly+nmx-lmz+lny-mnx}$$

or → $\frac{lx+my+nz}{0}$

IV

By taking new member;
~~$lx+my+nz=0$~~
$lxdx+mydy+nzdz=0$

By Integration; $\boxed{\frac{lx^2}{2}+m\frac{y^2}{2}+n\frac{z^2}{2}=c_2}$

So, $\phi\left\{\frac{x^2}{2}+\frac{y^2}{2}+\frac{z^2}{2}, \frac{lx^2}{2}+\frac{my^2}{2}+\frac{nz^2}{2}\right\}=0$

Q (2) Solve $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given that $u(0,y) = 3e^{-y} - e^{-5y}$ by method of Separation of variable.

Sol^n

$\Longrightarrow$ Given, $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ ------ (1)

let, trial sol^n of, ~~$4X'Y + XY' =$~~

$u(x,y) = X(x) \cdot Y(y)$ ------ (2)

$\xrightarrow{\text{diff}^n \text{ w.r.t } x}$ $\frac{\partial u}{\partial x} = X'Y$ ------ (3)

$\xrightarrow{\text{diff}^n \text{ w.r.t } y}$ $\frac{\partial u}{\partial y} = XY'$ ------ (4)

Putting the eq^n of (2), (3), (4) in eq^n (1):—

$$4X'Y + XY' = 3XY$$

$$4X'Y = 3XY - XY'$$

$$4X'Y = X(3Y - Y')$$

$$\underbrace{\frac{4X'}{X}}_{\mathrm{I}} = \underbrace{\frac{3Y - Y'}{Y}}_{\mathrm{II}} = \underbrace{K \text{ (let)}}_{\mathrm{III}}$$

$\xrightarrow{\text{By (I) \& (III) :—}}$ $\frac{X'}{X} = \frac{K}{4}$

By Integration, $\int \frac{X'}{X} = \int \frac{K}{4} dx$

$\log X = \frac{K}{4}x + \log c_1$

$$\log \frac{x}{c_1} = \frac{kx}{4}$$

So, $$\frac{x}{c_1} = e^{\frac{kx}{4}}$$

$$\Rightarrow x = c_1 e^{\frac{kx}{4}} \quad \text{------} (6)$$

again by II & III :—

$$\frac{3y - y'}{y} = k$$

$$\frac{3y}{y} - \frac{y'}{y} = k$$

By Integration, $$3 - \frac{y'}{y} = k$$

$$\cancel{\Rightarrow \int 3\,dy - \int \frac{y'}{y} = \int k\,dy}$$

$$\cancel{3y - \log y = ky + \log c_2}$$

$$\frac{y'}{y} = 3 - k$$

$$\int \frac{y'}{y} = \int (3-k)\,dy$$

$$\log y = (3-k)y + \log c_2$$

$$\frac{y}{c_2} = e^{(3-k)y}$$

$$y = c_2 e^{(3-k)y} \quad \text{------} (7)$$

Putting the values of (6) & (7) in (2)

$$u = x \cdot y$$

$$u = c_1 e^{\frac{kx}{4}} \cdot c_2 e^{(3-k)y}$$

$$\boxed{\therefore u = c_1 c_2 e^{\frac{kx}{4}} \cdot e^{(3-k)y}} \quad \text{—} (8)$$

Given that, $u(0,y) = 3e^{-y} - e^{-5y}$

By putting $u = 3e^{-y} - e^{-5y}$

at $x = 0$ in eq^n (8)

$$3e^{-y} - e^{-5y} = c_1c_2\, e^{(3-k)y}$$

Now equating, 1st term of left with Right side;

$$3e^{-y} = c_1c_2\, e^{(3-k)y}$$

$$\boxed{c_1c_2 = 3} \quad \& \quad -1 = 3-k$$

$$\boxed{k = 4}$$

again, equating, 2nd term of left with right side;

$$-e^{-5y} = c_1c_2\, e^{(3-k)y}$$

$$\boxed{c_1c_2 = -1} \; ; \quad -5 = 3-k$$

$$\boxed{k = 8}$$

So required Sol^n;

$$u = 3\,e^{x}\cdot e^{(3-4)y} + (-1)\,e^{\frac{8}{4}x}\cdot e^{(3-8)y}$$

$$u = 3e^{x}\cdot e^{-y} - e^{2x}\cdot e^{-5y}$$

$$u = 3e^{x-y} - e^{2x-5y}$$ Ans.

x ——— x

Case-II $\Rightarrow$ $F(x,y) = \sin(ax+by)$ or $\cos(ax+by)$

$D^2 \to -a^2$, $D'^2 \to -b^2$

$DD' \to -ab$

Que. $\Rightarrow$ $r - 2s + t = \sin(2x+3y)$

Solⁿ $\Rightarrow$ $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin(2x+3y)$

$(D^2 - 2DD' + D'^2)z = \sin(2x+3y)$

A.Eⁿ $\Rightarrow$ $D^2 - 2DD' + D'^2 = 0$ $\quad D \to m$, $D' \to 1$

$m^2 - 2m + 1 = 0$

$m = 1, 1$ $\quad$ roots are real & same

C.F. $\Rightarrow$ $f_1(y+x) + x f_2(y+x)$ — (1)

Now, $P.I. = \frac{1}{f(D,D')} F(x,y)$

$P.I. = \frac{1}{D^2 - 2DD' + D'^2} \sin(2x+3y)$

$\frac{1}{-4+12-9}\sin(2x+3y)$ $\quad \left\{ \begin{array}{l} D^2 \to -a^2 = -4 \\ D'^2 \to -b^2 = -9 \\ DD' \to -ab = -6 \end{array} \right.$

$\frac{1}{-13+12}\sin(2x+3y)$

$\frac{\sin(2x+3y)}{-1} = -\sin(2x+3y)$

Complete Solⁿ $\to$ $z = C.F. + P.I.$

$z = f_1(y+x) + x f_2(y+x) - \sin(2x+3y)$

Que ① A string is streched and fastened to two points l apart. Motion is started by displacing the string in the form $y = a\sin\frac{\pi x}{l}$ from which it is released at time $t=0$. Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t) = a\sin\left(\frac{\pi x}{l}\right)\cdot\cos\left(\frac{\pi ct}{l}\right)$

Sol^n The vibration of the string is given by

$$\boxed{\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}} \quad \text{--- (1)}$$

we have to apply Boundary and initial Conditions one by one such that;
As the end of the string are fixed for all time then $y(0,t) = 0$ --- (2)
& $y(l,t) = 0$ --- (3)

Since the transverse velocity of any point of the string is zero, $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$ —— 4

Also, $y(x,0) = y = a \sin \frac{\pi x}{l}$ —— 5

Now we have to solve, 1 subject to the boundary conditions 2 and 3 and initial condition 4 and 5

Since vibration of the string is periodic, therefore the soln of 1 is of the form

$$y(x,t) = \{C_1 \cos px + C_2 \sin px\} \cdot \{C_3 \cos cpt + C_4 \sin cpt\} \quad \text{— 6}$$

By 2 $\quad y(0,t) = C_1 [C_3 \cos cpt + C_4 \sin cpt]$ —— 7

here; $\quad C_3 \cos cpt + C_4 \sin cpt \neq 0$

so, $\boxed{C_1 = 0}$

put $C_1 = 0$ in eqn 6 :—

$$y(x,t) = C_2 \sin px \{C_3 \cos cpt + C_4 \sin cpt\} \quad \text{— 8}$$

Now by eqn 4 :— $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$

differentiating 8 w.r.t t :—

$$\frac{\partial y}{\partial t} = C_2 \sin px \{-C_3 \sin cpt\,(cp) + cp(C_4)\cos cpt\}$$

put $t = 0$

$$\left(\frac{\partial y}{\partial t}\right)_{t=0} = C_2 \sin px \{C_4 cp\} = 0$$

whence $C_2 C_4 cp = 0$

If $C_2 = 0$ in 8 will lead to the trivial soln $y(x,t) = 0$

classmate

Date

Page

Only Possiblity is that $c_4 = 0$

Thus (8) becomes $y(x,t) = c_2 c_3 \sin px \cos cpt$ — (9)

Now, by eqⁿ (3), ~~$y(l,t) =$~~

$y(l,t) = c_2 c_3 \sin pl \cdot \cos cpt = 0$, $\forall t$

since, $c_2 \neq 0$ & $c_3 \neq 0$

we have $\sin pl = 0$

$\sin pl = \sin n\pi$

$pl = n\pi \Rightarrow \boxed{p = \frac{n\pi}{l}}$

where n = an integer

Hence, (1) reduces to $y(x,t) = c_2 c_3 \sin \frac{n\pi x}{l} \cdot \cos \frac{n\pi ct}{l}$ — (10)

Finally imposing the last condition (5) :—

$y(x,0) = c_2 c_3 \sin \frac{n\pi x}{l} \cdot \cos \frac{n\pi c(0)}{l}$

$y(x,0) = c_2 c_3 \sin \frac{n\pi x}{l}$ ✓

let, $c_2 c_3 = a$, $n = 1$

Hence the required solⁿ is {By 10}

$$\boxed{y(x,t) = a \sin \frac{\pi x}{l} \cdot \cos \frac{\pi ct}{l}}$$ — (11)

Ans.

—x—

Que
Imp

If the chance that 1 of the 10 telephone lines is busy at an instant is 0.2

(a) What is the chance that 5 of the lines are busy.

(b) What is the ~~chance that 5 of the lines~~ most probable number of busy lines, and what is probability of this number.

(c) What is the probability that all the lines are busy.

Sol^n

p = prob. of 1 telephone lines is busy

$= 0.2$ (given)

$q = (1-p) = 0.8$

$n = 10$

classmate

Date

Page

(a) $P(x=5) = {}^{10}C_5 \, (0.2)^5 \, (0.8)^{10-5}$

$= {}^{10}C_5 \, (0.2)^5 \, (0.8)^5$

$= 0.026$

(b) Most probable no. of busy lines = Expectation (mean) $= np$

$= 10 \times 0.2 = 2$

probability of most probable no. is

$P(x=2) = {}^{10}C_2 \, (0.2)^2 \, (0.8)^8$

$=$

(c) $P(x=10) = {}^{10}C_{10} \, (0.2)^{10} \, (0.8)^0$

$=$

Que $\Rightarrow$ The bomb that a bomb dropped from a plane will strike the target is $1/5$ if six bombs are dropped find the prob. that

(a) exactly 2 will strike the target.

(b) At least 2 will strike the target.

Sol^n $\Rightarrow$ Prob. that a bomb dropped from a plane will strike the target $p = \frac{1}{5}$

classmate

Date

Page

$q = 1 - \frac{1}{5} = \frac{4}{5}$

$n = 6$

(a) exactly 2 :- $P(x=2) = {}^6C_2 \cdot \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^4$

$= 0.245$

(b) At least 2 :- $1 - [P_0 + P_1]$

$\Rightarrow 1 - \left[{}^6C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^6 + {}^6C_1 \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^5\right]$

$= 0.3447$

Que ⟹ Out of 800 families with 5 children each how many would you expect to have (a) 3 boys (b) 5 girls (c) either 2 or 3 boys. Assume equal prob. for boys & girls.

Sol^n ⟹ prob of girls and boys $= \frac{1}{2}$

$n = 5$

Total family $= 800$

(a) 3 boys $\rightarrow$ $P(X=3) = {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$

$= \frac{\lfloor 5}{\lfloor 3 \cdot \lfloor 2} \times \frac{1}{8} \times \frac{1}{4}$

$= \frac{10}{32}$

Since, Total 800 families $= \frac{10}{32} \times 800 = 250$

classmate

Date

Page

(b) 5 girls $\Rightarrow$ $p(x=5) = \left[{}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0\right] \times 800$

$= 25$

(c) either 2 or 3 boys $\Rightarrow$

$p(x = 2 \text{ or } 3) =$

$= 800\,[p(x=2) + p(x=3)]$

$= 800\left[{}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 + {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2\right]$

$= 500$

V. Imp Q Best fit of poisson dist to the set of observation

x:	0	1	2	3	4
f:	122	60	15	2	1

Sol^n $\Rightarrow$ Formula for Best fit :-

$$N\left[\frac{m^r e^{-m}}{r!}\right]$$

$N = \Sigma f_i = 200$

$m = \text{mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{60+30+6+4}{200}$

$m = 0.5$

$$\Rightarrow N\left[\frac{m^r e^{-m}}{r!}\right]$$

$$= 200\left[\frac{(0.5)^r e^{-0.5}}{r}\right]$$

Put $r = 0, 1, 2, 3, 4$

$f(x) = 121, 61, 15, 2, 0$

x or r :	0	1	2	3	4
f :	121	61	15	2	0

Que. Assuming that the diameters 1000 brass plugs taken consequentively from a machine normal distribution with mean 0.7515 c.m and S.D. 0.002 cm., how many of the plugs are approved diameter is 0.752 ± 0.004 cm.

Sol^n $\mu = 0.7515$ cm, $\sigma = 0.0020$ cm.

Given, diameter

(0.752 ± 0.004) cm.

Taking (+ve) $0.752 + 0.004 = 0.756 = a$

Taking (-ve) $0.752 - 0.004 = 0.748 = b$

we have, $P(\underset{a}{0.748} \le x \le \underset{b}{0.756}) = ?$

$$Z_1 = \frac{a-\mu}{\sigma} = \frac{0.7480 - 0.7515}{0.002} = -1.75$$

$$Z_2 = \frac{b-\mu}{\sigma} = \frac{0.756 - 0.7515}{0.0020} = 2.25$$

$P(Z) = F(Z_2) - F(Z_1) = F(2.25) - F(-1.75)$

$= 0.9477$

Total 1000 $\longrightarrow$ $0.9477 \times 1000 = 947.7$ Ans.

Que. In a normal Distribution 31% of the items are under 45 and 8% are over 64 find the mean and S.D. of the distribution.

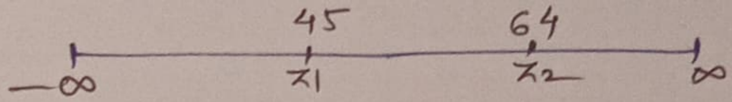
Sol^n $P(-\infty \le x \le 45) = 31\%$

$\Rightarrow P(-\infty \le x \le 45) = 0.31$ —— ①

$P(64 \le x \le \infty) = 8\%$

$\Rightarrow P(64 \le x \le \infty) = 0.08$ —— ②

Now we will convert variable in normal variable Z such that



$(-\infty \le z \le z_1) = 0.31$ — (3) (According to eq^n (1))

& $(z_2 \le z \le \infty) = 0.08$ — (4) (According to eq^n (2))

$P(z) = F(z_2) - F(z_1) = 0.31$. [By eq^n (3)]

$P(z) = F(z_1) - F(-\infty) = 0.31$

$F(z_1) = 0.31 - 0.5 = -0.19$

So, $\boxed{z_1 = -0.49}$ (by Antinormal)

Now, $P(z) = F(z_2) - F(z_1) = 0.08$ [By eq^n (4)]

$P(z) = F(\infty) - F(z_2) = 0.08$

$= 0.5 - F(z_2) = 0.08$

$F(z_2) = 0.5 - 0.08$

$\boxed{z_2 = 1.40}$ (By Antinormal)

Now, $z_1 = \dfrac{a-\mu}{\sigma} = \dfrac{45-\mu}{\sigma}$

$-0.49 = \dfrac{45-\mu}{\sigma} \Rightarrow \mu - 0.40\sigma = 45$ — (5)

& $z_2 = \dfrac{b-\mu}{\sigma} = \dfrac{64-\mu}{\sigma}$

$1.40 = \dfrac{64-\mu}{\sigma} \Rightarrow 1.40\sigma = 64 - \mu$

or, $\mu + 1.40\sigma = 64$ — (6)

By eq^n (5) & (6) :-

$\mu - 0.40\sigma = 45$

$\mu + 1.40\sigma = 64$

$$\boxed{\mu = 50 \quad \& \quad \sigma = 10}$$

Que Is the function defined as follows a density fu^n.

$$f(x) = \begin{cases} e^{-x} , & x \geq 0 \\ 0 , & x < 0 \end{cases}$$

(a) If so determine the Probability that the variate has this density will fall in the interval (1, 2)

(b) Also find the cummulative prob. fu^n. F(2).

Sol^n

Ist we will prove that Density fu^n,

(i) It is clear that $f(x) \geq 0$

and (ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$$= \int_{-\infty}^{0} + \int_{0}^{\infty}$$

$$0 + \int_0^{\infty} e^{-x}\, dx = 1$$

Date ___/___/___

Page ______

(8)

$$= \left[-e^{-x}\right]_0^\infty = 1$$

(a) $P(1 \leq x \leq 2) = \int_1^2 e^{-x}\,dx$

$$= \left(-e^{-x}\right)_1^2$$

$$= -e^{-2} + e^{-1} = -0.233$$

probability is +ve $= +0.233$ //.

(c) Cummulative Distribution

fun $= \int_{-\infty}^{x} f(x)\,dx$

$$F(2) = \int_{-\infty}^{2} f(x)\,dx$$

$$= \int_{-\infty}^{0} 0\,dx + \int_0^2 e^{-x}\,dx$$

$$= \left(-e^{-x}\right)_0^2$$

$$= -e^{-2} + 1$$

$$= 0.865$$ //.

—— * ——

V.V Imp

Q The Probability Density function

$$p(x) = y_0 e^{-|x|}, \quad -\infty \text{ to } \infty$$

Then Prove that $\boxed{y_0 = 1/2}$

also find

mean and variance.

Sol^n Mod function is Defined by

$$\left\{ |x| = \begin{array}{l} +x, x>0 \\ -x, x<0 \end{array} \right.$$

$$\boxed{e^{-|x|} = \begin{cases} e^{-x}, & x>0 \\ e^{x}, & x<0 \end{cases}}$$

Given that $p(x)$ is Density fu^n;

$$\Rightarrow \int_{-\infty}^{\infty} p(x)\,dx = 1$$

$$\Rightarrow \int_{-\infty}^{\infty} y_0 e^{-|x|}\,dx = 1$$

$$\Rightarrow \int_{-\infty}^{0} y_0 e^{+(x)}\,dx + \int_{0}^{\infty} y_0 e^{-(x)}\,dx = 1$$

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Date ___/___/___
Page ___

Since, $|x|$ is an even fu^n and range is $-\infty$ to ∞ then By Definite Integral

$$p(x) = 2\int_0^\infty y_0 e^{-x}\,dx = 1$$

$$\Rightarrow 2y_0\left[-e^{-x}\right]_0^\infty = 1$$

$$\Rightarrow 2y_0[0+1] = 1$$

$$\Rightarrow 2y_0 = 1$$

then $\boxed{y_0 = 1/2}$

Now, we have to find mean (μ) and Variance

By Formula

$$\boxed{\text{mean} = E(x) = \int_{-\infty}^{\infty} x\,P(x)\,dx}$$

$$E(x) = \int_{-\infty}^{0} y_0\, x e^{x}\,dx + \int_0^\infty y_0\, x e^{-x}\,dx$$

$$= y_0\left[xe^{x} - e^{x}\right]_{-\infty}^{0} + y_0\left[x(-e^{-x}) - (e^{-x})\right]_0^\infty$$

$$= y_0[-1+1] = 0$$

$$\boxed{E(x) = 0}$$

Note:- $\int_{-\infty}^{\infty} \underbrace{x}_{\text{odd } fu^n}\,\underbrace{P(x)}_{\text{even } fu^n}\,dx = 0 \times e = 0$

$= 0$

Date __/__/__

Page ____

$$\text{Variance} = E(x-\mu)^2 = \int_{-\infty}^{\infty} (x-\mu)^2 P(x)\,dx$$

$$= \int_{-\infty}^{0} (x-\mu)^2 y_0 e^{x}\,dx + \int_{0}^{\infty} (x-\mu)^2 y_0 e^{-x}\,dx$$

$$\boxed{\mu = 0}$$

$$= \int_{-\infty}^{0} x^2 y_0 e^{+x}\,dx + \int_{0}^{\infty} x^2 y_0 e^{-x}\,dx$$

$$= y_0\left[\left(x^2 e^{x} - 2xe^{x} + 2e^{x}\right)_{-\infty}^{0} + \left\{x^2(-e^{-x}) - 2x(e^{-x}) + 2(-e^{-x})\right\}_{0}^{\infty}\right]$$

$$\text{Var.} = 2$$

$$\boxed{\text{Put } y_0 = 1/2}$$

then, std Deviation

$$\sigma = \sqrt{2}$$

—x—

missing term

(1) Evaluate, $\Delta \tan^{-1}x$.

Soln ⇒ $\Delta \tan^{-1}x = \tan^{-1}(x+h) - \tan^{-1}x$

Formula :- $\tan^{-1}A - \tan^{-1}B = \tan^{-1}\left\{\frac{A-B}{1+AB}\right\}$

$$= \tan^{-1}\left[\frac{x+h-x}{1+(x+h)x}\right]$$

$$= \tan^{-1}\left(\frac{h}{1+x^2+hx}\right)$$ Ans

(2) $\Delta^2 \cdot ab^x$

Soln ⇒ $\Delta\left[\Delta ab^x\right]$

$\Delta\left[ab^{x+1} - ab^x\right]$

$\Rightarrow a\,\Delta\left[b^x \cdot b - b^x\right]$

$\Rightarrow a(b-1)\,\Delta\{b^x\}$

again,

$\Rightarrow a(b-1)\left[b^{x+1} - b^x\right]$

$\Rightarrow a(b-1)\,b^x(b-1)$

$\Rightarrow a(b-1)^2 b^x$ Ans

* [In algebraic term we always put $h=1$]

classmate
Date
Page

Que ⇒ From the following table, estimate the number of students who obtained marks b/w 40 & 45

marks:	30-40	40-50	50-60	60-70	70-80
No. of students:	31	42	51	35	31

Solⁿ ⇒

marks less than (x)	No. of Students (y)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$x_0 = 40$	$y_0 = 31$				
		42 (Δy_0)			
50	73		9 ($\Delta^2 y_0$)		
		51		-25 ($\Delta^3 y_0$)	
60	124		-16		-37 ($\Delta^4 y_0$)
		35		12	
70	159		-4		
		31			
80	190				

We have to find no. of students who scored marks b/w 40 & 45.
Given that marks less than 40 = no. of student 31
Now we will find no. of students who scored less than 45 marks.
i.e. $x = 45$ which is near value of 40
so, $x_0 = 40$ $\Rightarrow p = \frac{x - x_0}{h} = \frac{45-40}{10}$

$$\boxed{p = 0.5}$$

By using Newton's Forward interpolation Formula;

$$y_{45} = y_{40} + p\Delta y_{40} + \frac{p(p-1)}{2!}\Delta^2 y_{40} + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_{40} + \ldots$$

$$= 31 + 0.5 \times 42 + \frac{0.5(-0.5)}{2} \times 9 + \frac{0.5(0.5)(-1.5)}{6} \times (-25) + \ldots$$

$$= 47.87$$

The no. of students with marks less than 45 is 47.87

But the no. of students with marks less than 40 is 31

Hence the number of students getting marks b/w 40 & 45 $= 48 - 31 = 17$ Ans

Que ⇒ Use Stirling formula to find y for $x = 35$ from the following table :

x :	20	30	40	50
y :	512	439	346	243

Sol^n ⇒

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
	20	$512 = y_{-2}$			
			$\Delta y_{-2} = -73$		
				$\Delta^2 y_{-2} = -20$	
	30	$439 = y_{-1}$			$\Delta^3 y_{-2} = 10$
			$\Delta y_{-1} = -93$		
$x_0 =$	40	$346 = y_0$			
				$\Delta^2 y_{-1} = -10$	
			$\Delta y_0 = -103$		
	50	$243 = y_1$			

$x = 35$ (A/c to Que) , $x_0 = 40$, $h = 10$

then $p = \dfrac{x - x_0}{h} = \dfrac{35-40}{10} = -0.5$

By Stirling Formula :-

$$y_{35} = 346 + (-0.5)\left\{\frac{-103-93}{2}\right\} + \frac{(-0.5)^2}{2}(-10) + \frac{(-0.5)\{(-0.5)^2-1\}}{3!} \times \left\{\frac{-10+10}{2}\right\}$$

$$\boxed{y_{35} = 393.75}$$

Que ⟹ Apply Bessel's formula to obtain y_{25}, Given $y_{20} = 2854$, $y_{24} = 3162$, $y_{28} = 3544$, $y_{32} = 3992$.

Sol^n ⟹

x	p	y	Δy	$\Delta^2 y$	$\Delta^3 y$
20	-1	$2854 = y_{-1}$			
			$\Delta y_{-1} = 308$		
24	0	$3162 = y_0$		$\Delta^2 y_{-1} = 74$	$\Delta^3 y_{-1} = -8$
			$\Delta y_0 = 382$		
28	1	$3544 = y_1$		$\Delta^2 y_0 = 66$	
			$\Delta y_1 = 448$		
32	2	$3992 = y_2$			

$\left.\begin{aligned} x &= 25 \\ x_0 &= 24 \\ h &= 4 \end{aligned}\right\}$ $\quad p = \dfrac{x - x_0}{h} = \dfrac{25-24}{4} = \dfrac{1}{4}$

By Bessels' formula ;

$$y_{25} = 3162 + \frac{1}{4}(382) + \frac{\frac{1}{4}\left(\frac{1}{4}-1\right)}{2}\left\{\frac{74+66}{2}\right\} +$$

$$\frac{\left(\frac{1}{4}-\frac{1}{2}\right)\frac{1}{4}\left(\frac{1}{4}-1\right)}{3!} \times (-8)$$

$$= 3250.875 \quad \text{(approx)}$$

Relation B/w Difference Operators

Prove that,
1. $\Delta = E - 1$
2. $\nabla = 1 - E^{-1}$
3. $\delta = E^{1/2} - E^{-1/2}$
4. $\mu = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]$
5. $\left[E^{1/2} + E^{-1/2}\right](1+\Delta)^{1/2} = 2 + \Delta$
5. $E = e^{hD}$, where, $D = \frac{d}{dx}$

6. $\Delta = E\nabla = \delta E^{1/2}$

Proof ① $\Delta f(x) = f(x+h) - f(x)$

$= Ef(x) - f(x)$ $\left\{ \text{Here, } E(x) = f(x+h) \right\}$

$\Delta f(x) = f(x)\{E - 1\}$

So, $\boxed{\Delta = E - 1}$

② $\nabla f(x) = f(x) - f(x-h)$

$= f(x) - E^{-1}f(x)$

$= f(x)\{1 - E^{-1}\}$

So, $\boxed{\nabla = 1 - E^{-1}}$

③ $\delta f(x) = f\left(x + \frac{h}{2}\right) - f\left(x - \frac{h}{2}\right)$

$= E^{1/2}f(x) - E^{-1/2}f(x)$ —— ①

$= \left[E^{1/2} - E^{-1/2}\right]f(x)$

So, $\boxed{\delta = E^{1/2} - E^{-1/2}}$

④ $\mu\{f(x)\} = \frac{1}{2}\left[f\left(x+\frac{h}{2}\right) + f\left(x - \frac{h}{2}\right)\right]$

Average operator

$= \frac{1}{2}\left[E^{1/2}f(x) + E^{-1/2}f(x)\right]$ {By ①}

$$\mu f(x) = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right] f(x)$$

So, $\boxed{\mu = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]}$

(5) $E = e^{hD}$

By Definition of shifting op. (expand the fn By Taylor's thm)

$$Ef(x) = f(x+h)$$

$$= f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) - - - \infty$$

$$E\{f(x)\} = f(x) + hDf(x) + \frac{h^2}{2!}D^2 f(x) + \frac{h^3}{3!}D^3 f(x) + - - - \infty$$

$$= f(x)\left[1 + hD + \frac{h^2}{2!}D^2 + \frac{h^3}{3!}D^3 + - - - \infty\right]$$

So, $E = \underbrace{1 + hD + \frac{h^2D^2}{2!} + \frac{h^3D^3}{3!} + - - \infty}_{}$

This is expansion of e^{hD}

[* formula $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + - - - \infty$]

So, $\boxed{E = e^{hD}}$

(6) $\Delta = E\nabla = \delta E^{1/2}$

$\cancel{\Delta f(x) = f(x+h) - f(x)}$

$$\underline{E\nabla f(x)} = E\left[f(x) - f(x-h)\right]$$

$$= Ef(x) - Ef(x-h)$$

$$= f(x+h) - f(x)$$

$$= \underline{\Delta} f(x)$$

So, $\boxed{E\nabla = \Delta}$

$\cancel{\nabla f(x)}$ $\underline{\nabla E} f(x) = \nabla \{ f(x+h) \}$

$= f(x+h) - f(x)$

$= \underline{\Delta} f(x)$

So, $\boxed{\nabla E = \Delta}$

Now, $\underline{\delta E^{1/2}} f(x) = \delta f(x+\frac{h}{2})$

$= f(x+\frac{h}{2}+\frac{h}{2}) - f(x+\frac{h}{2}-\frac{h}{2})$

$= f(x+h) - f(x)$

$= \underline{\Delta} f(x)$

So, $\boxed{\delta E^{1/2} = \Delta}$

* All the Best *