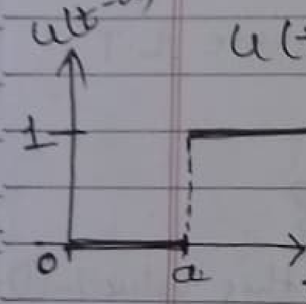
Unit Step function

Definition → The Unit function $u(t-a)$ is defined by as follows:

$$u(t-a)=\begin{cases}0 & \text{for } t<a\\ 1 & \text{for } t\geq a\end{cases}$$



where a is always +ve

$$\boxed{L\{u(t-a)\}=\frac{e^{-as}}{s}}$$

Properties :- (2nd shifting Property)

If $L\{f(t)\}=\bar{f}(s)$

then, $$\boxed{L\{f(t-a).u(t-a)\}=e^{-as}\,\bar{f}(s)}$$

~~Que 1 Find L.T. of~~ $\cancel{(t-1)^2\, u(t-1)}$

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1st Form

Find L.T. of

Que ⟹ $\sin t \cdot u(t-\pi)$

Sol^n ⟹ Changing in standard form of

$* \; L[(t-a)\, u(t-a)]$

$\sin t \cdot u(t-\pi)$

$\Rightarrow \sin(t-\pi+\pi) \cdot u(t-\pi)$

$\Rightarrow \sin[\pi+(t-\pi)] \cdot u(t-\pi)$ $\left(\sin(180+\theta) = -\sin\theta \right)$

$\Rightarrow -\sin(t-\pi) \cdot u(t-\pi)$

$\Rightarrow -L[\sin(t-\pi) \cdot u(t-\pi)]$

$\Rightarrow e^{-as}\bar{f}(s)$ (Formula)

$\Rightarrow - e^{-\pi s} \cdot \dfrac{1}{s^2+1}$

$\Rightarrow \dfrac{-e^{-\pi s}}{s^2+1}$ Ans.

Here $a=\pi$ and $f(t)=\sin t$
So $L\{f(t)\} = \dfrac{1}{s^2+1} = \bar{f}(s)$

Question in the form of Intervals

(2nd form)

$$F(t) = \begin{cases} f_1(t), & 0<t<a_1 \\ f_2(t), & a_1<t<a_2 \\ f_3(t), & a_2<t<a_3 \\ f_4(t), & a_3<t \end{cases}$$

$$f_1(t)\cdot u(t-0) + [f_2(t)-f_1(t)]\cdot u(t-a_1) + [f_3(t)-f_2(t)]\cdot u(t-a_2) + [f_4(t)-f_3(t)]\cdot u(t-a_3)$$

Que ① $f(t) = \begin{cases} t-1 & , 1<t<2 \\ 3-t & , 2<t<3 \end{cases}$

Find L.T. of unit step function;

Sol^n ⟹

$$f(t) = \begin{cases} t-1 & , 1<t<2 \\ 3-t & , 2<t<3 \end{cases}$$

$$f(t) = \begin{cases} 0 & ; 0<t<1 \\ t-1 & , 1<t<2 \\ 3-t & , 2<t<3 \\ 0 & , t>3 \end{cases}$$

$L[f(t-a) \cdot u(t-a)] = e^{-as} \bar{f}(s)$

$$= 0 \cdot u(t-0) + [t-1-0] \cdot u(t-1) + [3-t-(t-1)] \cdot u(t-2) + [0-(3-t)] \cdot u(t-3)$$

$$= (t-1)\, u(t-1) + (4-2t)\, u(t-2) + (t-3)\, u(t-3)$$

$$= (t-1)\, u(t-1) - 2(t-2)\, u(t-2) + (t-3)\, u(t-3)$$

$a=1$ $\quad a=2$ $\quad a=3$

$\left(L(t) = \frac{1}{s^2}\right)$

$$= \frac{e^{-s}}{s^2} - 2e^{-2s} \times \frac{1}{s^2} + e^{-3s} \times \frac{1}{s^2}$$

$$= \frac{e^{-s} - 2e^{-2s} + e^{-3s}}{s^2}$$

Ans