Langrange's Linear PDE.

The PDE of the form,
$Pp + Qq = R$ is the standard form of Linear PDE of order one is called "Langrange's LDE".

Here, P, Q, R are any fn of x, y, z.

<u>Working Rule</u>:— To Solve $Pp + Qq = R$ by Langrage's LDE:—

<u>Step ①</u> → $Pp + Qq = R$ (standard form)

<u>Step ②</u> → $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

<u>Step ③</u> → equate any 2 Independent terms each other and find the Soln

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① Solve, $xzp + yzq = xy$

Sol^n By Langrange's LDE :-

$$Pp + Qq = R$$

By comparing Given eq^n we get,

$$P = xz \quad , \quad Q = yz \quad \text{and} \quad R = xy$$

$$\frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{xy}$$

I II III (members)

Taking Ist two members :-

$$\frac{dx}{x\cancel{z}} = \frac{dy}{y\cancel{z}} \Rightarrow \frac{dx}{x} = \frac{dy}{y}$$

By Integrating, $\int \frac{dx}{x} = \int \frac{dy}{y}$

$$\log x = \log y + \log c_1 .$$

$$x = y c_1 \xrightarrow{or} \boxed{c_1 = \frac{x}{y}} \Rightarrow x = c_1 y$$

again, by equating last 2 members, $\frac{dy}{\cancel{y}z} = \frac{dz}{x\cancel{y}}$

$$\frac{dy}{z} = \frac{dz}{x}$$

Here, variable x is different from derivatives
So put $\underline{x = c_1 y}$

$$\frac{dy}{z} = \frac{dz}{c_1 y}$$

$$\Rightarrow c_1 y \, dy = z \, dz \Rightarrow \int c_1 y \, dy = \int z \, dz$$

$$c_1 \frac{y^2}{2} + \frac{z^2}{2} = c_2$$

again put $c_1 = x/y$

$$\frac{x}{y} \times \frac{y^2}{2} + \frac{z^2}{2} = c_2$$

$$\boxed{\frac{xy}{2} + \frac{z^2}{2} = c_2}$$

So final Ans is

$$\phi\left\{\frac{xy}{2} + \frac{z^2}{2}, \frac{x}{y}\right\} = 0$$

A.

(2) Solve,

$$yzp + zxq = xy$$

Sol^n $\Rightarrow$ $P = yz$, $Q = zx$, $R = xy$

$$\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$$

I II III

By Ist 2 members :-

$$\frac{dx}{y\cancel{z}} = \frac{dy}{\cancel{z}x} \Rightarrow \int x\,dx = \int y\,dy$$

$$\frac{x^2}{2} = \frac{y^2}{2} + c_1 \xrightarrow{or} \boxed{c_1 = \frac{x^2}{2} - \frac{y^2}{2}}$$

By last 2 members :-

$$\frac{dy}{z\cancel{x}} = \frac{dz}{\cancel{x}y}$$

$$\int y\,dy = \int z\,dz \Rightarrow \frac{y^2}{2} = \frac{z^2}{2} - c_2$$

$$\xrightarrow{or} \boxed{c_2 = \frac{y^2}{2} - \frac{z^2}{2}}$$

So, $\phi\left\{\frac{x^2}{2} - \frac{y^2}{2}, \frac{y^2}{2} - \frac{z^2}{2}\right\} = 0$ Ans

// Solve at last //

Que (3) $\Rightarrow$ Solve, $y^2p - xyq = x(z-2y)$

Sol^n $\Rightarrow$ $P = y^2$, $Q = -xy$, $R = x(z-2y)$

different

$$\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z-2y)}$$

I II III

By Ist 2 members $\Rightarrow$ $\frac{dx}{y^2} = \frac{dy}{-xy}$

$$\frac{dx}{y} = \frac{dy}{-x}$$

$$-x\,dx = y\,dy$$

By Integration; $-\int x\,dx = \int y\,dy$

$$-\frac{x^2}{2} = \frac{y^2}{2} - c_1$$ (our assumption)

$$\boxed{\frac{y^2}{2} + \frac{x^2}{2} = c_1}$$

By last 2 members;

$$\frac{dy}{-\cancel{x}y} = \frac{dz}{\cancel{x}(z-2y)}$$

$$(z-2y)\,dy = -y\,dz$$

$$\underbrace{z\,dy + y\,dz}_{} - 2y\,dy = 0$$

This is an exact Diffⁿ eqⁿ.

$$d(yz) - 2y\,dy = 0$$

By Integration, $yz - 2\frac{y^2}{2} = c_2$

$$\boxed{yz - y^2 = c_2}$$

So, $\phi\left\{\frac{y^2}{2} + \frac{x^2}{2},\ yz - y^2\right\} = 0$ Ans.

Q(4) Solve, $x^2p + y^2q = z^2$

Solⁿ ⟹ $P = x^2,\ Q = y^2,\ R = z^2$

$$\underset{\text{I}}{\frac{dx}{x^2}} = \underset{\text{II}}{\frac{dy}{y^2}} = \underset{\text{III}}{\frac{dz}{z^2}}$$

By Iˢᵗ 2 members;

$$\int \frac{dx}{x^2} = \int \frac{dy}{y^2}$$

$$-\frac{1}{x} = -\frac{1}{y} - c_1$$

then; $\boxed{c_1 = \frac{1}{x} - \frac{1}{y}}$

By last 2 members;

$$\frac{dy}{y^2} = \frac{dz}{z^2}$$

By Integration, $\int \frac{dy}{y^2} = \int \frac{dz}{z^2}$

$$-\frac{1}{y} = \frac{-1}{z} - c_2$$

So, $\boxed{c_2 = \frac{1}{y} - \frac{1}{z}}$

then, $\phi\left\{\frac{1}{x} - \frac{1}{y}, \frac{1}{y} - \frac{1}{z}\right\} = 0$ Ans

Que 3 Solve, $(y^2 + z^2 - x^2)p - 2xyq + 2zx = 0$

Sol^n → $P = y^2 + z^2 - x^2$, $Q = -2xy$, $R = 2zx$

* different

$$\underset{\text{I}}{\frac{dx}{y^2+z^2-x^2}} = \underset{\text{II}}{\frac{dy}{-2xy}} = \underset{\text{III}}{\frac{dz}{2zx}} \quad \text{—— (1)}$$

By (II) & (III) :—

$$\frac{dy}{-\cancel{2x}y} = \frac{dz}{\cancel{2x}z}$$

$$\int -\frac{dy}{y} = \int \frac{dz}{z}$$

$$-\log y = \log z - \log c_1$$

$$\log c_1 = \log z + \log y$$

So, $\boxed{c_1 = yz}$

~~Now, By members (I) & (III) :-~~

Now It is difficult to find another Answer, So we will solve such that

Again, Using x, y, z as multipliers, each fraction of (1) and we make a new member;

$$\cancel{\frac{dx}{y^2+z^2-x^2}} = \cancel{\frac{dz}{2zx}}$$
I III

~~It is difficult to calculate & find Soln,~~ So we multiply.

$$\frac{x\,dx + y\,dy + z\,dz}{-xy^2 - xz^2 - x^3}$$ IV (This is new member)

Now By equating IV th member with II or III,

such that,
$$\frac{x\,dx + y\,dy + z\,dz}{-x(y^2+z^2+x^2)} = \frac{dz}{2zx}$$
IV III

Now,
$$\frac{2(x\,dx + y\,dy + z\,dz)}{x^2+y^2+z^2} = \frac{-dz}{z}$$

Since we derivative of denominator is its numerator so we can write

By Integn,
$$\log(x^2+y^2+z^2) = -\log z + \log c_2$$
$$z(x^2+y^2+z^2) = c_2$$

So, $\phi\{yz, z(x^2+y^2+z^2)\} = 0$

Que (6) Solve, $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$

Soln Given eqn is in the form of Laugranges LDE :-

So,
$$\frac{dx}{x^2(y-z)} = \frac{dy}{y^2(z-x)} = \frac{dz}{z^2(x-y)} \quad \text{—(1)}$$

By Taking multiplier system;

$1/x^2, 1/y^2, 1/z^2$ multiply in each fraction of (1) and form a new member such that

dx dy dz

$$\frac{dx}{x^2(y-z)} = \frac{dy}{y^2(z-x)} = \frac{dz}{z^2(x-y)} =$$
$$\text{I} \qquad \text{II} \qquad \text{III}$$

$$\frac{\frac{1}{x^2}dx + \frac{1}{y^2}dy + \frac{1}{z^2}dz}{y-z+z-x+x-y}$$

or $$\frac{\frac{1}{x^2}dx + \frac{1}{y^2}dy + \frac{1}{z^2}dz}{0}$$
$$\text{IV}$$

By Taking new member;

$$\frac{1}{x^2}dx + \frac{1}{y^2}dy + \frac{1}{z^2}dz = 0$$

By Integration, $-\frac{1}{x} - \frac{1}{y} - \frac{1}{z} = -c_1$

or $\rightarrow$ $\boxed{\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = c_1}$

again, Taking another multipliers $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$

$$\frac{dx}{x^2(y-z)} = \frac{dy}{y^2(z-x)} = \frac{dz}{z^2(x-y)} = \frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{x(y-z)+y(z-x)+z(x-y)}$$
$$\text{I} \qquad \text{II} \qquad \text{III}$$

or, $$\frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{0}$$
$$\text{IV}$$

By Taking new member;

$$\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = 0$$

By Integration, $\log x + \log y + \log z = \log c_2$

$\boxed{xyz = c_2}$

So, $$\phi\left\{\frac{1}{x} + \frac{1}{y} + \frac{1}{z},\ xyz\right\} = 0$$

Ques (7) $(mz-ny)\frac{\partial z}{\partial x} + (nx-lz)\frac{\partial z}{\partial y} = ly-mx$

$(mz-ny)p + (nx-lz)q = ly-mx$

Sol → This is Langrange's LDE;

$$\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx}$$

By Taking multiplier, x, y, z

$$\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx} = \frac{xdx+ydy+zdz}{mxz-nxy+nxy-lyz+lzy-mxz}$$

or → $\frac{xdx+ydy+zdz}{0}$ IV

By Taking new member;

$xdx+ydy+zdz=0$

By Integration, $\boxed{\frac{x^2}{2}+\frac{y^2}{2}+\frac{z^2}{2}=c_1}$

again, By Taking multiplier, l, m, n.

$$\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx} = \frac{lx\,dx+my\,dy+nz\,dz}{mlz-nly+nmx-lmz+lny-mnx}$$

or → $\frac{lx+my+nz}{0}$ IV

By taking new member;

~~$lx+my+nz=0$~~

$lxdx+mydy+nzdz=0$

By Integration; $\boxed{\frac{lx^2}{2}+m\frac{y^2}{2}+n\frac{z^2}{2}=c_2}$

So, $\phi\left\{\frac{x^2}{2}+\frac{y^2}{2}+\frac{z^2}{2}, \frac{lx^2}{2}+\frac{my^2}{2}+\frac{nz^2}{2}\right\}=0$

Ans..

Que (8) $p\tan x + p\tan y = \tan z$

Sol^n^ This is Langrange's LPDE;

$$\frac{dx}{\tan x} = \frac{dy}{\tan y} = \frac{dz}{\tan z}$$

I II III

Taking 1^st^ 2 members;

$$\frac{dx}{\tan x} = \frac{dy}{\tan y}$$

By Integration; $\int \frac{dx}{\tan x} = \int \frac{dy}{\tan y}$

$$\log \sin x = \log \sin y + \log c_1$$

$$\log \sin x - \log \sin y = \log c_1$$

$$\boxed{\frac{\sin x}{\sin y} = c_1}$$

again By Taking last 2 members;

$$\frac{dy}{\tan y} = \frac{dz}{\tan z}$$

By Integration; $\int \frac{dy}{\tan y} = \int \frac{dz}{\tan z}$

$$\log \sin y = \log \sin z + \log c_2$$

$$\boxed{\frac{\sin y}{\sin z} = c_2}$$

Now, $\phi\left\{\frac{\sin x}{\sin y}, \frac{\sin y}{\sin z}\right\} = 0$ Ans.

Que (9) $x(y-z)p + y(z-x)q = z(x-y)$

Sol^n^ This is in the form of L.LPDE:-

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$$

Using multipliers 1,1,1 :-

$$= \frac{dx+dy+dz}{xy - xz + yz - xy + zx - yz} \quad \text{IV}$$ (This new member)

Or, $\frac{dx+dy+dz}{0}$

So, we get, $\int dx + \int dy + \int dz = 0$

$\boxed{x+y+z = c_1}$ —— (i)

Again, Using multipliers $1/x, 1/y, 1/z$;

then new member is

$$= \frac{1/x\,dx + 1/y\,dy + 1/z\,dz}{0}$$

So, we get, $\int 1/x\,dx + \int 1/y\,dy + \int 1/z\,dz = 0$ (new member)

$$\log x + \log y + \log z = \log c_2$$

or $\rightarrow$ $\boxed{xyz = c_2}$ —— (ii)

then,

$$\phi\{x+y+z,\ xyz\} = 0$$

Ans.

Different

Que (10) Solve, $(x^2-yz)p+(y^2-zx)q = z^2-xy$

Sol^n $\Rightarrow$ This is Laugranges L.PDE^n

$$\frac{dx}{x^2-yz} = \frac{dy}{y^2-zx} = \frac{dz}{z^2-xy}$$

Using multiplier systems;

$(1,-1,0)$, $(0,1,-1)$ & $(-1,0,1)$

$$\frac{dx-dy}{(x-y)(x+y+z)}_{IV} = \frac{dy-dz}{(y-z)(x+y+z)}_{V} = \frac{dz-dx}{(z-x)(x+y+z)}_{VI}$$

These are new members, which formed by multiplying $(1,-1,0)$, $(0,1,-1)$ & $(-1,0,1)$ in each fraction.

Taking 1st 2 members;

$$\frac{dx-dy}{(x-y)\cancel{(x+y+z)}} = \frac{dy-dz}{(y-z)\cancel{(x+y+z)}}$$

$$\int\frac{dx-dy}{x-y} = \int\frac{dy-dz}{y-z}$$

By Integration, $\log(x-y) = \log(y-z)+\log c_1$

$$\frac{x-y}{y-z} = c_1 \quad \text{--- (1)}$$

again, By last 2 members;

$$\frac{dy-dz}{(y-z)\cancel{(x+y+z)}} = \frac{dz-dx}{(z-x)\cancel{(x+y+z)}}$$

By Integration, $\int\frac{dy-dz}{y-z} = \int\frac{dz-dx}{z-x}$

$$\log(y-z) = \log(z-x) + \log c_2$$

$$\frac{y-z}{z-x} = c_2 \quad \text{——— (2)}$$

So, $\phi\left\{\frac{x-y}{y-z}, \frac{y-z}{z-x}\right\} = 0$ Ans.

Que (11) Solve, $px(z-2y^2) = (z-qy)(z-y^2-2x^3)$

Sol^n $\cancel{\frac{dx}{x(z-2y^2)} = \frac{dy}{\quad}}$

$$px(z-2y^2) + y(z-y^2-2x^3)q = z(z-y^2-2x^3)$$

Now, This is in the standard form of Langrange's LPDE :-

$$\frac{dx}{x(z-2y^2)} = \frac{dy}{y(z-y^2-2x^3)} = \frac{dz}{z(z-y^2-2x^3)}$$

$$\text{I} \qquad\qquad \text{II} \qquad\qquad \text{III}$$

Taking last 2 membes ;

$$\frac{dy}{y\cancel{(z-y^2-2x^3)}} = \frac{dz}{z\cancel{(z-y^2-2x^3)}}$$

$$\frac{dy}{y} = \frac{dz}{z}$$

By Integration, $\int \frac{dy}{y} = \int \frac{dz}{z}$

$$\log y = \log z + \log c_1$$

$$\frac{y}{z} = c_1 \quad \text{——— (1)}$$

Now Taking I st and III rd (or) you can take II nd and III rd also.

$$\frac{dx}{x(z-2y^2)} = \frac{dz}{z(z-y^2-2x^3)}$$

I III

$$\frac{dx}{x(z-\cancel{y^2}-2x^3)} = \frac{dz}{z(z-y^2-2x^3)}$$

Here, we put, $\frac{y}{z} = c_1$

or $\rightarrow$ $y = c_1 z$

$$\frac{dx}{x(z-2(c_1 z)^2)} = \frac{dz}{z(z-(c_1 z)^2-2x^3)}$$

$$\frac{dx}{x(z-2c_1^2 z^2)} = \frac{dz}{z(z-c_1^2 z^2-2x^3)}$$

$$\frac{dx}{x\cancel{z}(1-2c_1^2 z)} = \frac{dz}{\cancel{z}(z-c_1^2 z^2-2x^3)}$$

$$\frac{dz}{dx} = \frac{z-c_1^2 z^2-2x^3}{x(1-2c_1^2 z)}$$

* Here we want to form Linear diff^n eq^n of 1^{st} order

$$\frac{dy}{dx}+Py=Q$$

sol^n $\rightarrow$ $y(I.F) = \int Q(I.F)+C$

Or $\rightarrow$ $(1-2c_1^2 z)\frac{dz}{dx} = \frac{z-c_1^2 z^2}{x} - \frac{2x^3}{x}$

$$\left\{\begin{array}{l} \text{let, } u = z-c_1^2 z^2 \\ du = dz - c_1^2 \cdot 2z \\ du = (1-2c_1^2 z) \end{array}\right.$$

$$\frac{du}{dx} = \frac{u}{x} - 2x^2$$

This is Linear diff^n eq^n of 1^{st} order i $\rightarrow$

$$\frac{du}{dx} - \frac{1}{x}u = -2x^2$$

I.F $\rightarrow$ $e^{\int -\frac{1}{x}dx} = e^{-\log x} = x^{-1}$ or $\frac{1}{x}$

So sol^n $\rightarrow$ $u(I.F) = \int(-2x^2)(I.F)\,dx+$

$$\frac{u}{x} = \int -2x^2 \times \frac{1}{x}\,dx + C_2$$

$$\frac{u}{x} = \int -2x\,dx + C_2$$

$$\frac{u}{x} = -2\times\frac{x^2}{\cancel{x}} + c_2$$

$$\frac{u}{x} = -x^2 + c_2$$

Put, $u = z - c_1^2 z^2$

$$\frac{z - c_1^2 z^2}{x} + x^2 = c_2$$

again, put, $c_1 = y/z$

$$\frac{z - \frac{y^2}{z^2}\times z^2}{x} + x^2 = c_2$$

$$\frac{z - y^2}{x} + x^2 = c_2$$

Or, $\boxed{\frac{z}{x} - \frac{y}{x} + x^2 = c_2}$

Finally we get, $\phi\left\{\frac{y}{z}, \frac{z}{x} - \frac{y}{x} + x^2\right\} = 0$ Ans.

Method of Separation of Variable

In this method, we assume that the dependent variable is the product of 2 functions, each of which involves only one of the independent variables. As a consequence, 2 ordinary differential eqⁿ's are formed.

Q(1) Solve the diffⁿ eqⁿ, by Using method of separation of variable.

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$$

given that, $u(x,0) = 6e^{-3x}$

Solⁿ → Given, $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ ------(i)

Assume that the trial solⁿ of (i) is

Here, $X(x)$ is fⁿ of x only and $T(t)$ is fⁿ of t only → $u(x,t) = X(x)\cdot T(t)$ -----(ii)

diffⁿ w.r.t x:- $\frac{\partial u}{\partial x} = X'T$ ----(iii)

diffⁿ w.r.t t:- $\frac{\partial u}{\partial t} = XT'$ ----(iv)

By putting values of (ii), (iii) and (iv) in eqⁿ (i)

$$X'T = 2XT' + XT$$
$$X'T - XT = 2XT'$$
$$T(X' - X) = 2XT'$$
$$\underset{\text{I}}{\frac{X'-X}{X}} = \underset{\text{II}}{\frac{2T'}{T}} = \underset{\text{III}}{K \text{ (let)}}$$

By equating (I) and (III) :-

$$\frac{X'-X}{X} = K$$

$$\frac{X'}{X} - 1 = K$$

integrating w.r.t x:—

$$\int \frac{X'}{X} - \int dx = \int K\,dx$$

$$\log X - x = Kx + \log c_1$$

$$\log X = Kx + x + \log c_1$$

$$\log X = x(K+1) + \log c_1$$

So, $\frac{X}{c_1} = e^{(K+1)x} \Rightarrow X = c_1 e^{(K+1)x}$ —— (v)

again, By (II) and (III) :—

$$2\frac{T'}{T} = K$$

integrating w.r.t. t.

$$\cancel{2\log T}\; 2\int \frac{T'}{T} = \int K\,dt$$

$$2\log T = Kt + \log c_2$$

$$\log T = \frac{Kt}{2} + \frac{1}{2}\log c_2 \Rightarrow \log T = \frac{Kt}{2} + \log c_3$$

So, $\frac{T}{c_3} = e^{Kt/2} \Rightarrow T = c_3 e^{Kt/2}$ ---- (vi)

By putting the values of (v) & (vi) in eqn (ii) :—

$$u = X \cdot T$$

$$\boxed{u = c_1 c_3 e^{(K+1)x} \cdot e^{Kt/2}} \text{ ---- (vii)}$$

By Putting initial conditions;

$$u(x,0) = 6e^{-3x}$$

By putting $u = 6e^{-3x}$ when $t=0$ in (vii)

$$6e^{-3x} = e^{(K+1)x} \cdot e^{0}$$

$$6e^{-3x} = e^{(K+1)x}$$

By Comparing; $c_1c_3 = 6$ and

$-3 = k+1$

and $c_1 c_3 = 6$ $k = -4$

Put $k = -4$ in eqn (vii)

$$u = 6e^{(-4+1)x} \cdot e^{-4t/2}$$

$$u = 6e^{-3x} \cdot e^{-2t}$$

Ans

* *

(2) Solve $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given that $u(0,y) = 3e^{-y} - e^{-5y}$ by method of Separation of variable.

Solⁿ ⟹ Given, $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ ------ (1)

let, trial solⁿ of, ~~$4X'Y+XY'=$~~

$$u(x,y) = X(x) \cdot Y(y) \quad \text{------ (2)}$$

diffⁿ w.r.t x ⟶ $\frac{\partial u}{\partial x} = X'Y$ ------ (3)

diffⁿ w.r.t y ⟶ $\frac{\partial u}{\partial y} = XY'$ ------ (4)

Putting the eqⁿ of (2), (3), (4) in eqⁿ (1):—

$$4X'Y + XY' = 3XY$$

$$4X'Y = 3XY - XY'$$

$$4X'Y = X(3Y - Y')$$

$$\underbrace{\frac{4X'}{X}}_{\text{I}} = \underbrace{\frac{3Y - Y'}{Y}}_{\text{II}} = \underbrace{K \text{ (let)}}_{\text{III}}$$

By (I) & (III) :— ⟶ $\frac{X'}{X} = \frac{K}{4}$

By Integration, $\int \frac{X'}{X} = \int \frac{K}{4} dx$

$$\log X = \frac{Kx}{4} + \log c_1$$

$$\log \frac{x}{c_1} = \frac{kx}{4}$$

So, $$\frac{x}{c_1} = e^{\frac{kx}{4}}$$

$$\Rightarrow x = c_1 e^{\frac{kx}{4}} \quad \text{----- (6)}$$

again by II & III :-

$$\frac{3y - y'}{y} = k$$

$$\frac{3y}{y} - \frac{y'}{y} = k$$

By Integration, $$3 - \frac{y'}{y} = k$$

$$\cancel{\Rightarrow \int 3\,dy - \int \frac{y'}{y} = \int k\,dy}$$

$$\cancel{3y - \log y = ky + \log c_2}$$

$$\frac{y'}{y} = 3 - k$$

$$\int \frac{y'}{y} = \int (3-k)\,dy$$

$$\log y = (3-k)y + \log c_2$$

$$\frac{y}{c_2} = e^{(3-k)y}$$

$$y = c_2 e^{(3-k)y} \quad \text{----- (7)}$$

Putting the values of (6) & (7) in (2)

$$u = x \cdot y$$

$$u = c_1 e^{\frac{kx}{4}} \cdot c_2 e^{(3-k)y}$$

$$\boxed{u = c_1 c_2 e^{\frac{kx}{4}} \cdot e^{(3-k)y}} \quad \text{— (8)}$$

Given that, $u(0,y) = 3e^{-y} - e^{-5y}$

By putting $u = 3e^{-y} - e^{-5y}$ at $x = 0$ in eq^n (8)

$$3e^{-y} - e^{-5y} = c_1c_2\, e^{(3-k)y}$$

Now equating, 1st term of left with Right side;

$$3e^{-y} = c_1c_2\, e^{(3-k)y}$$

$$\boxed{c_1c_2 = 3} \quad \& \quad -1 = 3-k$$

$$\boxed{k=4}$$

again, equating, 2nd term of left with right side;

$$-e^{-5y} = c_1c_2\, e^{(3-k)y}$$

$$\boxed{c_1c_2 = -1}, \quad -5 = 3-k$$

$$\boxed{k=8}$$

So required Sol^n;

$$u = 3e^{\cancel{x}}\cdot e^{(3-4)y} + (-1)\, e^{\frac{8}{4}x}\cdot e^{(3-8)y}$$

$$u = 3e^{x}\cdot e^{-y} - e^{2x}\cdot e^{-5y}$$

$$u = 3e^{x-y} - e^{2x-5y}$$ Ans.

x ——— x