Normal Distribution (Continuous R.V.)

formula

$$p(X=x) = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} dx \quad \text{--- (1)}$$

But it is very difficult to evaluate. So we will change the variable the variable in normal variable. i.e. z.

Normal variable $z = \frac{x-\mu}{\sigma}$

Here, $\{\mu = 0$, $\{\mu = 1$

So, $P(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-z^2/2} dz$

Now It is very easy to solve,

$p(a < x < b) = \int_a^b f(x) dx$

$p(z_1 < z < z_2) = \int_{z_1}^{z_2} f(x) dz$

Here, formula $\boxed{z_1 = \frac{a-\mu}{\sigma}}$, formula $\boxed{z_2 = \frac{b-\mu}{\sigma}}$

and formula $F(z) = F(z_2) - F(z_1)$

Important Property :-
1) $F(-z) = -F(z)$
2) $F(\infty) = 0.5$
3) $F(0) = 0$

Que For a normally distributed variate with mean 1 and S.D. 3 find, the prob that,
(1) $3.43 \le x \le 6.19$
(2) $-1.43 \le x \le 6.19$

Soln Given, $\mu = 1$, $\sigma = 3$

(a) $\underset{a}{3.43} \le x \le \underset{b}{6.19}$

Converting in normal variable;

$z_1 \leq z \leq z_2$

$$z_1 = \frac{a-\mu}{\sigma} = \frac{3.43-1}{3} = 0.81$$

$$z_2 = \frac{b-\mu}{\sigma} = \frac{6.19-1}{3} = 1.73$$

$$P(z) = F(z_2) - F(z_1)$$
$$= F(1.73) - F(0.81)$$
$$= 0.4582 - 0.2910$$
$$= 0.1672 \quad \text{Ans.}$$

(b)

$-1.43 \leq x \leq 6.19$

a   b

$$z_1 = \frac{a-\mu}{\sigma} = \frac{-1.43-1}{3} = -0.81$$

$$z_2 = \frac{b-\mu}{\sigma} = \frac{6.19-1}{3} = 1.73$$

$$P(z) = F(z_2) - F(z_1)$$
$$= F(1.73) - F(-0.81)$$
$$= 0.4582 + 0.2910$$
$$= 0.7492 \quad \text{Ans.}$$

~~Sol~~ **Que** If z is normally distributed with mean 0 and variance 1, find

(a) $Pr(z \leq -1.64)$

(b) z_1, if $Pr(z \geq z_1) = 0.84$

Sol^n

$Pr(z_1 \leq z \leq -1.64) =$

$Pr(-\infty \leq z \leq -1.64) =$

$F(z_2) - F(z_1)$

$= F(-1.64) - F(-\infty)$

$= F(-1.64) + F(\infty)$

$= -0.4495 + 0.5$

$= 0.0505$

since, $\left\{ F(\infty) = 0.5 \right.$

(b) z_1, if $Pr(z \geq z_1) = 0.84$

$\therefore Pr(z_1 \leq z \leq \infty) = 0.84$

Here prob. is already given in the Que we have to find z_1,

$\cancel{F(z_2) - F(z_1) = 0.84}$

$F(\infty) - F(z_1) = 0.84$

$0.5 - F(z_1) = 0.84$

$F(z_1) = -0.34$

So, $z_1 = -0.99$ [By Anti normal Table]

Que ⟹ A Manufacturer of air-mail envelops knows from experience that the weight of the envelops is 'normally Distributed' with mean 1.95 gm and S.D 0.05 gm. about how many envelops weighting

1) 2 gm. or more

2) 2.05 gm. or more can be expected in a given Packet of 100 envelops.

Soln ⟹ Given, Mean $\mu = 1.95$ gm.

$\sigma = 0.05$ gm.

(1) 2 gm. or More :-

$P(\underset{a}{2} \leq x \leq \underset{b}{\infty}) = ?$

$$z_1 = \frac{a-\mu}{\sigma} = \frac{2-1.95}{0.05} = 1$$

$$z_2 = \frac{b-\mu}{\sigma} = \infty$$

$$p(z) = F(z_2) - F(z_1)$$
$$= F(\infty) - F(1)$$
$$= 0.5 - 0.3413$$
$$= 0.1587$$

There are 100 envelops, so

$$= 100 \times 0.1587$$
$$= 15.87 \quad \text{or} \quad 16 \text{ (approx)}$$

Sol^n

(2)

$$p(\underset{a}{2.05} \le x \le \underset{b}{\infty})$$

$$z_1 = \frac{a-\mu}{\sigma} = \frac{2.05 - 1.95}{0.05} = 2$$

$$z_2 = \frac{b-\mu}{\sigma} = \infty$$

$$p(z) = F(z_2) - F(z_1)$$
$$= F(\infty) - F(2)$$
$$= 0.5 - 0.4772$$
$$= 0.0228$$

Total 100 envelopes are there,

so 100×0.0228

$$= 2.28 \quad = 2 \text{ (approx)}$$

Que: The mean Hight of 500 students is 151 cm and the S.D 15 cm. assuming that the hight are normally distributed. Find how many students heights lies b/w 120 cm & 155 cm.

Sol^n

Mean $\mu = 151$ cm.

S.D. $\sigma = 15$ cm.

Firstly, $p(\underset{a}{120} \le x \le \underset{b}{155}) = ?$

classmate

Date

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$Z_1 = \frac{a-\mu}{\sigma} = \frac{120-151}{15} = -2.0667$

& $Z_2 = \frac{b-\mu}{\sigma} = \frac{155-151}{15} = 0.2667$

$P(Z) = F(Z_2) - F(Z_1)$
$= F(0.2667) - F(-2.0667)$
$= F(0.2667) + F(2.0667)$
$= 0.1026 + 0.4803$
$= 0.5829$

There are Total 500 students, so 500×0.5829
$= 291.45$
$= 291$ (approx)

Que ⟹ The μ and σ of the marks obtained by 1000 students in an exam are resp. 34.4 and 16.5, Assuming the normally distribution. Find the approximate no. of students expected to obtain marks b/w 30 and 60.

Sol^n ⟹ $\therefore \mu = 34.4$, $\sigma = 16.5$

we have to find

$P(\underset{a}{30} \le x \le \underset{b}{60}) = ?$

$Z_1 = \frac{30-34.4}{16.5} = -0.2667$

$Z_2 = \frac{60-34.4}{16.5} = 1.5515$

$P(Z) = F(Z_2) - F(Z_1)$
$= F(1.55) - F(-0.26)$
$= 0.4394 + 0.1026$
$= 0.5420$

There are 1000 students $\Rightarrow$ $1000 \times 0.5420 = 542$ Ans.

Que Assuming that the diameters 1000 brass plugs taken consequentively from a machine normal distribution with mean 0.7515 c.m and S.D. 0.002 cm., how many of the plugs are approved diameter is 0.752 ± 0.004 cm.

Solⁿ $\mu = 0.7515$ cm, $\sigma = 0.0020$ cm.

Given, diameter

(0.752 ± 0.004) cm.

Taking (+ve) $0.752 + 0.004 = 0.756 = a$

Taking (−ve) $0.752 - 0.004 = 0.748 = b$

we have, $P(\underset{a}{0.748} \le x \le \underset{b}{0.756}) = ?$

$$Z_1 = \frac{a-\mu}{\sigma} = \frac{0.7480 - 0.7515}{0.002} = -1.75$$

$$Z_2 = \frac{b-\mu}{\sigma} = \frac{0.756 - 0.7515}{0.0020} = 2.25$$

$$P(z) = F(z_2) - F(z_1) = F(2.25) - F(-1.75)$$
$$= 0.9477$$

Total 1000 $\longrightarrow$ $0.9477 \times 1000 = 947.7$ A.

Que In a normal Distribution 31 % of the items are under 45 and 8 % are over 64 find the mean and S.D. of the distribution.

Imp

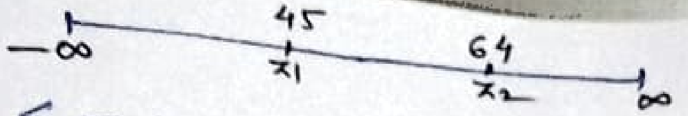
Solⁿ $P(-\infty \le x \le 45) = 31\%$

$\Rightarrow P(-\infty \le x \le 45) = 0.31$ —— (1)

$P(64 \le x \le \infty) = 8\%$

$\Rightarrow P(64 \le x \le \infty) = 0.08$ —— (2)

Now we will convert variable in normal variable Z such that



$(-\infty \le z \le z_1) = 0.31$ — ③ (According to eqⁿ ①)

& $(z_2 \le z \le \infty) = 0.08$ — ④ (According to eqⁿ ②)

$P(z) = F(z_2) - F(z_1) = 0.31$. [By eqⁿ ③]

$P(z) = F(z_1) - F(-\infty) = 0.31$

$F(z_1) = 0.31 - 0.5 = -0.19$

So, $\boxed{z_1 = -0.49}$ (by Antinormal)

Now, ~~$P(z) = F(z_2) - F(z_1) = 0.08$ [By eqⁿ ④]~~

$P(z) = F(\infty) - F(z_2) = 0.08$

$= 0.5 - F(z_2) = 0.08$

$F(z_2) = 0.5 - 0.08$

$\boxed{z_2 = 1.40}$ (By Antinormal)

Now, $z_1 = \frac{a-\mu}{\sigma} = \frac{45-\mu}{\sigma}$

$-0.49 = \frac{45-\mu}{\sigma} \Rightarrow \mu - 0.40\sigma = 45$ — ⑤

& $z_2 = \frac{b-\mu}{\sigma} = \frac{64-\mu}{\sigma}$

$1.40 = \frac{64-\mu}{\sigma} \Rightarrow 1.40\sigma = 64 - \mu$

or, $\mu + 1.40\sigma = 64$ — ⑥

By eqⁿ ⑤ & ⑥ :—

$$\mu - 0.40\sigma = 45$$
$$\mu + 1.40\sigma = 64$$

$$\boxed{\mu = 50 \quad \& \quad \sigma = 10}$$