(d) Find the rank correlation coefficient for the following data :

| x | 78 | 89 | 97 | 69 | 59 | 79 | 68 | 57 |
| y | 125 | 137 | 156 | 112 | 107 | 136 | 123 | 108 |

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B. E. (Third Semester) Examination, Nov.-Dec. 2015

(ET & T)

APPLIED MATHEMATICS-III

Time Allowed : Three hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : Part (a) of each question is compulsory carries 2 marks. Answer any two questions from (b), (c) and (d) from each question carries 7 marks.

1. (a) Write about Dirichlet's condition.

   (b) Prove that :

   $$x^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}, \ -\pi < x < \pi$$

Hence show :

(i) $\sum \dfrac{1}{n^2} = \dfrac{\pi^2}{6}$

(ii) $\dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + \cdots\cdots = \dfrac{\pi^2}{12}$

(c) Expand :

$$f(x) = \begin{cases} \dfrac{1}{4} - x, & \text{if } 0 < x < \dfrac{1}{2} \\[2mm] x - \dfrac{3}{4}, & \text{if } \dfrac{1}{2} < x < 1 \end{cases}$$

as the fourier series of sine terms.

(d) The following table gives the variations of periodic current over a period.

t sec.	0	T/6	T/3	T/2	2T/3	5T/6	T
A amp	1·98	1·30	1·05	1·30	−0·88	−0·25	1·98

Show that there is a direct current part of 0.75 amp in the variable current & obtain the amplitude of the first harmonics.

2. (a) Find Fourier sine & cosine integral.

(b) Find the Fourier cosine transform of e^{-x^2}.

(c) Solve the integral equation :

$$\int_0^\infty f(\theta)\cos\alpha\theta \; d\theta = \begin{cases} 1-\alpha, & 0 \le \alpha \le 1 \\ 0, & \alpha > 1 \end{cases}$$

Hence evaluate $\displaystyle\int_0^\infty \dfrac{\sin^2 t}{t^2} dt$.

(d) Write shifting property & modulation theorem.

3. (a) Write the definition of unit impulse function & their L.T.

(b) Find Laplace transform of :

$$\dfrac{1-\cos t}{t^2}$$

(c) Apply convolution theorem to evaluate :

$$L^{-1}\left[\frac{s}{\left(s^2+a^2\right)^2}\right]$$

(d) Solve by the transformation method :

$$ty'' + 2y' + ty = \cos t, \quad y(0) = 1$$

4. (a) Define Holomorphic function.

(b) If $f(z)$ is a regular function of z, prove that :

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left|f(z)\right|^2 = 4\left|f'(z)\right|^2$$

(c) Expand the following functions in Laurent's series :

$$F(z) = \frac{1}{(z+1)(z+2)}$$

in powers of $(z+1)$ for the range $0 < |z+1| < 2$.

(d) Evaluate :

$$\int_{-\infty}^{\infty} \frac{x^2\,dx}{\left(x^2+1\right)\left(x^2+4\right)}$$

5. (a) Define (+)ve & (−)ve correlation.

(b) The regression equations of two variables x and y are $x = 0.7y + 5.2$, $y = 0.3x + 2.8$. Find the means of the variables and the coefficient of correlation between them.

(c) The students got the following percentage of marks Mathematics and Mechanics are :

Roll no.	1	2	3	4	5	6	7	8	9	10
Marks in Mathematics	17	18	19	19	20	20	21	21	22	23
Marks in Mechanics	12	16	14	11	15	19	22	16	15	20

Calculate the coefficient of correlation.