Case - III $\quad f(x,y) = x^m y^n$

Que ① $(D^2 - 6DD' + 9D'^2)z = 12x^2 + 36xy$

Sol^n $(D^2 - 6DD' + 9D'^2)z = 0$

A. eq^n $D^2 - 6DD' + 9D'^2 = 0$

$m^2 - 6m + 9 = 0$ $\quad \begin{cases} D \to m \\ D' \to 1 \end{cases}$

$m = 3, 3$

C.F. $f_1(y+3x) + x f_2(y+3x)$

P.I. $\dfrac{1}{f(D,D')} 12x^2 + 36xy$

$$\frac{1}{D^2 - 6DD' + 9D'^2} 12x^2 + 36xy$$

$$\Rightarrow \frac{1}{D^2\left\{1 - 6\frac{D'}{D} + 9\frac{D'^2}{D^2}\right\}} 12x^2 + 36xy$$

$$\Rightarrow \frac{1}{D^2}\left\{1 - \left(6\frac{D'}{D} - 9\frac{D'^2}{D^2}\right)\right\}^{-1} 12x^2 + 36xy$$

$$\Rightarrow \frac{1}{D^2}\left[1 + \left(\frac{6D'}{D} - \frac{9D'^2}{D^2}\right) + \left(6\frac{D'}{D} - \frac{9D'^2}{D^2}\right)^2 + \cdots\right] 12x^2 + 36xy$$

** $(1+x)^n = 1 + nx + n(n-1)\dfrac{x^2}{2!} + n(n-1)(n-2)\dfrac{x^3}{3!} + \cdots \infty$

$$\Rightarrow \frac{1}{D^2}\left[(12x^2 + 36xy) + \left(\frac{6D'}{D} - \frac{9D'^2}{D^2}\right)(12x^2 + 36xy) + \left(\frac{6D'}{D} - \frac{9D'^2}{D^2}\right)^2 (12x^2 + 36xy) + \cdots\right]$$

$$\Rightarrow \frac{1}{D^2}\left[(12x^2 + 36xy) + \frac{6}{D} \times D'(12x^2 + 36xy) - \frac{9}{D^2} \times D'^2 (12x^2 + 36xy) + \frac{6^2}{D^2} \times D'(12x^2 + 36xy) + \frac{9}{D^4} \times D'^4 (12x^2 + 36xy) - 2\left(\frac{6D'}{D} - \frac{9D'^2}{D^2}\right)(12x^2 + 36xy) \cdots \infty\right.$$

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$$\Rightarrow \frac{1}{D^2}\Big[12x^2 - 36xy + \frac{6}{D}[36x] - \frac{9}{D^2}[0] + \frac{6}{D^2}[0] + \frac{9}{D^4}\times 0 - \frac{12}{D}\times D'(12x^2+36xy)\Big]$$

$$\Rightarrow \frac{1}{D^2}\Big[12x^2 - 36xy + 216x\int x\,dx - \frac{12}{D}\times 36x\Big]$$

$$\Rightarrow \frac{1}{D^2}\Big[12x^2 - 36xy + \overset{108}{\cancel{216}}\times\frac{x^2}{\cancel{2}} - 432\int x\,dx\Big]$$

$$\Rightarrow \frac{1}{D^2}\Big[12x^2 - 36xy + 108x^2 - \overset{216}{\cancel{432}}\times\frac{x^2}{\cancel{2}}\Big]$$

$$\Rightarrow \frac{1}{D^2}\Big[12x^2 - 36xy + 108x^2 - 216x^2\Big]$$

$$\Rightarrow \frac{1}{D^2}\Big[-96x^2 - 36xy\Big]$$

$$\Rightarrow \int\int -96x^2 - 36xy \; dx)\,dx$$

$$\Rightarrow \int\Big[-96\times\frac{x^2}{2} - 36y\times\frac{x^2}{2}\Big]dx$$

$$\Rightarrow -\overset{16}{\cancel{96}}\times\frac{x^3}{\cancel{3\times 2}} - \overset{12\;6}{36}y\times\frac{x^3}{\cancel{3\times 2}}$$

$$\Rightarrow -16x^3 - 6x^3y$$

$$Z = C.F + P.I.$$

Que $\Rightarrow$ $(D^2 - 2DD' + D'^2)z = x^3$

Sol^n $\Rightarrow$ $(D^2 - 2DD' + D'^2)z = 0$

A.eq^n $\Rightarrow$ $D^2 - 2DD' + D'^2 = 0$

$m^2 - 2m + 1 = 0$ $\left\{ \begin{array}{l} D \to m \\ D' \to 1 \end{array} \right.$

$m = 1, 1$

C.F. $\Rightarrow$ $f_1(y+x) + x f_2(y+x)$

P.I. $\Rightarrow$ $\frac{1}{f(D,D')} F(x,y)$

$\frac{1}{D^2 - 2DD' + D'^2} x^3$

$\frac{1}{D^2\left\{1 - 2\frac{D'}{D} + \frac{D'^2}{D^2}\right\}} x^3$

$\Rightarrow \frac{1}{D^2}\left[1 - \left(\frac{2D'}{D} - \frac{D'^2}{D^2}\right)\right]^{-1} x^3$

$\Rightarrow \frac{1}{D^2}\left[1 + \left(\frac{2D'}{D} - \frac{D'^2}{D^2}\right) + \left(\frac{2D'}{D} - \frac{D'^2}{D^2}\right)^2 + \left(\frac{2D'}{D} - \frac{D'^2}{D^2}\right)^3 + \text{-------}\right] x^3$

$\Rightarrow \frac{1}{D^2}\left[1 + \left(\frac{2D'}{D} - \frac{D'^2}{D^2}\right) + \left(\frac{4D'^2}{D^2} + \frac{D'^4}{D^4} - \frac{4D'^3}{D^3}\right) \frac{8D'^3}{D^3} + \text{----------}\right] x^3$

$\Rightarrow \frac{1}{D^2}\left[x^3 - \frac{2}{D} \times 0 - \frac{1}{D^2} \times 0 + \frac{4}{D^2} \times 0 \text{ ---}\right]$

$\Rightarrow \iint x^3\,dx\,dx = \int \frac{x^4}{4}\,dx$

$\Rightarrow \frac{x^5}{20}$ So, $z = C.F + PI$

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Que $\Rightarrow$ $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$

Sol^n $\Rightarrow$ $(D^2 + DD' - 6D'^2)z = y\cos x$

A.eq^n $\Rightarrow$ $D^2 + DD' - 6D'^2 = 0$

$m^2 + m - 6 = 0$ $\quad D \to m$

$m = -3, 2$ $\quad D' \to 1$

C.F. $\Rightarrow$ $f_1(y-3x) + f_2(y+2x)$

P.I. $\Rightarrow$ $\frac{1}{D^2 + DD' - 6D'^2} \cdot y\cos x$

By using General method (which is also known as failure case).

By 1^{st} factor $(D+3D')$

$\int F(x,y)\,dx$ $\quad$ let, $y - 3x = c$ then $y = c + 3x$

$\int F(x, c+3x)\,dx$

$\int y\cos x\,dx$

$\int \underset{u}{(c+3x)}\underset{v}{\cos x}\,dx$

$(c+3x)\sin x + 3\cos x$ $\quad$ again put, $c = y - 3x$

$(y - 3x + 3x)\sin x + 3\cos x$

$y\sin x + 3\cos x$ —— (1)

again, by 2^{nd} factor; $(D - 2D')$

$\int F(x,y)\,dx$ $\quad$ put, $y + 2x = c$ then $y = c - 2x$

$\int F(x, c-2x)\,dx$

$\int (y \sin x + 3\cos x)\, dx$ By eqⁿ ①

$\Rightarrow \int \left(\frac{c-2x}{4}\right) \sin x + 3\cos x \; dx$

$\Rightarrow \left[(c-2x)(-\cos x) - (-2)(-\sin x) + 3\sin x\right]$

again, put $c = y+2x$

$\Rightarrow \left[(y+\cancel{2x}-\cancel{2x})(-\cos x) - 2\sin x + 3\sin x\right]$

$\Rightarrow \left[-y\cos x + \sin x\right]$

So, $Z = C.F. + P.I.$

$Z = f_1(y-3x) + f_2(y+2x) + \left[\sin x - y\cos x\right]$

Ans

Que $(D^3 - D^2D' + DD'^2 - D'^3)Z = e^x \cos 2y$

Solⁿ $D^3 - D^2D' + DD'^2 - D'^3 = 0$

$m^3 - m^2 + m - 1 = 0$ $\left.\begin{matrix} D \to m \\ D' \to 1 \end{matrix}\right\}$

$m = -1, -1, 1$

C.F. $\rightarrow$ $f_1(y-x) + x f_2(y-x) + f_3(y+x)$

P.I. $\rightarrow$ $\dfrac{1}{D^3 - D^2D' + DD'^2 - D'^3} e^x \cos 2y$

By Using General method

By Taking 1^{st} factor, $(D+D')$

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let, Put $c = y - x \Rightarrow y = c + x$

$\int F(x,y)\,dx$

$\Rightarrow \int (x, c+x)\,dx$

$\Rightarrow \int e^{x} \cos 2y\,dx$

$\Rightarrow \int e^{x} \cos 2(c+x)\,dx$

$\Rightarrow \left[\frac{e^{x}}{1+4}\left\{\cos 2(c+x) + 2\sin 2(c+x)\right\}\right]$

$\Rightarrow \left[\frac{e^{x}}{5}\left\{\cos 2(c+x) + 2\sin 2(c+x)\right\}\right]$

$\Rightarrow \frac{e^{x}}{5}\left[\cos 2(y-x+x) + 2\sin 2(y-x+x)\right]$ Put, $c = y - x$

$\Rightarrow \frac{e^{x}}{5}\left[\cos 2y + 2\sin 2y\right]$ ------------ (1)

again, by 2^{nd} factor, $(D+D')$

$\Rightarrow \int F(x,y)\,dx$ Put, $c = y - x$
$y = c + x$

$\Rightarrow \int \frac{e^{x}}{5}\left\{\cos 2y + 2\sin 2y\right\}]\,dx$

$\Rightarrow \int \frac{e^{x}}{5}\left\{\cos 2(c+x) + 2\sin 2(c+x)\right\}\,dx$

$\Rightarrow \frac{1}{5}\left[\int e^{x}\cos 2(c+x)\,dx + \int e^{x}\sin 2(c+x)\,dx\right]$

$\Rightarrow \frac{1}{5}\left[\frac{e^{x}}{1+4}\left\{\cos 2(c+x) + 2\sin(c+x)\right\} + \frac{e^{x}}{1+4}\left\{\sin 2(c+x) - 2\cos(c+x)\right\}\right]$

$\Rightarrow \frac{1}{5}\left[\frac{e^{x}}{5}\left\{\cos 2(c+x) + 2\sin(c+x)\right\} + \frac{e^{x}}{5}\left\{\sin 2(c+x) - 2\cos(c+x)\right\}\right]$

$\Rightarrow \frac{1}{25}\left[e^{x}\left\{\cos 2(y-x+x) + 2\sin(y-x+x)\right\} + e^{x}\left\{\sin 2(y-x+x) - 2\cos(y-x+x)\right\}\right]$ {again put,

$\Rightarrow \frac{e^x}{25}\left[\cos 2y + 2\sin y + \sin 2y - 2\cos y\right]$ Ans

$z = C.F + P.I.$

Non-Homogeneous Fu^n

P.I. of Non-Hom fu^n is same as Hom. fu^n.
only C.F. is different.

In the Non-Hom. fu^n factor is in the form of $(D - mD' - a)$

m_1, m_2 are roots then, $\xrightarrow{C.F}$ $e^{ax} f_1(y + m_1 x) + e^{ax} f_2(y + m_2 x)$

Que $\Rightarrow$ $(D^2 - DD' + D' - 1)z = \cos(x + 2y)$

Sol^n $\Rightarrow$

$D^2 - DD' + D' - 1 = 0$

$(D^2 - 1) + D'(-D + 1) = 0$

$(D+1)(D-1) - D'(D-1) = 0$

$(D-1)(D - D' + 1) = 0$

Or $\Rightarrow$ $(D - 0D' - 1)(D - D' + 1) = 0$

Here factors are in the form of $(D - mD' - a)$ so, by comparing

$a = 1$ & $m = 0$ (by 1st factor) | $a = -1$, $m = 1$ (by 2nd factor)

then, $\xrightarrow{C.F.}$ $e^x[f_1(y + 0x)] + e^{-x}[f_2(y + x)]$

$\Rightarrow e^x f_1(y) + e^{-x} f_2(y + x)$

P.I. $\rightarrow \frac{1}{f(D,D')} F(x,y)$

$\frac{1}{D^2 - DD' + D' - 1} \cos(x+2y)$

put, $\left\{ \begin{array}{l} D^2 \rightarrow -1 \\ DD' \rightarrow -2 \end{array} \right.$

$\Rightarrow \frac{1}{-1+2+D'-1} \cos(x+2y)$

$\Rightarrow \frac{1}{D'} \cos(x+2y)$

$\Rightarrow \int \cos(x+2y)\,dy$ Integration w.r.t y

$\Rightarrow \frac{\sin(x+2y)}{2}$

So, $Z = C.F + P.I.$ Ans

(2) $[(D-D'-1)(D-D'-2)]z = e^{2x-y}$

Sol^n $\Rightarrow$ By comparing with factors $(D - mD' - a)$

By 1st factor :- $(D-D'-1)$ $m=1, a=1$

and By 2nd factor :- $(D-D'-2)$

$m=1, a=2$

C.F. $\rightarrow e^{x} f_1(y+x) + e^{2x} f_2(y+x)$

P.I. $\rightarrow \frac{1}{f(D,D')} F(x,y)$

$\frac{1}{(D-D'-1)(D-D'-2)} e^{2x-y}$ Put $D \rightarrow 2$, $D' \rightarrow -1$

$\frac{1}{(2+1-1)(2+1-2)} e^{2x-y}$

$\frac{1}{2} e^{2x-y}$

So, $Z = C.F + P.I.$ Ans.