Second Year B. Tech. -CSE

3rd semester Subject Code:AM102301

Subject Code	APPLIED MATHEMATICS-III	L = 3	T = 1	P = 0	Credits = 4
Evaluation Scheme	ESE	CT	TA	Total	ESE Duration
	100	20	30	150	3 Hours

Course Objectives	Course Outcomes
The objective of this course is to familiarize the prospective engineers with techniques in calculus of multivariable and infinite series expansion of continuous function as well as some statistical treatment of discrete functions. More precisely, the objectives are: • To investigate a thorough knowledge of partial differential equations which arise in mathematical descriptions of situations in engineering. • To develop the tool of Fourier series for learning advanced Engineering Mathematics. • To provide knowledge of Laplace transform of elementary functions including its properties and applications to solve ordinary differential equations. • To originate a thorough study about random quantities and their description in terms of their probability. • To provide a thorough understanding of interpolation.	**On successful completion of the course, the student will be able to:** **CO 1.** To have a thorough knowledge of PDE which arise in mathematical descriptions of situations in Engineering. **CO 2.** To make the students understand that Fourier series analysis is powerful methods where the formulas are integrals and to have knowledge of expanding periodic functions that explore variety of applications of Fourier series. **CO3.** To provide knowledge of Laplace transform of elementary functions including its properties and applications to solve ordinary differentials equations. **CO4.** To study about a quantity that may take any of a given range of values that can't be predicted as it is but can be described in terms of their probability **CO5.** To study the technique of estimating the values of a function for any intermediate value of the independent variable.

UNIT – I Partial differential equation: Formation, Solution by direct integration method, Linear equation of first order, Homogeneous linear equation with constant coefficients, Non-homogeneous linear equations, Method of separation of variables; Equation of vibrating string (wave equation). **[10 Hrs]**

UNIT – II Fourier Series- Euler's formula; Functions having point of discontinuity; Change of interval; Even and Odd function; Half range series; Harmonic Analysis. **[10Hrs]**

UNIT – III Laplace transform: Definition; Transform of elementary functions; Properties of Laplace transform; Inverse Laplace Transform (Method of partial fraction, using properties and Convolution theorem); Transform of Unit step function and Periodic functions; Application to the solution of ordinary differential equations. **[10Hrs]**

UNIT – IV Probability distributions: Random variable; Discrete and continuous probability distributions; Mathematical expectation; Mean, Variance and Moments; Moment generating functions; Probability distribution (Binomial, Poisson, and Normal distributions). **[10Hrs]**

SHRI SHANKARACHARYA TECHNICAL CAMPUS
SHRI SHANKARACHARYA GROUP OF INSTITUTIONS
BHILAI
(An Autonomous Institute affiliated to CSVTU,Bhilai)
Scheme of Examination and Syllabus 2021

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UNIT – V Interpolation with equal and unequal intervals: Finite difference, Newton's Forward and Backward Difference Formulae, Central Difference Formula, Stirling's Formula, Bessel's Formula, Langrange's Formula and Newton's Divided Difference Formula.

[10Hrs]

Text Books:

S. No.	Title	Authors	Publisher
1)	Higher Engineering Mathematics	Dr. B.S. Grewal	Khanna Publishers
2)	Numerical Methods in Engineering and Science	Dr. B.S. Grewal	Khanna Publishers
3)	Advanced Engineering Mathematics	Erwin Kreyszig	John Wiley & Sons
4)	Applied Engineering Mathematics	Madan Mohan Singh	BS Publications

Reference Books:

S. No.	Title	Authors	Publisher
1)	Calculus and Analytic geometry	G. B. Thomas and R. L. Finney	Pearson, Reprint
2)	Engineering Mathematics for first year	T. Veerarajan	Tata McGraw-Hill, New Delhi
3)	Higher Engineering Mathematics	B. V. Ramana	Tata McGraw Hill New Delhi
4)	A text book of Engineering Mathematics	N.P. Bali and Manish Goyal	Laxmi Publications

Dr. M M Singh, Chairman(BOS)

Laplace Transforms :

Transformation :— Transformation is a mathematical process by which one function converts into another function.

For example :— $D(\sin x) = \cos x$

$\downarrow$ odd fn. $\quad \downarrow$ even fn.

Here, D is Transformation operator.

Laplace Transforms :— Let $f(t)$ be a fn of t, $t \geq 0$, then Laplace Tr. is defined by $L[f(t)]$ which is defined by

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt \quad , t \geq 0$$

Here, s is a parameter which may be real or complex.

$$L[f(t)] = \bar{f}(s)$$

Laplace Tr. : developed by 'oliver Heaviside'.

Application of Laplace Transforms :—

① Laplace Trasform directly gives the Solution of Differential eqn.

② Electric signals → To solve problems of Electric signals.

Properties of L.T.

① Linearity property :— If c_1 and c_2 are constants and f and g are functions of t then

$$L[c_1 f(t) + c_2 f_2(t)] = c_1 L\{f_1(t)\} + c_2 L\{f_2(t)\}$$

* Existence of Laplace Transforms :-

The L.T. of $f(t)$, i.e. $\int_0^\infty e^{-st} f(t)\,dt$ exists for $s > a$ if

(i) $f(t)$ is continuous

(ii) $\lim\limits_{t \to \infty} e^{-at} f(t)$ is finite.

* Change of Scale Property :—

If $L[f(t)] = \bar{f}(s)$ then
$L[f(at)] = \frac{1}{a}\bar{f}\left(\frac{s}{a}\right)$

* I^{st} Shifting Property :—

If $L[f(t)] = \bar{f}(s)$ then
$L[e^{at} f(t)] = \bar{f}(s-a)$

Standard Formula for L.T.

① $\boxed{L(1) = \frac{1}{s}}$, $s>0$

$$L(1) = \int_0^\infty e^{-st} \cdot 1\, dt = \int_0^\infty e^{-st}\, dt$$

$$= \left[-\frac{e^{-st}}{s}\right]_0^\infty = \frac{1}{s} \qquad \text{(Here, } s>0\text{)}$$

② $L(t^n) = \int_0^\infty e^{-st}\, t^n\, dt$

$$= \int_0^\infty e^{-p}\left(\frac{p}{s}\right)^n \frac{dp}{s}$$

$$= \frac{1}{s^{n+1}}\int_0^\infty e^{-p} \cdot p^n\, dp$$

let $st = p$
or $t = \frac{p}{s}$
$dt = \frac{dp}{s}$
limits are 0 to ∞

$$= \frac{1}{s^{n+1}}\int_0^\infty e^{-p}\, p^{(n+1)-1}\, dp$$

By ~~Be~~ Gamma fu^n formula :—

$$\Gamma n = \int_0^\infty e^{-x} x^{n-1}\, dx$$

$$= \frac{1}{s^{n+1}} \Gamma n+1 \quad \text{or} \quad \frac{n!}{s^{n+1}} \quad , s>0$$

$$\boxed{L(t^n) = \frac{\Gamma n+1}{s^{n+1}} \quad \text{or} \quad \frac{n!}{s^{n+1}}}$$

(3) $L(e^{at}) = \int_0^\infty e^{-st} \cdot e^{at}\, dt$

$$= \int_0^\infty e^{-(s-a)t}\, dt$$

$$= \left[\frac{-e^{-(s-a)t}}{s-a} \right]_0^\infty = \frac{1}{s-a}$$

Here. $s>0$

$$\boxed{L(e^{at}) = \frac{1}{s-a}}$$

(4) $L\{\sin at\} = \int_0^\infty e^{-st} \sin at\, dt$

By Using Formula :- $\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2+b^2}\left[a \sin bx - b \cos bx\right]$

$$= \left[\frac{e^{-st}}{s^2+a^2} \left[-s\sin at - a\cos at \right] \right]_0^\infty$$

$$= \frac{-1}{s^2+a^2} \left\{ -a \right\} = \frac{a}{s^2+a^2}$$

$$\boxed{L(\sin at) = \frac{a}{s^2+a^2}}$$

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(5) $L(\cos at) = \int_0^\infty e^{-st} \cos at \, dt$

$$= \left\{ \frac{e^{-st}}{s^2+a^2}\left[-s\cos at + a\sin at\right] \right\}_0^\infty$$

$$= \frac{+s}{s^2+a^2}$$

$$\boxed{L(\cos at) = \frac{s}{s^2+a^2}}$$

(6) $L(\sinh at) = \int_0^\infty e^{-st} \sinh at \, dt$

Since, $\sinh at = \dfrac{e^{at} - e^{-at}}{2}$

$$= \int_0^\infty e^{-st}\left[\frac{e^{at} - e^{-at}}{2}\right] dt$$

$$= \frac{1}{2}\int_0^\infty \left\{ e^{-(s-a)t} - e^{-(s+a)t} \right\} dt$$

$$= \frac{1}{2}\left[-\frac{e^{-(s-a)t}}{s-a} + \frac{e^{-(s+a)t}}{s+a} \right]_0^\infty$$

$$= \frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a} \right]$$

$$= \frac{1}{2}\left[\frac{\cancel{s}+a - \cancel{s}+a}{s^2-a^2} \right]$$

$$= \frac{\cancel{2}a}{\cancel{2}(s^2-a^2)} = \frac{a}{s^2-a^2}$$

$$\boxed{L(\sinh at) = \frac{a}{s^2-a^2}}$$

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(7) $L(\cosh at) = \int_0^\infty e^{-st} \cosh at \, dt$

Since, $\cosh at = \dfrac{e^{at} + e^{-at}}{2}$

$= \int_0^\infty e^{-st} \left\{ \dfrac{e^{at} + e^{-at}}{2} \right\} dt$

$= \frac{1}{2} \int_0^\infty \left\{ e^{-(s-a)t} + e^{-(s+a)t} \right\} dt$

$= \frac{1}{2} \left[\dfrac{-e^{-(s-a)t}}{s-a} - \dfrac{e^{-(s+a)t}}{s+a} \right]_0^\infty$

$= \frac{1}{2} \left[\dfrac{1}{s-a} + \dfrac{1}{s+a} \right]$

$= \frac{1}{2} \left[\dfrac{s+\cancel{a} + s-\cancel{a}}{s^2-a^2} \right]$

$= \frac{1}{\cancel{2}} \times \dfrac{\cancel{2}s}{s^2-a^2} = \dfrac{s}{s^2-a^2}$

$$\boxed{L(\cosh at) = \dfrac{s}{s^2-a^2}}$$

Example of Linearity property

(1) $L\left[2e^{3t} + \frac{5}{2} \sin 5t\right] =$

$2L\{e^{3t}\} + \frac{5}{2} L\{\sin 5t\}$ by Linearity Prop.

$= 2\left[\dfrac{1}{s-3}\right] + \frac{5}{2} \left\{\dfrac{5}{s^2+25}\right\}$ Ans.

* [By Using formula, $L(e^{at}) = \dfrac{1}{s-a}$ & $L(\sin at) = \dfrac{a}{s^2+a^2}$]

Some Questions

① $\cos(at+b)$ Find L.T. of given fu^n.

Sol^n ⟹ let, $f(t) = \cos(at+b)$ Taking L.T. operator on both sides

$$L[f(t)] = L[\cos(at+b)]$$

$$L\{f(t)\} = L[\cos at \cdot \cos b - \sin at \cdot \sin b]$$

* (by Using formula, $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$)

$$= \cos b\{L(\cos at)\} - \sin b\{L(\sin at)\}$$

{By linearity prop}

$$= \cos b\left[\frac{s}{s^2+a^2}\right] - \sin b\left[\frac{a}{s^2+a^2}\right]$$

$$= \frac{1}{s^2+a^2}\{s\cos b - a\sin b\}$$ Ans.

② $(\sin t - \cos t)^2$ Find L.T.

Sol^n ⟹ let, $f(t) = (\sin t - \cos t)^2$

Taking L.T. operator on both sides,

$$L\{f(t)\} = L[(\sin t - \cos t)^2]$$

$$= L[\sin^2 t + \cos^2 t - 2\sin t\cos t]$$

$$= L(\sin^2 t) + L(\cos^2 t) - L[\sin 2t]$$

*

(By Using formula $2\sin A \cdot \cos B = \sin 2A$)

or, $= L[\sin^2 t + \cos^2 t] - L[\sin 2t]$

$= L(1) - L(\sin 2t)$

$= \frac{1}{s} - \frac{2}{s^2+4}$

or, $\frac{s^2+4-2s}{s(s^2+4)}$ Ans..

(3) find L.T. of $\cos^3 2t$

Sol^n let $f(t) = \cos^3 2t$

Taking L.T. operator on both sides.

$L\{f(t)\} = L[\cos^3 2t]$

By Using formula, $\cos^3 x = \frac{1}{4}[3\cos x + \cos 3x]$

$L[f(t)] = L[\cos^3 2t]$

$= L\frac{1}{4}[3\cos 2t + \cos 6t]$

$= \frac{3}{4} L(\cos 2t) + \frac{1}{4} L(\cos 6t)$

(By Linearity Prop.)

$= \frac{3}{4}\left[\frac{s}{s^2+4}\right] + \frac{1}{4}\left[\frac{s}{s^2+6}\right]$

$= \frac{s}{4}\left[\frac{3}{s^2+4} + \frac{3}{s^2+6}\right]$

Ans..

H.W: Find L.T. of $\sin^3 2t$.

use this formula $\rightarrow$ $\sin^3 x = \frac{1}{4}\left[3\sin x - \sin 3x\right]$

Q. Find L.T. of $L\{\sin\sqrt{t}\}$.

Sol^n $\rightarrow$ We have to solve $L\{\sin\sqrt{t}\}$ by using expansion of $\sin\sqrt{t}$.

expansion is $\rightarrow$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - - - \infty$$

So, $$\sin\sqrt{t} = \sqrt{t} - \frac{(\sqrt{t})^3}{3!} + \frac{(\sqrt{t})^5}{5!} - \frac{(\sqrt{t})^7}{7!} + - - \infty$$

Taking L.T. operator on both sides;

$$L[\sin\sqrt{t}] = L(t^{1/2}) - \frac{1}{6}L(t^{3/2}) + \frac{1}{120}L(t^{5/2}) - - - - - - \infty$$

$$= \frac{\Gamma\frac{1}{2}+1}{s^{3/2}} - \frac{1}{6}\frac{\Gamma\frac{3}{2}+1}{s^{5/2}} + \frac{1}{120}\frac{\Gamma\frac{5}{2}+1}{s^{7/2}} - - - - - - \infty$$

since, $\Gamma n+1 = n\Gamma n$ so,

$$= \frac{\frac{1}{2}\Gamma\frac{1}{2}}{s^{3/2}} - \frac{1}{6}\frac{\frac{3}{2}\Gamma\frac{3}{2}}{s^{5/2}} + \frac{1}{120}\frac{\frac{5}{2}\Gamma\frac{5}{2}}{s^{7/2}} - - - - - - \infty$$

since, $\left(\Gamma\frac{1}{2} = \sqrt{\pi}\right)$

$$= \frac{\frac{1}{2}\sqrt{\pi}}{s^{3/2}} - \frac{1}{4}\frac{\Gamma 1+\frac{1}{2}}{s^{5/2}} + \frac{1}{120}\frac{\frac{5}{2}\cdot\Gamma 1+\frac{3}{2}}{s^{7/2}} - - - - - \infty$$

$$= \frac{\sqrt{\pi}}{2s^{3/2}} - \frac{1}{4}\frac{\frac{1}{2}\Gamma\frac{1}{2}}{s^{5/2}} + \frac{1}{120}\times\frac{5}{2}\times\frac{3}{2}\frac{\Gamma 3/2}{s^{7/2}} - - - - - \infty$$

$$= \frac{\sqrt{\pi}}{2s^{3/2}} - \frac{\sqrt{\pi}}{8s^{5/2}} + \frac{\Gamma 3/2}{32 s^{7/2}} - - - - \infty$$

$$= \frac{\sqrt{\pi}}{2s^{3/2}} - \frac{\sqrt{\pi}}{8s^{5/2}} + \frac{\Gamma 1+1/2}{32 s^{7/2}} - - - - - \infty$$

$$= \frac{\sqrt{\pi}}{2s^{3/2}} - \frac{\sqrt{\pi}}{8s^{5/2}} + \frac{\frac{1}{2}\Gamma\frac{1}{2}}{32 s^{7/2}} - - - - \infty$$

$$= \frac{\sqrt{\pi}}{2s^{3/2}} - \frac{\sqrt{\pi}}{8s^{5/2}} + \frac{\sqrt{\pi}}{64 s^{7/2}} - - - - \infty$$

$$= \frac{\sqrt{\pi}}{2s^{3/2}} \left[1 - \frac{1}{4s} + \frac{1}{2!}\left(\frac{1}{4s}\right)^2 - \frac{1}{3!}\left(\frac{1}{4s}\right)^3 - - - - - - - \infty \right]$$

(We know that the expansion of e^{-x} is

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} - - - - - - - - \infty \quad)$$

$$= \frac{\sqrt{\pi}}{2s^{3/2}} \left[e^{-1/4s} \right]$$

Ans..

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