Que $\Rightarrow$ $L^{-1}\left(\frac{1}{s^3-a^3}\right)$

Sol^n $\frac{1}{s^3-a^3} = \frac{1}{(s-a)(s^2+as+a^2)}$

By Partial fraction;

$$\frac{1}{(s-a)(s^2+as+a^2)} = \frac{A}{s-a} + \frac{Bs+C}{s^2+as+a^2} \quad \text{-------- (1)}$$

$$\frac{1}{(s-a)(s^2+as+a^2)} = \frac{A(s^2+as+a^2)+(s-a)(Bs+C)}{(s-a)(s^2+as+a^2)}$$

$$1 = A(s^2+as+a^2)+(s-a)(Bs+C) \quad \text{------ (2)}$$

Put $s = a \rightarrow$

$$1 = A(a^2+a^2+a^2) \Rightarrow 1 = 3a^2A \Rightarrow \boxed{A = \frac{1}{3a^2}}$$

put $A = \frac{1}{3a^2}$ in eq^n (2) ; —

$$1 = \frac{1}{3a^2}\left[s^2+as+a^2\right] + (s-a)(Bs+C)$$

$$1 = \frac{1}{3a^2}(s^2+as+a^2) + Bs^2 + Cs - Bas - Ca$$

$$0.s + 0s^2 + 1 = s^2\left(\frac{1}{3a^2}+B\right) + s\left(\frac{1}{3a}+C-Ba\right) + \frac{1}{3}$$

Comparing coeffi. of s^2 with LHS : —

$$\frac{1}{3a^2}+B = 0 \Rightarrow \boxed{B = \frac{-1}{3a^2}}$$

again, comparing coeff. of s with LHS.

$$\frac{1}{3a}+C-Ba = 0 \Rightarrow \frac{1}{3a}+C-\left(\frac{-1}{3a^2}\right)a = 0$$

$$\Rightarrow \boxed{C = \frac{-2}{3a}}$$

By Putting values of A, B, C in (1)

$$\frac{1}{(s-a)(s^2+as+a^2)} = \frac{1}{3a^2(s-a)} + \frac{-\frac{1}{3a^2}s - \frac{2}{3a}}{s^2+as+a^2}$$

$$= \frac{1}{3a^2(s-a)} - \frac{s}{3a^2(s^2+as+a^2)} - \frac{2}{3a(s^2+as+a^2)}$$

(3)

These 2 terms are not in standard form of Inverse L.T.

$$s^2+as+a^2 = s^2+as+\left(\frac{1}{4}+\frac{3}{4}\right)a^2$$

(split)

$$= \underbrace{s^2+\left(\frac{1}{2}a\right)^2+2\times s\times\frac{a}{2}}+\left(\frac{\sqrt{3}}{2}a\right)^2$$

$$= \left(s+\frac{a}{2}\right)^2+\left(\frac{\sqrt{3}}{2}a\right)^2$$

$$\Rightarrow \frac{1}{(s-a)(s^2+as+a^2)} = \frac{1}{3a^2(s-a)} - \frac{s}{3a^2\left\{\left(s+\frac{a}{2}\right)^2+\left(\frac{\sqrt{3}a}{2}\right)^2\right.} - \frac{2}{3a}\frac{1}{(s+a/2)^2+\left(\frac{\sqrt{3}a}{2}\right)^2}$$

$$= \frac{1}{3a^2(s-a)} - \frac{s+\frac{a}{2}-\frac{a}{2}}{3a^2\left\{\left(s+\frac{a}{2}\right)^2+\left(\frac{\sqrt{3}a}{2}\right)^2\right.} - \frac{2}{3a}\frac{1}{\left(s+\frac{a}{2}\right)^2+\left(\frac{\sqrt{3}a}{2}\right)^2}$$

$$= \frac{1}{3a^2(s-a)} - \frac{s+a/2}{3a^2\left\{\left(s+\frac{a}{2}\right)^2+\left(\frac{\sqrt{3}a}{2}\right)^2\right\}} + \frac{a/2}{3a^2\left\{\left(s+\frac{a}{2}\right)^2+\left(\frac{\sqrt{3}a}{2}\right)^2\right\}} - \frac{2}{3a}\frac{1}{\left\{(s+a/2)^2+\left(\frac{\sqrt{3}a}{2}\right)^2\right\}}$$

(2) $$= \frac{1}{3a^2(s-a)} - \frac{s+a/2}{3a^2\left\{(s+a/2)^2+\left(\frac{\sqrt{3}a}{2}\right)^2\right.} + \frac{1}{6a}\frac{1}{(s+a/2)+\left(\frac{\sqrt{3}a}{2}\right)^2} - \frac{2}{3a}\frac{1}{\left\{(s+a/2)^2+\left(\frac{\sqrt{3}a}{2}\right)^2\right\}}$$

Inverse L.T.

$$= \frac{1}{3a^2}e^{at} - \frac{1}{3a^2}e^{-at/2}\cos\frac{\sqrt{3}at}{2} + \frac{1}{6a}\times\frac{e^{-\frac{at}{2}}}{\frac{\sqrt{3}a}{2}}\sin\frac{\sqrt{3}at}{2} - \frac{2}{3a}e^{-at/2}\times\frac{1}{\frac{\sqrt{3}a}{2}}\sin\frac{\sqrt{3}at}{2}$$

(3) $$= \frac{1}{3a^2}e^{at} - \frac{1}{3a^2}e^{-at/2}\cos\frac{\sqrt{3}}{2}at + \frac{1}{3\sqrt{3}a^2}e^{-\frac{at}{2}}\sin\frac{\sqrt{3}}{2}at - \frac{4}{3\sqrt{3}a^2}e^{-at/2}\sin\frac{\sqrt{3}at}{2}$$

$$= \frac{e^{at}}{3a^2} - \frac{1}{3a^2}e^{-at/2}\cos\frac{\sqrt{3}}{2}at - \frac{3}{3\sqrt{3}a^2}e^{-at/2}\sin\frac{\sqrt{3}}{2}at$$

$$= \frac{e^{at}}{3a^2} - \frac{1}{3a^2}e^{-at/2}\cos\frac{\sqrt{3}}{2}at - \frac{1}{\sqrt{3}a^2}e^{-at/2}\sin\frac{\sqrt{3}}{2}at$$

(4)

5