(>100) Poisson Distribution (Discrete R.V.)

1) The events which are rare.
2) which have a large no. of trials.

$$\boxed{P(X=r) = \frac{m^r e^{-m}}{r!}}$$

m = mean = np
r = prob of Success
n = no. of trials. (large)

Que → If a random var. has a poisson dist. such that $P(1) = P(2)$ find
① mean ② $P(4)$

Solⁿ ① → $P(1) = P(2)$

$$\frac{m e^{-m}}{1!} = \frac{m^2 e^{-m}}{2!}$$

$$\boxed{m=2}$$

Solⁿ ② →

$$P(4) = \frac{(2)^4 e^{-2}}{4!} = \frac{16 \times e^{-2}}{4\times 3\times 2} = \frac{2}{3} e^{-2}$$

$$= 0.09022$$

Que → x is a poisson dist. and it is found that the prob. that $x=2$ is $2/3$ of the prob. that $x=1$. Find the prob that $x=0$ and the prob. that $x=3$, what is the prob. that x exceeds 3 ?

Solⁿ →

$$P(X=2) = \frac{2}{3} P(X=1)$$

$$\frac{m^2 e^{-m}}{2!} = \frac{2}{3} \frac{m^1 e^{-m}}{1!}$$

$$m = \frac{4}{3}$$

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$$P(x=0) = \frac{\left(\frac{4}{3}\right)^0 e^{-4/3}}{0!} = 0.2636$$

$$P(x=1) = \frac{\left(\frac{4}{3}\right)^1 e^{-4/3}}{1!} = 0.1041$$

exceeds more than 3 $\rightarrow P(X>3)$

strictly Greater than

$$= 1 - [P_0 + P_1 + P_2 + P_3]$$

$$= 1 - \left[\left(\frac{4}{3}\right)^0 \frac{e^{-4/3}}{0!} + \left(\frac{4}{3}\right)^1 \frac{e^{-4/3}}{1!} + \left(\frac{4}{3}\right)^2 \frac{e^{-4/3}}{2!}\right]$$

$$= 0.1506$$

Que ⟹ A certain screw making machine produces on average of 2 defective screws out of 100, and packs them in boxes of 500. find the prob. that a box contains 15 defective screws.

Solve ⟹ Prob. of defectives $p = \frac{2}{100}$

$p = 0.02$

$n = 500$

mean = $np = m = 0.02 \times 500 = 10$

let r be the prob. of defectives

$$P(x=15) = \frac{m^r e^{-m}}{r!} = \frac{(10)^{15} e^{-10}}{15!}$$

$$= 0.0347$$

Que ⟹ A manufacturer knows that the condensor he makes on the average 1% defectives he packs them in boxes of 100, what is the prob. that a box picked at random will contain 3 or more faulty condensors.

Sol^n $\Rightarrow$ $p = \frac{1}{100} = 0.01$

$n = 100$

$np = 100 \times 0.01 = 1$

$P(x \geq 3) = 1 - [P_0 + P_1 + P_2]$

at least 3

$= 1 - \left[\frac{(1)^0 e^{-1}}{0!} + \frac{(1)^1 e^{-1}}{1!} + \frac{(1)^2 e^{-1}}{2!} \right]$

$= 0.08$

Imp

Que $\Rightarrow$ A car hire firm has 2 cars which it hires out day by day. The no. of demands for a car on each day is distributed as a poisson dis. with mean 1.5 calculate the proportion of day.

rare cases

(a) on which is no demand.

(b) on which demand is refused.

Sol^n $\Rightarrow$ mean $m = 1.5$

(a) prob. on which there is no demand

$P(x=0) = \frac{(1.5)^0 e^{-1.5}}{0!} = 0.2231$

(b) Prob. on which demand is refused

$= 1 - [\text{Prob. of all demands}]$

$= 1 - [\underbrace{P(x=0)}_{\text{0 demand}} + \underbrace{P(x=1)}_{\text{demand of 1 car}} + \underbrace{P(x=2)}_{\text{demand of 2 cars}}]$

$= 0.1913$

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Que ⇒ The incidence of occupational disease in an industry in such that the work-men have a 10% chance of suffering from it what is the prob. that in a grp of 7; 5 or more will suffer from it.

Sol^n ⇒ prob of suffering disease is

$$p = \frac{10}{100} = 0.1$$

$n = 7$ ⟶ mean $np = 7 \times 0.1$.

Mean $m = \frac{7}{10}$

$P(x \geq 5) = P(x=5) + P(x=6) + P(x=7)$

at least 5

$$= \frac{\left(\frac{7}{10}\right)^5 e^{-7/10}}{5!} + \frac{\left(\frac{7}{10}\right)^6 e^{-7/10}}{6!} + \frac{\left(\frac{7}{10}\right)^7 (e^{-7/10})}{7!}$$

$$= 0.0008$$

Que ⇒ If the prob. of a bad reaction from a certain infection is 0.001 determine the chance that out of 2000 individuals more than two will get a bad reaction.

Sol^n ⇒ mean $m = np = 2000 \times 0.001 = 2$

Prob. that more than 2 will get a bad reaction $= 1 - (P_0 + P_1)$

$$= 1 - \left[\frac{2^0 \times e^{-2}}{0!} + \frac{2^1 \times e^{-2}}{1!}\right]$$

$=$

V. IMP Q. Best fit of poisson dist to the set of Observation

x:	0	1	2	3	4
f:	122	60	15	2	1

Sol^n ⟹ Formula for Best fit :—

$$N\left[\frac{m^r e^{-m}}{r!}\right]$$

$N = \Sigma f_i = 200$

$m = \text{mean} = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{60+30+6+4}{200}$

$m = 0.5$

$$\Rightarrow N\left[\frac{m^r e^{-m}}{r!}\right]$$

$$= 200\left[\frac{(0.5)^r e^{-0.5}}{r}\right]$$

Put $r = 0, 1, 2, 3, 4$

$f(x) = 121, 61, 15, 2, 0$

x or r :	0	1	2	3	4
f :	121	61	15	2	0

Normal Distribution (Continuous R.V)

Formula $P(X=x) = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} dx$ --- ①

But it is very difficult to evaluate. So we will change the variable in normal variable. i.e. Z.

Normal variable $Z = \frac{x-\mu}{\sigma}$ Here, $\left\{\begin{matrix}\mu=0\\ \mu=1\end{matrix}\right.$

So, $P(Z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-Z^2/2} dz$

Now It is very easy to Solve,

$P(a<x<b) = \int_a^b f(x)dx$

$P(Z_1<Z<Z_2) = \int_{Z_1}^{Z_2} f(z)\,dz$

Here, formula $\boxed{Z_1 = \frac{a-\mu}{\sigma}}$, formula $\boxed{Z_2 = \frac{b-\mu}{\sigma}}$

and formula $\boxed{F(Z) = F(Z_2) - F(Z_1)}$

Important Property :-
1) $F(-z) = -F(z)$
2) $F(\infty) = 0.5$
3) $F(0) = 0$

Que → For a normally distributed variate with mean 1 and S.D. 3 find, the prob that,

① $3.43 \le x \le 6.19$

② $-1.43 \le x \le 6.19$

Sol^n → Given, $\mu = 1$, $\sigma = 3$

ⓐ $\underset{a}{3.43} \le x \le \underset{b}{6.19}$

Converting in normal variable,

$z_1 \leq z \leq z_2$

$z_1 = \frac{a-\mu}{\sigma} = \frac{3.43-1}{3} = 0.81$

$z_2 = \frac{b-\mu}{\sigma} = \frac{6.19-1}{3} = 1.73$

$P(z) = F(z_2) - F(z_1)$

$= F(1.73) - F(0.81)$

$= 0.4582 - 0.2910$

$= 0.1672$ Ans.

(b)

$-1.43 \leq x \leq 6.19$
a b

$z_1 = \frac{a-\mu}{\sigma} = \frac{-1.43-1}{3} = -0.81$

$z_2 = \frac{b-\mu}{\sigma} = \frac{6.19-1}{3} = 1.73$

$P(z) = F(z_2) - F(z_1)$

$= F(1.73) - F(-0.81)$

$= 0.4582 + 0.2910$

$= 0.7492$ Ans.

Que If z is normally distributed with mean 0 and variance 1, find

(a) $Pr(z \leq -1.64)$