Sequential Entropy Pooling Heuristics

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Abstract

This article introduces two sequential heuristics that are designed to overcome some of the practical limitations of the Entropy Pooling (EP) method. Both heuristics repeatedly apply EP to sequentially arrive at the posterior probability and usually lead to significantly better solutions than the original approach. In some cases, the sequential heuristics coincide with the original method, while they automatically ensure logical consistency in others. They are also able to solve interesting and practically relevant problems that the original approach simply cannot. The new approach is coined Sequential Entropy Pooling (SeqEP). Given the benefits of the new method, this article argues that it should become the standard for future EP applications.

Documented Python code that replicates the results of the original approach is available in the open-source package fortitudo.tech. More information about the package can be found on https://os.fortitudo.tech.

Keywords: Entropy Pooling, relative entropy, Kullback-Leibler divergence, change of measure, market views, stress-tests, Monte Carlo simulation, nonlinear convex optimization, heuristic algorithms, Python Programming Language.

1 Introduction

The Entropy Pooling (EP) method introduced by Meucci (2008) is very powerful for incorporating views and stress-testing general possibly analytically unknown return, price, and risk factor distributions. For practical feasibility purposes, EP is formulated as a minimization of a convex nonlinear function subject to linear equality and inequality constraints. This formulation makes it easy to solve the problem quickly and reliably using nonlinear convex optimization technologies.

However, commonly interesting views on, e.g., variances, skewness, kurtosis, and correlations cannot be formulated as linear constraints on the posterior probabilities in full generality. Implementations of these views require us to fix other parameters in order to formulate them as linear constraints on the posterior probabilities. The original suggestion is to fix parameters to their prior values when necessary. However, there are several problems with this approach, e.g., in the worst case it can lead to logical inconsistencies, but in most cases it simply introduces some potentially strong implicit views.

The purpose of this article is to introduce two sequential heuristics that repeatedly apply the original EP method and usually lead to significantly better solutions. The new approach is coined Sequential Entropy Pooling (SeqEP). It is important to underline that there are no heuristics in relation to solving the original EP problem. Sequential Entropy Pooling is heuristic in the sense that it tries to arrive at approximate solutions to the fully general relative entropy (RE) minimization problem, which allows the constraints on posterior probabilities to be nonlinear functions. The original EP approach with some parameters fixed to their prior values can similarly be seen as a heuristic to the fully general problem and will therefore serve as a natural benchmark for the performance of the new method.

It is perhaps natural to ask whether one can have a view on, e.g., the variance without simultaneously having a view on the mean, but this article will not go into philosophical considerations of this kind. It is however important to underline that SeqEP automatically ensures logical consistency in the case where one has a view on, e.g., US and European equity expected returns simultaneously with a view on the variance of some basket of the two.

The rest of this article is organized as follows: Section 2 gives a recap of the original EP method, Section 3 presents the sequential heuristics, why they are expected to work, and some of their features, Section 4 is a case study with simulated data, and Section 5 is a conclusion.

2 Entropy Pooling recap

Deviating from the notation and starting point in Meucci (2008), the market's prior distribution is defined in terms of the matrix $R \in \mathbb{R}^{S \times I}$ of simulations with associated joint probability vector $p \in \mathbb{R}^S$. The columns in R are denoted R_i and represent the marginal distribution of return, price, or risk factor i, $i = 1, 2, ..., I$. Each row in R represents a joint realization of the I returns, prices, and risk factors.

To arrive at a posterior probability vector $q \in \mathbb{R}^S$ that incorporates the user's views or stress-tests, the EP method adjusts the probabilities of the joint realizations by solving the minimum relative entropy problem with linear constraints

$$q = \underset{x}{\operatorname{argmin}} \left\{ x^T \left(\ln x - \ln p \right) \right\} \tag{1}$$

subject to

$$Gx \leq h \quad \text{and} \quad Ax = b,$$

with Lagrangian

$$\mathcal{L}(x,\lambda,\nu) = x^T(\ln x - \ln p) + \lambda^T(Gx - h) + \nu^T(Ax - b).$$

Meucci (2008) shows that the solution is given by

$$x(\lambda,\nu) = \exp\left\{\ln p - \iota - G^T\lambda - A^T\nu\right\}, \tag{2}$$

with ι being an S-dimensional vector of ones. The solution (2) illustrates that positivity constraints on scenario probabilities $x \geq 0$ are automatically satisfied and can therefore be omitted.

Note that (1) is a function of the number of scenarios S and therefore potentially very high-dimensional. On the other hand, the Lagrange dual function

$$\mathcal{G}(\lambda,\nu) = \mathcal{L}(x(\lambda,\nu),\lambda,\nu)$$

is only a function of the Lagrange multipliers λ and ν and therefore has dimension equal to the number of views in addition to a Lagrange multiplier for the requirement that posterior probabilities sum to one. Meucci (2008) therefore proposes to solve the dual problem given by

$$(\lambda^\star,\nu^\star) = \underset{\lambda\geq 0,\nu}{\operatorname{argmax}}\ \mathcal{G}(\lambda,\nu)$$

and subsequently recover the solution to the original/primal problem (1) by computing

$$q = x(\lambda^\star,\nu^\star).$$

One of the main limitations of the EP method is that views are specified as linear constraints $Gx \leq h$ and $Ax = b$ on the posterior probabilities. This is not a problem for views on, e.g., unconditional means where a view on the expected value of i can be formulated simply as

$$R_i^T x \gtreqless \tilde{\mu}_i,$$

using $\gtreqless$ to indicate that the view can be an equality or inequality in one of the two directions. However, views on, e.g., the variance of i given by the constraint

$$\left(R_i^T \odot R_i^T\right)x - \left(R_i^T x\right)^2 \gtreqless \tilde{\sigma}_i^2 \tag{3}$$

are clearly nonlinear in the posterior probabilities x, with $\odot$ denoting the element-wise Hadamard product.

To overcome the nonlinearity issue, we have to fix the value of the second term in (3) such that views on variances reduce to

$$\left(R_i^T \odot R_i^T\right)x - \mu^2 \gtreqless \tilde{\sigma}_i^2$$

and become linear in x. To be logically consistent, this approach requires a simultaneous equality view on the expected value of i, i.e., we must add the view

$$R_i^T x = \mu.$$

The approach above naturally raises the question of how we determine an appropriate value for μ. If the user has a view on the expected value of i given by $\tilde{\mu}_i$ simultaneously with a view on the variance of i, the question is immediately answered with $\mu = \tilde{\mu}_i$. If the user does not have a view on the expected value of i, a suggestion is to set $\mu = R_i^T p$, i.e., to the expected value of the prior distribution. This is the suggestion given by Meucci (2008). But can we do better? Yes, sometimes!

3 Sequential Entropy Pooling

Let $\mathcal{C}_i,\ i \in \{0, 1, 2, \ldots\}$, denote the class of parameters that require i other parameters from some or all of the classes $\mathcal{C}_j,\ j = 0, 1, \ldots, i-1$, to be fixed in order to be expressed as linear constraints on the posterior probabilities. Many commonly interesting parameters can be characterized by these classes. For example, means belong to $\mathcal{C}_0$, variances belong to $\mathcal{C}_1$ (mean fixed), skewness and kurtosis belong to $\mathcal{C}_2$ (mean and variance fixed), and correlations belong to $\mathcal{C}_4$ (two means and two variances fixed). Finally, let $\bar{\mathcal{C}}$ denote the class of parameters that can be formulated as linear constraints on the posterior probabilities but do not belong to any $\mathcal{C}_i$. Hence, $\mathcal{C} = \left\{\mathcal{C}_0, \mathcal{C}_1, \mathcal{C}_2, ..., \bar{\mathcal{C}}\right\}$ is the class of all parameters that can be formulated as linear constraints on the posterior probabilities, with or without fixing other parameters.

A particular set of views $\mathcal{V}$ can be partitioned in a similar way, i.e., $\mathcal{V} = \left\{\mathcal{V}_0, \mathcal{V}_1, ..., \mathcal{V}_I, \bar{\mathcal{V}}\right\}$ with each $\mathcal{V}_i$ being the set of views on parameters that belong to $\mathcal{C}_i$, and $\bar{\mathcal{V}}$ being the set of views on parameters that belong to $\bar{\mathcal{C}}$. The main idea of Sequential Entropy Pooling is to process views according to this partition, carry forward the updated parameters $\theta_i,\ i = 0, 1, ..., I$, and use them to set fixed values when specifying the views in $\mathcal{V}_j,\ j = i+1, i+2, ..., I$, and $\bar{\mathcal{V}}$. More specifically, EP is sequentially applied to the sequence of views with increasing cardinality given by $\mathcal{V}^0 = \{\mathcal{V}_0\}, \mathcal{V}^1 = \{\mathcal{V}_0, \mathcal{V}_1\}, ..., \mathcal{V} = \left\{\mathcal{V}_0, \mathcal{V}_1, ..., \mathcal{V}_I, \bar{\mathcal{V}}\right\}$. Note that the final set in this sequence contains all views $\mathcal{V}$, so the final posterior probabilities are guaranteed to satisfy all views, assuming that the views are feasible for the scenarios in R of course.

With the partitioning of the views established, the remaining question is which prior probability to use in the sequential processing. There are two natural choices in this regard. One is to use the original prior probability vector p, while the other is to use the updated posterior probabilities $q_0, q_1, ..., q_I$ associated with the updated parameters $\theta_0, \theta_1, ..., \theta_I$. This choice is exactly the difference between the two heuristics. Algorithm 1 (H1) uses the original probability vector p in all iterations, while Algorithm 2 (H2) uses the updated posterior probabilities $q_0, q_1, ..., q_I$, except the first iteration where p is used. The two heuristics usually lead to practically the same posterior probabilities, but H1 is slightly better when measured by the RE, as each RE minimization step is against the original probability vector p, while H2 is usually slightly faster. Hence, H2 can be used if computation time is a crucial factor, while H1 is recommended for all other purposes.

Before presenting the sequential heuristics, some additional definitions must be established. For convenience, we define $q_{-1} = p$ and $\theta_{-1} = \theta_{prior}$. By $EP\left(\mathcal{V}^i, \theta, r\right)$ we mean that the EP method presented in Section 2 is applied to the set of views $\mathcal{V}^i$ using the parameter values in θ as fixed values when necessary and $r \in \mathbb{R}^S$ as the prior probability vector. Finally, $f\left(R, r\right)$ denotes the function that computes updated parameter values.

The two sequential heuristics are given by Algorithm 1 (H1) and Algorithm 2 (H2) below.

Algorithm 1 (H1)

for $i \in \{0, 1, ..., I\}$
    if $\mathcal{V}_i \neq \emptyset$, compute $q_i = EP\left(\mathcal{V}^i, \theta_{i-1}, p\right)$ and $\theta_i = f\left(R, q_i\right)$
    else $q_i = q_{i-1}$ and $\theta_i = \theta_{i-1}$
if $\bar{\mathcal{V}} \neq \emptyset$, compute $q = EP\left(\mathcal{V}, \theta_I, p\right)$
else $q = q_I$
return q

Algorithm 2 (H2)

for $i \in \{0, 1, ..., I\}$
    if $\mathcal{V}_i \neq \emptyset$, compute $q_i = EP\left(\mathcal{V}^i, \theta_{i-1}, q_{i-1}\right)$ and $\theta_i = f\left(R, q_i\right)$
    else $q_i = q_{i-1}$ and $\theta_i = \theta_{i-1}$
if $\bar{\mathcal{V}} \neq \emptyset$, compute $q = EP\left(\mathcal{V}, \theta_I, q_I\right)$
else $q = q_I$
return q

3.1 Why are the sequential heuristics expected to work?

To understand why the sequential heuristics are expected to work, it is important to realize that they usually impose weaker implicit views than the original approach, which simply uses prior values when necessary. For example, in the first sequential update using the views in $\mathcal{V}^0$, the update is completely correct for the case where the user only has views that belong to the class $\mathcal{C}_0$, because we do not need the additional flexibility that fully general nonlinear constraints give us. If the user has additional views that do not belong to $\mathcal{C}_0$, these can of course affect the final values of the parameters in θ_0, which are used as fixed values in the subsequent iteration of the sequential heuristics, but this effect tends to be small, i.e., views on parameters that belong to $\mathcal{C}_i$ tend to affect other parameters that belong to $\mathcal{C}_i$ more than views on parameters that belong to $\mathcal{C}_j$ with $i \neq j$. It is probably possible to construct simulations where these statements are not true and purposefully exploit the design of the sequential heuristics, but this hypothesis is likely to be true in practical applications with scenarios that resemble real-world markets and views on commonly interesting parameters.

3.2 What are some other benefits?

Imagine a situation with two instruments with prior means of 5% and 10% and you having a rank view that the mean of the first asset should be higher than the mean of the second asset in addition to a volatility view on both of the assets. This is a perfectly valid and practically relevant view, but this problem is unsolvable with the original EP method that simply fixes means to their prior values when specifying volatility views. With the sequential heuristics, this problem is easy to solve because the means are updated first and then subsequently fixed for the volatility views.

Other benefits are more of a convenience type, but these are nonetheless important for practical applications because they minimize the probability that views with logical inconsistencies are specified. For example, a 50/50 basket of the two instruments from before has a prior expected value of 7.5%. Now imagine that you have a view that their expected values should be 4% and 8%, respectively, so the posterior mean of the 50/50 basket is 6%. If you additionally have a view on, e.g., the variance of the basket, the sequential heuristics will automatically ensure that the expected value is correctly set to 6% when specifying the variance view, while the original approach requires you to specify this manually in order to avoid logical inconsistencies, i.e., an infeasible set of constraints on the posterior probabilities.

3.3 When do the heuristics coincide with the original approach?

It should be easy to convince oneself that the posterior probabilities of H1 coincide with the original approach for a case where all views are fully specified. By fully specified we mean, e.g., having equality views on both the expectations and variances of two assets as well as an equality or inequality view on the correlation between the two assets. In this case, no additional information is obtained about the parameters that are used as fixed values when specifying subsequent views in the sequential heuristics, i.e., the final view specification will end up being exactly the same as the original approach.

Since H1 and H2 usually lead to almost the same posterior probabilities, the above statement will also be true for H2 most of the time. However, significant deviations might occur for H2 as the RE minimization is performed against the running prior probabilities $p, q_0, q_1, ..., q_I$ and not always the initial prior probability p. Finally, the sequential heuristics will both coincide with the original approach in the cases where the user has views that belong only to one $\mathcal{C}_i$ or only to $\bar{\mathcal{C}}$.

4 Case study

The case study[1] uses simulated data with parameters given by the Danish common return expectations for the 2nd half of 2021[2] and an assumption that returns follow a log-normal distribution. However, the approach applies to general and possibly analytically unknown distributions because the EP method operates on joint scenarios, which could have been historical realizations or Monte Carlo simulations from a more sophisticated simulation model.

For the simulation, we generate $S = 10,000$ joint scenarios for the $I = 10$ asset classes covered by the common return expectations. Some prior statistics are shown in Table 1, and the prior correlation matrix is reported in Appendix A.

	Mean	Volatility	Skewness	Kurtosis
Gov & MBS	-0.7%	3.2%	0.10	3.02
Corp IG	-0.4%	3.4%	0.11	3.11
Corp HY	1.9%	6.1%	0.17	2.97
EM Debt	2.7%	7.5%	0.22	3.06
DM Equity	6.4%	14.9%	0.40	3.15
EM Equity	8.0%	26.9%	0.77	4.10
Private Equity	13.7%	27.8%	0.72	3.76
Infrastructure	5.9%	10.8%	0.31	3.19
Real Estate	4.3%	8.1%	0.23	3.09
Hedge Funds	4.8%	7.2%	0.20	3.05

Table 1: Prior statistics of the simulated returns.

[1]https://github.com/fortitudo-tech/fortitudo.tech/blob/main/examples/2_EntropyPooling.ipynb
[2]https://www.afkastforventninger.dk/en/common-return-expectations/

We use an illustrative set of views $\mathcal{V}$ given by: mean of Private Equity is 10%, volatility of EM Equity is less than or equal to 20%, skewness of DM Equity is less than or equal to -0.75, kurtosis of DM Equity is greater than or equal to 3.5, and correlation between Corp HY and EM Debt is 50%.

Posterior statistics are reported in the following tables for H1, H2, and the original EP method together with the relative effective number of scenarios (ENS) introduced by Meucci (2012) and RE[3], while posterior correlation matrices are reported in Appendix A. For all tables, the views are marked in bold. For Table 4, which reports the results of the original EP approach, the parameters that must be fixed to their prior values in order for us to be able to implement the views as linear constraints on the posterior probabilities are marked with an underscore.

Table 2 and Table 3 illustrate that the practical differences between H1 and H2 are minor. Since H1 is still marginally better, it is advised to use this heuristic. The average computation time for H1 in this particular case was 0.03 seconds, while the average for H2 was 0.02 seconds. For most practical applications, this difference in computation time is insignificant. The computation time is also feasible for larger simulations of, e.g., $S = 100,000$ scenarios with an average of around 0.25 seconds for both H1 and H2 using an Intel Core i7-11800H @ 2.30GHz CPU.

	Mean	Volatility	Skewness	Kurtosis
Gov & MBS	-0.6%	3.2%	0.07	2.97
Corp IG	-0.4%	3.3%	0.12	3.11
Corp HY	1.6%	5.5%	-0.05	3.04
EM Debt	2.6%	7.0%	0.12	3.08
DM Equity	5.0%	13.1%	**-0.75**	**3.50**
EM Equity	5.5%	**20.0%**	0.00	3.20
Private Equity	**10.0%**	23.4%	0.18	3.24
Infrastructure	5.5%	10.5%	0.26	3.11
Real Estate	3.8%	7.9%	0.14	3.05
Hedge Funds	4.2%	6.5%	-0.49	3.71

Table 2: Posterior statistics for Algorithm 1 (H1) with $ENS = 81.89\%$ and $RE = 19.98\%$.

	Mean	Volatility	Skewness	Kurtosis
Gov & MBS	-0.6%	3.2%	0.07	2.97
Corp IG	-0.4%	3.3%	0.12	3.11
Corp HY	1.6%	5.5%	-0.05	3.04
EM Debt	2.6%	7.0%	0.12	3.08
DM Equity	5.0%	13.1%	**-0.75**	**3.50**
EM Equity	5.5%	**19.9%**	0.00	3.20
Private Equity	**10.0%**	23.4%	0.18	3.24
Infrastructure	5.5%	10.5%	0.26	3.11
Real Estate	3.8%	7.9%	0.14	3.05
Hedge Funds	4.2%	6.5%	-0.49	3.71

Table 3: Posterior statistics for Algorithm 2 (H2) with $ENS = 81.77\%$ and $RE = 20.13\%$.

[3]The relation between ENS and RE is given by $ENS = \exp\{-RE\}$. Both numbers are reported for convenience.

	Mean	Volatility	Skewness	Kurtosis
Gov & MBS	-0.6%	3.2%	0.06	2.91
Corp IG	-0.5%	3.4%	0.14	3.12
Corp HY	1.9%	6.1%	-0.06	2.97
EM Debt	2.7%	7.5%	0.13	3.07
DM Equity	6.4%	14.9%	**-0.75**	**3.50**
EM Equity	8.0%	**20.0%**	-0.22	3.34
Private Equity	**10.0%**	24.3%	0.12	3.17
Infrastructure	5.7%	10.6%	0.28	3.16
Real Estate	3.7%	8.0%	0.12	3.02
Hedge Funds	4.6%	7.0%	-0.62	3.81

Table 4: Posterior statistics for the original EP approach with $ENS = 70.92\%$ and $RE = 34.36\%$.

When compared to Table 2 and Table 3, Table 4 clearly illustrates the implicit views of the original approach, and these are the cause of the lower ENS / higher RE. This shows how the sequential heuristics benefit from using the updated parameters $\theta_0, \theta_1, ..., \theta_I$ instead of simply prior values.

Note that EP can be used directly in combination with CVaR portfolio optimization as explained by Vorobets (2022b). For a case study where EP is used in combination with CVaR optimization of a derivatives portfolio, see Vorobets (2022a). Finally, for case studies where EP including the sequential heuristics is combined with Bayesian networks for causal views and stress-testing, see Vorobets (2023).

5 Conclusion

This article introduces Sequential Entropy Pooling (SeqEP), which is designed to overcome some of the practical limitations imposed by the requirement that constraints are linear in the posterior probabilities and to improve on the original EP heuristic of always using prior values when necessary. This article also gives an explanation of why the sequential heuristics are expected to work in practice and presents a case study that confirms the fundamental hypothesis by illustrating the superiority of the new algorithms. The new algorithms can be used as direct substitutes in all applications of the original EP method, e.g., in cases with different view confidences and multiple users as described by Meucci (2008) and Vorobets (2023).

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Appendix A

	1	2	3	4	5	6	7	8	9	10
(1, Gov & MBS)	100%	60%	0%	30%	-20%	-10%	-30%	-10%	-20%	-20%
(2, Corp IG)	60%	100%	50%	60%	10%	20%	10%	10%	10%	30%
(3, Corp HY)	0%	50%	100%	60%	60%	69%	59%	30%	30%	70%
(4, EM Debt)	30%	60%	60%	100%	40%	59%	30%	20%	20%	40%
(5, DM Equity)	-20%	10%	60%	40%	100%	69%	79%	40%	40%	80%
(6, EM Equity)	-10%	20%	69%	59%	69%	100%	69%	30%	39%	79%
(7, Private Equity)	-30%	10%	59%	30%	79%	69%	100%	39%	49%	79%
(8, Infrastructure)	-10%	10%	30%	20%	40%	30%	39%	100%	40%	40%
(9, Real Estate)	-20%	10%	30%	20%	40%	39%	49%	40%	100%	50%
(10, Hedge Funds)	-20%	30%	70%	40%	80%	79%	79%	40%	50%	100%

Table 5: Prior correlation matrix.

	1	2	3	4	5	6	7	8	9	10
(1, Gov & MBS)	100%	61%	1%	34%	-21%	-10%	-33%	-9%	-19%	-22%
(2, Corp IG)	61%	100%	48%	60%	5%	15%	4%	7%	9%	27%
(3, Corp HY)	1%	48%	100%	**50%**	52%	61%	51%	25%	25%	63%
(4, EM Debt)	34%	60%	**50%**	100%	30%	50%	15%	14%	14%	28%
(5, DM Equity)	-21%	5%	52%	30%	100%	62%	74%	36%	36%	76%
(6, EM Equity)	-10%	15%	61%	50%	62%	100%	60%	24%	34%	73%
(7, Private Equity)	-33%	4%	51%	15%	74%	60%	100%	36%	46%	74%
(8, Infrastructure)	-9%	7%	25%	14%	36%	24%	36%	100%	37%	36%
(9, Real Estate)	-19%	9%	25%	14%	36%	34%	46%	37%	100%	47%
(10, Hedge Funds)	-22%	27%	63%	28%	76%	73%	74%	36%	47%	100%

Table 6: Posterior correlation matrix for the sequential heuristics H1 and H2.

	1	2	3	4	5	6	7	8	9	10
(1, Gov & MBS)	100%	60%	-2%	35%	-23%	-10%	-34%	-10%	-20%	-24%
(2, Corp IG)	60%	100%	51%	63%	9%	20%	7%	9%	11%	29%
(3, Corp HY)	-2%	51%	100%	**50%**	57%	64%	55%	27%	27%	67%
(4, EM Debt)	35%	63%	**50%**	100%	31%	51%	16%	16%	15%	29%
(5, DM Equity)	-23%	9%	57%	31%	100%	66%	76%	37%	38%	79%
(6, EM Equity)	-10%	20%	64%	51%	66%	100%	62%	27%	36%	75%
(7, Private Equity)	-34%	7%	55%	16%	76%	62%	100%	38%	47%	76%
(8, Infrastructure)	-10%	9%	27%	16%	37%	27%	38%	100%	39%	38%
(9, Real Estate)	-20%	11%	27%	15%	38%	36%	47%	39%	100%	49%
(10, Hedge Funds)	-24%	29%	67%	29%	79%	75%	76%	38%	49%	100%

Table 7: Posterior correlation matrix for the original EP method.