Maths Lecture DSA

Prime Nos:-
(Number that is divisible by 1 and itself is a prime number)

2, 3, 5, 7, 13, ------

To find 13 :-

~~1~~, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, ~~13~~

From the above table check if any number is dividing 13 & the remainder is zero.

```
for (int i=2; i<n; i++)
{
    if(n%i==0)
        Not prime
}
Prime
```

Another example:-

1 × 36
2 × 18
3 × 12
4 × 9
6 × 6
9 × 4
12 × 3
18 × 2
36 × 1

Hence, only these checks for numbers ≤ sqrt(N)

Repeating hence ignore

Sieve

Q.

N = 40

0 → False
x → True

0, 1

2 3 ~~4~~ 5 ~~6~~ 7 ~~8~~ ~~9~~ ~~10~~

11 ~~12~~ 13 ~~14~~ ~~15~~ ~~16~~ 17 ~~18~~ 19 ~~20~~

~~21~~ ~~22~~ 23 ~~24~~ 25 ~~26~~ ~~27~~ ~~28~~ 29 ~~30~~

31 ~~32~~ ~~33~~ ~~34~~ 35 ~~36~~ 37 ~~38~~ ~~39~~ ~~40~~

```
for(i=2; i*i<=n; i++)
  if(!primes[i])
    for(j=i*2; j<=n; j+=i)
      primes[j] = true;
```

False in array means prime.

Time complexity

$$\frac{n}{2} + \frac{n}{3} + \frac{n}{5} + \frac{n}{7} + \cdots$$

$$n\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots\right)$$

$$\Pi$$

Harmonic progression for primes

$\log(\log N)$

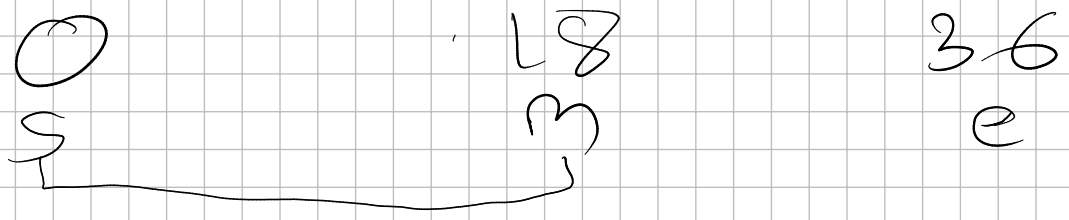
Total time complexity:-

$O(N * \log(\log N))$

Q2 Finding the square root of a number:-

n = 36

we will apply binary search because the numbers are sorted.




```
if ( m * m > n)
    e = m - 1;
else
    s = m + 1;
```

Sqrt (40) = 6.32 ?

above way

Root = 6.1

= 6.2

= 6.3 Ans

= 6.4

↳ same thing for 0.001

0.01

0.1/10 = 0.01

0.01/10 = 0.001

Factors of a number

$n = 20 \Rightarrow 1, 2, 4, 5, 10, 20$

$20 \% 1 \Rightarrow 20 \times 1 = 20$ ✓

$20 \% 2 \Rightarrow 10 \times 2 = 20$ ✓

$20 \% 4 \Rightarrow 5 \times 4 = 20$ ✓

$20 \% 5 \Rightarrow 4 \times 5 = 20$

$20 \% 10 \Rightarrow 2 \times 10 = 20$

$20 \% 20 \Rightarrow 1 \times 20 = 20$

Repeated

Properties of modulo (%):-

- $(a+b) \% m = ((a \% m) + (b \% m)) \% m$
- $(a-b) \% m = ((a \% m) - (b \% m) + m) \% m$
- $(a*b) \% m = ((a \% m) * (b \% m)) \% m$
- $\left(\frac{a}{b}\right) \% m = ((a \% m) * (b^{-1} \% m)) \% m$

$b^{-1} \% m \Rightarrow$ Multiplicative modulo inverse (mmi)

Ex:- $(6 * y) \% 7 = 1$

$y = mmi$ for 6 & $y = 6$

$(6 \times 6) \% 7 = 36 \% 7 = 1$

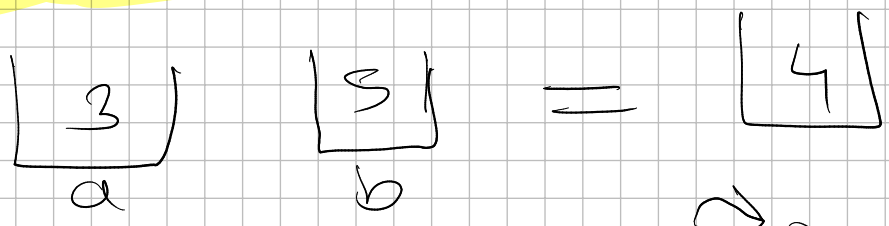
$mmi = b^{-1} \% m$ means that

b & m & co-prime

Co-Prime :- Both numbers common factors

- $(a + m) \% m = a \% m$
- $m \% m = 0 \quad \forall \; x \in +ve$ Integers

Die-hard example



1st → $(a, b) = (0, 0) \rightarrow (3, 0) \rightarrow (0, 3)$

2nd → $(0, 3) \rightarrow (3, 3) \rightarrow (1, 5) \rightarrow (1, 0)$

$(0, 1)$

3rd → $(0, 1) \rightarrow (3, 1) \rightarrow (0, 4)$

Ans

HCF (Highest common Factor)

HCF (4, 18) = 2

1, 2, 4 ← 4

→ 18

1, 2, 3, 6, 9, 18

HCF (3, 9) = 3

1, 3 ← 3

→ 9

1, 3

Highest common factor of both the digits.

LCM (Least common Multiple)

lcm (a, b) = min. no. divisible by both a & b

lcm (2, 4) = 4

lcm (3, 7) = 21