CSL 467: Homework #2

Due on Friday, August 29, 2014

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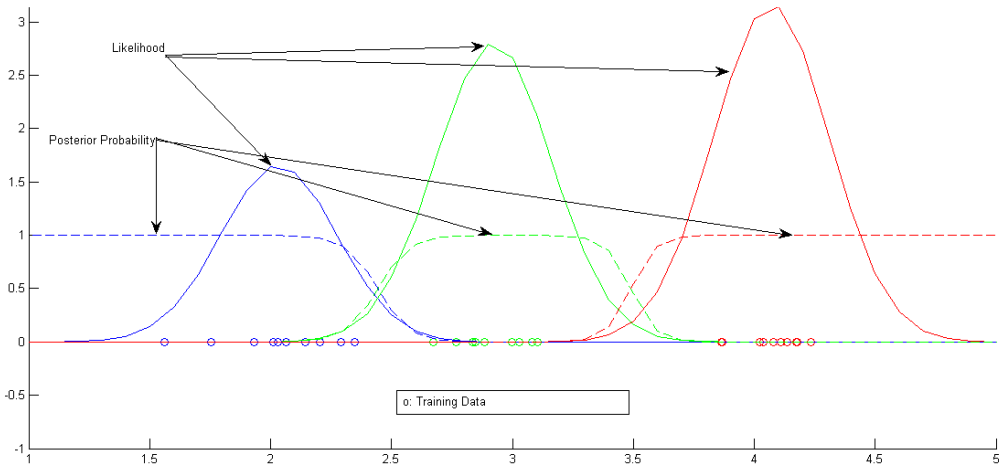
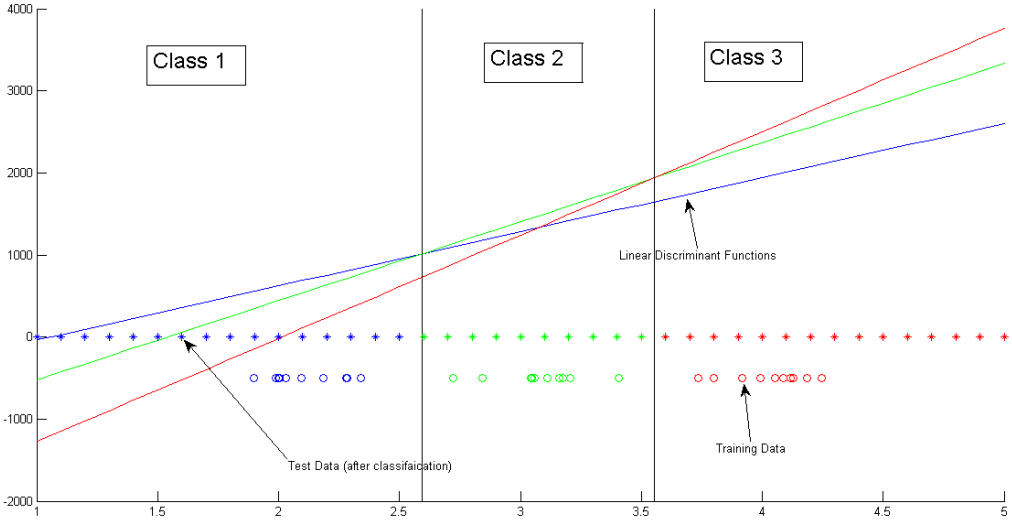
Problem 1

(a)


Likelihood
Posterior Probability
o: Training Data

(a)


Class 1
Class 2
Class 3
Linear Discriminant Functions
Test Data (after classifaication)
Training Data

Problem 2

After introducing regularization term $||w^2||$,

$J(w) = \frac{1}{N}\sum_{n=1}^{N}(-y_n log(g(x_n)) - (1-y_n)log(1-g(x_n))) - \frac{\lambda}{2N}\sum_{j=1}^{m} w_j^2$

where $g(x_n) = \frac{1}{1+x^{-w^T x_n}}$

Now, we will derive the weight update equation:
Since for $j = 0$, w_j^2 term is not there:

$\frac{\partial J(w)}{\partial w_o} = \frac{1}{N}\sum_{n=1}^{N}(g(x_n - y_n))x_{d_n}$

For $j = 1\, to\, m$,

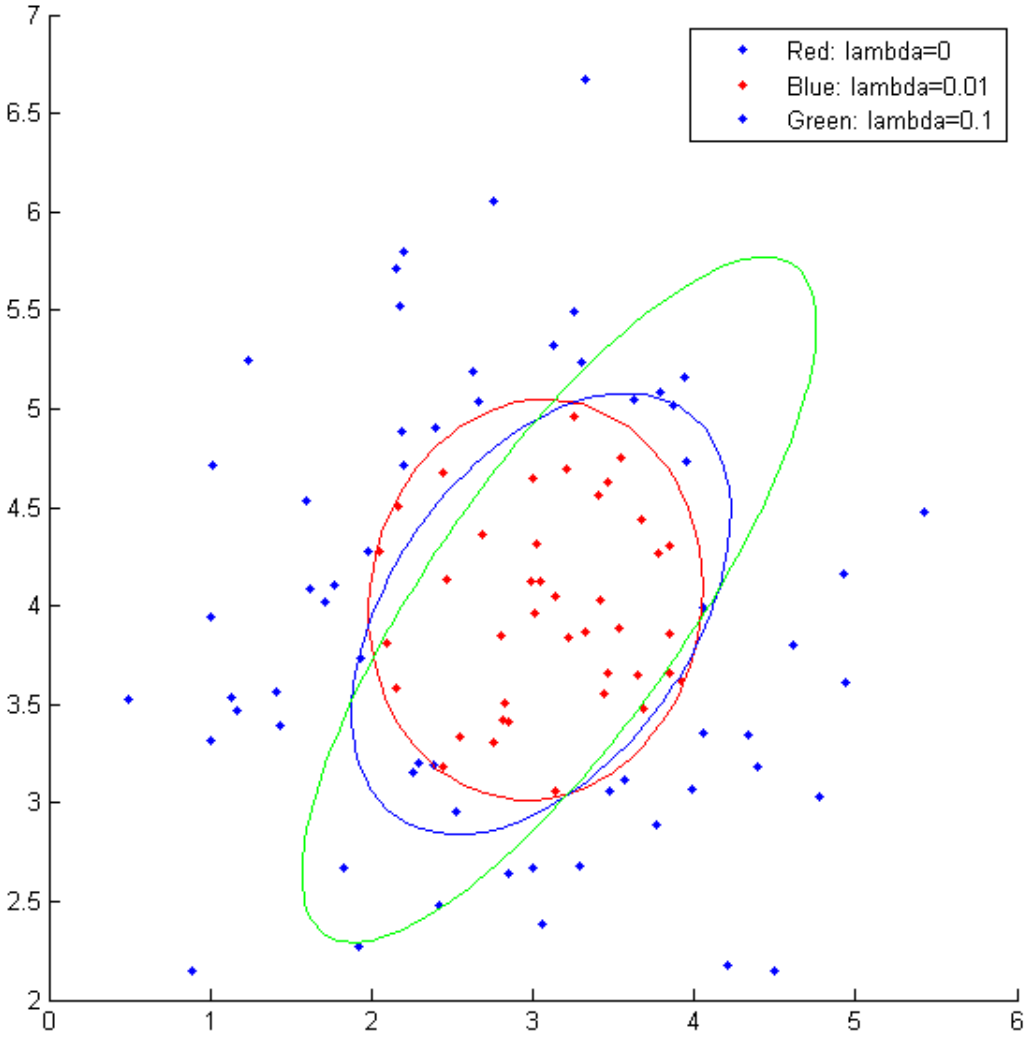
$\frac{\partial J(w)}{\partial w_i} = \frac{1}{N}\sum_{n=1}^{N}(g(x_n - y_n))x_{d_n} + \frac{\partial}{\partial w_i}(\frac{\lambda}{2N}\sum_{j=1}^{m} w_j^2) \qquad i = 1, 2, \ldots m$

$\frac{\partial J(w)}{\partial w_i} = \frac{1}{N}\sum_{n=1}^{N}(g(x_n - y_n))x_{d_n} + \frac{\lambda w_i}{N}$

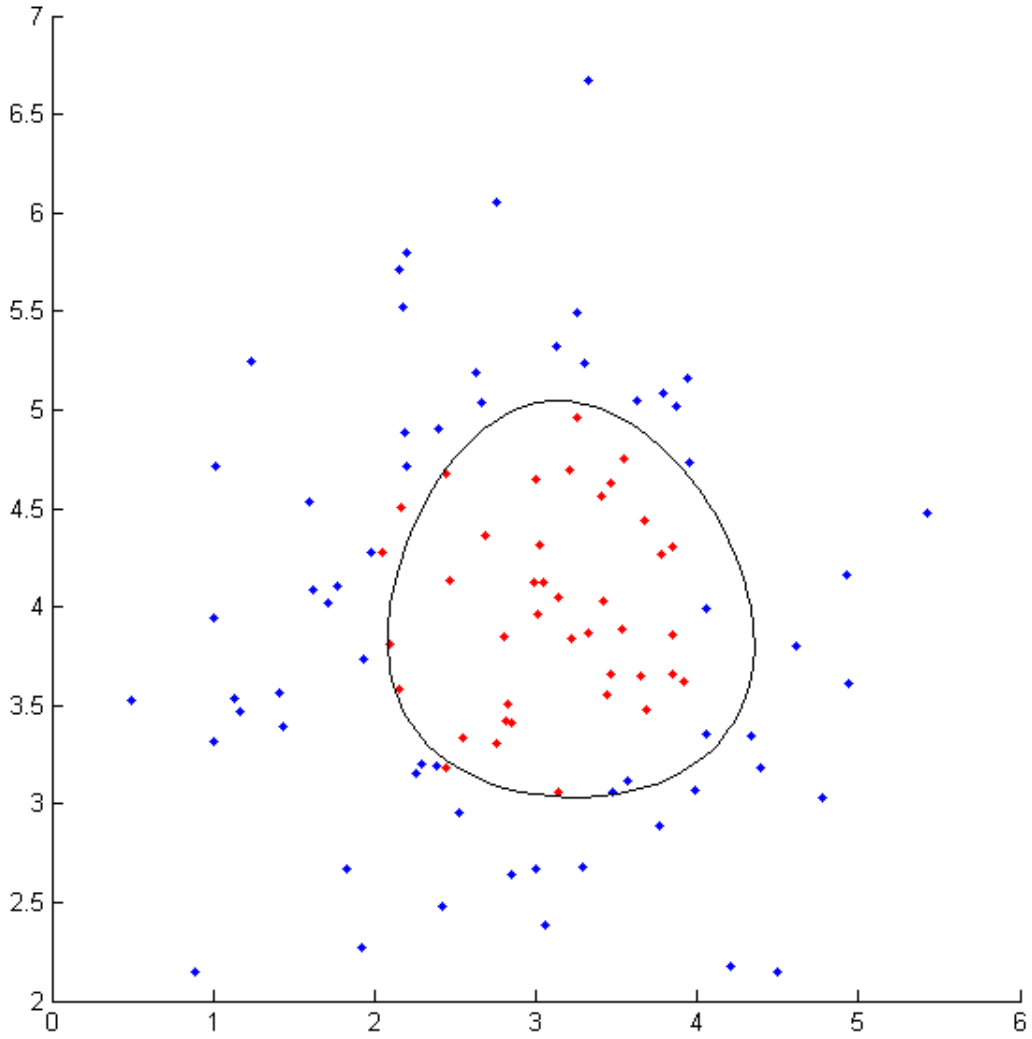
Now, $w_i = w_i + \eta\frac{\partial J(w)}{\partial w_i}$

Problem 3

Contour plot for logistic regression. All contours correspond to regression value 0.5. Contour in red corresponds to regularization parameter (λ) = 0 . Contour in blue corresponds to regularization parameter (λ) = 0.01 . Contour in green corresponds to regularization parameter (λ) = 0.1 .



Contour plot for Linear Discriminant. Regression value is 0.28 .



Problem 4

(a)

The probability of student to get a in the class is $\frac{e^{\beta_0+\beta_1 x_1+\beta_2 x_2}}{1+e^{\beta_0+\beta_1 x_1+\beta_2 x_2}}$

Given,
β_0 = -8
β_1 = 0.05

β_2 = 1
x_1 = 5
x_2 = 7.5
$\therefore$ Probablity $P(x_1, x_2) = \frac{e^{-8+0.05*5+1*7.5}}{1+e^{-8+0.05*5+1*7.5}}$

$\implies$ Probablity $P(x_1, x_2) = 0.43782349911$

(b)

Student's GPA is 7.5 and we already know $\beta_0, \beta_1, \beta_2$.
He need to have 60% chance of getting A grade.
Rewriting the formula for Probability,

$\frac{P(x_1,x_2)}{1-P(x_1,x_2)} = e^{\beta_0+\beta_1 x_1+\beta_2 x_2}$

$\frac{0.6}{1-0.6} = e^{-8+0.05*x_1+1*7.5}$

$ln(1.5) = -8 + 0.05 * x_1 + 1 * 7.5$

$0.4054651081 = -0.5 + 0.05 * x_1$

$x_1 = \frac{0.9054651081}{0.05}$

$x_1 = 18.1093021622$

$\therefore$ He will need to study approx. 18.1 hours