Relative Predictors and Logistic Model for Future Power

1 Construct relative predictors (US vs. UK)

Let t index years. For any variable X, define the *log ratio*

$$x_t \equiv \ln \frac{X_t^{\mathrm{US}}}{X_t^{\mathrm{UK}}}.$$

Based on available data, we construct the following relative predictors:

Military power ratio. Let S_t^{US} and S_t^{UK} denote the US and UK shares of global military capacity:

$$S_t^{\mathrm{US}} = \frac{\mathrm{MilCap}_t^{\mathrm{US}}}{\sum_i \mathrm{MilCap}_t^i}, \quad S_t^{\mathrm{UK}} = \frac{\mathrm{MilCap}_t^{\mathrm{UK}}}{\sum_i \mathrm{MilCap}_t^i},$$

where MilCap_t^i represents military capacity for country i at time t. Then:

$$m_t \equiv \ln \frac{S_t^{\mathrm{US}}}{S_t^{\mathrm{UK}}} \quad \text{(log ratio of global military shares).}$$

This captures relative position in the global military hierarchy rather than simple bilateral comparison.

Education ratio.

$$e_t \equiv \ln \frac{\mathrm{Edu}_t^{\mathrm{US}}}{\mathrm{Edu}_t^{\mathrm{UK}}}.$$

Food price volatility ratio. Let σ_t^{US} and σ_t^{UK} denote the volatility of food prices in the US and UK respectively:

$$v_t \equiv \ln \frac{\sigma_t^{\mathrm{US}}}{\sigma_t^{\mathrm{UK}}} \quad \text{(higher } \Rightarrow \text{ more volatile food prices in US vs UK).}$$

Exchange rate volatility. Let σ_t^{FX} denote the volatility of the US/UK exchange rate.

Add lags to reduce simultaneity: use $x_{t-\ell}$ with $\ell \in \{1, 2\}$.

2 Define a binary "future power" target

Pick a horizon h (e.g., 5 or 10 years). Let the *relative power score* be R_t:

- **GDP leader (PTT-lite):** $R_t \equiv \ln\bigl(\mathrm{GDP}_t^{\mathrm{US}}/\mathrm{GDP}_t^{\mathrm{UK}}\bigr)$.
- **Composite (optional):** $R_t = w_m\, m_t + w_e\, e_t + w_v\, v_t + w_\sigma\, \sigma_t^{\mathrm{FX}}$ (standardize components, set w by PCA or equal weights).

  Two binary targets:

$$\begin{aligned}\textbf{(A) Leader at } t+h: &\quad Y_t^{(h)} = \mathbf{1}\{R_{t+h} \ge 0\} \quad (=1 \text{ if US leads at } t+h).\\ \textbf{(B) Overtake in } (t, t+h]: &\quad Y_t^{(h)} = \mathbf{1}\{R_t < 0 \ \wedge\ R_{t+h} \ge 0\}.\end{aligned}$$

(A) predicts who leads at horizon h; (B) predicts overtake risk within the window. With two countries and ∼110 years, (A) is simpler and more stable.

3 Logistic specification

Let $X_t = \bigl[m_{t-\ell},\, e_{t-\ell},\, v_{t-\ell},\, \sigma_{t-\ell}^{\mathrm{FX}}\bigr]$. Include a smooth time trend $f(t)$ (e.g., cubic spline) to absorb global drift:

$$\Pr\bigl(Y_t^{(h)} = 1 \mid X_t\bigr) = \Lambda\Bigl(\beta_0 + \beta_m\, m_{t-\ell} + \beta_e\, e_{t-\ell} + \beta_v\, v_{t-\ell} + \beta_\sigma\, \sigma_{t-\ell}^{\mathrm{FX}} + f(t)\Bigr),$$

where $\Lambda(z) = \frac{1}{1+e^{-z}}$.

Interpretation. $\beta_m > 0$: higher US military share relative to UK raises the probability the US GDP advantage increases. $\beta_e > 0$: higher US education levels relative to UK raise that probability. β_v: higher food price volatility in US vs UK may indicate structural instability (sign ambiguous). β_σ: exchange rate volatility effects on future GDP advantage (sign ambiguous).

Trend. $f(t)$: natural cubic spline with 3–5 df. With only two countries, do not include country fixed effects (collinear).

Estimation. No weights needed; use robust SEs (HC1).

4 Output: a predictive variable of future relative power

The model returns

$$\hat{q}_t^{(h)} \equiv \widehat{\Pr}\bigl(Y_t^{(h)} = 1 \mid X_t\bigr) \in [0,1],$$

interpretable as the probability the US is the leader at $t+h$ (or the overtake risk in the hazard version).

If a smooth *power score* is preferred, map the same index via a rescaled logistic:

$$S_t^{(h)} \equiv 2\,\hat{q}_t^{(h)} - 1 \in [-1,1],$$

so > 0 favors US, < 0 favors UK.

5 Practical fitting details (tiny panel)

- **Horizon/lag grid:** try $h \in \{5, 10\}$, $\ell \in \{1, 2\}$.
- **Regularization:** with ∼100 obs, add ridge penalty:

$$\max_{\beta} \sum_t \ell\big(\beta; Y_t, X_t\big) - \lambda \|\beta\|_2^2.$$

- **Validation:** time-based CV (rolling origin) to avoid look-ahead.
- **Robustness:** swap m_t across military expenditure shares (MilEx) and military personnel shares (MilPer); test composite measures combining both.

6 Minimal data pipeline (pseudocode)

1. **Build relative predictors:**

$$m_t = \ln \frac{S_t^{\mathrm{US}}}{S_t^{\mathrm{UK}}},$$

   where $S_t^{\mathrm{US}} = \mathrm{MilCap}_t^{\mathrm{US}} / \sum_i \mathrm{MilCap}_t^i$ and $S_t^{\mathrm{UK}} = \mathrm{MilCap}_t^{\mathrm{UK}} / \sum_i \mathrm{MilCap}_t^i$,

$$e_t = \ln \frac{\mathrm{Edu}_t^{\mathrm{US}}}{\mathrm{Edu}_t^{\mathrm{UK}}}, \quad v_t = \ln \frac{\sigma_t^{\mathrm{US}}}{\sigma_t^{\mathrm{UK}}}, \quad \sigma_t^{\mathrm{FX}} = \text{exchange rate volatility.}$$

2. **Create target $Y_t^{(h)}$:**

$$R_{t+h} = g_{t+h} \quad \text{(or composite)}, \qquad Y_t^{(h)} = \mathbf{1}\{R_{t+h} \geq 0\},$$

   then drop the last h years.

3. **Assemble design** with lag ℓ, add spline $f(t)$.
4. **Fit** logistic (optionally ridge), **predict** $\hat{q}_t^{(h)}$.
5. **Use** $\hat{q}_t^{(h)}$ as the future power predictor; date "inflections" where $\hat{q}_t^{(h)}$ crosses 0.5 or where $\Delta \hat{q}_t^{(h)}$ peaks.