Problem A: Design of a non-powered hydrofoil surfboard

Hydrofoil surfboard is a novel water sports equipment that harnesses the power of water flow. By utilizing the design of hydrofoils, it converts the energy from water flow into propulsive force, enabling individuals to glide swiftly on the surface of the water. The inspiration for hydrofoil surfboard's design stems from bird wings, with its wing shape and structure meticulously crafted to achieve optimal propulsion efficiency.

The structure of a hydrofoil surfboard consists of three main components:foils, and tail wings. The board body serves as the core of the entire surfboard, with its length and width determining stability and glide speed. Hydrofoils are crucial parts of board that determine propulsion efficiency. Their shape and size are designed based on principles of fluid mechanics to achieve optimal propulsion effects. Tail wings are used to control the direction and stability of the surfboard, also designed according to principles of fluid mechanics.

Please establish mathematical models based on relevant knowledge of mechanics to analyze and answer the following questions:

1) How do the width and length of the surfboard affect its stability and speed?
2) How should the hydrofoil section be designed to achieve optimal propulsion?
3) How should the tail fin be designed direction and maintain stability of the surfboard?
4) Considering the influence of ocean waves, how should a driver control in order to minimize effort and achieve faster speeds?

Problem B Mathematical Modeling of Wildfire Evacuation in Maui, Hawaii

Maui, Hawaii, located in the central Pacific, is renowned for its stunning natural landscapes and diverse vegetation, making it a popular tourist destination. However, the island faces increased wildfire

risks due to factors such as drought, hurricanes, vegetation changes, and a scarcity of firefighters. To ensure the safety of residents and tourists, there is a need to develop a wildfire evacuation model for planning evacuation strategies during wildfire events.

Requirement 1: Model Establishment

Considering the geographical information, meteorological data, and population distribution in Maui, construct a mathematical model to describe the wildfire propagation and evacuation process. The model should incorporate the effects of factors like drought, hurricanes, and vegetation changes.

Requirement 2: Wildfire Propagation

Using the model, estimate the speed and extent of wildfire propagation under different conditions. Take into account factors such as wind direction, vegetation density, and temperature affecting wildfire spread.

Requirement 3: Evacuation Strategy

Devise an evacuation strategy to minimize the risk of casualties among affected populations. Consider evacuation routes, evacuation points, and traffic flow in different regions.

Requirement 4: Firefighter Resource Allocation

Given the scarcity of firefighter resources, design a firefighter deployment strategy to effectively contain the spread of wildfires. Determine the optimal allocation of firefighters.

Requirement 5: Evacuation Time

Based on the model's results, estimate the time required for residents and tourists in different areas to safely evacuate. Consider factors such as traffic congestion.

Requirement 6: Risk Assessment

Develop a risk assessment model that considers the impact of factors such as drought, hurricanes, and vegetation changes on wildfire risk. Establish an early warning system based on this model to notify residents and tourists in advance.

Considering these factors, students will need to apply mathematical modeling and analytical skills to address the wildfire evacuation problem in Maui, Hawaii, ensuring the safety of island residents and visitors during wildfire events.

Problem C Nowcasting of Severe Convective Precipitation

China has a vast territory and complex natural conditions, resulting in a wide variety of catastrophic weather types and significant regional differences. Among them, severe convective weather such as thunderstorms, strong winds, hail, tornadoes, and short-term heavy rainfall are the most serious types of catastrophic weather that cause economic losses and endanger life safety[1]. Taking 2022 as an example, the number of deaths, missing persons, and direct economic losses caused by wind and hail disasters caused by strong convective weather in China accounted for 73% and 69% respectively. Due to the characteristics of sudden and localized strong convective weather, short life history, and severe disasters, short-term (0-12 hours) and near (0-2 hours) forecasts are often difficult in weather forecasting.

The traditional approach prediction of severe convective weather mainly relies on observational data such as radar, combined with storm identification and tracking techniques for radar extrapolation prediction. This method obtains the radar reflectivity factor at future times through extrapolation, and further uses the empirical relationship between the radar reflectivity factor and precipitation (i.e. Z-R relationship) to estimate the precipitation at future times [2]. In recent years, with the accumulation of big data and the development of computer computing power, artificial intelligence and deep learning technologies have developed rapidly. Deep learning methods are data-driven methods that theoretically improve their performance as the amount of training data increases, making them very suitable for short-term and imminent prediction fields with a large accumulation of radar observation data. At present, there are two main types of short-term and imminent prediction models based on deep learning internationally. One type is based on Convolutional Neural Networks (CNNs), such as U-Net and other models [3]; Another type is based on Recurrent Neural Networks (RNNs), such as ConvLSTM, DGMR, and other models [4,5].

Raindrops are subject to air resistance during falling, and they can be flat spherical or Mantou shaped, and generally the larger the raindrop, the flatter it is. Therefore, the reflection characteristics of raindrops on electromagnetic waves with horizontal polarization (electric field vibration direction in the horizontal plane) and vertical polarization (electric field vibration direction in the vertical plane) are different. Traditional radar can only transmit and receive electromagnetic waves in one polarization direction, while the new dual polarization radar can simultaneously transmit and receive electromagnetic waves in both horizontal and vertical polarization directions. It can obtain information such as the size, phase state, and water content of precipitation particles based on the difference in intensity and phase relationship of the echoes in both polarization directions [6], which is collectively

referred to as microphysical information. In recent years, research has shown that the microphysical information reflected by dual polarization radar variables includes key information such as the evolution state and spatial dynamic structure of convective systems [7,8]. Therefore, the application of dual polarization radar variables is theoretically of great significance for predicting strong convection.

In order to better apply dual polarization radar to improve short-term and imminent forecasts of severe convective precipitation, please answer the following questions:

1) How to effectively apply dual polarization variables to improve strong convection forecasting is still a key and difficult problem in current meteorological forecasting. Please use the data provided in the question to establish a mathematical model that can extract microphysical feature information from dual polarization radar data for strong convection near prediction. The input for the near forecast is the radar observations （Z_H 、Z_{DR}、K_{DP}） from the previous hour (10 frames), and the output is the Z_H forecast from the following hour (10 frames).
2) Please use the Z_H, Z_{DR} and precipitation data provided in the question to design an appropriate mathematical model and use Z_H and Z_{DR} for quantitative precipitation estimation. The model inputs are Z_H and Z_{DR}, and the output is precipitation. (Note: The algorithm cannot use K_{DP} variables.)

Explanation of terms:

1) **Dual polarization radar**: A new type of weather detection radar that can provide richer physical information than traditional radar. It obtains information on the size, phase state, and moisture content of precipitation particles by measuring their reflection of electromagnetic waves in both horizontal and vertical directions. These information, collectively known as microphysical information, can help us better predict severe convective weather. The three most commonly used variables for dual polarization radar are: 1) Z_H, the horizontal reflectivity factor, which refers to the echo intensity in the horizontal direction, usually expressed in dBZ, mainly reflecting the strength of precipitation; 2) Z_{DR}, differential reflectance, refers to the difference in echo intensity between horizontal and vertical directions, mainly reflecting the size of precipitation particles in the observation area; 3) K_{DP}, differential phase shift, refers to the phase difference between the horizontal and vertical echoes caused by precipitation particles per unit distance, mainly reflecting the liquid water content.
2) **Z-R relationship:** The empirical relationship between radar reflectivity and precipitation, usually expressed as $R = a * Z^b$, where R is precipitation, Z is radar reflectivity, and a and b are empirical parameters, which usually vary in different regions and precipitation types.

Attachment data:

1. NJU-CPOL dual polarization radar data:
   https://box.nju.edu.cn/f/16bbb37458d3443dbf9f/?dl=1
2. Precipitation grid data:
   https://box.nju.edu.cn/f/076f5aeb2ec64b87bde8/?dl=1

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Problem D "Slow down, you move too fast…"

Tompkins County Sheriff's Office plays a major role[1] in enforcing traffic laws on the roads in and around Ithaca, NY. Prevention of speeding and of "driving while intoxicated" (DWI) are among their main priorities since both of these significantly increase the risk of accidents and the likelihood of severe outcomes when such accidents occur.

Your team's task is to advise the Road Patrol Division within the Sheriff's Office on the best use of their resources to achieve this. While DWI screening is mostly performed around holidays, the detection of speeding motorists is a daily responsibility, which Patrol Division officers perform as a part of their regular duties. On any given day, there might be up to three Sheriff's Deputies spending some portion of their shift on traffic-related activities.

We ask your team to address the following list.

(1) Scheduling speed check activities. Right now, the locations and times/dates for speed checks are often selected in an ad-hoc fashion, partly based on anecdotal evidence of speeding behavior (& data on past accidents), but also heavily influenced by Deputies' individual schedules and other responsibilities. If the Patrol Division wanted to switch to more centralized planning, what algorithm would you suggest they should follow?

(2) How well does it work? In developing this schedule, your main goal is not to catch every single violation, but rather to improve the overall safety of driving on local roads. Develop an approach for assessing the effectiveness of your scheduling algorithm.

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[1] Enforcing traffic rules around Ithaca is a joint responsibility of several organizations: New York State Police, Tompkins County Sheriff's Office, and police departments of separate municipalities & even colleges (Ithaca College & Cornell). But for the purposes of this problem, we focus on the activities of the Sheriff's Office only.

(3) Is it fair? As you know, ensuring fairness and impartiality of law enforcement activities is a significant concern in our society. Provide a criterion for measuring the fairness of your scheduling and address its relationship with the effectiveness.

(4) How robust is it? Keep in mind that Deputies have many other responsibilities which might take precedence, making it necessary to suspend speed checks on specific days or at specific locations. Quantify the effects of such (relatively-common) disruptions on your proposedscheduling approach. If robustness is the real goal, how should your scheduling algorithm change to achieve it?

(5) Lasting effects? In many cases, getting a ticket (or even a warning) for a traffic violation alters a driver's future behavior. Moreover, even witnessing another car pulled over makes most motorists less likely to engage in risky driving behavior in the near future. Develop and validate a model for the lasting power of such effects, which might also be relevant to your main "optimal scheduling" approach.

(6) Asymmetric information? Waze and similar phone apps allow drivers to share information about current driving conditions and ongoing police activities. This can decrease the effectiveness of speed checks, and your proposed assessment should reflect this, possibly also affecting your scheduling recommendations.

(7) Traffic Calming by other means? NYS Department of Transportation (DOT) and the County Highway Department also have important roles in improving the road safety. Movable speed-warning signs (which alert approaching drivers to their actual speed and the relevant speed limit) are among their effective "traffic calming" measures. Assuming that Tompkins County has a total of 4 such movable signs, provide recommendations on where and when they should be deployed given your (already developed) schedule of speed checks by Sheriff's Deputies. Can you improve the effectiveness by developing both schedules jointly?

In addition to your detailed manuscript, please write a one-page executive summary addressed to Undersheriff Jennifer K. Olin. Please make sure to explain your main proposals and their advantages, but also the possible drawbacks along with limitations of your modeling approach.

Please note: Law enforcement officers are quite busy – please don't reach out to them to ask for clarifications or additional data. But feel free to use whatever data you find online; for example:

Sheriff's Office Activity Logs for October, September, and August 2022.
(Data for other months is also available here – please search for "Activity Logs".)

New York State Traffic Safety Statistical Repository (TSSR)
Contains a lot of data on types of tickets issued + statistics on who issued them to whom for what and when. E.g., select "Ticket
Reports" → "Speeding tickets" → "TOMPKINS COUNTY" + relevant year(s) on the right. Note that the NY State troopers are significantly more active in speed limit enforcement, but they usually do not work within municipalities, which have their own police departments. So, most of the speed limit enforcement within Ithaca is currently done by Sheriff's Office. Also worth noting: while we don't have statistics on speeding in particular, only about 2/3 of all traffic stops by Sheriff's Office result in warnings rather than citations/tickets and thus are not reflected in the above database.

A crowd-sourced list of a few common speed check locations around Ithaca, NY
Note that this is list is not exhaustive – many other locations are also used in practice.

Except for highways or "NY State Routes", the default speed limit on regular streets in Ithaca and most surrounding communities is 30 miles per hour. E.g., you can find a list of exceptions here for the City of Ithaca and here for the nearby Village of Lansing.

The list of all accidents recorded in Tompkins County in 2022 (up to 11/01/22).
Note that the "contributing factor code = 19" means that some speeding was involved. The list of other "contributing factor codes" can be found here.

Problem E Optimal design for the ventilation and dust removal systems

The ventilation and dust removal system will purify the workshop during the production process. The ventilation and dust removal system is generally divided into two parts: the first part is the exhausting system, which has fans, pipes of appropriate size and injection hoods at the dust generating position. The fan promotes the air flow and inhales the inlet valves and pipes, and releases dust out of the production workshop; one part is the inlet fan system, which has fan and pipes of appropriate sizes

at appropriate locations to import outdoor air into the workshop.

The evaluation index of ventilation and dust removal system is indoor dust concentration. Adjusting the exhaust volumes of outlet air and inlet air, the air pressure in the workshop is maintained and the indoor dust concentration is ensured to be lower the safe level. In order to achieve those engineering targets, mathematical models are needed to answer the following questions:

(1) Establish a mathematical model and algorithm to determine the functional relationship between the volume flow rate of the air at the rectangular inlet cover of the exhausting duct and the volume flow rate of the air at a certain distance away from the cover;

(2) Establish a mathematical model and algorithm to determine the functional relationship between the volume flow rate of the air at the circular discharge outlet of the inlet fan system and the flow rate of the air at a certain distance away from the outlet;

(3) If there is only one dust source, one exhausting fan system and one inlet fan system in the workshop. The workshop is 10 meters long, 5 meters wide, and 6 meters high. The pollution source is located in the center of the workshop, where the height is 1.2 meters, the length is 1 meter, and the width is 1 meter. The inlet cover of the exhausting fan is located 0.5 meters above the pollution source, and its size is consistent with the cross-sectional area of the pollution source. Establish mathematical models and algorithms to determine the optimal air inlet volume and air exhausting volume to achieve a safe dust concentration;

(4) Regarding the workshop in question (3), if the air outlet of the inlet fan protrudes 0.1 meters from the inner wall and is circular. Establish a mathematical model and algorithm, select one of the four interior walls to place the outlet position of the inlet fan, and determine the optimal diameter, height and horizontal position of the outlet duct.

Problem F Brexit

Brexit was the nickname for "British exit" from the EU, the economic and policy union of which the U.K. had been a member since 1973.

In 2015, the Conservative Party called for the referendum. Most of the pro-Brexit voters were older, working-class residents of England's countryside. They were afraid of the free movement of immigrants and refugees, claiming in the process that citizens of poorer countries were taking jobs and benefits. Small businesses were also frustrated by EU fees. Others felt leaving the EU would create jobs. Many felt the U.K. paid more into the EU than it received. Those who voted to stay in the EU primarily lived in London, Scotland, and Northern Ireland. They liked free trade with the EU, and claimed most EU immigrants were young and eager to work. Most felt that leaving the EU would

damage the U.K.'s global status.

That changed on June 23, 2016, when the U.K. voted to leave the EU. The residents decided that the benefits of free trade weren't enough to offset the costs of free movement of immigration. The vote was 17.4 million in favor of leaving vs. 16.1 million who voted to remain.

At the last moment before Christmas Eve 2020, and about a week before the end of the transition period, the EU and the UK finally reached a package agreement, including the bilateral trade agreement after Brexit. This marks a new page in the relationship between Britain and the EU.

Requirement 1 Collect relevant data and establish models to prove the inevitability of Brexit.

Requirement 2 Brexit changes the nature of the formal relationship between the U.K. and the EU and throws London's place as a global financial center into question. It creates new trade restrictions between the U.K. and the rest of Europe and limits the ability of British citizens to move as freely around the EU. These are only a few of the significant effects of Brexit, and the full impact has yet to unfold. Please establish an appropriate model to assess the current impact of Brexit on the UK as a whole.

Requirement 3 Please establish a model based on various indicators of the UK before and after Brexit to analyze the possibility of entering the EU in the future.