Elements of Macroeconomics TA Session 5: Assignment 4

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Slides on https://github.com/Haruki-Shibuya/TA

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Announcement

- Midterm 2 will be returned next Monday.
- Grading allocation
- Q1: Haruki hshibuy1@jh.edu
- Q2: Shiqi h.q@jh.edu
- Q3: Qingyuan qfang6@jh.edu

- We review Assignment 4 today

Q1

Category	CPI (Dec 2004)	Retired Person Weight	College Student Weight
Housing	190.7	0.1	0.05
Food	188.9	0.15	0.15
Transportation	164.8	0.05	0.2
Medical Care	314.9	0.6	0
Education	112.6	0	0.4
Recreation	108.5	0.1	0.2
Total		**1**	**1**

Q1(a)

a) Calculate the overall CPI for the retired person and for the college student by multiplying the CPI for each of the categories by the relative importance of that category to the individual and then summing each of the categories. ↵

Q1(a)

Category	CPI (Dec 2004)	Retired Person Weight	Retired Person Contribution	College Student Weight	College Student Contribution
Housing	190.7	0.1	190.7 × 0.1 = 19.07	0.05	190.7 × 0.05 = 9.535
Food	188.9	0.15	188.9 × 0.15 = 28.335	0.15	188.9 × 0.15 = 28.335
Transportation	164.8	0.05	164.8 × 0.05 = 8.24	0.2	164.8 × 0.2 = 32.96
Medical Care	314.9	0.6	314.9 × 0.6 = 188.94	0	0
Education	112.6	0	0	0.4	112.6 × 0.4 = 45.04
Recreation	108.5	0.1	108.5 × 0.1 = 10.85	0.2	108.5 × 0.2 = 21.7
Total		**1**	**255.435**	**1**	**137.57**

Q1(b)

b) The CPI for all items in December 2004 was 190.3. How do your calculations for a CPI for the retired person and the college student compare to the overall CPI?

- CPI(retired) > overall CPI > CPI(student)

Q2

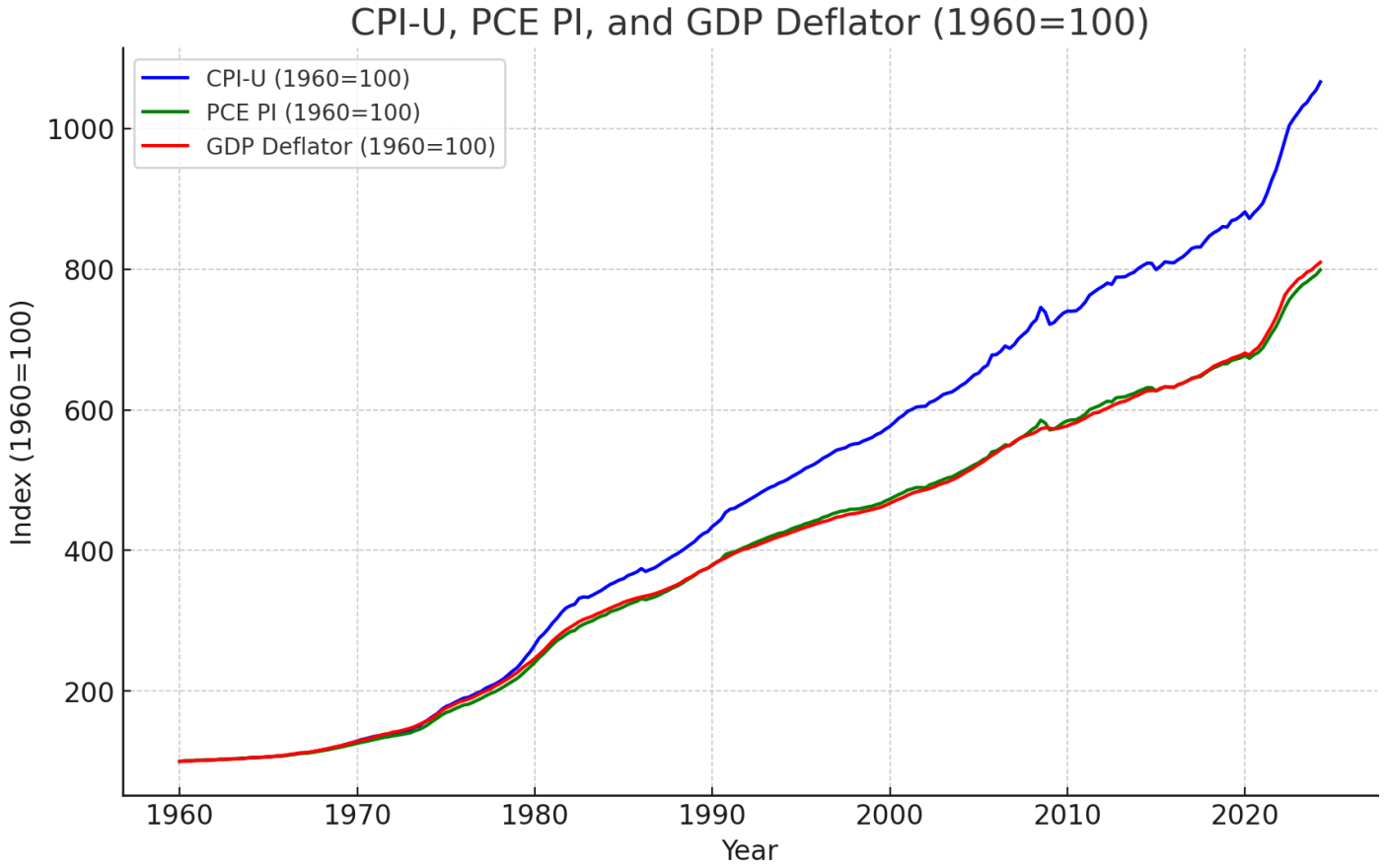
Year	GDP Deflator	CPI	PCE
2002	104.1	179.9	77.9
2003	106.0	184.0	79.8
2004	108.3	188.9	82.1

Q2(a)

a) For each price index, calculate the inflation rate from 2002 to 2003 and from 2003 to 2004.

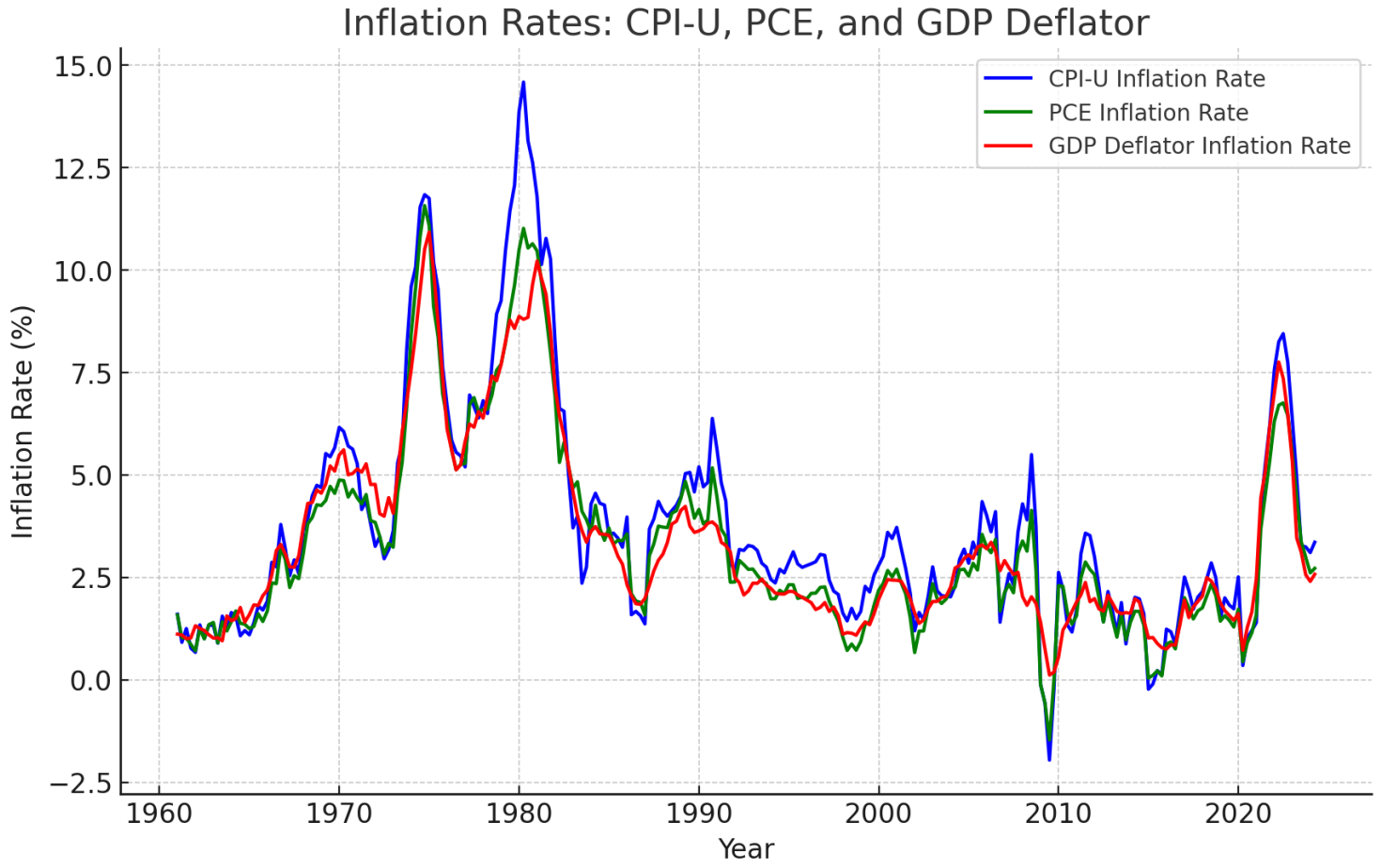
Year Range	Price Index	Index in Previous Year	Index in Current Year	Inflation Rate Formula	Inflation Rate (%)
2002-2003	GDP Deflator	104.1	106.0	$\frac{106.0-104.1}{104.1} \times 100$	1.825%
2003-2004	GDP Deflator	106.0	108.3	$\frac{108.3-106.0}{106.0} \times 100$	2.17%
2002-2003	CPI	179.9	184.0	$\frac{184.0-179.9}{179.9} \times 100$	2.28%
2003-2004	CPI	184.0	188.9	$\frac{188.9-184.0}{184.0} \times 100$	2.66%
2002-2003	PCE	77.9	79.8	$\frac{79.8-77.9}{77.9} \times 100$	2.44%
2003-2004	PCE	79.8	82.1	$\frac{82.1-79.8}{79.8} \times 100$	2.88%

Q2 (b)



- CPI inflation >> PCE inflation ≈ GDP deflator inflation

Q2 (b)



- CPI inflation >> PCE inflation ≈ GDP deflator inflation

Q2 (b)

- CPI is a **Laspeyres Price Index:**

$$\text{CPI}_t = \frac{\sum_{i \in S_{\text{basc}}} (\text{Price of good } i \text{ in period } t \times \text{Quantity of good } i \text{ in base period})}{\sum_{i \in S_{\text{basc}}} (\text{Price of good } i \text{ in base period} \times \text{Quantity of good } i \text{ in base period})}$$

- Requitement: quantities must be fixed over time to focus on price changes
- CPI uses the quantities consumed in the base period
- But normally, consumers avoid buying a good whose price went up
- So, CPI overstates inflation in a sense but still used for the CoL adjustment
- Cf. Chained CPI

Q2 (b)

- GDP deflator is a **Paasche Price Index:**

$$\text{GDP Deflator}_{t,t-1} = \frac{\sum_{i \in S_t} (\text{Price of good } i \text{ in period } t \times \text{Quantity of good } i \text{ in period } t)}{\sum_{i \in S_t} (\text{Price of good } i \text{ in period } t-1 \times \text{Quantity of good } i \text{ in period } t)}$$

- It uses the quantities produced in the current period
- GDP deflator understates inflation in a sense
- Cf. chained GDP deflator

Q2 (b)

- PCE PI is a chained Fisher index:

$$\text{PCE PI}_t = F_{t,t-1} \times F_{t-1,t-2} \times F_{t-2,t-3} \times \cdots$$

- where F_t **is a Fisher index** is a geometric mean of a Laspeyres and Paasche price index**:**

$$\text{Fisher Ideal Index}_{t,t-1} = \sqrt{\text{Laspeyres Index}_{t,t-1} \cdot \text{Paasche Index}_{t,t-1}}$$

- Designed to avoid fixed-basket biases
- Note: we can compute chained CPI or GDP deflator too

Q2 (b)

b) Explain 3 factors that drive a wedge between the GDP deflator and the CPI, and PCE and CPI.

GDP deflator = nominal GDP/real GDP

CPI = cost of basket / cost of basket in base year

PCE = nominal consumption/ real consumption

Wedges:

1. GDP deflator only includes domestically produced goods and services, while CPI includes imported goods. Example: imports from China got cheaper over time which decreased CPI, but not GDP deflator

Q2 (b)

b) Explain 3 factors that drive a wedge between the GDP deflator and the CPI, and PCE and CPI.

GDP deflator = nominal GDP/real GDP
CPI = cost of basket / cost of basket in base year
PCE = nominal consumption/ real consumption

2. CPI captures the average consumption of a urban household of 4 while PCE includes aggregate consumption of the economy (including luxury goods). Example: Super Yachts got more expensive over time. CPI does not pick it up.

Q2 (b)

b) Explain 3 factors that drive a wedge between the GDP deflator and the CPI, and PCE and CPI.

GDP deflator = nominal GDP/real GDP

CPI = cost of basket / cost of basket in base year

PCE = nominal consumption/ real consumption

3. GDP deflator includes investment and government spending while PCE only uses consumption.
   Example: Military goods got more expensive which is part of G, but not C

Q2 (b)

b) Explain 3 factors that drive a wedge between the GDP deflator and the CPI, and PCE and CPI.

GDP deflator = nominal GDP/real GDP

CPI = cost of basket / cost of basket in base year

PCE = nominal consumption/ real consumption

4. PCE is chain-weighted

Q2 (b)

b) Explain 3 factors that drive a wedge between the GDP deflator and the CPI, and PCE and CPI.

GDP deflator = nominal GDP/real GDP

CPI = cost of basket / cost of basket in base year

PCE = nominal consumption/ real consumption

5. PCE includes purchases that were paid for by the business/govt on behalf of HH

- E.g., Medicare/Medicaid, health insurance provided by employer

Q2 (c)

c) Explain why using CPI to measure cost of living may be more relevant than using the GDP deflator or the PCE index.↵

↵

CPI is measured on a consumption basket which should mirror the consumption behavior of a urban household of 4 in the US. Therefore, if we think about cost of living, we want to know what this average household can afford. GDP and PCE include many more items/ or put a bigger weight on products which this household is not consuming anyway. Therefore, CPI might be more relevant.↵

Q2 (d)

Use the information in the below to answer the following questions.

Year	CPI
1996	156.9
1997	160.5
2000	172.2
2024	314.4

d) Amanda was paid a minimum wage of $4.75 an hour in 1996 and $5.15 in 1997, did the value of her income keep pace with the change in cost of living? If not, what should the wage per hour have been?

Q2 (d)

- Wage growth factor = $5.15/$4.75=1.084
- CPI growth factor = 160.5/156.9=1.023
- Wage growth outpaced the inflation
- Wage of $4.75 × 1.023=$4.86 or higher would keep pace with CoL

Q2 (e)

e) Amanda was paid a minimum wage of $3.35 an hour in 1983 and she was paid a minimum wage of $5.15 in year 2000. Assuming the base year was 1983, did the value of her income keep pace with the change in cost of living? If not, what should the wage per hour have been?

- Wage growth factor = $\$5.15/\$3.35 = 1.537$
- CPI growth factor = $172.2/100 = 1.722$
- CPI growth outpaced the wage growth
- Wage of $\$3.35 \times 1.722 = \$5.77$ or higher would keep pace with CoL

Q2 (f)

f) Assuming the base year was 1983, what should be the equivalent amount of the minimum wage in 2024?

$$Minimum\ wage_{2024} = \frac{314.4}{100} \times 3.35 = \$10.53$$

Q3 (a)

Question 3

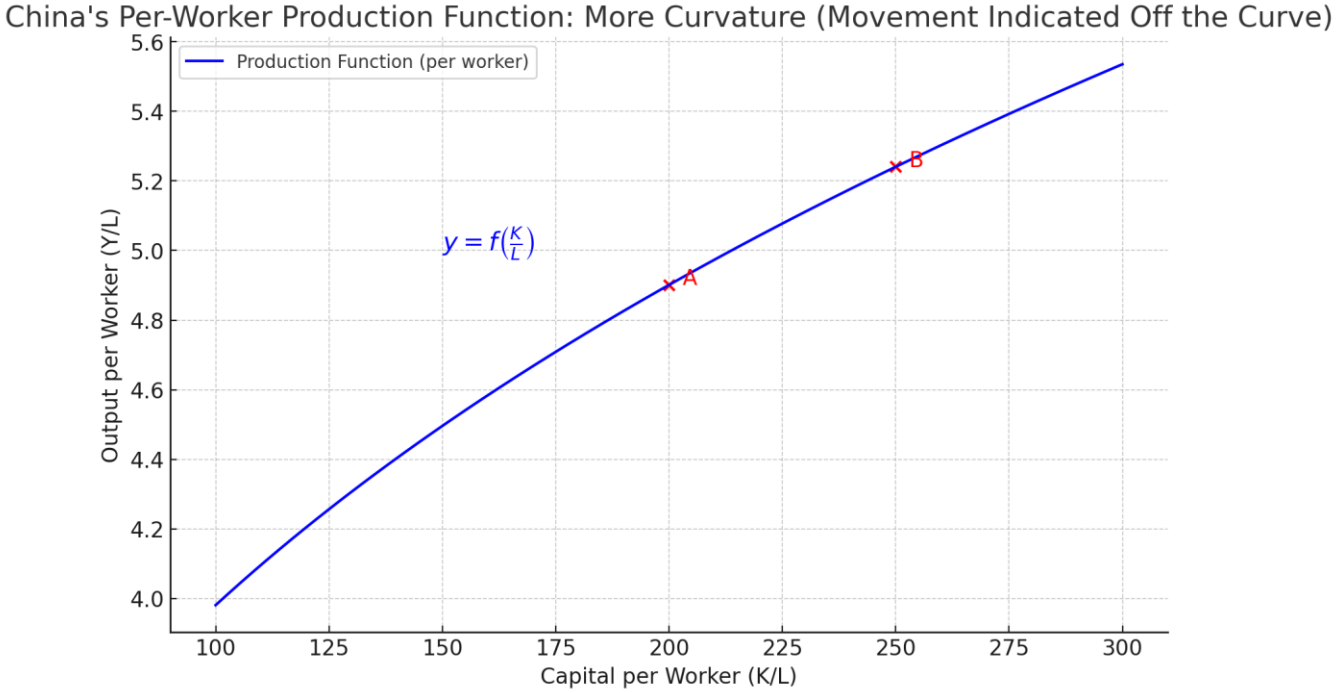
a) Which of the following will result in a movement along China's per-worker production function, and which will result in a shift of China's per-worker production function? Briefly explain, include graph(s) in your explanation.

i. Capital per hour worked increases from 200 yuan per hour worked to 250 yuan per hour worked.

ii. The Chinese government doubles its spending on support for university research.

iii. A reform of the Chinese school system results in more highly trained Chinese workers.

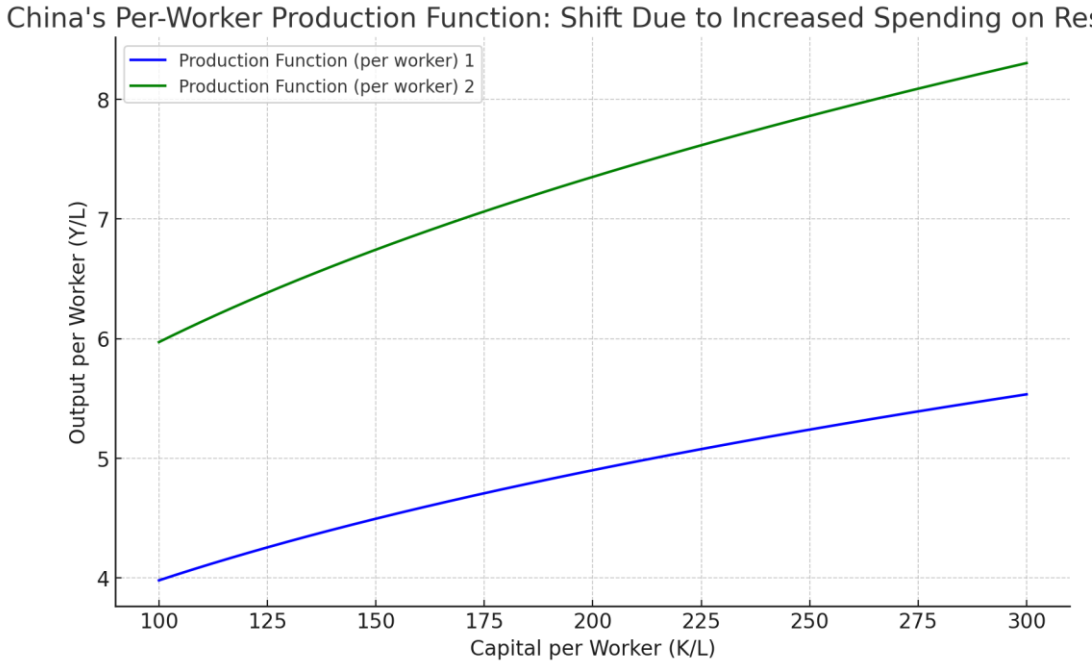
Q3 (a)-(i)



- Typical production function: $Y = AK^{\alpha}L^{1-\alpha}$
- $\frac{Y}{L} = A\left(\frac{K}{L}\right)^{\alpha}$ or $y = Ak^{\alpha}$

Q3 (a)-(ii)

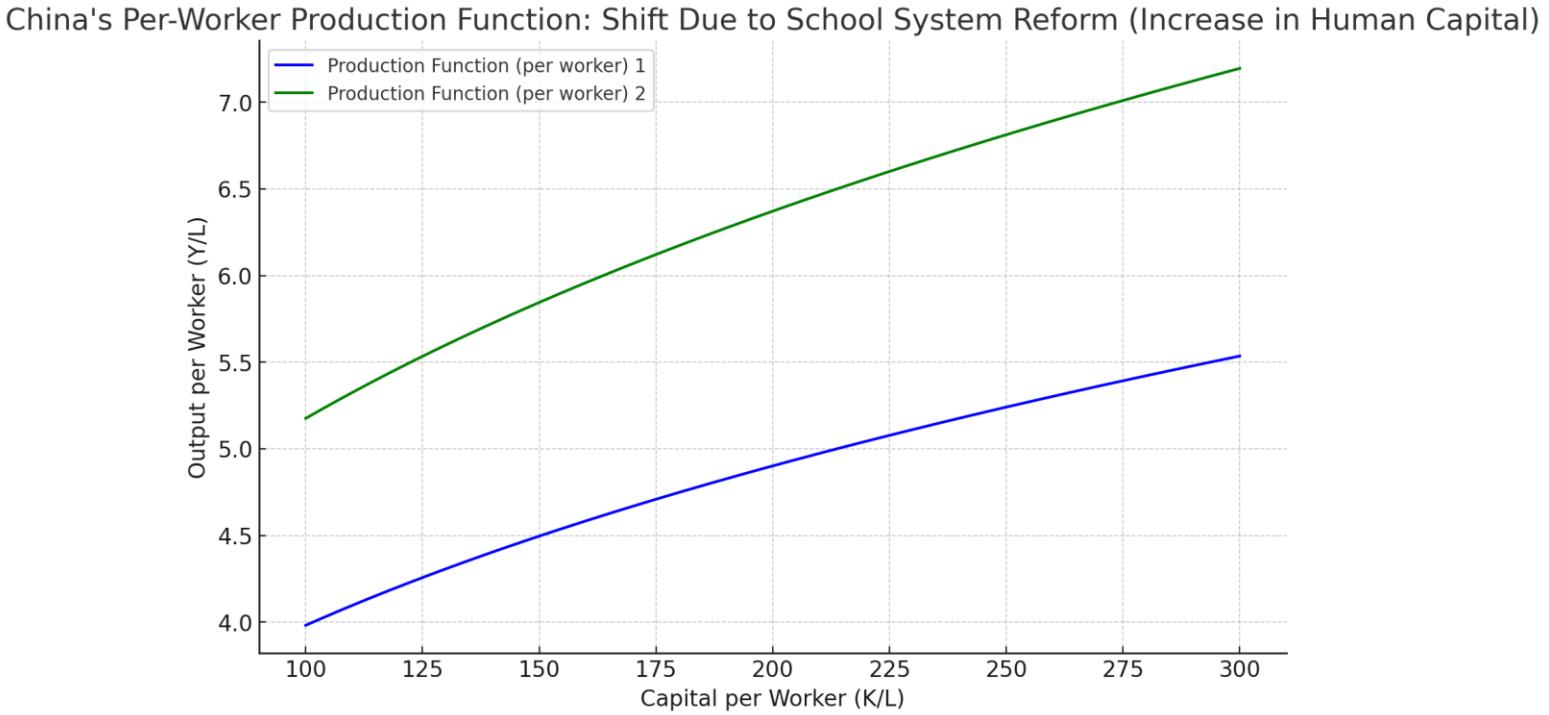
ii. The Chinese government doubles its spending on support for university research.↵



- $y = Ak^{\alpha}$
- A ↑

Q3 (a)-(iii)

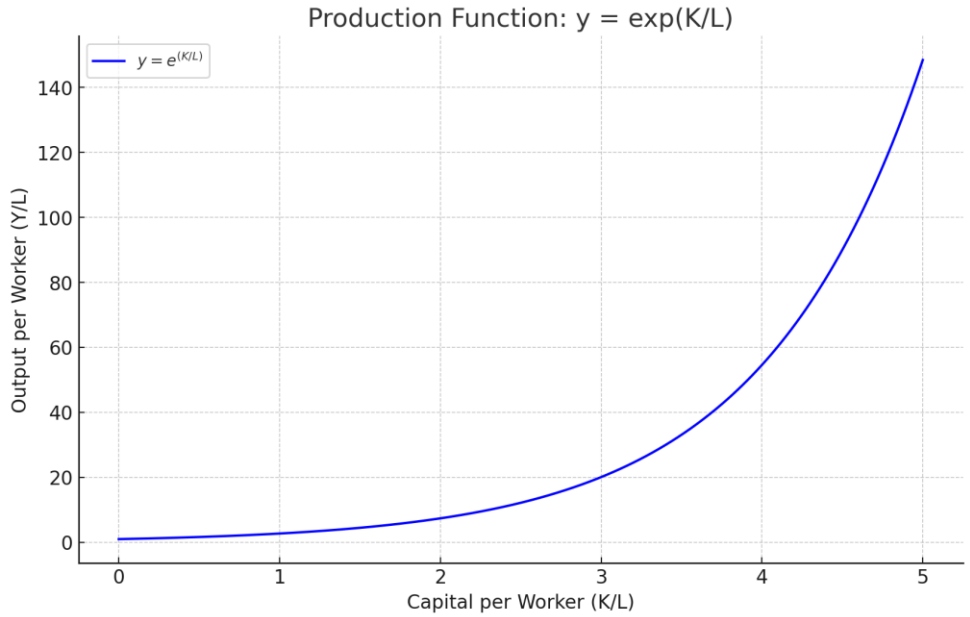
iii. A reform of the Chinese school system results in more highly trained Chinese workers.↵



- $y = Ak^{\alpha}$ and A ↑
- Or $Y = AK^{\alpha}(EL)^{1-\alpha} \Rightarrow y = Ak^{\alpha}E^{1-\alpha}$ and E ↑

Q3 (b)

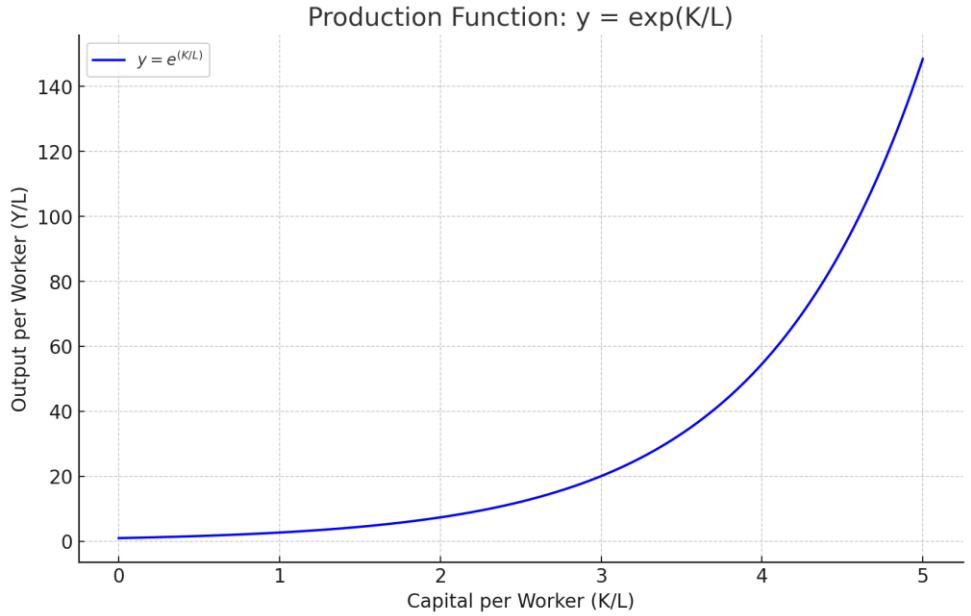
b) If the per worker production function were shaped as shown in the following graph, what would be the implications for economic growth of a country that was accumulating increasing quantities of capital per hour worked? Briefly explain.



- Example: $y = f(k) = Ae^{\alpha k}$
- Not common, empirically

Q3 (b)

b) If the per worker production function were shaped as shown in the following graph, what would be the implications for economic growth of a country that was accumulating increasing quantities of capital per hour worked? Briefly explain.↵



- f”(k)>0 is required. i.e., increasing marginal return to capital-labor ratio

Q4

Assume the following long-term statistics for the Loco economy.
The working age population grows 1.8% per year.
The rate of labor force participation declines 0.5% per year.
Labor productivity climbs at 2% per year
Year-end 2030: Unemployment = 3.5%,

	LF level	employ. level	Unemp. level	unemp. rate
2030	100	96.5	3.5	3.5%
2031				
2032				
2033				

Q4

- %ΔWAP = 1.8% (Working Age Population)
- %ΔLFPR = -0.5% (Labor Force Participation Rate)
- %ΔLP = 2% (Labor Productivity)

	LF level	employ. level	Unemp. level	unemp. rate
2030	100	96.5	3.5	3.5%
2031				
2032				
2033				

Q4 (A)

A. What is an estimate of the economy's long term sustainable growth rate? Show work. ↵

- $LTSG := \%\Delta LF + \%\Delta LP$
- $LF_t = WAP_t \times LFPR_t \Rightarrow \log(LF_t) = \log(WAP_t) + \log(LFPR_t)$
- $\Rightarrow \dot{LF}_t/LF_t = \dot{WAP}_t/WAP_t + \dot{LFPR}_t/LFPR_t$, where $\dot{X}_t := \frac{\mathrm{d}}{\mathrm{dt}}\mathrm{X_t}$
- $\Leftrightarrow$ %LF=%ΔWAP+%ΔLFPR
- So, $LTSG = \%\Delta WAP + \%\Delta LFPR + \%\Delta LP = 1.8 - 0.5 + 2.0 = 3.3\%$

Q4(B)(a)

B. Assume population, participation, and labor productivity advance at their long-term trajectories in 2031. Assume the economy grows by 4.3%, in 2031

a. How fast did employment grow in 2031? Show work,

- $Y_t = L_t \times \frac{Y_t}{L_t} = L_t \times LP_t \Rightarrow \log(Y_t) = \log(L_t) + \log(LP_t)$
- $\Rightarrow \dot{Y}_t/Y_t = \dot{L}_t/L_t + \dot{LP}_t/LP_t$, where $\dot{X}_t := \frac{\mathrm{d}}{\mathrm{dt}} \mathrm{X_t}$
- Hence, $\%\Delta Y = \%\Delta L + \%\Delta LP$
- Assume $\%\Delta \mathrm{L} = \%\Delta \mathrm{E}$, then $\%\Delta \mathrm{E} = \%\Delta \mathrm{Y} - \%\Delta \mathrm{LP} = 4.3\% - 2.0\% = 2.3\%$

Q4(B)(b)

b. What was the unemployment rate, year-end 2031? Show work ↵
(Hint: use the table provided above to help solve question)↵

- $U_{rate\ 2031} = \frac{U\_2031}{LF\_2031}$
- $LF_{2031} = LF_{2030} * (1 + 0.013) = 100 * 1.013 = 101.3$
- $E_{2031} = \mathrm{E}_{2030} \times (1 + 0.023) = 96.5 * 1.023 = 98.7195$
- $U_{2031} = LF_{2031} - E_{2031} = 101.3 - 98.7195 = 2.5805$
- $U_{rate\ 2031} = \frac{2.5805}{101.3} = 2.547$

Q5

Question 5

Use the following macroeconomic model to questions that follow (**Show your calculations**)

$C = C_0 + bY$	(consumption function)
$I = I_0$	(planned investment function)
$G = G_0$	(government purchases function)
$NX = X_0 - M_0$	(net export function)

Q5 (a)

a) Write down the aggregate expenditure function. ↵

↵

$AE = C + I\ (planned) + G + NX$ ↵

$AE = C_0 + bY + I_0 + G_0 + X_0 - M_0$ ↵

Q5(b)

b) Suppose that autonomous consumption is $1,500, government purchases are $1,250, planned investment spending is $1,250, net exports are -$250, and the MPC is 0.75. What is the value (in terms of Y) of aggregate expenditure?

$$AE = C_0 + bY + I_0 + G_0 + X_0 - M_0$$
$$= 1500 + 0.75Y + 1250 + 1250 - 250$$
$$= 3750 + 0.75Y$$

Q5(c)

c) For GDP of $20,000, what is the value of the unintended change in inventories? What is the implication for changes in GDP in the subsequent period? Explain.

AE=3750 + 0.75(20000)= 18,750

$I_u = Y - AE = 20000 - 18750 = 1250$

Output > AE. Implication: unplanned inventory accumulation => reduce production next period => Y will decrease, and Unemployment will increase

- $Y = C+I(actual)+G+NX$
- $AE = C+I(planned)+G+NX$
- $Y - AE = I(actual) - I(planned) = I\_u$

Q5 (d)

d) Derive the macroeconomic equilibrium, Y as a function of expenditure variables.

In equilibrium, $Y = AE$

$(1\text{-}b)Y = C_0 + I + G + NX$

$$Y^* = \frac{1}{1-b}[C_0 + I_0 + G_0 + NX]$$

Q5 (e)

e) Calculate the spending multiplier. What is the equilibrium GDP? What is the unintended change in inventories at this output level?

$$Spending\ mutiplier = \frac{1}{1-b} = \frac{1}{1-0.75} = 4$$

$$Y^* = \frac{1}{1-b}[C_0 + I_0 + G_0 + NX] = 15000$$

In equilibrium, $I_u = 0$

- **Spending multiplier** measures the impact of a change in autonomous spending (government expenditures, investments, or exports) on GDP

Taking into account the feedback loop C ↑ ⇒Y ↑ ⇒C ↑ …

Q5 (f)

f) For GDP of $10,000, what is the value of aggregate expenditure, and what is the value of the unintended change in inventories? Show your calculations.

AE=3750 + 0.75(10000)= 11,250

I_u = Y – AE = 10000 – 11250 = -1250

Output < AE. Implication: unplanned decrease in inventory => increase in production next period => Y will increase, and Unemployment will decrease

Q5 (g)

g) Suppose that autonomous consumption is 500, government purchases are $1,000, planned investment spending is $1,250, net exports are -$250, and the MPC is 0.8. What is equilibrium GDP?↵

$$Y^* = \frac{1}{1-b}[C_0 + I_0 + G_0 + NX] = \frac{1}{1-0.8}[500 + 1250 + 1000 - 250] = 12500$$ ↵