Elements of Macroeconomics TA Session 8: Assignment 6&7

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Slides on https://github.com/Haruki-Shibuya/TA

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Outline

- Most of the questions in Assignment 6 are similar to Midterm Exam 3
- Today I'll cover Assignment 7

Q1

- Consider an economy described by the following equations:
- $C = C_0 + bY_d$ (b is MPC)
- $I= I_0$
- $G = G_0$
- $T = tY$
- $TR= TR_0$
- where Y is GDP, (Y_d is disposable income), C is consumption, I is investment, G is government purchases, T is taxes, and TR is transfer payment from government to households.

Q1(a)

- (a)Derive the equation for equilibrium GDP.

- Recall $Y = C + I + G = C\_0 + bY\_d + I\_0 + G\_0$
- Since $Y\_d = Y - T + TR = Y - tY + TR\_0$,
- $Y=C\_0+b(Y - tY + TR\_0) + I\_0 + G\_0$,
- We obtain $Y^* = 1/[1 - b(1 - t)][C\_0 + bTR\_0 + I\_0 + G\_0]$

Q1(b)

- b) Write down the equations for the government purchases multiplier and transfer payment multiplier

- Recall Y*= $1/[1-b(1-t)][C\_0+bTR\_0+I\_0+G\_0]$

- Government purchases multiplier :
- M_G = $\partial Y*/\partial G\_0 \; = \; 1/[1-MPC*(1-t)]$

- Transfer payment multiplier :
- M_TR = $\partial Y*/\partial TR\_0 \; = MPC/[1-MPC*(1-t)]$

Q1(c)

- c) Assuming C0 = 100, MPC=0.75, TR=40, I=500, G0 = 125, tax rate is 20%, calculate
- i) equilibrium GDP, ii) government purchases spending multiplier iii) transfer payment multiplier

- 1. $Y* = \left(\frac{1}{[1-0.75+0.75*0.2]}\right) * (100 + 0.75 * 40 + 500 + 125) = 1887.5$
- 2. $M\_G = \frac{1}{[1-0.75*(0.8)]} = 2.5$
- 3. $M\_TR = 0.75/[1 - 0.75 * (0.8)] = 1.875$

Q1(c)

- Interpretation:

- M_G = $1 + MPC(1-t) + [MPC * (1-t)]^2 + \cdots$
- M_TR = $MPC(1-t) + [MPC * (1-t)]^2 + [MPC * (1-t)]^3 + \cdots$

- Government purchases have a higher multiplier effect

Q1(d)

- d) If the economy were at full employment, GDP would be 2,000. Assuming no change in monetary policy, what change in government purchases would restore full employment?

- Recall $\Delta Y* = 1/[1 - MPC * (1 - t)]\, \Delta G\_0 = 2.5 \Delta G\_0$
- Hence to fill the gap, $2000\text{-}1877.5 = 2.5\Delta G\_0 \Rightarrow \Delta G\_0 = 45$ is necessary
- assuming horizontal SRAS for all the questions.
- (i.e., no price change.)

Q1(f)

- f) If the economy were at full employment, GDP would be 2,000. Assuming no change in monetary policy, what change in taxes would restore full employment?
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- Multiplier_T = $\partial Y^*/\partial T\_0$ = - MPC/(1-MPC) = -3
- ΔY^* = -3ΔT_0
- Hence, to fill the gap, 2000-1887.5=-3ΔT_0 $\Rightarrow$ T_0 = -37.5 is necessary

Q2 (a)

- MPC: 0.6, real GDP:$bn 500, income tax rate=0.2. Fill the "?"s below

Rounds	Change in G or C	Change in real GDP	Change in taxes	Change in YD
		(billions of dollars)		
1	$\Delta G =$ \$10.00	\$10.00	\$2.00	\$8.00
2	$\Delta C =$ \$4.80	4.80	0.96	3.84
3	$\Delta C =$?	?	?	?
4	$\Delta C =$?	?	?	?
5	$\Delta C =$?	?	?	?
6	$\Delta C =$?	?	?	?
7	$\Delta C =$?	?	?	?
8	$\Delta C =$?	?	?	?
9	$\Delta C =$?	?	?	?
10	$\Delta C =$?	?	?	?

Q2 (a)

- MPC: 0.6, real GDP:$bn 500, income tax rate=0.2,
- Recall:
- $Y = C_0 + MPC * (1 - t) * Y + I + G$
- $\Rightarrow$ Y*= $[1/(MPC * (1 - t))](C\_0 + I + G)$
- $\Rightarrow \partial Y^*/\partial G = 1 + MPC(1 - t) + [MPC * (1 - t)]^2 + \cdots$
- $= 1 + 0.48 + 0.48^2 + 0.48^3 \ldots$
- $\Delta Y^* = (1 + 0.48 + 0.23 + 0.11 + \cdots)\Delta G$

Q2 (a)

- $\Delta Y^* = (1 + \ 0.48 \ + 0.23 + 0.11 + \cdots)$*\$10bn

Rounds	Delta G or Delta C	Delta Y	Delta T	Delta YD
1	10.00	10.00	2.00	8.00
2	4.80	4.80	0.96	3.84
3	2.30	2.30	0.46	1.84
4	1.11	1.11	0.22	0.88
5	0.53	0.53	0.11	0.42
6	0.25	0.25	0.05	0.20
7	0.12	0.12	0.02	0.10
8	0.06	0.06	0.01	0.05
9	0.03	0.03	0.01	0.02
10	0.01	0.01	0.00	0.01
total changes		**19.22**	**3.84**	**15.37**

Q2 (a)

- MPC: 0.6, real GDP:$bn 500, income tax rate=0.2,
- Y_d* = (1-t)Y* = $[(1-t)/(1-MPC(1-t))](C\_0+I+G)$
- $\Rightarrow \partial Y\_d^*/\partial G = (1-\mathrm{t})\{1+\ MPC(1-t)+\ [MPC(1-t)]^2+\cdots\}$
- $= 0.8(1+\ 0.48\ +\ 0.48^2+0.48^3\ldots)$
- ΔYd* = (0.8+ 0.384+1.84+0.88+…)ΔG

Q2 (a)

- $\Delta Yd^* = (0.8 + 0.384 + 1.84 + 0.88 + \ldots)*\$10bn$

Rounds	Delta G or Delta C	Delta Y	Delta T	Delta YD
1	10.00	10.00	2.00	8.00
2	4.80	4.80	0.96	3.84
3	2.30	2.30	0.46	1.84
4	1.11	1.11	0.22	0.88
5	0.53	0.53	0.11	0.42
6	0.25	0.25	0.05	0.20
7	0.12	0.12	0.02	0.10
8	0.06	0.06	0.01	0.05
9	0.03	0.03	0.01	0.02
10	0.01	0.01	0.00	0.01
total changes		**19.22**	**3.84**	**15.37**

Q2 (a)

- MPC: 0.6, real GDP:$bn 500, income tax rate=0.2,
- $T^* = tY^* = [t/(1 - MPC(1-t))](C\_0 + I + G)$
- $\Rightarrow \partial T^*/\partial G = t\{1 + MPC(1-t) + [MPC(1-t)]^2 + \cdots\}$
- $= 0.2(1 + 0.48 + 0.48^2 + 0.48^3 \ldots)$
- $\Delta T^* = (0.2 + 0.96 + 0.46 + 0.22 + \ldots)\Delta G$

Q2 (a)

- $\Delta T^* = (0.2 + 0.96 + 0.46 + 0.22 + \ldots) * \$10\text{bn}$

Rounds	Delta G or Delta C	Delta Y	Delta T	Delta YD
1	10.00	10.00	2.00	8.00
2	4.80	4.80	0.96	3.84
3	2.30	2.30	0.46	1.84
4	1.11	1.11	0.22	0.88
5	0.53	0.53	0.11	0.42
6	0.25	0.25	0.05	0.20
7	0.12	0.12	0.02	0.10
8	0.06	0.06	0.01	0.05
9	0.03	0.03	0.01	0.02
10	0.01	0.01	0.00	0.01
total changes		**19.22**	**3.84**	**15.37**

Q2 (a)

- MPC: 0.6, real GDP:\$bn 500, income tax rate=0.2,
- $C^* = Y^* - I - G = \left[\frac{1}{1-MPC(1-t)}\right] C_0 + \left[\frac{MPC(1-t)}{1-MPC(1-t)}\right](I + G)$
- $\Rightarrow \partial C^*/\partial G = MPC(1-t) + [MPC(1-t)]^2 + [MPC(1-t)]^3 + \cdots$
- $= 0.48 + 0.48^2 + 0.48^3 \ldots$
- $\Delta C^* = (0.48 + 0.23 + 0.11 + \ldots)\Delta G$
- The effect on consumption starts from the 2nd round

Q2(a)

- ΔG=\$10bn in 1st round,
- $\Delta C^* = (0.48 + 0.23 + 0.11 + \ldots) * \$10\text{bn}$ in later rounds

Rounds	Delta G or Delta C	Delta Y	Delta T	Delta YD
1	10.00	10.00	2.00	8.00
2	4.80	4.80	0.96	3.84
3	2.30	2.30	0.46	1.84
4	1.11	1.11	0.22	0.88
5	0.53	0.53	0.11	0.42
6	0.25	0.25	0.05	0.20
7	0.12	0.12	0.02	0.10
8	0.06	0.06	0.01	0.05
9	0.03	0.03	0.01	0.02
10	0.01	0.01	0.00	0.01
total changes		19.22	3.84	15.37

Q2(a)

- a) What is the total change in real GDP after the 10 rounds?→$19.22bn
- What is the value of the multiplier?→$\frac{1}{1-MPC*(1-t)} = \frac{1}{1-0.6*0.8} =$ 1.92307692
- What would you expect the total change in real GDP to be, based on the multiplier formula? → $19.2307692 bn
- How do your two answers compare? → Close. (The 10-round sum is a fairly accurate approximation for the sum of infinite series.)

Q2(b)

- Now MPC=0.75 and income tax rate=0.1. Redo the computations.

Rounds	Delta G or Delta C	Delta Y	Delta T	Delta YD
1	10.00	10.00	2.00	8.00
2	6.75	6.75	0.68	6.08
3	4.56	4.56	0.46	4.10
4	3.08	3.08	0.31	2.77
5	2.08	2.08	0.21	1.87
6	1.40	1.40	0.14	1.26
7	0.95	0.95	0.09	0.85
8	0.64	0.64	0.06	0.57
9	0.43	0.43	0.04	0.39
10	0.29	0.29	0.03	0.26
total changes		**30.17**	**6.03**	**24.13**

Multiplier = 1/(1-MPC*(1-t)) = 1/(1-0.75*0.9)= 3.077

Expected Delta Y = 30.77

Q3(a)

- Calculate the change in government purchases of goods and services necessary to close the recessionary or inflationary gaps in the following cases. Assume that the SRAS curve is horizontal so that the change in real GDP arising from a shift of the aggregate demand curve equals the size of the shift of the curve.

- (a) real GDP: $100 bn, potential output: $160 bn, income tax rate: 0.2, and the MPC: 0.75.
- $\partial Y^*/\partial G = \frac{1}{[1-MPC(1-t)]} = 2.5 \Rightarrow \Delta Y^* = 2.5\Delta G$
- $160 - 100 = 2.5\Delta G \Rightarrow \Delta G = 24\ (\$bn)$

Q3(b)

- (b) real GDP: \$250 bn, potential output: \$200 bn, income tax rate: 0.1, and the MPC: 0.5.
- $\partial Y^*/\partial G = \frac{1}{[1-MPC(1-t)]} = 1.82 \Rightarrow \Delta Y^* = 1.82\Delta G$
- $200 - 250 = 1.82\Delta G \Rightarrow \Delta G = -27.5\ (\$bn)$

Q3(c)

- (b) real GDP: \$180 bn, potential output: \$100 bn, income tax rate: 0.25, and the MPC: 0.8.
- $\partial Y^*/\partial G = \frac{1}{[1-MPC(1-t)]} = 2.5 \Rightarrow \Delta Y^* = 2.5\Delta G$
- $100 - 180 = 2.5\Delta G \Rightarrow \Delta G = -32\ (\$bn)$