

# Elements of Macroeconomics TA

## Session 8:

### Assignment 6&7

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Slides on <https://github.com/Haruki-Shibuya/TA>

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# Outline

- Most of the questions in Assignment 6 are similar to Midterm Exam 3
- Today I'll cover Assignment 7

# Q1

- Consider an economy described by the following equations:
- $C = C_0 + bY_d$  (b is MPC)
- $I = I_0$
- $G = G_0$
- $T = tY$
- $TR = TR_0$
- where Y is GDP, ( $Y_d$  is disposable income), C is consumption, I is investment, G is government purchases, T is taxes, and TR is transfer payment from government to households.

# Q1(a)

- (a) Derive the equation for equilibrium GDP.
- Recall  $Y = C + I + G = C_0 + bY_d + I_0 + G_0$
- Since  $Y_d = Y - T + TR = Y - tY + TR_0$ ,
- $Y = C_0 + b(Y - tY + TR_0) + I_0 + G_0$ ,
- We obtain  $Y^* = 1/[1 - b(1 - t)][C_0 + bTR_0 + I_0 + G_0]$

# Q1(b)

- b) Write down the equations for the government purchases multiplier and transfer payment multiplier
- Recall  $Y^* = 1/[1 - b(1 - t)][C_0 + bTR_0 + I_0 + G_0]$
- Government purchases multiplier :
- $M_G = \partial Y^* / \partial G_0 = 1/[1 - MPC * (1 - t)]$
- Transfer payment multiplier :
- $M_{TR} = \partial Y^* / \partial TR_0 = MPC/[1 - MPC * (1 - t)]$

## Q1(c)

- c) Assuming  $C_0 = 100$ ,  $MPC=0.75$ ,  $TR=40$ ,  $I=500$ ,  $G_0 = 125$ , tax rate is 20%, calculate
- i) equilibrium GDP, ii) government purchases spending multiplier iii) transfer payment multiplier

- 1.  $Y^* = \left( \frac{1}{[1-0.75+0.75*0.2]} \right) * (100 + 0.75 * 40 + 500 + 125) = 1887.5$

- 2.  $M_G = \frac{1}{[1-0.75*(0.8)]} = 2.5$

- 3.  $M_{TR} = 0.75/[1 - 0.75 * (0.8)] = 1.875$

# $Q_1(c)$

- Interpretation:
- $M_G = 1 + MPC(1 - t) + [MPC * (1 - t)]^2 + \dots$
- $M_{TR} = MPC(1 - t) + [MPC * (1 - t)]^2 + [MPC * (1 - t)]^3 + \dots$
- Government purchases have a higher multiplier effect

## Q1(d)

- d) If the economy were at full employment, GDP would be 2,000. Assuming no change in monetary policy, what change in government purchases would restore full employment?
- Recall  $\Delta Y^* = 1/[1 - MPC * (1 - t)] \Delta G_0 = 2.5 \Delta G_0$
- Hence to fill the gap,  $2000 - 1877.5 = 2.5 \Delta G_0 \Rightarrow \Delta G_0 = 45$  is necessary
- assuming horizontal SRAS for all the questions.
- (i.e., no price change.)



## Q1(f)

- f) If the economy were at full employment, GDP would be 2,000. Assuming no change in monetary policy, what change in taxes would restore full employment?
- 
- $\text{Multiplier}_T = \partial Y^* / \partial T_0 = -\text{MPC} / (1 - \text{MPC}) = -3$
- $\Delta Y^* = -3\Delta T_0$
- Hence, to fill the gap,  $2000 - 1887.5 = -3\Delta T_0 \Rightarrow T_0 = -37.5$  is necessary

# Q2 (a)

- MPC: 0.6, real GDP:\$bn 500, income tax rate=0.2. Fill the “?”s below

Rounds	Change in G or C	Change in real GDP (billions of dollars)	Change in taxes	Change in YD
1	$\Delta G = \$10.00$	\$10.00	\$2.00	\$8.00
2	$\Delta C = \$4.80$	4.80	0.96	3.84
3	$\Delta C = ?$	?	?	?
4	$\Delta C = ?$	?	?	?
5	$\Delta C = ?$	?	?	?
6	$\Delta C = ?$	?	?	?
7	$\Delta C = ?$	?	?	?
8	$\Delta C = ?$	?	?	?
9	$\Delta C = ?$	?	?	?
10	$\Delta C = ?$	?	?	?

## Q2 (a)

- MPC: 0.6, real GDP:\$bn 500, income tax rate=0.2,
- Recall:
- $Y = C_0 + MPC * (1 - t) * Y + I + G$
- $\Rightarrow Y^* = [1 / (1 - MPC * (1 - t))] (C_0 + I + G)$
- $\Rightarrow \partial Y^* / \partial G = 1 + MPC(1 - t) + [MPC * (1 - t)]^2 + \dots$
- $= 1 + 0.48 + 0.48^2 + 0.48^3 \dots$
- $\Delta Y^* = (1 + 0.48 + 0.23 + 0.11 + \dots) \Delta G$

## Q2 (a)

■  $\Delta Y^* = (1 + 0.48 + 0.23 + 0.11 + \dots) * \$10\text{bn}$

Rounds	Delta G or Delta C	Delta Y	Delta T	Delta YD
1	10.00	10.00	2.00	8.00
2	4.80	4.80	0.96	3.84
3	2.30	2.30	0.46	1.84
4	1.11	1.11	0.22	0.88
5	0.53	0.53	0.11	0.42
6	0.25	0.25	0.05	0.20
7	0.12	0.12	0.02	0.10
8	0.06	0.06	0.01	0.05
9	0.03	0.03	0.01	0.02
10	0.01	0.01	0.00	0.01
total changes		19.22	3.84	15.37

## Q2 (a)

- MPC: 0.6, real GDP: \$bn 500, income tax rate=0.2,
- $Y_d^* = (1-t)Y^* = [(1-t)/(1-MPC(1-t))](C_0 + I + G)$
- $\Rightarrow \partial Y_d^* / \partial G = (1-t)\{1 + MPC(1-t) + [MPC(1-t)]^2 + \dots\}$
- $= 0.8(1 + 0.48 + 0.48^2 + 0.48^3 \dots)$
- $\Delta Y_d^* = (0.8 + 0.384 + 1.84 + 0.88 + \dots) \Delta G$

## Q2 (a)

■  $\Delta Y_d^* = (0.8 + 0.384 + 1.84 + 0.88 + \dots) * \$10\text{bn}$

Rounds	Delta G or Delta C	Delta Y	Delta T	Delta YD
1	10.00	10.00	2.00	8.00
2	4.80	4.80	0.96	3.84
3	2.30	2.30	0.46	1.84
4	1.11	1.11	0.22	0.88
5	0.53	0.53	0.11	0.42
6	0.25	0.25	0.05	0.20
7	0.12	0.12	0.02	0.10
8	0.06	0.06	0.01	0.05
9	0.03	0.03	0.01	0.02
10	0.01	0.01	0.00	0.01
total changes		19.22	3.84	15.37

## Q2 (a)

- MPC: 0.6, real GDP: \$bn 500, income tax rate=0.2,
- $T^* = tY^* = [t/(1 - MPC(1 - t))](C_0 + I + G)$
- $\Rightarrow \partial T^*/\partial G = t\{1 + MPC(1 - t) + [MPC(1 - t)]^2 + \dots\}$
- $= 0.2(1 + 0.48 + 0.48^2 + 0.48^3 \dots)$
- $\Delta T^* = (0.2 + 0.96 + 0.46 + 0.22 + \dots) \Delta G$

## Q2 (a)

■  $\Delta T^* = (0.2 + 0.96 + 0.46 + 0.22 + \dots) * \$10\text{bn}$

Rounds	Delta G or Delta C	Delta Y	Delta T	Delta YD
1	10.00	10.00	2.00	8.00
2	4.80	4.80	0.96	3.84
3	2.30	2.30	0.46	1.84
4	1.11	1.11	0.22	0.88
5	0.53	0.53	0.11	0.42
6	0.25	0.25	0.05	0.20
7	0.12	0.12	0.02	0.10
8	0.06	0.06	0.01	0.05
9	0.03	0.03	0.01	0.02
10	0.01	0.01	0.00	0.01
total changes		19.22	3.84	15.37



## Q2 (a)

- MPC: 0.6, real GDP: \$bn 500, income tax rate=0.2,
- $C^* = Y^* - I - G = \left[ \frac{1}{1-MPC(1-t)} \right] C_0 + \left[ \frac{MPC(1-t)}{1-MPC(1-t)} \right] (I + G)$
- $\Rightarrow \partial C^* / \partial G = MPC(1-t) + [MPC(1-t)]^2 + [MPC(1-t)]^3 + \dots$
- $= 0.48 + 0.48^2 + 0.48^3 \dots$
- $\Delta C^* = (0.48 + 0.23 + 0.11 + \dots) \Delta G$
- The effect on consumption starts from the 2<sup>nd</sup> round

## Q2(a)

- $\Delta G = \$10\text{bn}$  in 1<sup>st</sup> round,
- $\Delta C^* = (0.48 + 0.23 + 0.11 + \dots) * \$10\text{bn}$  in later rounds

Rounds	Delta G or Delta C	Delta Y	Delta T	Delta YD
1	10.00	10.00	2.00	8.00
2	4.80	4.80	0.96	3.84
3	2.30	2.30	0.46	1.84
4	1.11	1.11	0.22	0.88
5	0.53	0.53	0.11	0.42
6	0.25	0.25	0.05	0.20
7	0.12	0.12	0.02	0.10
8	0.06	0.06	0.01	0.05
9	0.03	0.03	0.01	0.02
10	0.01	0.01	0.00	0.01
total changes		19.22	3.84	15.37

## Q2(a)

- a) What is the total change in real GDP after the 10 rounds? → \$19.22bn
- What is the value of the multiplier? →  $\frac{1}{1-MPC*(1-t)} = \frac{1}{1-0.6*0.8} = 1.92307692$
- What would you expect the total change in real GDP to be, based on the multiplier formula? → \$19.2307692 bn
- How do your two answers compare? → Close. (The 10-round sum is a fairly accurate approximation for the sum of infinite series.)

## Q2(b)

- Now  $MPC=0.75$  and income tax rate  $=0.1$ . Redo the computations.

Rounds	Delta G or Delta C	Delta Y	Delta T	Delta YD
1	10.00	10.00	2.00	8.00
2	6.75	6.75	0.68	6.08
3	4.56	4.56	0.46	4.10
4	3.08	3.08	0.31	2.77
5	2.08	2.08	0.21	1.87
6	1.40	1.40	0.14	1.26
7	0.95	0.95	0.09	0.85
8	0.64	0.64	0.06	0.57
9	0.43	0.43	0.04	0.39
10	0.29	0.29	0.03	0.26
total changes		30.17	6.03	24.13

$$\text{Multiplier} = 1/(1-MPC*(1-t)) = 1/(1-0.75*0.9) = 3.077$$

$$\text{Expected Delta Y} = 30.77$$

## Q3(a)

- Calculate the change in government purchases of goods and services necessary to close the recessionary or inflationary gaps in the following cases. Assume that the SRAS curve is horizontal so that the change in real GDP arising from a shift of the aggregate demand curve equals the size of the shift of the curve.
- (a) real GDP: \$100 bn, potential output: \$160 bn, income tax rate: 0.2, and the MPC: 0.75.
- $\partial Y^* / \partial G = \frac{1}{[1 - MPC(1 - t)]} = 2.5 \Rightarrow \Delta Y^* = 2.5 \Delta G$
- $160 - 100 = 2.5 \Delta G \Rightarrow \Delta G = 24 (\$bn)$

## Q3(b)

- (b) real GDP: \$250 bn, potential output: \$200 bn, income tax rate: 0.1, and the MPC: 0.5.
- $\partial Y^* / \partial G = \frac{1}{[1 - MPC(1 - t)]} = 1.82 \Rightarrow \Delta Y^* = 1.82 \Delta G$
- $200 - 250 = 1.82 \Delta G \Rightarrow \Delta G = -27.5 (\$bn)$

## Q3(c)

- (b) real GDP: \$180 bn, potential output: \$100 bn, income tax rate: 0.25, and the MPC: 0.8.
- $\partial Y^* / \partial G = \frac{1}{[1 - MPC(1 - t)]} = 2.5 \Rightarrow \Delta Y^* = 2.5 \Delta G$
- $100 - 180 = 2.5 \Delta G \Rightarrow \Delta G = -32 (\$bn)$