DCIT 403 DESIGNING INTELLIGENT AGENTS

SESSION 4 – ROBOTS KINEMATICS

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SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

DEPARTMENT OF COMPUTER SCIENCE

Session Overview

Objectives

Students should be able to understand the transformational, motion and displacement characteristics of robots.

Session Outline

The key topics to be covered in the session are as follows :

- Robots transformation
- Uniform Kinematics
- Inverse Kinematics

Reading List

Saeed B. Niku, (), Intro to Robotics, analysis control and applications, 2nd Edition

Topic One

ROBOTS KINEMATICS

Robot Kinematics: Position Analysis

INTRODUCTION

- Forward Kinematics:
  to determine where the robot's hand is?
  (If all joint variables are known)

- Inverse Kinematics:
  to calculate what each joint variable is?
  (If we desire that the hand be located at a particular point)

ROBOTS AS MECHANISM

- Multiple type robot have multiple DOF.
  (3 Dimensional, open loop, chain mechanisms)

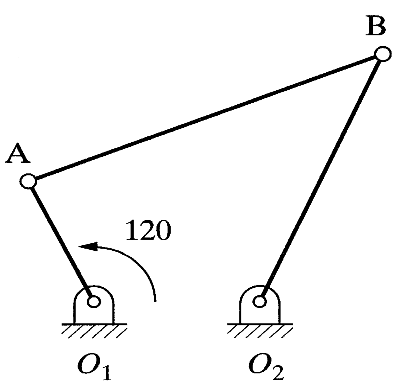


Fig. 4.1 A one-degree-of-freedom closed-loop four-bar mechanism

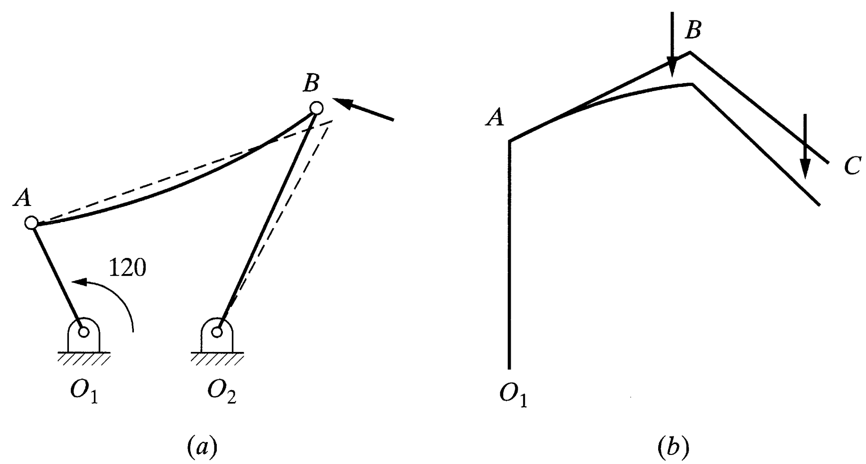


Fig. 4.2 (a) Closed-loop versus (b) open-loop mechanism

Robot Kinematics: Position Analysis

MATRIX REPRESENTATION

Representation of a Point in Space

- A point P in space :
  3 coordinates relative to a reference frame

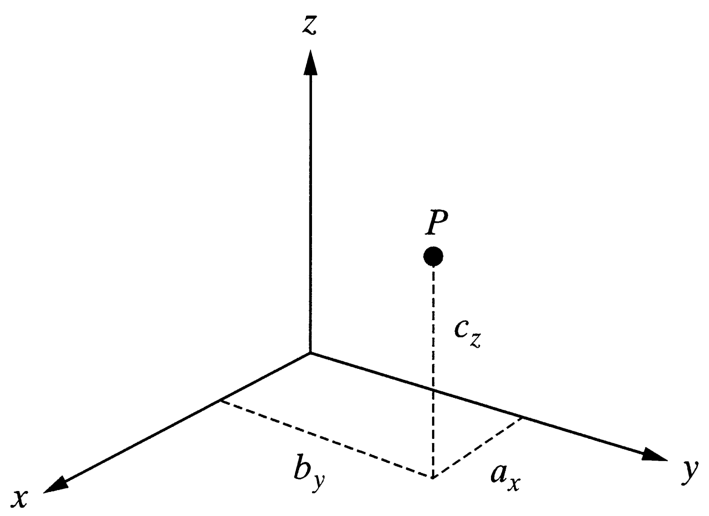


Fig. 4.3 Representation of a point in space

$$P = a_x \hat{i} + b_y \hat{j} + c_z \hat{k}$$

Robot Kinematics: Position Analysis

MATRIX REPRESENTATION

Representation of a Vector in Space

- A Vector *P* in space :
  3 coordinates of its tail and of its head

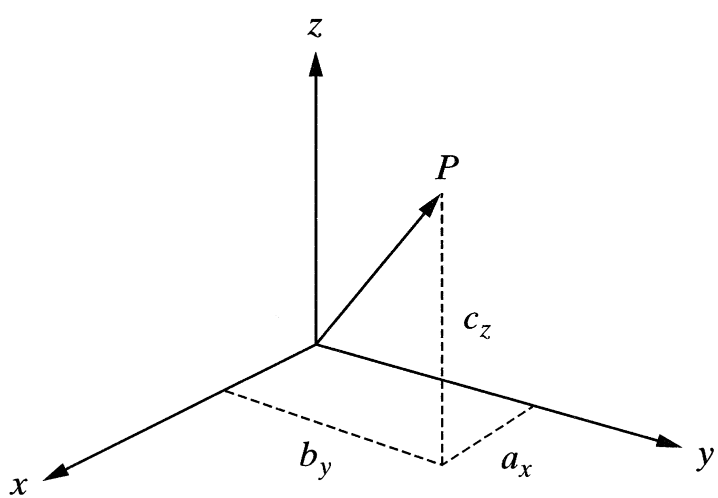


Fig. 4.4 Representation of a vector in space

$$\overline{P} = a_x \hat{i} + b_y \hat{j} + c_z \hat{k}$$

$$\overline{P} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Robot Kinematics: Position Analysis

MATRIX REPRESENTATION

Representation of a Frame at the Origin of a Fixed-Reference Frame

- Each Unit Vector is mutually perpendicular. : normal, orientation, approach vector

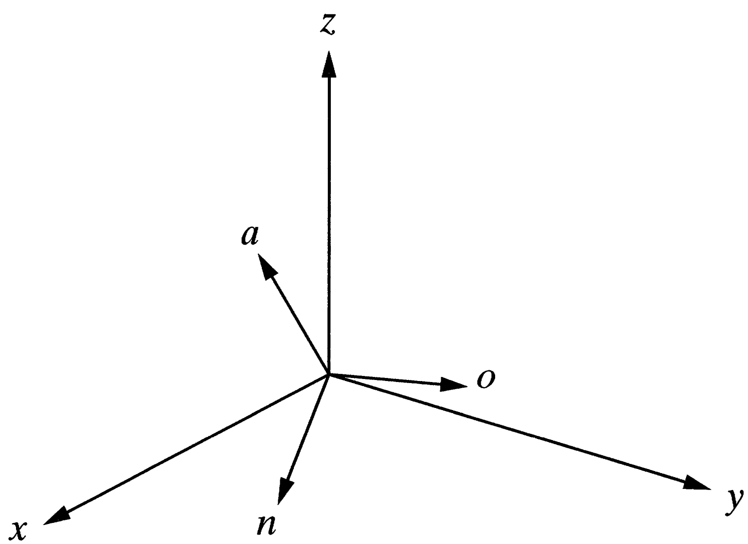


Fig. 4.5 Representation of a frame at the origin of the reference frame

$$F = \begin{bmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{bmatrix}$$

Robot Kinematics: Position Analysis

MATRIX REPRESENTATION

Representation of a Frame in a Fixed Reference Frame

- Each Unit Vector is mutually perpendicular. :
  normal, orientation, approach vector

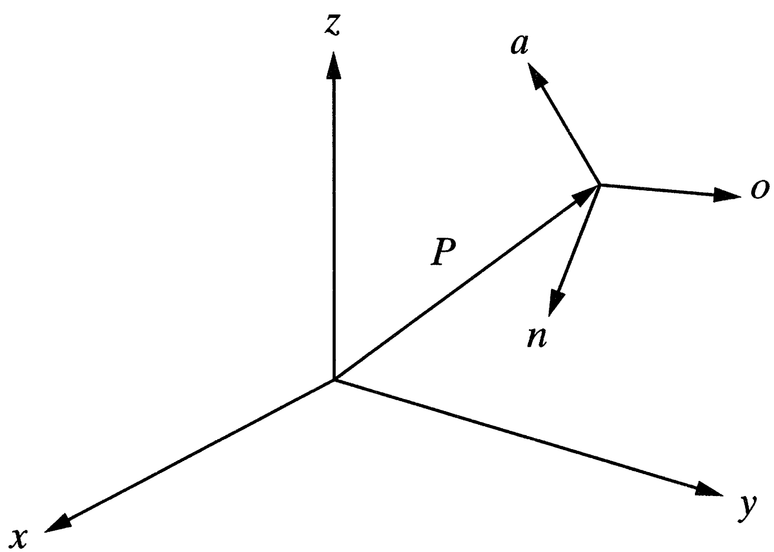


Fig. 4.6 Representation of a frame in a frame

$$F = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Robot Kinematics: Position Analysis

MATRIX REPRESENTATION

Representation of a Rigid Body

- An object can be represented in space by attaching a frame to it and representing the frame in space.

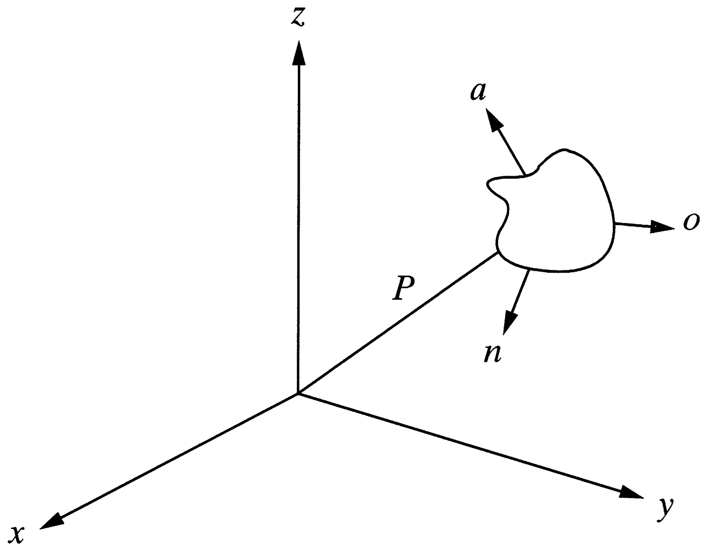


Fig. 2.8 Representation of an object in space

$$F_{object} = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Robot Kinematics: Position Analysis

HOMOGENEOUS TRANSFORMATION MATRICES

- A transformation matrices must be in square form.

  - It is much easier to calculate the inverse of square matrices.
  - To multiply two matrices, their dimensions must match.

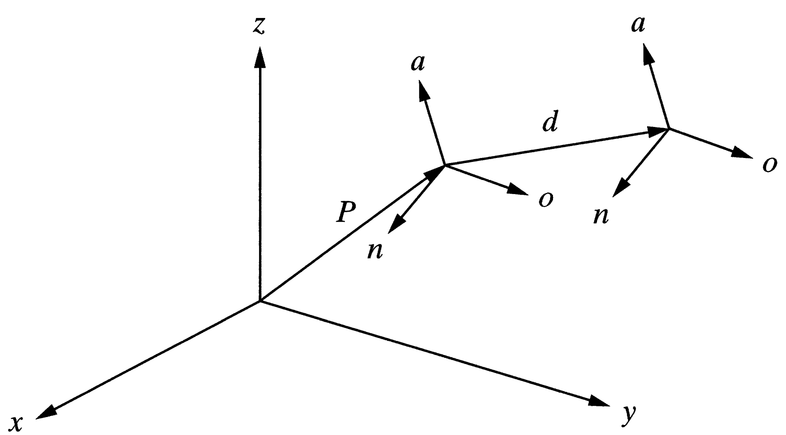
$$F = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Robot Kinematics: Position Analysis

REPRESENTATION OF TRANSFORMATINS

Representation of a Pure Translation

- A transformation is defined as making a movement in space.
  - A pure translation.
  - A pure rotation about an axis.
  - A combination of translation or rotations.



$$T = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fig. 4.9 Representation of an pure translation in space

Robot Kinematics: Position Analysis

REPRESENTATION OF TRANSFORMATINS

Representation of a Pure Rotation about an Axis

- **Assumption :** The frame is at the origin of the reference frame and parallel to it.

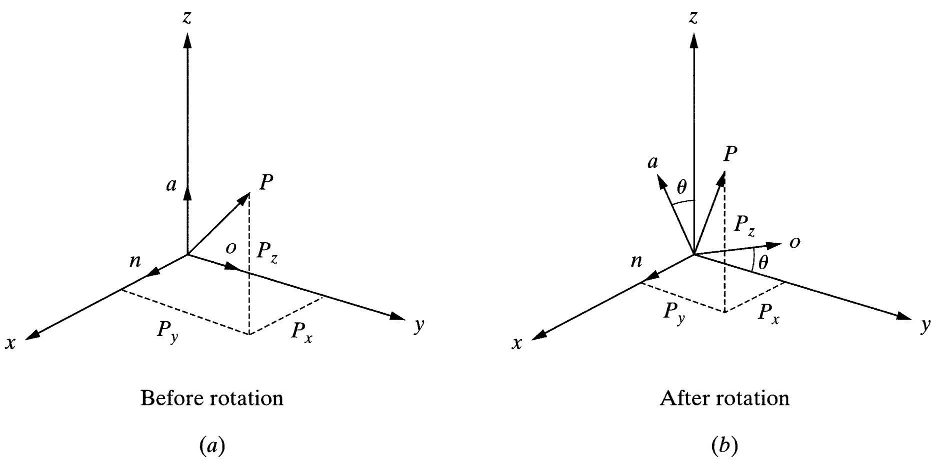


Fig. 4.10 Coordinates of a point in a rotating frame before and after rotation.

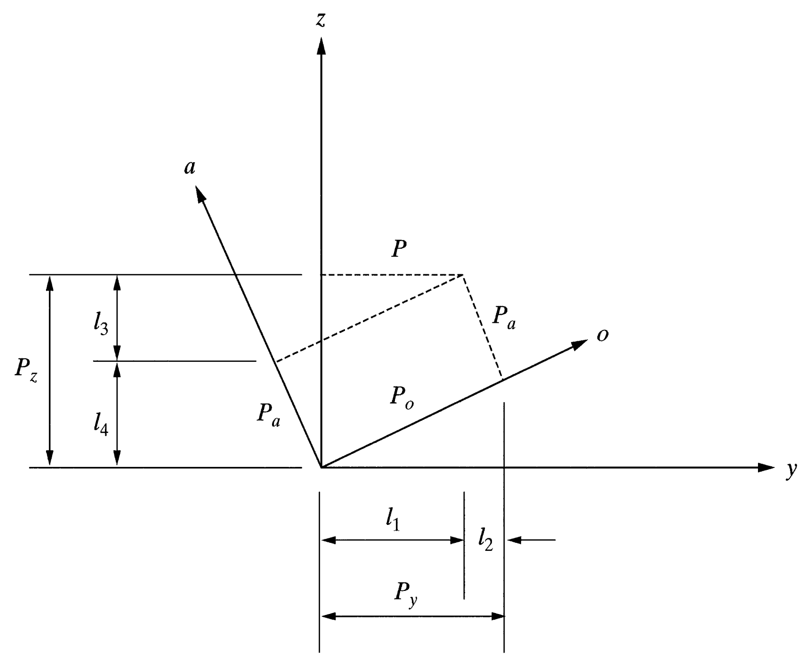


Fig. 4.11 Coordinates of a point relative to the reference frame and rotating frame as viewed from the *x*-axis.

Robot Kinematics: Position Analysis

REPRESENTATION OF TRANSFORMATINS

Representation of Combined Transformations

- A number of successive translations and rotations….

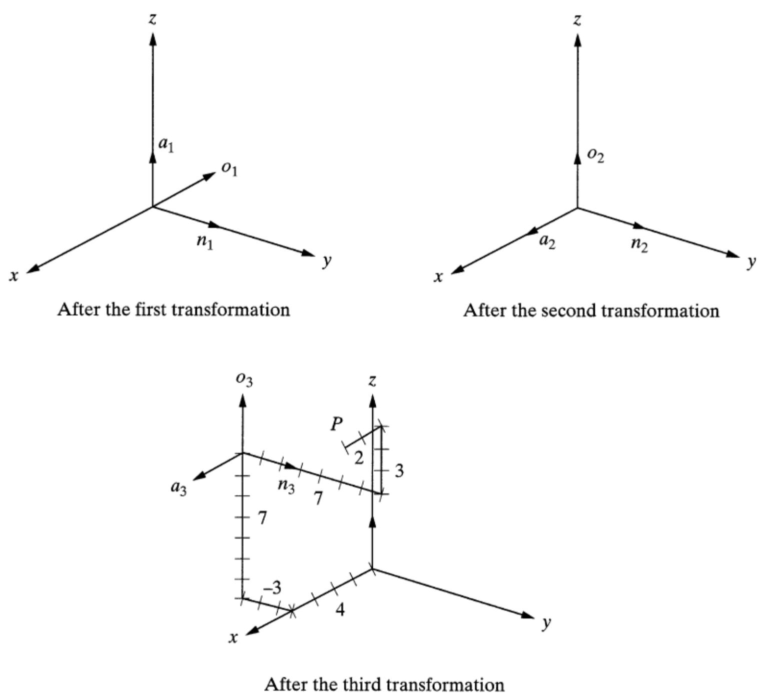


Fig. 4.13 Effects of three successive transformations

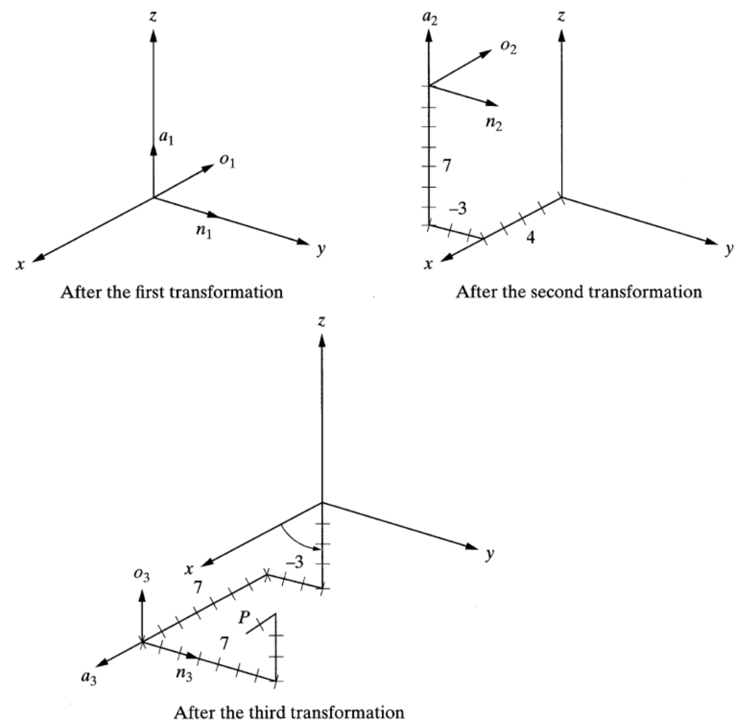


Fig. 4.14 Changing the order of transformations will change the final result

2.5 REPRESENTATION OF TRANSFORMATINS

2.5.5 Transformations Relative to the Rotating Frame

◆ Example 2.8

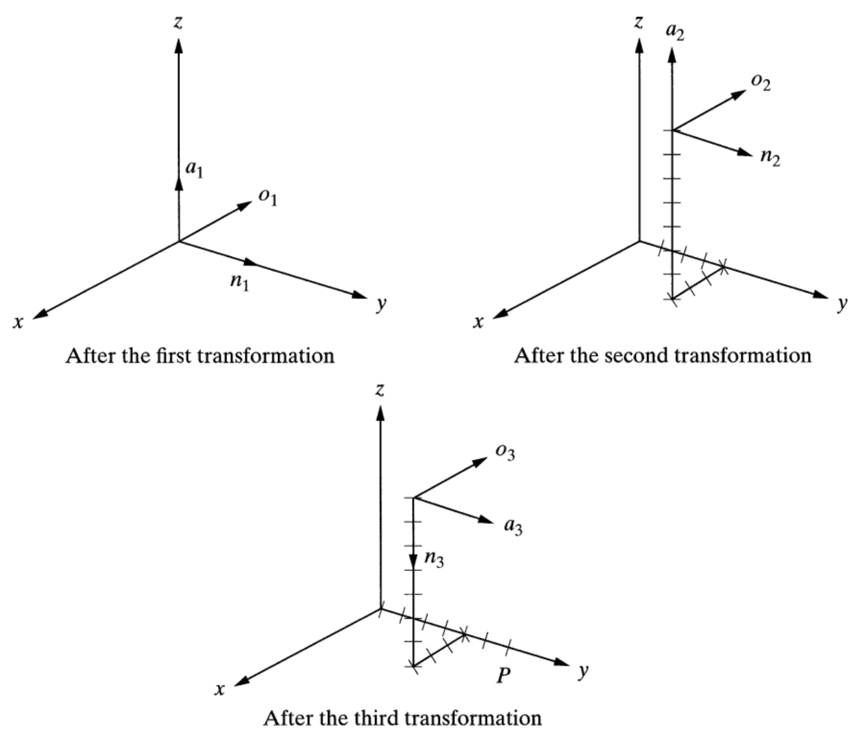


Fig. 2.15 Transformations relative to the current frames.

2.6 INVERSE OF TRANSFORMATION MATIRICES

- Inverse of a matrix calculation steps :
  - Calculate the determinant of the matrix.
  - Transpose the matrix.
  - Replace each element of the transposed matrix by its own minor(adjoint matrix).
  - Divide the converted matrix by the determinant.

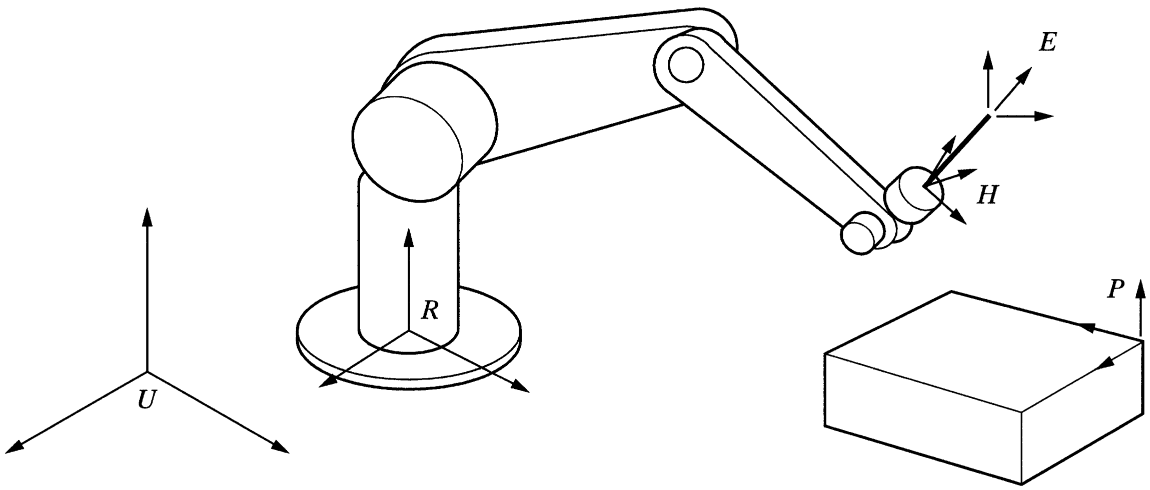


Fig. 2.16 The Universe, robot, hand, part, and end effecter frames.

Robot Kinematics: Position Analysis

FORWARD AND INVERSE KINEMATICS OF ROBOTS

- Forward Kinematics Analysis:
  - Calculating the position and orientation of the hand of the robot.
  - If all robot joint variables are known, one can calculate where the robot is at any instant.

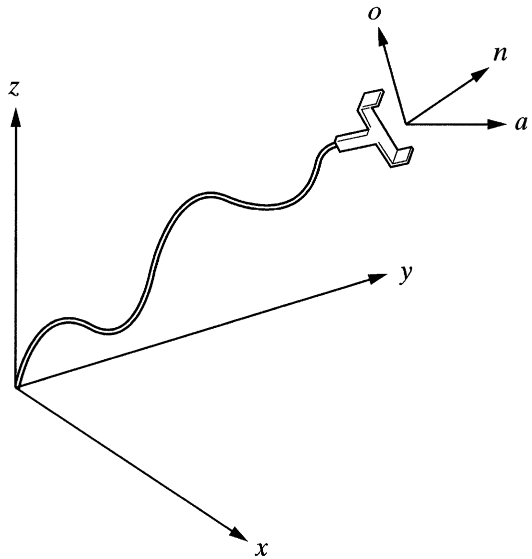


Fig. 4.17 The hand frame of the robot relative to the reference frame.

Robot Kinematics: Position Analysis

FORWARD AND INVERSE KINEMATICS OF ROBOTS

Forward and Inverse Kinematics Equations for Position

- Forward Kinematics and Inverse Kinematics equation for position analysis :

  (a) Cartesian (gantry, rectangular) coordinates.
  (b) Cylindrical coordinates.
  (c) Spherical coordinates.
  (d) Articulated (anthropomorphic, or all-revolute) coordinates.

Robot Kinematics: Position Analysis

FORWARD AND INVERSE KINEMATICS OF ROBOTS

Forward and Inverse Kinematics Equations for Position

(a) Cartesian (Gantry, Rectangular) Coordinates

- IBM 7565 robot
  - All actuator is linear.
  - A gantry robot is a Cartesian robot.

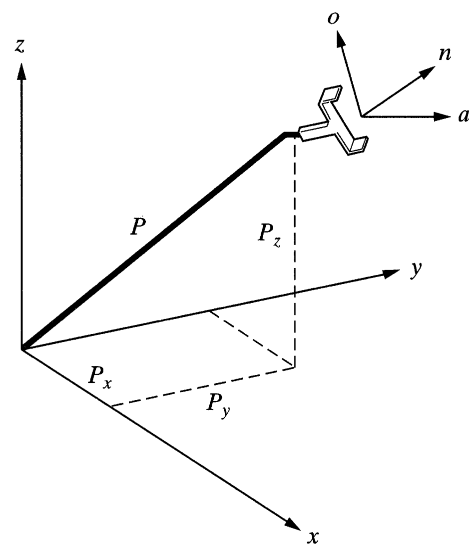


Fig. 4.18 Cartesian Coordinates.

$$
{}^{R}T_{P} = T_{cart} = \begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Robot Kinematics: Position Analysis

FORWARD AND INVERSE KINEMATICS OF ROBOTS

Forward and Inverse Kinematics Equations for Position

(b) Cylindrical Coordinates

- 2 Linear translations and 1 rotation
  - translation of r along the x-axis
  - rotation of α about the z-axis
  - translation of l along the z-axis

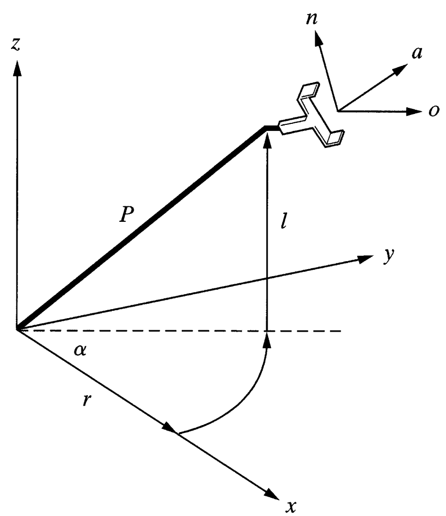


Fig. 4.19 Cylindrical Coordinates.

$$^{R}T_{P} = T_{cyl}(r,\alpha,l) = \text{Trans}(0,0,l)\text{Rot}(z,\alpha)\text{Trans}(r,0,0)$$

$$^{R}T_{P} = T_{cyl} = \begin{bmatrix} C\alpha & -S\alpha & 0 & rC\alpha \\ S\alpha & C\alpha & 0 & rS\alpha \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Robot Kinematics: Position Analysis

FORWARD AND INVERSE KINEMATICS OF ROBOTS

Forward and Inverse Kinematics Equations for Position

(c) Spherical Coordinates

- 2 Linear translations and 1 rotation
  - translation of r along the z-axis
  - rotation of β about the y-axis
  - rotation of γ along the z-axis

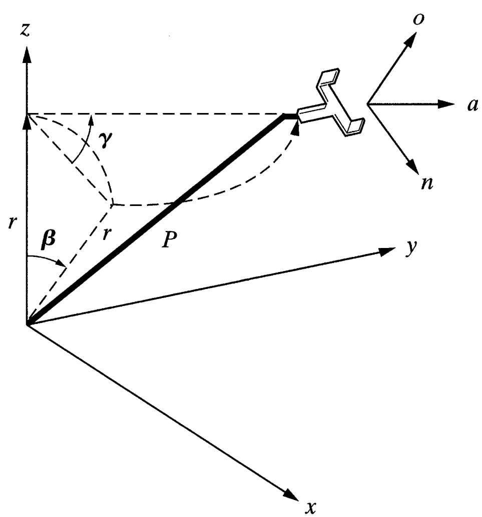


Fig. 4.20 Spherical Coordinates.

$$ {}^{R}T_{P} = T_{sph}(r,\beta,l) = \text{Rot}(z,\gamma)\text{Rot}(y,\beta)\text{Trans}(0,0,\gamma) $$

$$ {}^{R}T_{P} = T_{sph} = \begin{bmatrix} C\beta \cdot C\gamma & -S\gamma & S\beta \cdot C\gamma & rS\beta \cdot C\gamma \\ C\beta \cdot S\gamma & C\gamma & S\beta \cdot S\gamma & rS\beta \cdot S\gamma \\ -S\beta & 0 & C\beta & rC\beta \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Robot Kinematics: Position Analysis

FORWARD AND INVERSE KINEMATICS OF ROBOTS

Forward and Inverse Kinematics Equations for Position

(d) Articulated Coordinates

- 3 rotations -> Denavit-Hartenberg representation

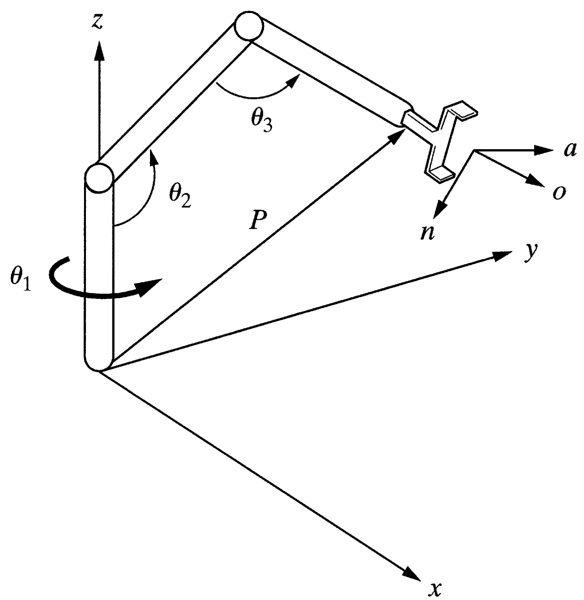


Fig. 4.21 Articulated Coordinates.

Robot Kinematics: Position Analysis

FORWARD AND INVERSE KINEMATICS OF ROBOTS

Forward and Inverse Kinematics Equations for Orientation

- Roll, Pitch, Yaw (RPY) angles
- Euler angles
- Articulated joints

Robot Kinematics: Position Analysis

FORWARD AND INVERSE KINEMATICS OF ROBOTS

Forward and Inverse Kinematics Equations for Orientation

(a) Roll, Pitch, Yaw(RPY) Angles

- Roll: Rotation of ϕ_a about $\overline{a}$-axis (z-axis of the moving frame)
- Pitch: Rotation of ϕ_o about $\overline{o}$-axis (y-axis of the moving frame)
- Yaw: Rotation of ϕ_n about $\overline{n}$-axis (x-axis of the moving frame)

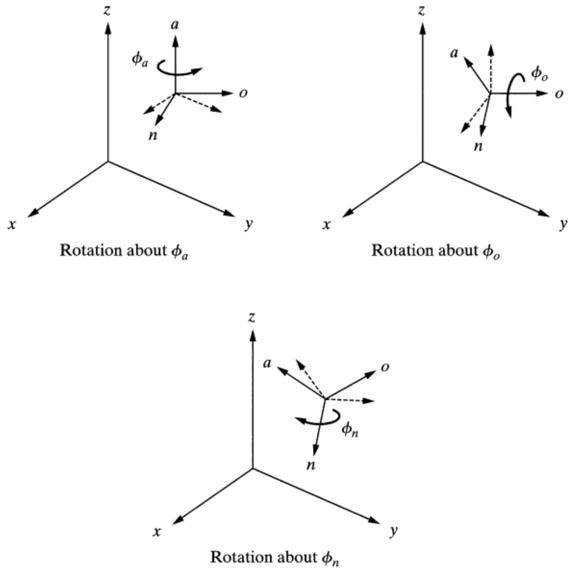


Fig. 2.22 RPY rotations about the current axes.

Robot Kinematics: Position Analysis

FORWARD AND INVERSE KINEMATICS OF ROBOTS

Forward and Inverse Kinematics Equations for Orientation

(b) Euler Angles

- Rotation of ϕ about $\overline{a}$-axis (z-axis of the moving frame) followed by
- Rotation of θ about $\overline{o}$-axis (y-axis of the moving frame) followed by
- Rotation of ψ about $\overline{a}$-axis (z-axis of the moving frame).

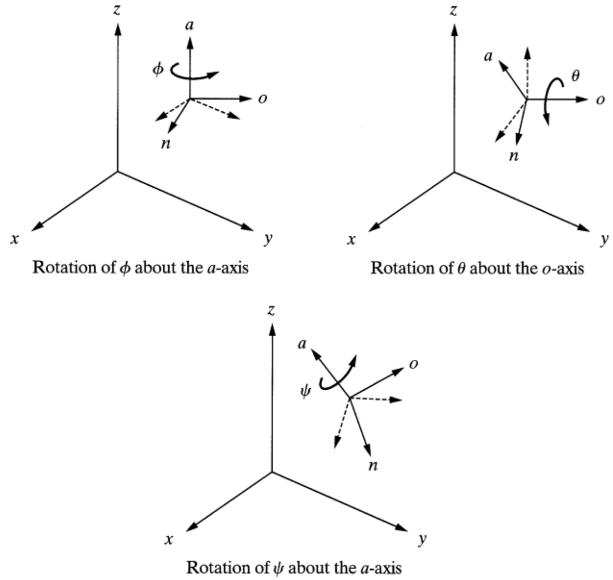


Fig. 4.24 Euler rotations about the current axes.

Robot Kinematics: Position Analysis

FORWARD AND INVERSE KINEMATICS OF ROBOTS

Forward and Inverse Kinematics Equations for Orientation

(c) Articulated Joints

Robot Kinematics: Position Analysis

FORWARD AND INVERSE KINEMATICS OF ROBOTS

Forward and Inverse Kinematics Equations for Orientation

- Assumption : Robot is made of a Cartesian and an RPY set of joints.

$$^{R}T_{H} = T_{cart}(P_x, P_y, P_z) \times RPY(\phi_a, \phi_o, \phi_n)$$

- Assumption : Robot is made of a Spherical Coordinate and an Euler angle.

$$^{R}T_{H} = T_{sph}(r, \beta, \gamma) \times Euler(\phi, \theta, \psi)$$

$$\downarrow$$

Another Combination can be possible……

Denavit-Hartenberg Representation

Robot Kinematics: Position Analysis

DENAVIT-HARTENBERG REPRESENTATION OF FORWARD KINEMATIC EQUATIONS OF ROBOT

- Denavit-Hartenberg Representation :

  @ Simple way of modeling robot links and joints for any robot configuration, regardless of its sequence or complexity.

  @ Transformations in any coordinates is possible.

  @ Any possible combinations of joints and links and all-revolute articulated robots can be represented.

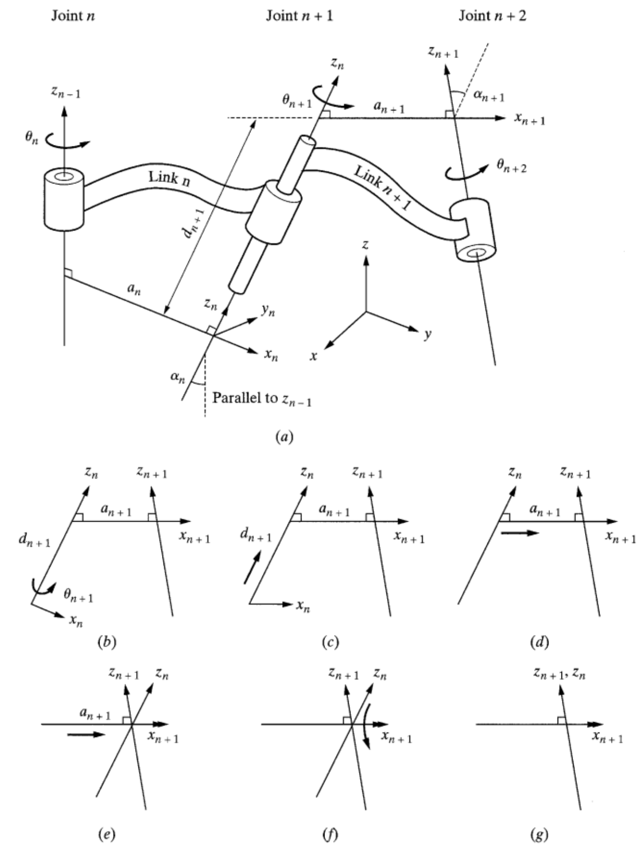


Fig. 2.25 A D-H representation of a general-purpose joint-link combination

Robot Kinematics: Position Analysis

DENAVIT-HARTENBERG REPRESENTATION OF FORWARD KINEMATIC EQUATIONS OF ROBOT

- Denavit-Hartenberg Representation procedures:

  Start point:

  Assign joint number *n* to the first shown joint.

  Assign a local reference frame for each and every joint before or after these joints.

  Y-axis does not used in D-H representation.

Robot Kinematics: Position Analysis

DENAVIT-HARTENBERG REPRESENTATION OF FORWARD KINEMATIC EQUATIONS OF ROBOT

- Procedures for assigning a local reference frame to each joint:
  - ⁎ All joints are represented by a *z*-axis.
    (right-hand rule for rotational joint, linear movement for prismatic joint)
  - ⁎ The common normal is one line mutually perpendicular to any two skew lines.
  - ⁎ Parallel *z*-axes joints make a infinite number of common normal.
  - ⁎ Intersecting *z*-axes of two successive joints make no common normal between them(Length is 0.).

Robot Kinematics: Position Analysis

DENAVIT-HARTENBERG REPRESENTATION OF FORWARD KINEMATIC EQUATIONS OF ROBOT

- Symbol Terminologies :
  - θ : A rotation about the *z*-axis.
  - *d* : The distance on the *z*-axis.
  - *a* : The length of each common normal (Joint offset).
  - α : The angle between two successive *z*-axes (Joint twist)

Only θ and *d* are joint variables.

Robot Kinematics: Position Analysis

DENAVIT-HARTENBERG REPRESENTATION OF FORWARD KINEMATIC EQUATIONS OF ROBOT

- The necessary motions to transform from one reference frame to the next.

  (I) Rotate about the z_n-axis an able of θ_{n+1}. (Coplanar)

  (II) Translate along z_n-axis a distance of d_{n+1} to make x_n and x_{n+1} colinear.

  (III) Translate along the x_n-axis a distance of a_{n+1} to bring the origins of x_{n+1} together.

  (IV) Rotate z_n-axis about x_{n+1} axis an angle of α_{n+1} to align z_n-axis with z_{n+1}-axis.

Robot Kinematics: Position Analysis

THE INVERSE KINEMATIC SOLUTION OF ROBOT

- Determine the value of each joint to place the arm at a desired position and orientation.

$$^{R}T_H = A_1 A_2 A_3 A_4 A_5 A_6$$

$$= \begin{bmatrix} C_1(C_{234}C_5C_6 - S_{234}S_6) & C_1(-C_{234}C_5C_6 - S_{234}C_6) & & \\ -S_1S_5C_6 & +S_1S_5C_6 & C_1(C_{234}S_5) + S_1C_5 & C_1(C_{234}a_4 + C_{23}a_3 + C_2a_2) \\ S_1(C_{234}C_5C_6 - S_{234}S_6) & S_1(-C_{234}C_5C_6 - S_{234}C_6) & & \\ +C_1S_5C_6 & -C_1S_5C_6 & S_1(C_{234}S_5) - C_1C_5 & S_1(C_{234}a_4 + C_{23}a_3 + C_2a_2) \\ S_{234}C_5C_6 + C_{234}S_6 & -S_{234}C_5C_6 + C_{234}C_6 & S_{234}S_5 & S_{234}a_4 + S_{23}a_3 + S_2a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Robot Kinematics: Position Analysis

THE INVERSE KINEMATIC SOLUTION OF ROBOT

$$A_1^{-1} \times \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_1^{-1}[RHS] = A_2 A_3 A_4 A_5 A_6$$

$$\begin{bmatrix} C_1 & S_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ S_1 & -C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_2 A_3 A_4 A_5 A_6$$

Robot Kinematics: Position Analysis

THE INVERSE KINEMATIC SOLUTION OF ROBOT

$$\theta_1 = \tan^{-1}\left(\frac{p_y}{p_x}\right)$$

$$\theta_2 = \tan^{-1}\frac{(C_3a_3 + a_2)(p_z - S_{234}a_4) - S_3a_3(p_xC_1 + p_yS_1 - C_{234}a_4)}{(C_3a_3 + a_2)(p_xC_1 + p_yS_1 - C_{234}a_4) + S_3a_3(P_z - S_{234}a_4)}$$

$$\theta_3 = \tan^{-1}\left(\frac{S_3}{C_3}\right)$$

$$\theta_4 = \theta_{234} - \theta_2 - \theta_3$$

$$\theta_5 = \tan^{-1}\frac{C_{234}(C_1a_x + S_1a_y) + S_{234}a_z}{S_1a_x - C_1a_y}$$

$$\theta_6 = \tan^{-1}\frac{-S_{234}(C_1n_x + S_1n_y) + S_{234}n_z}{-S_{234}(C_1o_x + S_1o_y) + C_{234}o_z}$$

Robot Kinematics: Position Analysis

INVERSE KINEMATIC PROGRAM OF ROBOTS

- A robot has a predictable path on a straight line,
- Or an unpredictable path on a straight line.

  * A predictable path is necessary to recalculate joint variables.
    (Between 50 to 200 times a second)
  * To make the robot follow a straight line, it is necessary to break the line into many small sections.
  * All unnecessary computations should be eliminated.

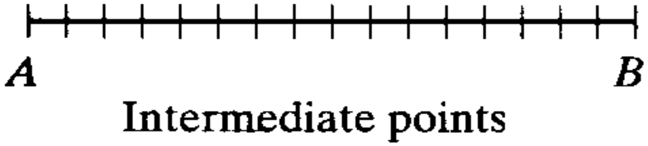


Fig. 2.30 Small sections of movement for straight-line motions

Robot Kinematics: Position Analysis

DEGENERACY AND DEXTERITY

∴ Degeneracy : The robot looses a degree of freedom and thus cannot perform as desired.

- When the robot's joints reach their physical limits, and as a result, cannot move any further.
- In the middle point of its workspace if the *z*-axes of two similar joints becomes colinear.

∴ Dexterity : The volume of points where one can position the robot as desired, but not orientate it.

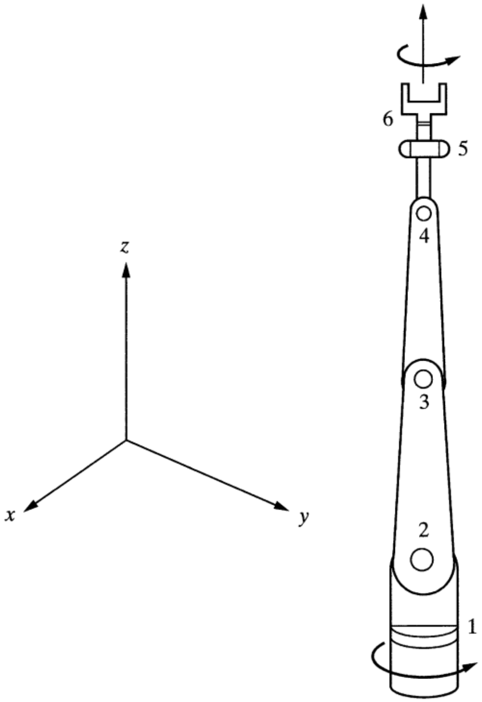


Fig. 4.31 **An example of a robot in a degenerate position.**

Robot Kinematics: Position Analysis

THE FUNDAMENTAL PROBLEM WITH D-H REPRESENTATION

$\therefore$ Defect of D-H presentation : D-H cannot represent any motion about the *y*-axis, because all motions are about the *x*- and *z*-axis.

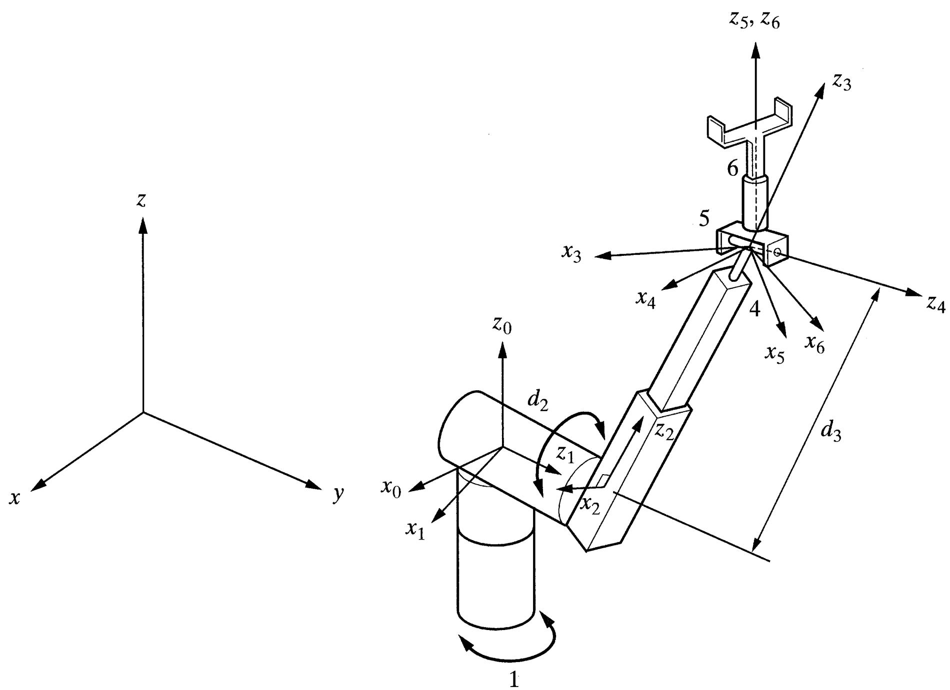


Fig. 4.31 The frames of the Stanford Arm.

TABLE 2.3 THE PARAMETERS TABLE FOR THE STANFORD ARM

#	θ	d	a	α
1	θ_1	0	0	-90
2	θ_2	d_1	0	90
3	0	d_1	0	0
4	θ_4	0	0	-90
5	θ_5	0	0	90
6	θ_6	0	0	0