CORE MATHEMATICS FOR SENIOR HIGH SCHOOLS

NEW INTERNATIONAL EDITION
(Sixth Edition)

CHRISTIAN A. HESSE, BSc, MPhil
Lecturer of Mathematics and Statistics
Methodist University College Ghana

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PREFACE

This book is intended to encourage an understanding and appreciation of core mathematics at the Senior High School level in West Africa. Providing appropriate solutions to examination problems is of particular importance in the study of mathematics. As a mathematics lecturer, the author has discovered the weaknesses and shortcomings of students in the handling of examination questions. Subsequently, to guide students in answering typical questions in core mathematics as set out in recent examinations, the writer has paid particular attention to those areas of the syllabus, which many students find difficult.

A prominent feature of this book is the inclusion of many examples. Each example is carefully selected to illustrate the application of a particular mathematical technique and or interpretation of results. Another feature is that each chapter has an extensive collection of exercises. It is important that students have several exercises to practice.

This book is therefore designed to help students to:

1. acquire the basic skills and understanding which is vital to examination success.
2. appreciate the use of mathematics as a tool for analysis and effective thinking.
3. discover order, patterns and relations.
4. communicate their thoughts through symbolic expressions and graphs.
5. develop mathematical abilities useful in commerce, industry and public service.

I have gone to great lengths to make this text both pedagogically sound and error-free. If you have any suggestions, or find potential errors, please contact the writer at **akrongh@yahoo.com**.

C. A. Hesse
January, 2011

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