

Introduction to CMOS VLSI Design

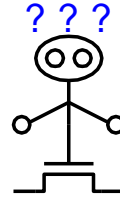
Lecture 5: Logical Effort

Outline

- ☐ Introduction
- ☐ Delay in a Logic Gate
- ☐ Multistage Logic Networks
- ☐ Choosing the Best Number of Stages
- ☐ Example
- ☐ Summary

Introduction

- ❑ Chip designers face a bewildering array of choices
 - What is the best circuit topology for a function?
 - How many stages of logic give least delay?
 - How wide should the transistors be?
- ❑ Logical effort is a method to make these decisions
 - Uses a simple model of delay
 - Allows back-of-the-envelope calculations
 - Helps make rapid comparisons between alternatives
 - Emphasizes remarkable symmetries



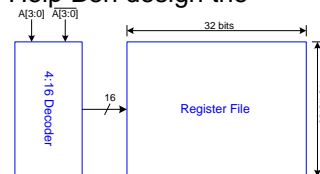
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Example

- ❑ Ben Bitdiddle is the memory designer for the Motoroil 68W86, an embedded automotive processor. Help Ben design the decoder for a register file.
- ❑ Decoder specifications:
 - 16 word register file
 - Each word is 32 bits wide
 - Each bit presents load of 3 unit-sized transistors
 - True and complementary address inputs $A[3:0]$
 - Each input may drive 10 unit-sized transistors
- ❑ Ben needs to decide:
 - How many stages to use?
 - How large should each gate be?
 - How fast can decoder operate?



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Delay in a Logic Gate

- Express delays in process-independent unit

$$d = \frac{d_{abs}}{\tau}$$

$$\tau = 3RC$$

≈ 12 ps in 180 nm process

40 ps in 0.6 μm process

Delay in a Logic Gate

- Express delays in process-independent unit

$$d = \frac{d_{abs}}{\tau}$$

- Delay has two components

$$d = f + p$$

Delay in a Logic Gate

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- ❑ Effort delay $f = gh$ (a.k.a. stage effort)
 - Again has two components

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Delay in a Logic Gate

- ❑ Express delays in process-independent unit

$$d = \frac{d_{abs}}{\tau}$$

- ❑ Delay has two components

$$d = f + p$$

- ❑ Effort delay $f = gh$ (a.k.a. stage effort)
 - Again has two components

- ❑ g : logical effort

- Measures relative ability of gate to deliver current
- $g \equiv 1$ for inverter

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Delay in a Logic Gate

- ❑ Express delays in process-independent unit

$$d = \frac{d_{abs}}{\tau}$$

- ❑ Delay has two components

$$d = f + p$$

- ❑ Effort delay $f = gh$ (a.k.a. stage effort)

- Again has two components

- ❑ h : electrical effort = C_{out} / C_{in}

- Ratio of output to input capacitance

- Sometimes called fanout

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Delay in a Logic Gate

- ❑ Express delays in process-independent unit

$$d = \frac{d_{abs}}{\tau}$$

- ❑ Delay has two components

$$d = f + p$$

- ❑ Parasitic delay p

- Represents delay of gate driving no load

- Set by internal parasitic capacitance

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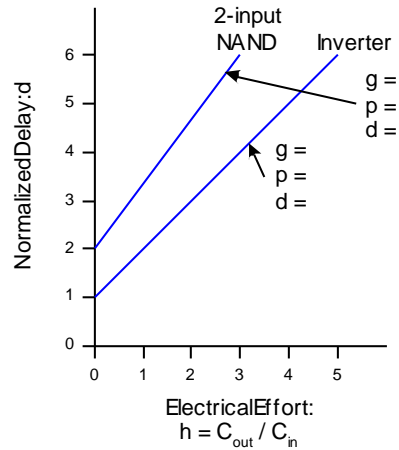
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Delay Plots

$$d = f + p$$

$$= gh + p$$



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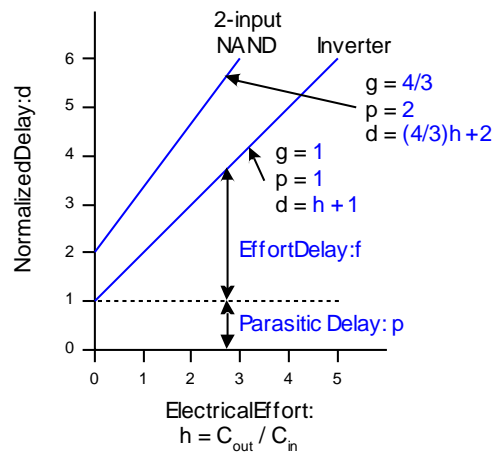
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Delay Plots

$$d = f + p$$

$$= gh + p$$

□ What about NOR2?



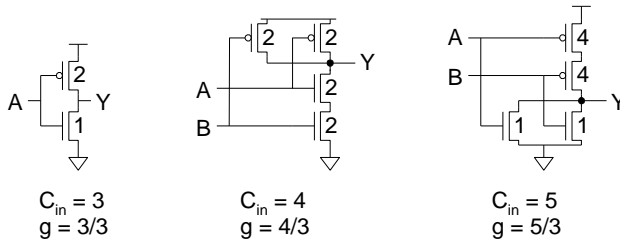
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Computing Logical Effort

- ❑ DEF: *Logical effort is the ratio of the input capacitance of a gate to the input capacitance of an inverter delivering the same output current.*
- ❑ Measure from delay vs. fanout plots
- ❑ Or estimate by counting transistor widths



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Catalog of Gates

- ❑ Logical effort of common gates

Gate type	Number of inputs				
	1	2	3	4	n
Inverter	1				
NAND		4/3	5/3	6/3	$(n+2)/3$
NOR		5/3	7/3	9/3	$(2n+1)/3$
Tristate / mux	2	2	2	2	2
XOR, XNOR		4, 4	6, 12, 6	8, 16, 16, 8	

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Catalog of Gates

- Parasitic delay of common gates
 - In multiples of p_{inv} (≈ 1)

Gate type	Number of inputs				
	1	2	3	4	n
Inverter	1				
NAND		2	3	4	n
NOR		2	3	4	n
Tristate / mux	2	4	6	8	2n
XOR, XNOR		4	6	8	

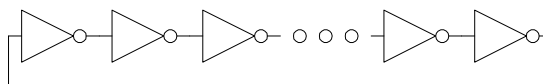
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Example: Ring Oscillator

- Estimate the frequency of an N-stage ring oscillator



Logical Effort: $g =$

Electrical Effort: $h =$

Parasitic Delay: $p =$

Stage Delay: $d =$

Frequency: $f_{osc} =$

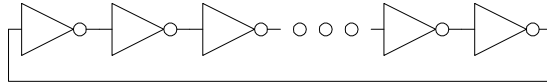
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Example: Ring Oscillator

- Estimate the frequency of an N-stage ring oscillator



Logical Effort: $g = 1$

Electrical Effort: $h = 1$

Parasitic Delay: $p = 1$

Stage Delay: $d = 2$

Frequency: $f_{osc} = 1/(2 \cdot N \cdot d) = 1/4N$

31 stage ring oscillator in
0.6 μm process has
frequency of ~ 200 MHz

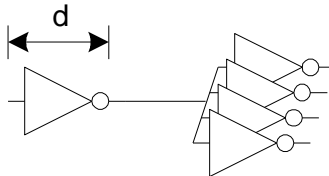
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Example: FO4 Inverter

- Estimate the delay of a fanout-of-4 (FO4) inverter



Logical Effort: $g =$

Electrical Effort: $h =$

Parasitic Delay: $p =$

Stage Delay: $d =$

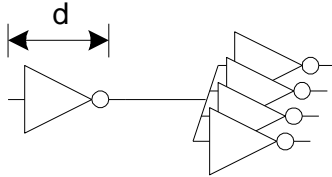
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Example: FO4 Inverter

- Estimate the delay of a fanout-of-4 (FO4) inverter



Logical Effort: $g = 1$

Electrical Effort: $h = 4$

Parasitic Delay: $p = 1$

Stage Delay: $d = 5$

The FO4 delay is about

200 ps in 0.6 μm process

60 ps in a 180 nm process

$f/3$ ns in an f μm process

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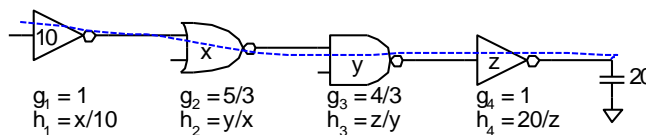
Multistage Logic Networks

- Logical effort generalizes to multistage networks

- Path Logical Effort $G = \prod g_i$

- Path Electrical Effort $H = \frac{C_{\text{out-path}}}{C_{\text{in-path}}}$

- Path Effort $F = \prod f_i = \prod g_i h_i$



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Multistage Logic Networks

❑ Logical effort generalizes to multistage networks

❑ *Path Logical Effort* $G = \prod g_i$

❑ *Path Electrical Effort* $H = \frac{C_{out-path}}{C_{in-path}}$

❑ *Path Effort* $F = \prod f_i = \prod g_i h_i$

❑ Can we write $F = GH$?

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Paths that Branch

❑ No! Consider paths that branch:

$G =$

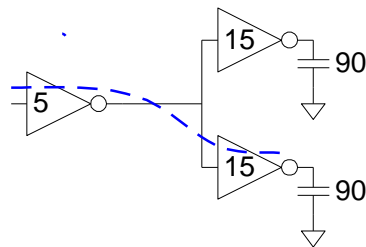
$H =$

$GH =$

$h_1 =$

$h_2 =$

$F = GH?$



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Paths that Branch

- ❑ No! Consider paths that branch:

$$G = 1$$

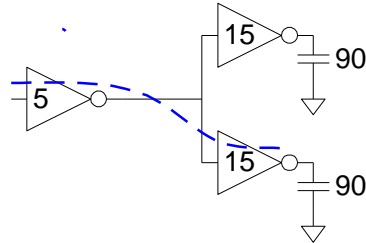
$$H = 90 / 5 = 18$$

$$GH = 18$$

$$h_1 = (15 + 15) / 5 = 6$$

$$h_2 = 90 / 15 = 6$$

$$F = g_1 g_2 h_1 h_2 = 36 = 2GH$$



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Branching Effort

- ❑ Introduce *branching effort*
 - Accounts for branching between stages in path

$$b = \frac{C_{\text{on path}} + C_{\text{off path}}}{C_{\text{on path}}}$$

$$B = \prod b_i$$

Note:

$$\prod h_i = BH$$

- ❑ Now we compute the path effort
 - $F = GBH$

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Multistage Delays

- ❑ Path Effort Delay $D_F = \sum f_i$
- ❑ Path Parasitic Delay $P = \sum p_i$
- ❑ Path Delay $D = \sum d_i = D_F + P$

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Designing Fast Circuits

$$D = \sum d_i = D_F + P$$

- ❑ Delay is smallest when each stage bears same effort

$$\hat{f} = g_i h_i = F^{\frac{1}{N}}$$

- ❑ Thus minimum delay of N stage path is

$$D = NF^{\frac{1}{N}} + P$$

- ❑ This is a **key** result of logical effort
 - Find fastest possible delay
 - Doesn't require calculating gate sizes

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Gate Sizes

- How wide should the gates be for least delay?

$$\hat{f} = gh = g \frac{C_{out}}{C_{in}}$$

$$\Rightarrow C_{in_i} = \frac{g_i C_{out_i}}{\hat{f}}$$

- Working backward, apply capacitance transformation to find input capacitance of each gate given load it drives.
- Check work by verifying input cap spec is met.

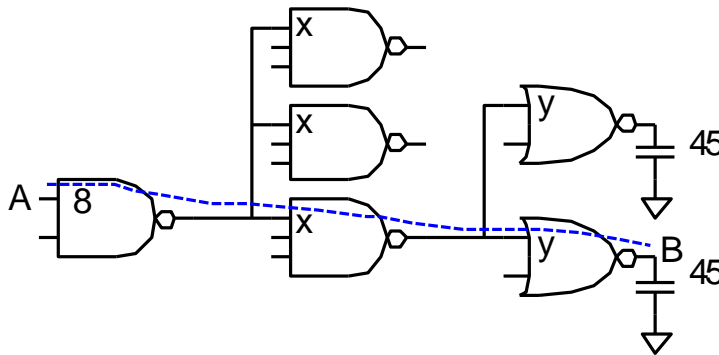
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Example: 3-stage path

- Select gate sizes x and y for least delay from A to B

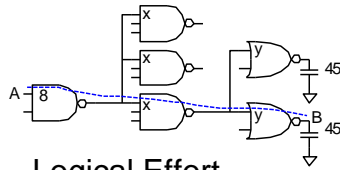


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Example: 3-stage path



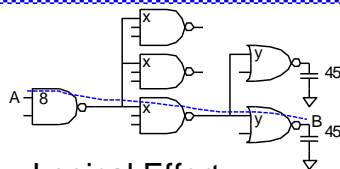
Logical Effort	$G =$
Electrical Effort	$H =$
Branching Effort	$B =$
Path Effort	$F =$
Best Stage Effort	$\hat{f} =$
Parasitic Delay	$P =$
Delay	$D =$

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Example: 3-stage path



Logical Effort	$G = (4/3) * (5/3) * (5/3) = 100/27$
Electrical Effort	$H = 45/8$
Branching Effort	$B = 3 * 2 = 6$
Path Effort	$F = GBH = 125$
Best Stage Effort	$\hat{f} = \sqrt[3]{F} = 5$
Parasitic Delay	$P = 2 + 3 + 2 = 7$
Delay	$D = 3 * 5 + 7 = 22 = 4.4 \text{ FO4}$

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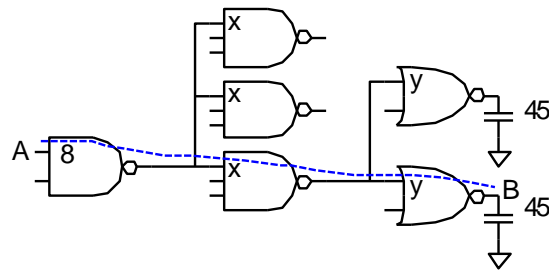
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Example: 3-stage path

- Work backward for sizes

$y =$

$x =$



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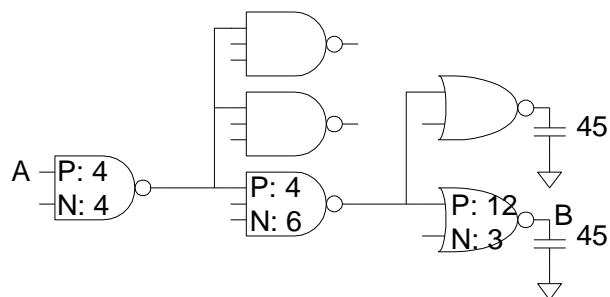
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Example: 3-stage path

- Work backward for sizes

$$y = 45 * (5/3) / 5 = 15$$

$$x = (15 * 2) * (5/3) / 5 = 10$$



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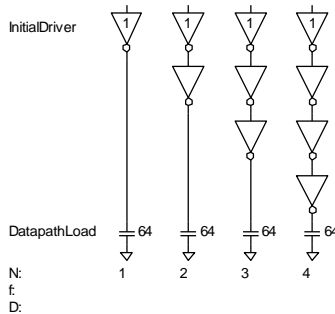
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Best Number of Stages

- ❑ How many stages should a path use?
 - Minimizing number of stages is not always fastest
- ❑ Example: drive 64-bit datapath with unit inverter

D =



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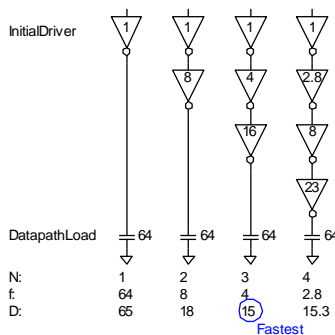
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Best Number of Stages

- ❑ How many stages should a path use?
 - Minimizing number of stages is not always fastest
- ❑ Example: drive 64-bit datapath with unit inverter

$$D = NF^{1/N} + P$$

$$= N(64)^{1/N} + N$$



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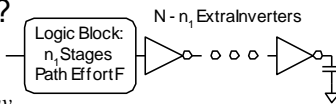
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Derivation

- Consider adding inverters to end of path
 - How many give least delay?

$$D = NF^{\frac{1}{N}} + \sum_{i=1}^{n_1} p_i + (N - n_1) p_{inv}$$



$$\frac{\partial D}{\partial N} = -F^{\frac{1}{N}} \ln F^{\frac{1}{N}} + F^{\frac{1}{N}} + p_{inv} = 0$$

- Define best stage effort $\rho = F^{\frac{1}{N}}$

$$p_{inv} + \rho(1 - \ln \rho) = 0$$

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Best Stage Effort

- $p_{inv} + \rho(1 - \ln \rho) = 0$ has no closed-form solution
- Neglecting parasitics ($p_{inv} = 0$), we find $\rho = 2.718$ (e)
- For $p_{inv} = 1$, solve numerically for $\rho = 3.59$

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Sensitivity Analysis

- ❑ How sensitive is delay to using exactly the best number of stages?



- ❑ $2.4 < \rho < 6$ gives delay within 15% of optimal
 - We can be sloppy!
 - I like $\rho = 4$

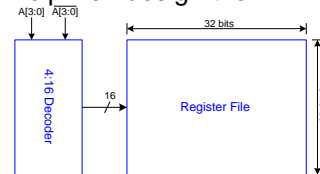
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Example, Revisited

- ❑ Ben Bitdiddle is the memory designer for the Motorola 68W86, an embedded automotive processor. Help Ben design the decoder for a register file.



- ❑ Decoder specifications:
 - 16 word register file
 - Each word is 32 bits wide
 - Each bit presents load of 3 unit-sized transistors
 - True and complementary address inputs $A[3:0]$
 - Each input may drive 10 unit-sized transistors
- ❑ Ben needs to decide:
 - How many stages to use?
 - How large should each gate be?
 - How fast can decoder operate?

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Example, Revisited

Truth Table:

INPUTS				OUTPUTS															
A ₃	A ₂	A ₁	A ₀	Y ₁₅	Y ₁₄	Y ₁₃	Y ₁₂	Y ₁₁	Y ₁₀	Y ₉	Y ₈	Y ₇	Y ₆	Y ₅	Y ₄	Y ₃	Y ₂	Y ₁	Y ₀
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	1	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
1	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$$Y_0 = A_0 \cdot A_1 \cdot A_2 \cdot A_3'$$

$$Y_1 = A_0 \cdot A_1' \cdot A_2 \cdot A_3$$

$$Y_2 = A_0 \cdot A_1' \cdot A_2 \cdot A_3'$$

$$Y_3 = A_0 \cdot A_1 \cdot A_2 \cdot A_3$$

$$Y_4 = A_0 \cdot A_1 \cdot A_2' \cdot A_3'$$

$$Y_5 = A_0 \cdot A_1 \cdot A_2' \cdot A_3$$

$$Y_6 = A_0 \cdot A_1 \cdot A_2 \cdot A_3'$$

$$Y_7 = A_0 \cdot A_1 \cdot A_2 \cdot A_3$$

$$Y_8 = A_0 \cdot A_1' \cdot A_2' \cdot A_3'$$

$$Y_9 = A_0 \cdot A_1' \cdot A_2' \cdot A_3$$

$$Y_{10} = A_0 \cdot A_1' \cdot A_2 \cdot A_3'$$

$$Y_{11} = A_0 \cdot A_1' \cdot A_2 \cdot A_3$$

$$Y_{12} = A_0 \cdot A_1 \cdot A_2' \cdot A_3'$$

$$Y_{13} = A_0 \cdot A_1 \cdot A_2' \cdot A_3$$

$$Y_{14} = A_0 \cdot A_1 \cdot A_2 \cdot A_3'$$

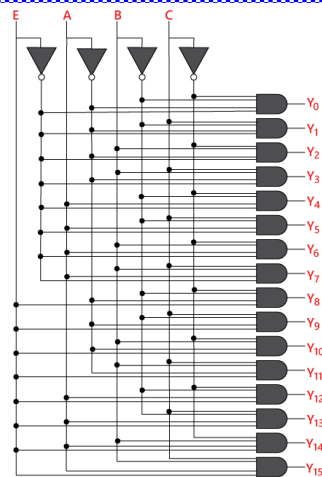
$$Y_{15} = A_0 \cdot A_1 \cdot A_2 \cdot A_3$$

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Example, Revisited



5: Logical Effort

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Number of Stages

- ❑ Decoder effort is mainly electrical and branching

Electrical Effort: $H =$

Branching Effort: $B =$

- ❑ If we neglect logical effort (assume $G = 1$)

Path Effort: $F =$

Number of Stages: $N =$

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Number of Stages

- ❑ Decoder effort is mainly electrical and branching

Electrical Effort: $H = (32 \cdot 3) / 10 = 9.6$

Branching Effort: $B = 8$

- ❑ If we neglect logical effort (assume $G = 1$)

Path Effort: $F = GBH = 76.8$

Number of Stages: $N = \log_4 F = 3.1$

- ❑ Try a 3-stage design

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Gate Sizes & Delay

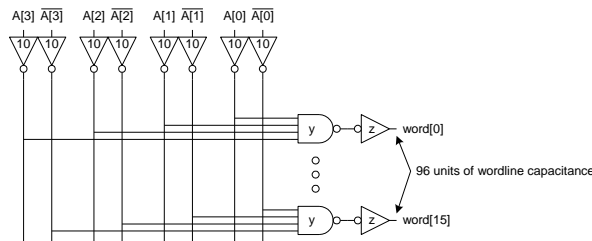
Logical Effort: $G =$

Path Effort: $F =$

Stage Effort: $\hat{f} =$

Path Delay: $D =$

Gate sizes: $z =$ $y =$



5: Logical Effort

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Gate Sizes & Delay

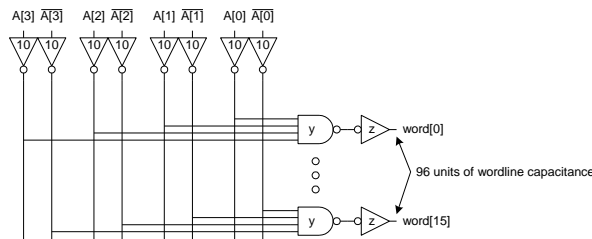
Logical Effort: $G = 1 * 6/3 * 1 = 2$

Path Effort: $F = GBH = 154$

Stage Effort: $\hat{f} = F^{1/3} = 5.36$

Path Delay: $D = 3\hat{f} + 1 + 4 + 1 = 22.1$

Gate sizes: $z = 96 * 1/5.36 = 18$ $y = 18 * 2/5.36 = 6.7$



5: Logical Effort

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Comparison

- ❑ Compare many alternatives with a spreadsheet

Design	N	G	P	D
NAND4-INV	2	2	5	29.8
NAND2-NOR2	2	20/9	4	30.1
INV-NAND4-INV	3	2	6	22.1
NAND4-INV-INV-INV	4	2	7	21.1
NAND2-NOR2-INV-INV	4	20/9	6	20.5
NAND2-INV-NAND2-INV	4	16/9	6	19.7
INV-NAND2-INV-NAND2-INV	5	16/9	7	20.4
NAND2-INV-NAND2-INV-INV-INV	6	16/9	8	21.6

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Review of Definitions

Term	Stage	Path
number of stages	1	N
logical effort	g	$G = \prod g_i$
electrical effort	$h = \frac{C_{out}}{C_{in}}$	$H = \frac{C_{out-path}}{C_{in-path}}$
branching effort	$b = \frac{C_{on-path} + C_{off-path}}{C_{on-path}}$	$B = \prod b_i$
effort	$f = gh$	$F = GBH$
effort delay	f	$D_F = \sum f_i$
parasitic delay	p	$P = \sum p_i$
delay	$d = f + p$	$D = \sum d_i = D_F + P$

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Method of Logical Effort

- 1) Compute path effort
- 2) Estimate best number of stages
- 3) Sketch path with N stages
- 4) Estimate least delay
- 5) Determine best stage effort

$$F = GBH$$

$$N = \log_4 F$$

$$D = NF^{\frac{1}{N}} + P$$

$$\hat{f} = F^{\frac{1}{N}}$$

- 6) Find gate sizes

$$C_{in_i} = \frac{g_i C_{out_i}}{\hat{f}}$$

5: Logical Effort

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TSMC

The image shows two pages from the TSMC 0.18um 1P6M design kit. The left page is for the inverter (NOT gate) and the right page is for the NAND gate. Both pages include a logic symbol, a table of functions, and a table of parasitic data (P, C, D) for various input and output loads.

$$t_{pd} = (25.3 + 14.6)/2ps + (3.6 \text{ fF/gate})(h \text{ gates})((4.53 + 2.37)ns/pF) = (20 + 12.4h)ps$$

$$t_{pd} = (31.3 + 19.5)/2ps + (4.2 \text{ fF/gate})(h \text{ gates})((4.53 + 2.84)ns/pF) = (25.4 + 15.5h)ps$$

5: Logical Effort

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Limits of Logical Effort

- ❑ Chicken and egg problem
 - Need path to compute G
 - But don't know number of stages without G
- ❑ Simplistic delay model
 - Neglects input rise time effects
- ❑ Interconnect
 - Iteration required in designs with wire
- ❑ Maximum speed only
 - Not minimum area/power for constrained delay

5: Logical Effort

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Summary

- ❑ Logical effort is useful for thinking of delay in circuits
 - Numeric logical effort characterizes gates
 - NANDs are faster than NORs in CMOS
 - Paths are fastest when effort delays are ~ 4
 - Path delay is weakly sensitive to stages, sizes
 - But using fewer stages doesn't mean faster paths
 - Delay of path is about $\log_4 F$ FO4 inverter delays
 - Inverters and NAND2 best for driving large caps
- ❑ Provides language for discussing fast circuits
 - But requires practice to master

5: Logical Effort

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