Joint DOA Estimation and Source Signal Tracking With Kalman Filtering and Regularized QRD RLS Algorithm

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***Abstract*—In this brief, we present a nontraditional approach for estimating and tracking signal direction-of-arrival (DOA) using an array of sensors. The proposed method consists of two stages: in the first stage, the sources modeled by autoregressive (AR) processes are estimated by the celebrated Kalman filter, and in the second stage, the efficient QR-decomposition-based recursive least square (QRD-RLS) technique is employed to estimate the DOAs and AR coefficients in each observed time interval. The AR-modeled sources can provide useful temporal information to handle cases such as the number of sources being larger than the number of antennas. In addition, the symmetric array enables one to transfer a complex-valued nonlinear problem to a real-valued linear one, which can reduce the computational complexity. Simulation results demonstrate the superior performance of the algorithm for estimating and tracking DOA under different scenarios.**

***Index Terms*—Autoregressive (AR) model, direction-of-arrival (DOA) estimation and tracking, Kalman filter (KF), QR-decomposition, recursive least square (RLS).**

I. Introduction

DIRECTION-OF-arrival (DOA) estimation and tracking techniques for multiple targets using an antenna array have been of great interest in a variety of commercial and military applications such as communications, sonar, air traffic control, and electronic reconnaissance areas [1]. Many algorithms, such as beamforming-based techniques [2], subspace-based techniques [3], and sparsity-based techniques [4], have been proposed to solve the DOA estimation problem. Most of these techniques rely on subspace or eigen-based information that can be derived from the eigenvalue decomposition of the covariance matrix of sample outputs or from the singular value decomposition of the array data matrix. This has also motivated researchers to develop subspace-based tracking techniques for moving targets (see, e.g., [5, Part XII]). Instead of updating the whole eigenspace using new data as they arrive, subspace tracking techniques only deal with the signal or noise subspace to reduce the computational complexity and storage requirements. An efficient class of methods is the project approximation subspace tracking (PAST) method and its variants [6]. The PAST method recursively finds the signal subspace by minimizing the weighted least square (LS) error between the received data and the projection approximation constructed from the current subspace estimate. In order to track fast-varying subspace, a Kalman filter (KF)-based subspace tracking method for estimating the subspace recursively using prior information of the state dynamics has been recently proposed [7]. Although subspace-based methods are attractive for their good performance and low complexity, they suffer from at least two limitations. First, since subspace-based techniques require the dimension of the signal subspace to be less than that of the whole space, it is impossible to handle the case in which the number of sources is equal to or larger than that of antennas. Second, the subspace-based techniques are developed according to the principle that the more snapshots and sensors there are, the better the performance. Unfortunately, it is not true for the moving targets due to the fact that such targets can cause a spread in the array spatial spectrum, and it is much more important to increase the number of array sensors [8].

The aim of this brief is to develop an efficient DOA estimation and tracking scheme tackling some of the aforementioned limitations. The basic idea of the proposed method is to exploit the dynamics of the source signals and utilize the KF in tracking the source statistics. In particular, we assume that the sources are characterized by autoregressive (AR) models. Speech and communication signals, for instance, are often recognized as AR processes [9]. In fact, DOA estimation and tracking has an very important application in microphone array processing of speech signals [10]. It also finds many applications in mobile communication systems [11]. These practical applications have strongly motivated us to study DOA estimation and tracking of a class of source signals that can be modeled by AR processes. In our method, the source steering vectors and AR coefficients are recursively estimated by using a simple QR-decomposition-based recursive least square (QRD-RLS) algorithm and KF. During each time interval, we first employ the dynamic model of the time-varying AR sources and the KF along with the estimated AR coefficients and DOAs in the previous interval to predict the temporal state information of each target. Then, a QRD-based RLS algorithm is used to update the AR coefficients and DOAs by means of the relationships between the sources and the AR coefficients or DOAs. The

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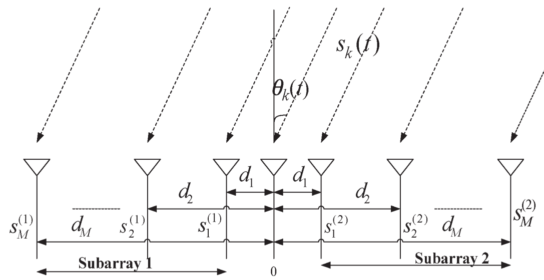


Fig. 1. Geometry of the sensor array.

aforementioned procedure can be done iteratively to further improve the accuracy.

II. Signal and System Model

Suppose that there are K narrow-band moving sources with wavelength λ, which impinge onto a symmetric linear array, as shown in Fig. 1. The whole array is assumed to be divided into two subarrays with interelement spacings $d_1, d_2, \ldots, d_M$. Here, we assume $d_1 \leq \lambda/2$ to make sure that the estimation of DOAs is unambiguous, and then, the sensor element at the origin is used as reference.

We assume that all the sources are independent order-L_k AR processes, and the complex envelope at time t of the kth AR source is

$$x_k(t) = \sum_{l_k=1}^{L_k} a_{l_k}^k(t) x_k(t-l_k) + v_k(t) \tag{1}$$

where $\nu_k(t)$ is the excitation of the AR process, which is a white Gaussian noise with zero mean and variance $\sigma_{v_k}^2(t)$, and $a_{l_k}^k(t)$ $(k=1,2,\ldots,K)$ are real-valued AR coefficients [12], [13]. Note that we have made here an assumption that AR coefficients are real, and it is possible to extend this technique for complex AR coefficients by small modifications. Equation (1) can be rewritten as the following state-space representation:

$$\mathbf{x}_k(t) = \mathbf{F}_k(t)\mathbf{x}_k(t-1) + \mathbf{v}_k(t) \tag{2}$$

where $\mathbf{F}_k(t) = [\mathbf{a}_k^T(t)[\mathbf{I}_{(L_k-1)}\mathbf{0}_{(L_k-1)\times 1}]^T]^T$ with $\mathbf{a}_k(t) = a_1^k(t)\cdots a_{L_k}^k(t)$, and $\mathbf{v}_k(t) = [v_k(t)\mathbf{0}_{1\times(L_k-1)}]^T$. Here, superscript T denotes matrix transpose. By using (2), the K AR sources can be written as the following state equation:

$$\begin{aligned}\mathbf{x}(t) = \begin{bmatrix}\mathbf{x}_1(t)\\ \cdots \\ \mathbf{x}_K(t)\end{bmatrix} &= \begin{bmatrix}\mathbf{F}_1(t)\mathbf{0}\\ \cdots \\ \mathbf{0}\mathbf{F}_K(t)\end{bmatrix}\begin{bmatrix}\mathbf{x}_1(t-1)\\ \cdots \\ \mathbf{x}_K(t-1)\end{bmatrix} + \begin{bmatrix}\mathbf{v}_1(t)\\ \cdots \\ \mathbf{v}_K(t)\end{bmatrix}\\ &= \mathbf{F}(t)\mathbf{x}(t-1) + \mathbf{v}(t)\end{aligned} \tag{3}$$

where $\mathbf{F}(t) \in \mathbb{R}^{(\sum_{k=1}^K L_k)\times(\sum_{k=1}^K L_k)}$ is a block diagonal matrix, and $\mathbf{x}(t)$ and $\mathbf{v}(t)$ are $\sum_{k=1}^K L_k$-dimensional complex vectors. Since $v_k(t)$ $(k=1,\ldots,K)$ are independent, the covariance matrix of $\mathbf{v}(t)$ is a block diagonal matrix, which can be expressed as $\mathbf{Q}(t) = \mathrm{diag}(\mathbf{Q}_1(t),\ldots,\mathbf{Q}_K(t))$.

At time t, let $\theta_1(t),\ldots,\theta_K(t)$ denote the DOAs of the K targets according to the broadside of the array, and let $y_m^{(i)}(t)$ $(m=1,\ldots,M;\ i=1,2)$ denote the complex signals of the mth sensor of the ith subarray with $y_0(t)$ being the data received by the reference element. Then, the observed output complex signals of the $2M+1$ sensors, denoted by $\mathbf{y}(t) \in \mathbb{C}^{2M+1}$, can be written as

$$\begin{aligned}\mathbf{y}(t) &\triangleq [\,\mathbf{y}_1^T(t) \quad y_0(t) \quad \mathbf{y}_2^T(t)\,]^T\\ &= [\,\mathbf{A}_1^T(\boldsymbol{\theta}(t)) \quad \mathbf{1} \quad \mathbf{A}_2^T(\boldsymbol{\theta}(t))\,]^T \mathbf{s}(t) + \mathbf{e}(t)\\ &\triangleq \mathbf{A}(\boldsymbol{\theta}(t))\mathbf{s}(t) + \mathbf{e}(t)\end{aligned} \tag{4}$$

where $\mathbf{y}_i(t) = [y_1^{(i)}(t),\ldots,y_M^{(i)}(t)]^T (i=1,2)$ is an M-dimensional vector of complex signals at the ith subarray outputs, $\mathbf{s}(t) = [x_1(t)\ \ x_2(t)\ \ \cdots\ \ x_K(t)]^T = \boldsymbol{\Gamma}\mathbf{x}(t)$ is a K-dimensional vector of the target sources with $\boldsymbol{\Gamma}$ being a $K \times \sum_{k=1}^K L_k$-dimensional selection matrix whose entries are 1 on the position $(k, \sum_{p=1}^k L_p - L_k + 1)$ $(k=1,2,\ldots,K)$ and 0 elsewhere, and $\mathbf{e}(t) = [e_M^{(1)}(t)\cdots e_1^{(1)}(t)e_0(t)e_1^{(2)}(t)\cdots e_M^{(2)}(t)]^T$ is a $(2M+1)$-dimensional vector of complex white measurement noises with zero-mean and covariance matrix $\mathbf{R}(t)$. Finally, $\mathbf{A}(\theta(t))$ is the $(2M-1)\times K$ array manifold matrix of the whole array, and $\mathbf{A}_i(\theta(t))$ $(i=1,2)$ is the $M \times K$ array manifold matrix of the ith subarray as given by

$$\mathbf{A}_i(\boldsymbol{\theta}(t)) = \begin{bmatrix} e^{-(-1)^i j d_M \alpha_1(t)} & e^{-(-1)^i j d_M \alpha_2(t)} & \cdots & e^{-(-1)^i j d_M \alpha_K(t)}\\ \vdots & \vdots & \vdots & \vdots\\ e^{-(-1)^i j d_1 \alpha_1(t)} & e^{-(-1)^i j d_1 \alpha_2(t)} & \cdots & e^{-(-1)^i j d_1 \alpha_K(t)}\end{bmatrix} \tag{5}$$

where $\alpha_k(t) = 2\pi \sin\theta_k(t)/\lambda$, and the arrangement of d_m in the reverse order of m is for convenience.

It is easy to verify that $\mathbf{A}_1(\theta(t)) = (\mathbf{A}_2(\theta(t)))^*$, where superscript $*$ denotes complex conjugate. Therefore, the matrix $\mathbf{A}(\theta(t))$ in (4) can be rewritten as

$$\mathbf{A}(\boldsymbol{\theta}(t)) = \left[\mathbf{A}_1^T(\boldsymbol{\theta}(t))\ \mathbf{1}_{1\times M}^T\ \mathbf{J}_M\mathbf{A}_1^H(\boldsymbol{\theta}(t))\right]^T \tag{6}$$

where $\mathbf{J}_M$ is an $M\times M$ antidiagonal matrix with all entries equal to 1. It can be seen that each column of matrix $\mathbf{A}(\theta(t))$ is conjugate symmetric.

III. KF and AR Coefficient Estimation

Herein, we assume that both $\theta(t)$ and the AR coefficients are slowly time varying in the observed interval [14]. In particular, we assume that over each time interval, the change in both $\theta(t)$ and $\mathbf{F}(t)$ is small enough so that

$$\begin{aligned}\boldsymbol{\theta}(t) \approx \boldsymbol{\theta}(n\tau), \quad \mathbf{a}_k(t) \approx \mathbf{a}_k(n\tau), \qquad t \in ((n-1)\tau, n\tau]\\ (n=1,2,\ldots;\ k=1,\ldots,K).\end{aligned} \tag{7}$$

It is also assumed that there are N snapshots or signal samples available to process the received data and estimate the AR coefficients and DOAs over each interval $((n-1)\tau, n\tau]$. Consequently, the N snapshots in the nth time interval can be approximately expressed as

$$\mathbf{y}(n,j) = \mathbf{A}(\boldsymbol{\theta}(n))\mathbf{s}(n,j) + \mathbf{e}(n,j), \quad j=1,\ldots,N. \tag{8}$$

Note that (8) can be considered as the discrete-time version of (4) in the nth interval. To simplify the expressions, we will use $\mathbf{y}(j), \mathbf{A}(n), \mathbf{s}(j)$, and $\mathbf{e}(j)$ instead of $\mathbf{y}(n,j), \mathbf{A}(\theta(n)), \mathbf{s}(n,j)$, and $\mathbf{e}(n,j)$ in the sequel.

As mentioned earlier, the proposed algorithm is to explore the dynamics of the source signals and jointly estimates the DOA and AR parameters of the source signals. To this end, we shall track the source state vector $\mathbf{x}(t)$ using KF given initial DOAs and then update the DOAs using a regularized QRD-RLS algorithm [15]. In general, the KF has an optimal minimum mean squares state estimator for a linear state-space system, whereas the regularized QRD-RLS algorithm has good numerical stability and small estimation variance.

The required state-space model at the nth interval can be obtained from the state dynamical equation in (3) and the sensor measurement equations in (8) as follows:

$$\mathbf{x}(j) = \mathbf{F}(n)\mathbf{x}(j-1) + \mathbf{v}(j) \tag{9}$$

$$\mathbf{y}(j) = \mathbf{A}(n)\mathbf{\Gamma}\mathbf{x}(j) + \mathbf{e}(j) \tag{10}$$

where the system matrices $\mathbf{F}$ and $\mathbf{A}$ depend only on n and can be considered as constants in each interval, and $\mathbf{v}(j)$ and $\mathbf{e}(j)$ are assumed to be uncorrelated, i.e., $E[\mathbf{v}(p)\mathbf{e}^T(q)] = \mathbf{0}$, for all p and q. Then, the state of the system can be recursively estimated using the KF [16] as

$$\hat{\mathbf{x}}(j|j) = \hat{\mathbf{x}}(j|j-1) + \mathbf{K}(j)\left(\mathbf{y}(j) - \mathbf{A}(n-1)\mathbf{\Gamma}\hat{\mathbf{x}}(j|j-1)\right) \tag{11}$$

where the "hat" denotes the estimate, $\hat{\mathbf{x}}(j|j)$ is the update or *a posteriori* estimate, and $\hat{\mathbf{x}}(j|j-1) = \mathbf{F}(n-1)\hat{\mathbf{x}}(j-1|j-1)$ is the best estimate prior to assimilating the measurement $\mathbf{y}(j)$, and $\mathbf{K}(j)$ is the Kalman gain matrix to be determined by the following Kalman recursion [16]:

$$\mathbf{P}(j|j-1) = \mathbf{F}(n-1)\mathbf{P}(j-1|j-1)\mathbf{F}^T(n-1) + \mathbf{Q}(j-1) \tag{12}$$

$$\begin{aligned}\mathbf{K}(j) = {} & \mathbf{P}(j|j-1)\mathbf{\Gamma}^T\mathbf{A}^H(n-1) \\ & \times\left[\mathbf{A}(n-1)\mathbf{\Gamma}\,\mathbf{P}(j|j-1)\mathbf{\Gamma}^T\mathbf{A}^H(n-1) + \mathbf{R}(j)\right]^{-1}\end{aligned} \tag{13}$$

$$\mathbf{P}(j|j) = \left(\mathbf{I}_{\left(\sum_{k=1}^{K} L_k\right)} - \mathbf{K}(j)\mathbf{A}(n-1)\mathbf{\Gamma}\right)\mathbf{P}(j|j-1) \tag{14}$$

where $\mathbf{P}(j|j-1)$ and $\mathbf{P}(j|j)$ are the error covariance matrices associated with the *a priori* estimate $\hat{\mathbf{x}}(j|j-1)$ and the *a posteriori* estimate $\hat{\mathbf{x}}(j|j)$, respectively. Since all states of the system have been estimated by (11), it is easy to separate the sources, giving the kth AR source expressed in the following linear regression model:

$$x_k(j) = \mathbf{x}_k^T(j-1)\mathbf{a}_k(n) + v_k(j) \tag{15}$$

where $\mathbf{a}_k(n)$ is the AR coefficients for the nth interval to be estimated. Since $\mathbf{a}_k(n)$ is real valued while $x_k(j)$ and $v_k(j)$ are complex valued, we can transform (15) to a real-domain expression to reduce the computational load by separating the real and imaginary parts, leading to

$$z_k(j) = \mathbf{B}_k^T(j)\mathbf{a}_k(n) + \mathbf{w}_k(j) \tag{16}$$

where

$$z_k(j) = \begin{bmatrix}\mathrm{Re}\left[x_k(j)\right] \\ \mathrm{Im}\left[x_k(j)\right]\end{bmatrix} \quad \mathbf{B}_k(j) = \begin{bmatrix}\mathrm{Re}\left[\mathbf{x}_k^T(j-1)\right] \\ \mathrm{Im}\left[\mathbf{x}_k^T(j-1)\right]\end{bmatrix}^T$$

$$\mathbf{w}_k(j) = \begin{bmatrix}\mathrm{Re}\left[v_k(j)\right] \\ \mathrm{Im}\left[v_k(j)\right]\end{bmatrix}.$$

We now estimate $\mathbf{a}_k(n)$ by employing the regularized QRD-RLS algorithm described in [15, Table I]. As compared with RLS, QRD-RLS has a better performance, lower arithmetic complexity, and better numerical stability in finite word-length implementation. In the next section, we will consider the tracking and estimation of the DOAs.

IV. DOA Estimation and Tracking

We implement the DOA estimation in the real-valued domain [17] in view of its excellent accuracy and substantial reduction of the computational burden over conventional DOA estimation techniques. In particular, we derive a real-valued DOA estimation and tracking method based also on the regularized QRD-RLS algorithm. First, let us introduce an odd unitary transformation defined by [17]

$$\mathbf{T}_{2M+1} = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{I}_M & \mathbf{0}_{M\times 1} & i\mathbf{I}_M \\ \mathbf{0}_{1\times M} & \sqrt{2} & \mathbf{0}_{1\times M} \\ \mathbf{J}_M & \mathbf{0}_{M\times 1} & -i\mathbf{J}_M\end{bmatrix}. \tag{17}$$

By left multiplying (10) by the unitary transformation in (17), one gets

$$\begin{aligned}\mathbf{y}_T(j) & \triangleq \mathbf{T}_{2M+1}^H\mathbf{A}(n)\mathbf{\Gamma}\mathbf{x}(j) + \mathbf{T}_{2M+1}^H\mathbf{e}(j) \\ & \triangleq \Bigg[y_T^{-M}(j),\ y_T^{-M+1}(j),\ldots, \\ & \qquad \sum_{k=1}^{K} x_k(j),\ldots,y_T^{M-1}(j),\ y_T^{M}(j)\Bigg]^T.\end{aligned} \tag{18}$$

Let $\mathbf{A}_T(n) \triangleq \mathbf{T}_{2M+1}^H\mathbf{A}(n)$. It is easy to show that $\mathbf{A}_T(n)$ is a real-valued matrix as given by

$$\mathbf{A}_T(n) = \sqrt{2}\begin{bmatrix}\mathrm{Re}\left[\mathbf{A}_1(n)\right] \\ \mathbf{1}_{1\times M} \\ \mathrm{Im}\left[\mathbf{J}_M\mathbf{A}_1(n)\right]\end{bmatrix} \tag{19}$$

where

$$\begin{aligned}&\mathrm{Re}\left[\mathbf{A}_1(n)\right] \\ &= \begin{bmatrix}\cos\left(d_M\alpha_1(n)\right) & \cos\left(d_M\alpha_2(n)\right) & \cdots & \cos\left(d_M\alpha_K(n)\right) \\ \vdots & \vdots & \vdots & \vdots \\ \cos\left(d_1\alpha_1(n)\right) & \cos\left(d_1\alpha_2(n)\right) & \cdots & \cos\left(d_1\alpha_K(n)\right)\end{bmatrix}\end{aligned}$$

$$\begin{aligned}&\mathrm{Im}\left[\mathbf{J}_M\mathbf{A}_1(n)\right] \\ &= \begin{bmatrix}\sin\left(d_1\alpha_1(n)\right) & \sin\left(d_1\alpha_2(n)\right) & \cdots & \sin\left(d_1\alpha_K(n)\right) \\ \vdots & \vdots & \vdots & \vdots \\ \sin\left(d_M\alpha_1(n)\right) & \sin\left(d_M\alpha_2(n)\right) & \cdots & \sin\left(d_M\alpha_K(n)\right)\end{bmatrix}.\end{aligned}$$

Using (18) and $\mathbf{A}_T(n)$, we can find that each element of $\mathbf{y}_T(j)$ can be described by a linear regression model as

$$y_T^m(j) = (\mathbf{\Gamma x}(j))^T \left(\mathbf{A}_T^m(n)\right)^T \qquad (m = \pm 1,\ \pm 2, \ldots, \pm M) \tag{20}$$

where $\mathbf{A}_T^m(n)$ denotes the mth row of matrix $\mathbf{A}_T(n)$. Note that the autocovariance matrix of $\mathbf{e}_T(j)$ is the same as those of $\mathbf{e}(j)$ due to its unitary invariance. Similar to the estimation of the AR coefficients, we can again employ the QRD-RLS algorithm to estimate all the row vectors of $\mathbf{A}_T(n)$ except for the middle one, namely, $\mathbf{A}_T^m(n)$, $m = \pm 1, \pm 2, \ldots, \pm M$. Furthermore, we can combine $\mathbf{A}_T^{+m}(n)$ and $\mathbf{A}_T^{-m}(n)$ as one group, giving

$$\text{Group } m: \begin{Bmatrix} \sin\left(d_m\alpha_1(n)\right) & \cdots & \sin\left(d_m\alpha_K(n)\right) \\ \cos\left(d_m\alpha_1(n)\right) & \cdots & \cos\left(d_m\alpha_K(n)\right) \end{Bmatrix}$$
$$(m = 1,\ 2, \cdots, M). \tag{21}$$

Obviously, one could exploit either the sine or cosine vector of group 1 to obtain the DOA estimate, but this sine or cosine calculation may fail to obtain the DOA estimate when the argument of the inverse sine or cosine function is greater than unity. To overcome this limitation, we can combine the sine and cosine vectors to get a new set of tangent vector for each group, i.e.,

$$\text{Group } m: \ \{\tan\left(d_m\alpha_1(n)\right) \cdots \tan\left(d_m\alpha_K(n)\right)\}$$
$$(m = 1,\ 2, \ldots, M). \tag{22}$$

Note that there may be a small disadvantage in (22) compared with (21) if the sine and cosine parts of (21) are decoupled. However, as the same estimated sources $\mathbf{x}(j)$ should always produce the same index, there is no need to deal with the pair-matching problem in our method. From (21) and (22), we see that each group can be used to estimate the DOA, implying that we can achieve DOA estimation only by using two receivers and sensors. Therefore, our proposed method is also suitable for the case where the number of receivers is much less than that of sensors and sources. As is well known, for $d_m > \lambda/2$, $m = 2, \ldots, M$, an ambiguity may happen due to the inability of the tangent function in dealing with more than 180° where the arc-tangent function is to select the right DOA from $|d_m\alpha_1(n)| > \pi/2$. Herein, we employ a technique similar to that suggested in [18] to handle this ambiguity problem. According to the assumption $d_1 \leq \lambda/2$, we can obtain the rough DOA estimate without ambiguity by using the first group of data. There exist multiple values for the same tangent value, i.e.,

$$\begin{aligned} \chi_{m,k}^{l_m} &= \chi_{m,k} + l_m\lambda/2d_m \ \lceil -2(1+\chi_{m,k})d_m/\lambda \rceil \leq l_m \\ &\leq \lfloor 2(1-\chi_{m,k})d_m/\lambda \rfloor \\ \chi_{m,k} &\triangleq \sin\left(\theta_k(n)\right), \qquad m = 2, \ldots, M;\ k = 1, \ldots, K \end{aligned} \tag{23}$$

where $\lfloor \bullet \rfloor$ and $\lceil \bullet \rceil$ denote the smallest integer greater than "$\bullet$" and the largest integer less than "$\bullet$", respectively. Thus, the unambiguous estimate for the kth angle can be obtained as

$$\hat{\theta}_{(m),k}(n) = \tan^{-1}\left(\arg\min_{\chi_k^{l_m}} \left| \sin^{-1}\left(\chi_{m,k}^{l_m}\right) - \hat{\theta}_{(m-1),k}(n) \right| \right). \tag{24}$$

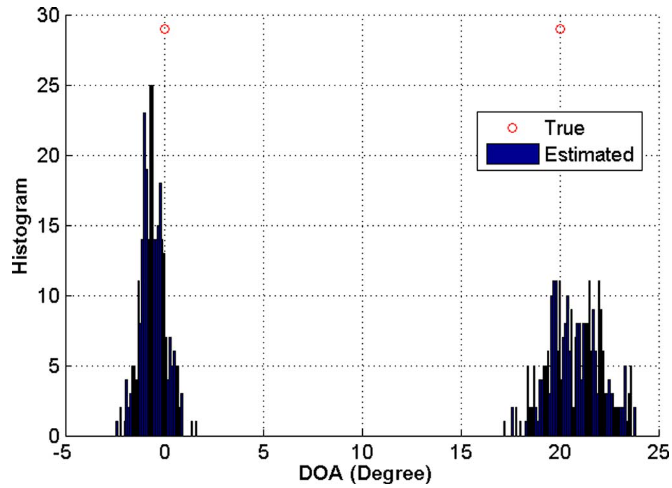


Fig. 2. Histogram of DOA estimates for two AR sources of DOA at [0° and 20°] using two sensors with $d_1 = \lambda/4$.

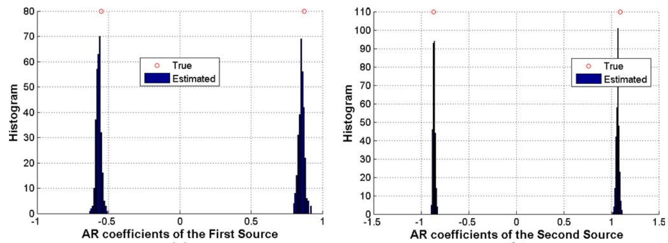
where $\hat{\theta}_{(m),k}(n)$ is the mth estimated value according to the unambiguous $(m-1)$th estimate. Note that it is also possible to use two subarrays to estimate the DOAs without requiring the reference element, in which case an even unitary transformation matrix [17] should be utilized.

V. Simulation Results

Here, some simulation results are presented to show the effectiveness of the proposed DOA estimation and tracking method. Here, we assume that the AR model order of the source signals is fixed and known. In practical applications, it can be determined by standard model order estimation techniques. Note that the reference sensor is not used to carry out DOA estimation in the following simulations in order to reduce the cost of the array system for practical considerations.

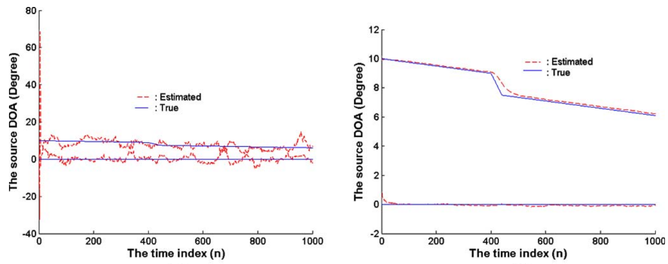
Example 1—Performance of Joint AR Coefficients and DOA Estimation: In this example, the sensor array consists of three isotropic antennas spaced by half a wavelength, i.e., $d_1 = \lambda/4$. The sources are two second-order AR stationary signals with coefficients $\mathbf{a}_1 = [\,0.872 \quad -0.550\,]$ and $\mathbf{a}_2 = [\,1.096 \quad -0.870\,]$, and each has a signal-to-noise ratio (SNR) of 30 dB. Here, the SNR of the kth signal is defined as the ratio of the kth signal power to the average power of noise $\mathbf{e}(t)$. The DOAs are chosen to be 0° and 20°. The other parameters are chosen as follows: the initial guess for the two DOAs is 5° and 25°, the initial AR coefficients are $[\,0.772 \quad -0.450\,]$ and $[\,0.96 \quad -0.77\,]$, and $N = 30$ snapshots are used. As shown in Figs. 2 and 3, the proposed method can estimate the DOAs of the two sources and the AR coefficients very well using only two sensors spaced by half-wavelength, which is almost impossible for conventional subspace-based method. This confirms the effectiveness of the proposed method in exploiting the dynamic information of the AR-modeled sources.

Example 2—Performance of DOA Tracking: To assess the DOA tracking performance of the proposed method, we set the simulation conditions as follows: AR coefficients are the same as those in Example 1, and herein, these values are assumed to be known previously, and SNR is set at 10 dB. Five sensors with $d_1 = \lambda/2$ and $d_2 = \lambda$ are used. Fig. 4 depicts the DOA tracking result of the proposed method in comparison with that obtained using the PAST method in [6] when the DOAs of the



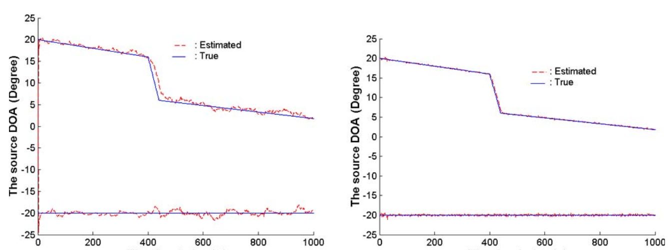
(a) (b)

Fig. 3. Histogram of AR coefficient estimation for two AR sources with DOA at $[0^\circ$ and $20^\circ]$ and AR coefficients $[0.872\ \ -550]$ and $[1.096\ \ -0.870^\circ]$ using two sensors with $d_1 = \lambda/4$.



(a) (b)

Fig. 4. Tracking two sources: one fixed DOA of 0° and the other moving from 10° to 6.1° with $N = 3$ for each interval. (a) PAST method. (b) Proposed method.



(a) (b)

Fig. 5. Tracking two sources: one fixed DOA of -20° and the other moving from 20° to 1.8° with $N = 30$ for each interval. (a) PAST method. (b) Proposed method.

two sources are 0° and -20° and $N = 3$. Fig. 5 shows the simulation results of both the proposed method and the PAST method for the DOA that is governed by

$$\mathrm{DOA} = \begin{cases} 10 - n/400 \text{ and } 20 - n/100, & n \in [1\ 400] \\ 9 - 1.5(n-400)/40 & \\ \quad \text{and } 16 - 2.5(n-400)/10, & n \in [401\ 440] \\ 7.5 - 2.5(n-440)/1000 & \\ \quad \text{and } 6 - 7.5(n-440)/1000, & n \in [441\ 1000] \end{cases}$$

when $N = 30$. Both of the initial DOAs are 2° and 12°. From Fig. 4, we see that the subspace-based method using PAST is unable to resolve the closely spaced angles in this case, whereas the proposed method yields satisfactory tracking results. Fig. 5 shows that although the PAST method can track the separated DOAs, it is hard to track the fast moving target, whereas our method can handle this situation satisfactorily. Two possible explanations are 1) our method takes advantage of the temporal information and 2) the subspace swap and leakage between the signal and noise subspaces degrades considerably the performance of the subspace-based methods in the case of closely spaced DOAs. It is also clear that the more snapshots used, the better performance for fast moving targets can be achieved. Moreover, through a large number of simulations, we have also found that the tracking performance is not sensitive to the initial guess of DOA values.

VI. Conclusion

In this brief, we have proposed an efficient DOA estimation and tracking method based on KF and the QRD-RLS algorithm. Due to the state information underlain by the AR model, we have been able to exploit the elegant KF to obtain the temporal information and then the efficient QRD-RLS to estimate the DOA and AR coefficients. A unitary transformation is also used to implement DOA estimation in the real-valued domain, where M real-valued vector groups are calculated for estimating the DOAs. Finally, simulation results have shown that our proposed method yields an excellent DOA estimation performance and a better DOA tracking performance than the subspace-based techniques do.

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