

## Sender–Receiver Exercise 7: Reading for Receivers

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# 1 Sender–Receiver Exercise: IS LINEAR TIME is unsolvable

The goals of this exercise are:

- to develop your skills at understanding, distilling, and communicating proofs and the conceptual ideas in them,
- to practice reductions for proving unsolvability, and gain more intuition for what kinds of problems about programs are unsolvable.

To prepare for this exercise as a Receiver, you should try to understand the theorem statement and definition in Section 1.1 below, and review the material from the previous sections of this chapter. Your partner Sender will communicate the proof of Theorem 1.1.

## 1.1 The Result

Rice’s Theorem (Theorem 25.16 in the textbook) says that all nontrivial problems about the input–output behavior of programs (i.e. about a program’s semantics) are unsolvable. Here we will see an example of a computational problem that is *not* about the input–output behavior of programs but is nevertheless unsolvable:

**Input:** A RAM program  $P$

**Output:** yes if  $P$  has running time  $O(n)$ , no otherwise

**Computational Problem IS LINEAR TIME**

As usual, the statement “ $P$  has running time  $O(n)$ ” means that there are constants  $c$  and  $n_0$  such that for all  $n \geq n_0$  and all inputs  $x$  of length at most  $n$ ,  $P(x)$  halts within  $c \cdot n$  steps. Note that the constants  $c$  and  $n_0$  are allowed to depend on  $P$ .

**Theorem 1.1.** IS LINEAR TIME *is unsolvable*.

**The approach to proving unsolvability.**

**The reduction.**

**Correctness of the reduction.**