Section 2: Search

CS 182 - Artificial Intelligence

A **search problem** can be defined by the tuple (S, A, R, C, S_0, G), where

- S represents the set of possible states
- A represents the set of possible actions, which may vary from state to state
- $R : S \times A \to S$ is the transition model, which represents the result of actions from a state
- $C : S \times A \to \mathbb{R}$ represents the cost of taking an action
- S_0 represents the starting state
- G represents the goal test, the set of goal states

Together, these parameters implicitly form a state space graph, where nodes are possible states and edges are actions connecting states.

Tree search is a method of exploring a state space graph by expanding out potential plans. Tree search builds a tree of searched states and maintains a fringe of partial plans under consideration. The objective is to find a plan that reaches the goal state, and it is ideal to try to expand as few nodes as possible on the state space graph.

Graph search extends tree search to account for state space graphs with cycles by keeping a set of visited nodes. These visited nodes are not explored again, avoiding repeated work.

Uninformed Search:

- Depth-First Search (DFS) maintains the fringe as a stack and expands the "deepest" nodes first.
- Breadth-First Search (BFS) maintains the fringe as a queue and expands the "shallowest" nodes first.
- Uniform Cost Search (UCS) maintains the fringe as a priority queue ordered by the cumulative cost of a path. UCS is optimal for all edge costs > 0.

Informed Search:

- Greedy Search maintains the fringe as a priority queue ordered by a heuristic function $h(x)$ that maps states to estimated distance to goal.
- A* Search extends greedy search and maintains the fringe as a priority queue ordered by summing the cumulative cost of a path to state x, $g(x)$, and the heuristic at x, $h(x)$ aka $f(x) = g(x) + h(x)$.

Psuedocode for both Tree and Graph search are shown below. Note: the frontier referenced below is the fringe. Think about how using a different data structure for the fringe would allow each of the algorithms to execute the above search algorithms.

```
function TREE-SEARCH(problem) returns a solution, or failure
  initialize the frontier using the initial state of problem
  loop do
     if the frontier is empty then return failure
     choose a leaf node and remove it from the frontier
     if the node contains a goal state then return the corresponding solution
     expand the chosen node, adding the resulting nodes to the frontier
```

```
function GRAPH-SEARCH(problem) returns a solution, or failure
  initialize the frontier using the initial state of problem
  initialize the explored set to be empty
  loop do
     if the frontier is empty then return failure
     choose a leaf node and remove it from the frontier
     if the node contains a goal state then return the corresponding solution
     add the node to the explored set
     expand the chosen node, adding the resulting nodes to the frontier
        only if not in the frontier or explored set
```

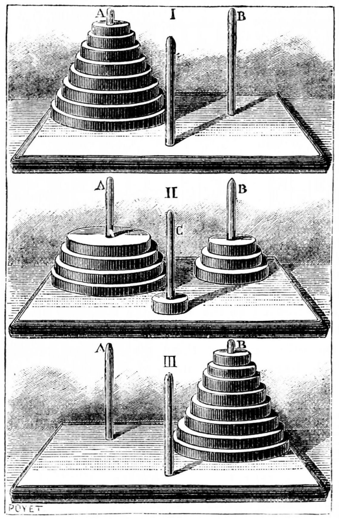
Heuristics:

A heuristic function is used to estimate the cost of reaching a goal from a state in the state space.

A heuristic h is admissible if it never overestimates the cost of reaching the goal, i.e. $\forall x, h(x) \leq h^*(x)$ where h^* is the optimal cost to reach the goal.

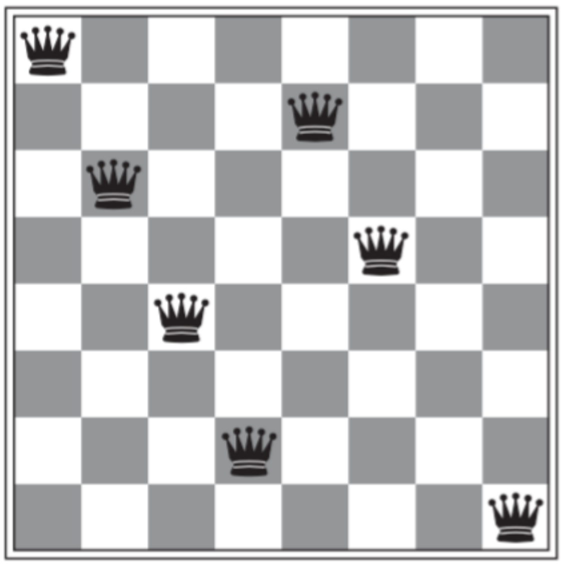
A heuristic h is consistent if it never overestimates the estimated distance to the goal of a neighbor plus the cost of reaching the neighbor, i.e. $\forall x, h(x) \leq c(x, x') + h(x')$

1. Consider the Tower of Hanoi puzzle, where a stack of disks must be moved from the leftmost peg to the rightmost peg, while maintaining the invariant that no disk may be placed on top of a smaller disk. Describe each element of a search problem for solving this puzzle. How many possible states are in this representation? (Image: Popular Science Monthly Vol. 26)



2. Consider the 8 queens puzzle, where queens must be placed on a chessboard so they do not attack each other. Describe each element of a search problem for solving this puzzle. How many possible states are in this representation? (AIMA)

Figure 1: A nearly-complete 8-queens puzzle solution



3. Can you think of another, more compact representation of the state space? (AIMA)

4. You are programming a holiday shopping robot that will drive from store to store in order to buy all the gifts on your shopping list. You have a set of N gifts $G = \{g_1, g_2, ...g_N\}$ that must be purchased. There are M stores, $S = \{s_1, s_2, ...s_M\}$ each of which stocks a known inventory of items: we write $g_k \in s_i$ if store s_i stocks gift g_k. Shops may cover more than one gift on your list and will never be out of the items they stock. Your home is the store s_1, which stocks no items.

   The actions you will consider are travel-and-buy actions in which the robot travels from its current location s_i to another store s_j in the fastest possible way and buys whatever items remaining on the shopping list that are sold at s_j. The time to travel-and-buy from s_i to s_j is $t(s_i, s_j)$. You may assume all travel-and-buy actions represent shortest paths, so there is no faster way to get between s_i and s_j via some other store. The robot begins at your home with no gifts purchased. You want it to buy all the items in as short a time as possible and return home.

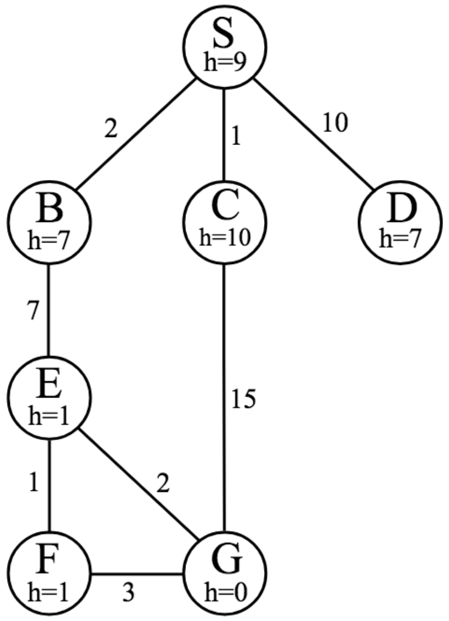
   For this planning problem, you use a state space where each state is a pair (s, u) where s is the current location and u is the set of unpurchased gifts on your list (so $g \in u$ indicates that gift g has not yet been purchased). (Berkeley CS 188, Fall 2011)

   How large is the state space in terms of the quantities defined above?

5. You have waited until very late to do your shopping, so you decide to send an swarm of R robot minions to shop in parallel. Each robot moves at the same speed, so the same store-to-store times apply. The problem is now to have all robots start at home, end at home, and for each item to have been bought by at least one robot (you dont have to worry about whether duplicates get bought). Hint: consider that robots may not all arrive at stores in sync. (Berkeley CS 188, Fall 2011)

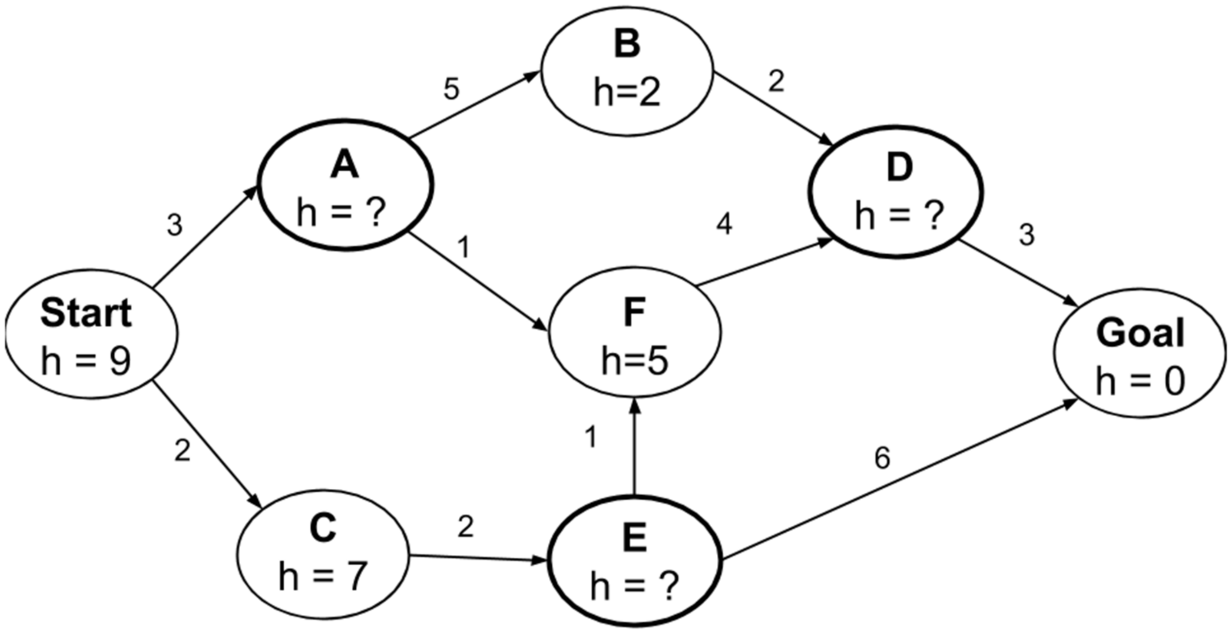
   Give a minimal state space for this search problem:

6. Consider the search graph shown below. S is the start state and G is the goal state. All edges are bidirectional. For each of the following graph search strategies, give the path that would be returned, or write none if no path will be returned. If there are any ties, assume alphabetical tiebreaking (i.e., nodes for states earlier in the alphabet are expanded first in the case of ties).
(Berkeley CS 188, Fall 2011)



- DFS
- BFS
- UCS
- Greedy Search
- A* Search

7. Consider the state space graph shown below in which some of the states are missing a heuristic value. (Berkeley CS 188, Spring 2015)



Determine the possible range for each missing heuristic value so that the heuristic is admissible.

Additionally, for what range of each heuristic value is the heuristic consistent?

8. The heuristic path algorithm (Pohl, 1977) is a best-first search in which the evaluation function is $f(n) = (2 - w)g(n) + wh(n)$. (AIMA)

   For what values of w is this optimal, assuming that h is admissible?

   What kind of search does this perform for $w = 0$, $w = 1$, and $w = 2$?

9. For the Tower of Hanoi puzzle from before, come up with an admissible heuristic function.