Section 8: Robot Motion Planning

CS 182 - Artificial Intelligence

1 Definitions

To make trajectory plans in robotics, we distinguish between two spaces:

- The **task space** of a robot is the standard, Cartesian space in which it moves. It is usually defined by 3-dimensional (x, y, z) coordinates (but if one of the dimensions is irrelevant, such as for a Roomba, we can also define the task space by just (x, y)).
- The **configuration space** of a robot is the n-dimensional space of joint angles that fully defines the positioning of the robot. The number n can be thought of as the number of degrees of freedom.

The relationship between a point in the task space and a point in the configuration space is determined by the structure and geometry of the robot. In the forward direction, **forward kinematics** is the mapping of a robot configuration to an **end-effector** (hand) position in task space. Conversely, **inverse kinematics** maps a point in task space to a robot configuration.

Since the path-planning algorithms we discussed are based on random sampling, there are no deterministic guarantees of completeness and optimality that were discussed for search algorithms. Instead, we consider **probabilistic completeness** and **probabilistic optimality**, which ask whether algorithms are complete or optimal in the limit that the number of samples approaches infinity.

2 Algorithms

Rapidly-exploring Random Tree (RRT): This algorithm accepts a start and goal location. To build a path, it iteratively: (1) samples states $s \in S$ until it finds one that is collision-free; (2) finds closest state $s_c \in T$; (3) extends s_c toward s; (4) adds resulting state s to T; (5) repeats until T contains a path from s_0 to s_{goal}. This algorithm is probabilistically complete, and searches for feasbile paths. A variant includes **goal-directed sampling**, which augments the sampling step (1). Instead of just sampling randomly, we sample the goal with a (small) probability p. This makes it more likely that the tree actually reaches the goal, rather than waiting until it "stumbles" into it.

Bi-directional RRT: This a variant on RRT which grows two trees – one starting from the start state and one from the goal state – and alternates between the two of them for sampling and extending. This version tends to perform better on difficult "bugtrap" problems.

RRT*: While RRT is very widely used, it searches for feasible but not optimal paths. RRT* is a variant on RRT that has guarantees of probabilistic optimality. The algorithm iteratively: sample state $s \in S$ until one is found that is collision-free; (2) find closest state $s_c \in T$; (3) extend s_c toward s resulting in state s'; (4) find all $s_{near} \subseteq T$ within a distance d to s'; (5) find $s_{min} \in s_{near}$ that has the lowest path cost to $s_0 \to s_{min} \to s'$; (6) add edge $s_{min} \to s'$ to T; (7) check path cost through s' to all states in $s \in s_{near}$, if any are lower than existing path cost to s, then "rewire" tree to include edge $s' \to s$; (8) repeat until maximum iterations reached and T contains a path from s_0 to s_{goal}.

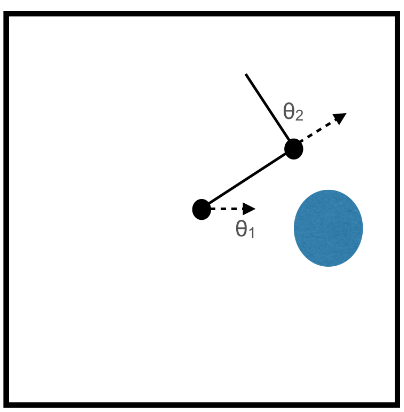
Probabilistic Roadmap (PRM): The previous RRT algorithms have been **single-query**: they are designed around one specific set of start and goal states. PRM is designed to be **multi-query**: it builds a graph of **milestones** that can be re-used for multiple path-planning problems. The algorithm first builds the PRM offline: (1) samples configurations by picking points at random; (2) tests sampled configurations for collision; (3) retains collision-free configurations as milestones; (4) links each milestone with straight paths to its nearest neighbors; (5) retains collision-free links as local paths. Then online one: (6) includes the start and goal as milestones; (7) connects the start and goal to its nearest neighbors; (8) searches the PRM graph (using graph search) for a path from s to g.

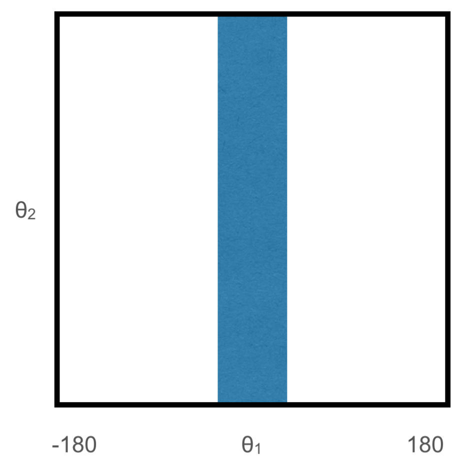
3 Problems

Exercise 1

Consider the following "elbow" robot in 2-space, tethered to the center of the square. For simplicity, assume only the "joint" attached to the central point can collide with the circle. Map out the configuration space in the adjacent square, including the collision boundary with the circle.

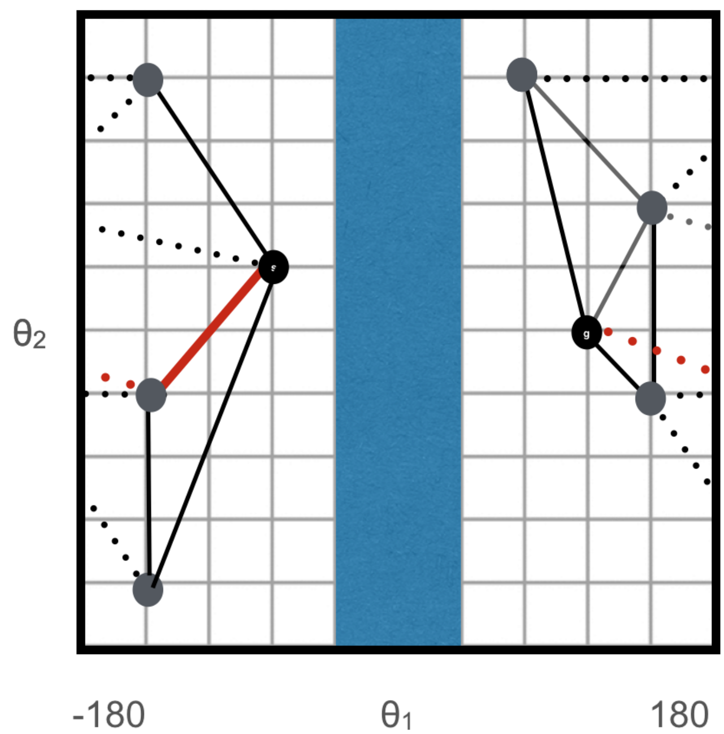
(Hint: think about the angle of the "shoulder" and "elbow" of the robot)





Exercise 2

Consider the "elbow" robot from above. Assume the following points are sampled for PRM, with the given start and goal nodes. Create a graph for the PRM using the 3-nearest neighbors approach, and find the shortest path from s to g.



Exercise 3

Assume the following figure shows the first 7 states sampled, in order, through RRT and RRT*. Trace out the tree formed by the first 7 steps of RRT, and RRT*. For RRT* assume a radius of 2.99 for checking better paths and a maximum extension distance of 2.25.

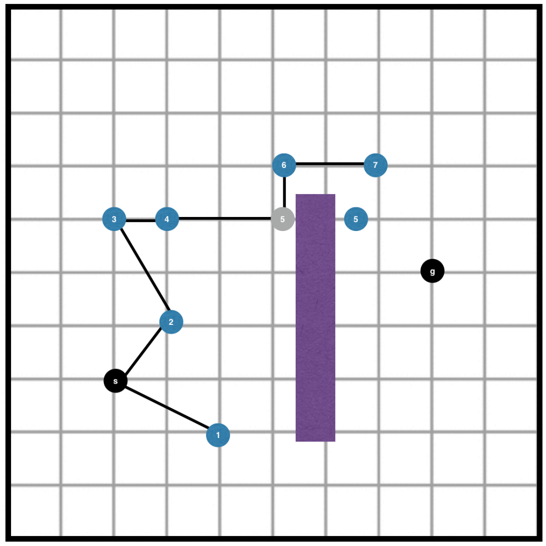


Figure 1: RRT

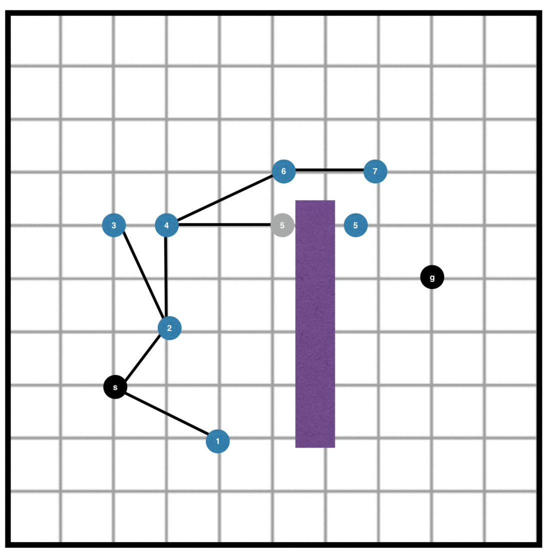


Figure 2: RRT*