Section 4: CSPs and Local Search

CS 182 - Artificial Intelligence

CSPs are formalized as a triple $\langle X, D, C \rangle$.

1. $X = \{X_1, \ldots, X_n\}$: A set of variables in the problem
2. $D = \{D_1, \ldots, D_n\}$: The domains for these variables
3. $C = \{C_1, \ldots, C_m\}$: Constraints

Constraints encode the limits on the values/domains for each variable contingent on the values/domains of other variables.

One way to solve a CSP is through **backtracking search** which is simply DFS with two changes:

1. Fix the variable ordering ($X_1 = D_1 \rightarrow X_2 = D_2 \equiv X_2 = D_2 \rightarrow X_1 = D_1$).
2. Check constraints as you go and backtrack if you violate any.

One way to optimize this process is to use the most constrained variable (the variable with the minimum remaining values) and assign the least constraining variable. Another optimization is to use **forward checking** to do a one step check for inconsistent solutions at each assignment. Finally one can use **arc consistency** to check the entire chain of implications of one assignment.

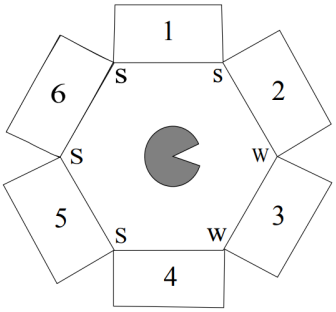
Optimization is the process of computing an optimal solution (e.g., min cost, max reward) to some mathematical problem. Optimization is often used to solve complex problems as it casts a real world problem into a simple math problem which can be solved via off-the-shelf solvers. Unfortunately, many optimization problems are NP-complete. Therefore, globally optimal solutions cannot often be found and instead locally optimal solutions are searched for from some (ideally) informed initial condition. **Local Search Algorithms** include:

1. **Hill Climbing** is the simplest approach. Given a state s, generate all successors s' and move to the best one. If there is no better successor, return the current state. A variant of this allows for **random restarts** to try to escape local minimums.
2. **Simulated Annealing** picks a random successor state at each step. If the random successor is better, the state moves. If it is worse, it is accepted with probability $\exp(\Delta E/T)$. The algorithm starts with large T (exploration) and decreases it over time.
3. In **Beam Search**, we choose k random initial states. All successors for all k states are expanded. Among all of the generated states, the best k states are chosen.
4. **Genetic Algorithms** start with a set of possible solutions. Inspired by natural selection, at each iteration, a biased sample (based on a measure of "fitness") is taken from the current solution set. A new solution set is then constructed by "crossing over" (aka combining parts of) pairs of sampled solutions. "Mutation" (aka random changes to solutions) are also added during iterations. These algorithms can avoid local minima but may take many iterations to converge to a good solution.
5. **Gradient Descent** relies on an assumption that we are minimizing a locally convex (or maximizing a locally concave) function. It takes the gradient, the partial derivatives, of the function which points in the direction of maximum descent (ascent) and moves in that direction iteratively, $x_{t+1} = x_t - \alpha \nabla f(x)$, until it reaches a local minimum (maximum) where $\nabla f(x) = 0$. **Stochastic Gradient Descent** approximates the gradient through sampling and is otherwise the same. $x_{t+1} = x_t - \alpha \left(f(x + w) - f(x)\right) w$ for $w \sim \mathcal{N}(0, \Sigma)$. These are both used in continuous domains.

Practice Problems

1. Pacman is trapped![1] He is surrounded by mysterious corridors, each of which leads to either a pit (P), a ghost (G), or an exit (E). In order to escape, he needs to figure out which corridors, if any, lead to an exit and freedom, rather than the certain doom of a pit or a ghost.

   The one sign of what lies behind the corridors is the wind: a pit produces a strong breeze (S) and an exit produces a weak breeze (W), while a ghost doesn't produce any breeze at all. Unfortunately, Pacman cannot measure the strength of the breeze at a specific corridor. Instead, he can stand between two adjacent corridors and feel the max of the two breezes. For example, if he stands between a pit and an exit he will sense a strong (S) breeze, while if he stands between an exit and a ghost, he will sense a weak (W) breeze. The measurements for all intersections are shown in the figure below. Also, while the total number of exits might be zero, one, or more, Pacman knows that two neighboring squares will not both be exits.



   Pacman models this using variables X_i for each corridor i and domains P, G, and E.

   (a) State the binary and/or unary constraints for this CSP (either implicitly or explicitly).

   (b) Cross out / remove the values from the domains of the variables that will be deleted by forward checking from the initial constraints (part a). What about arc consistency?

X_1	P	G	E
X_2	P	G	E
X_3	P	G	E
X_4	P	G	E
X_5	P	G	E
X_6	P	G	E

   (c) According to minimum remaining values (MRV), which variable or variables could the solver assign first?

   (d) Assume that Pacman knows that $X_6 = G$. List all the solutions of this CSP or write none if no solutions exist.

[1] From Berkeley Section material

2. Consider the graph with 8 nodes A_1, A_2, A_3, A_4, H, T, F_1, F_2. A_i is connected to A_{i+1} for all $i \in [1, 2, 3]$, each A_i is connected to H, H is connected to T, and T is connected to each F_i. Find a 3-coloring of this graph by hand using the following strategy: backtracking with conflict-directed backjumping, the variable order A_1, H, A_4, F_1, A_2, F_2, A_3, T, and the value order R, G, B.

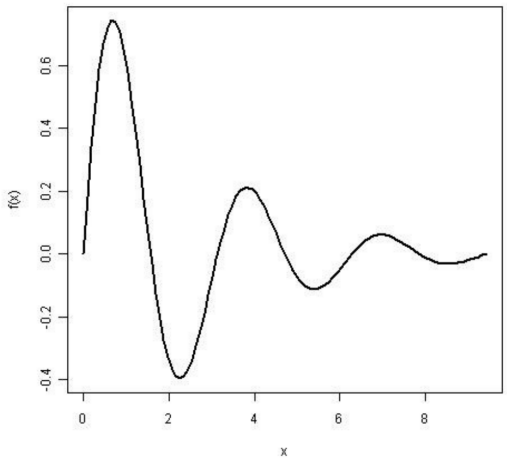
3. Consider the problem of placing k knights on an $n \times n$ chessboard such that no two knights are attacking each other, where k is given and $k \leq n^2$.

   (a) Choose a CSP formulation. In your formulation, what are the variables? What is the domain for each variable?

   (b) What are the constraints?

   (c) Now consider the problem of putting as many knights as possible on the board without any attacks. Explain how to solve this with local search by defining appropriate actions and a sensible objective function. **Note**: Don't try to solve this problem, only lay out the strategy.

4. Given the function shown in the Figure below, what are the solution for hill climbing and simulated annealing with the starting points $x = 1$, $x = 4$ and $x = 5$? Additionally give the solution for a Beam Search with $k = 3$ and starting points 2, 4 and 6.[2]



5. Give the name of the algorithm that results from each of the following special cases:

   (a) Local beam search with $k = 1$.

   (b) Local beam search with one initial state and no limit on the number of states retained.

   (c) Simulated annealing with $T = \infty$ at all times.

   (d) Genetic algorithm with population size $N = 1$.

[2] from University of Hildesheim

6. In (neural network) machine translation, you have a fully differentiable function f with a set of continuous parameters $\theta \in \mathbb{R}^N$ that predicts an output sequence $\mathbf{y}$ from an input sequence $\mathbf{x}$. You have a corpus with sentences in English ($\mathbf{X}$) and corresponding translations in Spanish ($\mathbf{Y}$). Your goal is to find a set of parameters θ that maximizes $p(\mathbf{y}|\mathbf{x}, \theta)$.

   (a) Formalize this problem as a local search problem. What is a state? What are possible successor states?

   (b) Which algorithm from lecture could we use to find the best state? Is there a problem with this algorithm?

   Now that you successfully found a good θ, you are given an unknown sentence $\mathbf{x}$ and the task to translate it. You realize that while you can successfully assign a score to a pair of sentences $\mathbf{x}$ and $\mathbf{y}$, you still a way need to find the best $\mathbf{y}$.

   (c) Formalize the problem of finding the best sequence of output words $\mathbf{y} = w_1, \ldots, w_l$ as a search problem. What is a state? What is a successor state? What is a goal state? Note that "word" in this context can also mean a special character.

   (d) How many states do you need to expand at each search step?

   A property of f is that the score of each word in the output depends not only on the input $\mathbf{x}$, but also on all previously generated words in $\mathbf{y}$.

   (e) What does property this imply for your search algorithm choice?

   (f) Suggest a search algorithm to tackle this problem, and discuss possible drawbacks of using this approach over others.