Section 6: Reinforcement Learning

CS 182 - Artificial Intelligence

We use **Reinforcement Learning (RL)** when we have a Markov Decision Process (MDP) for which we know:

- S: set of states
- A: set of actions (for any given state)
- s_0: start state
- s_g: goal state (or set of goal states)

but do not know:

- $T : S \times A \to S$: transition map (or model)
- $R : S \times A \to \mathbb{R}$: reward function

The goal of reinforcement learning is to learn a **policy** $\pi : S \to A$ which returns the best action for any given state in the problem. In the process of looking for the policy, RL algorithms often calculate:

- $V : S \to \mathbb{R}$: **value function**, which stores the utility of every state
- $Q : S \times A \to \mathbb{R}$: **Q-value function**, which stores the value of every (state, action) pair

In **Passive Reinforcement Learning**, an agent has a fixed policy, and learns about the environment while executing that policy. In lecture, we discussed two algorithms:

1. **Direct evaluation** runs many "experiments", or "simulations", resulting from following a given policy π, and keeps track of the total discounted rewards of the visited states. Afterwards, the reward instances for each state are averaged to determine the state's value. This could be written as

   $$V(s) = \frac{1}{n} \sum_i (R(s_i) + \gamma R(s_i') + \gamma^2 R(s_i'') + ...)$$

   where s_i, s_i', s_i'', ..., are the states visited in the ith simulation. Direct evaluation can also be referred to as a **Monte Carlo** approach, since it depends on averaging over many randomized simulations or samples.

2. **Temporal difference learning** iteratively runs experiments following a given policy. With each transition, a sample-based Bellman update is made to the value of the visited state (both equations are computing the same thing):

   $$V^\pi(s) \leftarrow V^\pi(s) + \alpha[R(s, \pi(s), s') + \gamma V^\pi(s') - V^\pi(s)]$$

   $$V^\pi(s) \leftarrow (1 - \alpha) V^\pi(s) + \alpha[R(s, \pi(s), s') + \gamma V^\pi(s')]$$

   We can think of this algorithm can be thought of as sample-based policy evaluation. Remember policy evaluation was:

   $$V^\pi(s) \leftarrow \sum_{s'} T(s, \pi_k(s), s')[R(s, \pi(s), s') + \gamma V^\pi(s')]$$

   In Q-learning we are still executing some policy π we just don't know what the transition model is and thus can't compute all of the possible transitions to sum over. Instead we are simply computing one of the values in the sum and making a weighted update of our current value to incorporate the new information.

In **Active Reinforcement Learning**, and agent chooses actions and tries to find an optimal policy while learning about the environment. The primary algorithm used in active RL is **Q-learning**. Similarly to TD learning, Q-learning makes sample-based Bellman updates, but to Q-values instead of values:

$$Q(s,a) \leftarrow Q(s,a) + \alpha[R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)]$$

$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + \alpha[R(s,a,s') + \gamma \max_{a'} Q(s',a')]$$

Since we are learning Q-values and not values we can extract the current estimate of the optimal policy by doing a one-step argmaximization over the Q-value immediately at any time:

$$\pi(s) = arg\max_{a} Q(s,a)$$

Some algorithmic details and variations:

1. In active algorithms, we must choose actions to take from states. One approach is **ϵ-greedy action selection**: with (small) probability ϵ, actions are chosen randomly; with (large) probability $1-\epsilon$, we choose the current optimal action. This encourages exploration of the state space and is generally helpful in practice. Note that this is called off-policy learning as we learn the exact Q-values but execute with a different policy, the ϵ-greedy policy.

2. How can we evaluate the choice of parameters or exploration functions in Q-learning? One approach is to measure **regret** – the difference between expected rewards and final optimal rewards.

3. In linear **approximate Q-learning** we approximate the Q-function with a set of functions:

   $$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \cdots + w_m f_m(s,a)$$

   This is often also called **feature based** Q-learning. As the functions are supposed to represent higher level features of the state space. For example, for PACMAN they could be the number of ghosts, the distance to the nearest ghost, the distance to the nearest food, etc. The advantage here is that we simply need to learn the correct weights (and thus we have a smaller amount of things to learn vs. the full tabular standard method where we have to learn a Q-value for every single state-action pair). The disadvantage is that the quality of the Q-values is highly dependent on the quality of the functions. We now also need to do a two step update, first of the Q-values using the current weights and new information, and then an update of the weights due to the new information.

   $$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + \alpha[R(s,a,s') + \gamma \max_{a'} Q(s',a')]$$

   $$w_i \leftarrow w_i + \alpha f_i(s,a)[R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)]$$

Algorithmic parameters:

- α: learning rate (determines the integration of new information into current estimates)
- γ: discount rate (determines how quickly rewards decay with time)

Practice Problems

1. Suppose we are Q-learning with ϵ-greedy action selection, starting with $\epsilon = 0.2$. When is it a good idea to...
   - keep ϵ constant over time?
   - decrease ϵ to 0 over time?
   - increase ϵ over time?

2. Suppose we are Q-learning. When is it a good idea to...
   - set $\alpha = 0$?
   - set $\alpha = 1$?
   - set $\alpha = 0.2$ and decrease it to 0 over time?

3. What would the returned value assignments be for Direct Evaluation over a deterministic MDP? Can you find a simple upper bound on the number of experiments that need to be run by the algorithm for the value function to converge? What if the MDP is not deterministic?

4. When using features to represent the Q-function, is it guaranteed that the feature-based Q-learning finds the same optimal Q^* as would be found when using a tabular representation for the Q-function?

5. Why is temporal difference (TD) learning of Q-values (Q-learning) superior to TD learning of values?

6. Recall a problem from last week's section: Consider an MDP having three states, (1, 2, 3), where State 3 is a terminal state. In States 1 and 2, there are two possible actions: A and B. Unlike last week, we no longer know the rewards or transition model ahead of time.

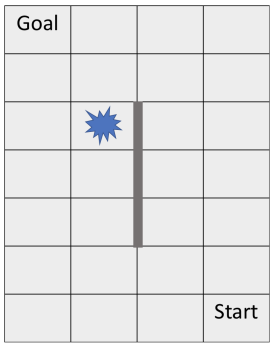
   Suppose we decide to run Q-learning on this problem with $\alpha = 0.2$ and a 0.9 discount ($\gamma = 0.9$). All Q-values are initialized to 0. While running one episode/simulation within the algorithm, we get the following (s, a, s', r) samples (in order):

   (State 1, Action B, State 1, reward = -1)
   (State 1, Action A, State 1, reward = -1)
   (State 1, Action A, State 2, reward = -2)
   (State 2, Action B, State 2, reward = -2)
   (State 2, Action B, State 3, reward = 0)

   Run Q-learning based on these samples. How are the Q-values updated?

   How would the solution be different if we chose all actions following the optimal policy?

7. Consider the Pacman-inspired game board below. Suppose that every action has a 80% chance of moving in the desired direction, and a 10% chance of moving in each of two perpendicular directions (though we only learn this transition function by executing actions in the world). The lower left-hand square is numbered (0, 0), the start square is (3, 0), and the goal square is (0, 6). The goal of this game is for Pacman to navigate to the exit (reward of 100) without touching the dangerous explosion square at (1, 4), which terminates the game with a reward of −100. Pacman cannot cross the thick wall in the middle of the board; an action causing a wall collision would make Pacman stay in place.



Let's say we choose to solve this problem with feature-based Q-learning, with the features:
f_1 = the Manhattan distance to the goal
f_2 = 0 or 1 indicating whether the explosion square is in 1-square radius of Pacman.

(a) Which gameboard squares will be equivalent to the state defined by ($f_1 = 4$, $f_2 = 1$)?

(b) What are the advantages and disadvantages of using this feature representation?