Lecture 18: Variational Autoencoders

CS109B Data Science 2

Pavlos Protopapas, Mark Glickman, and Chris Tanner

Outline

Motivation for Variational Autoencoders (VAE)

Mechanics of VAE

Separability of VAE

The math behind everything

Generative models

Outline

Motivation for Variational Autoencoders (VAE)

Mechanics of VAE

Separability of VAE

The math behind everything

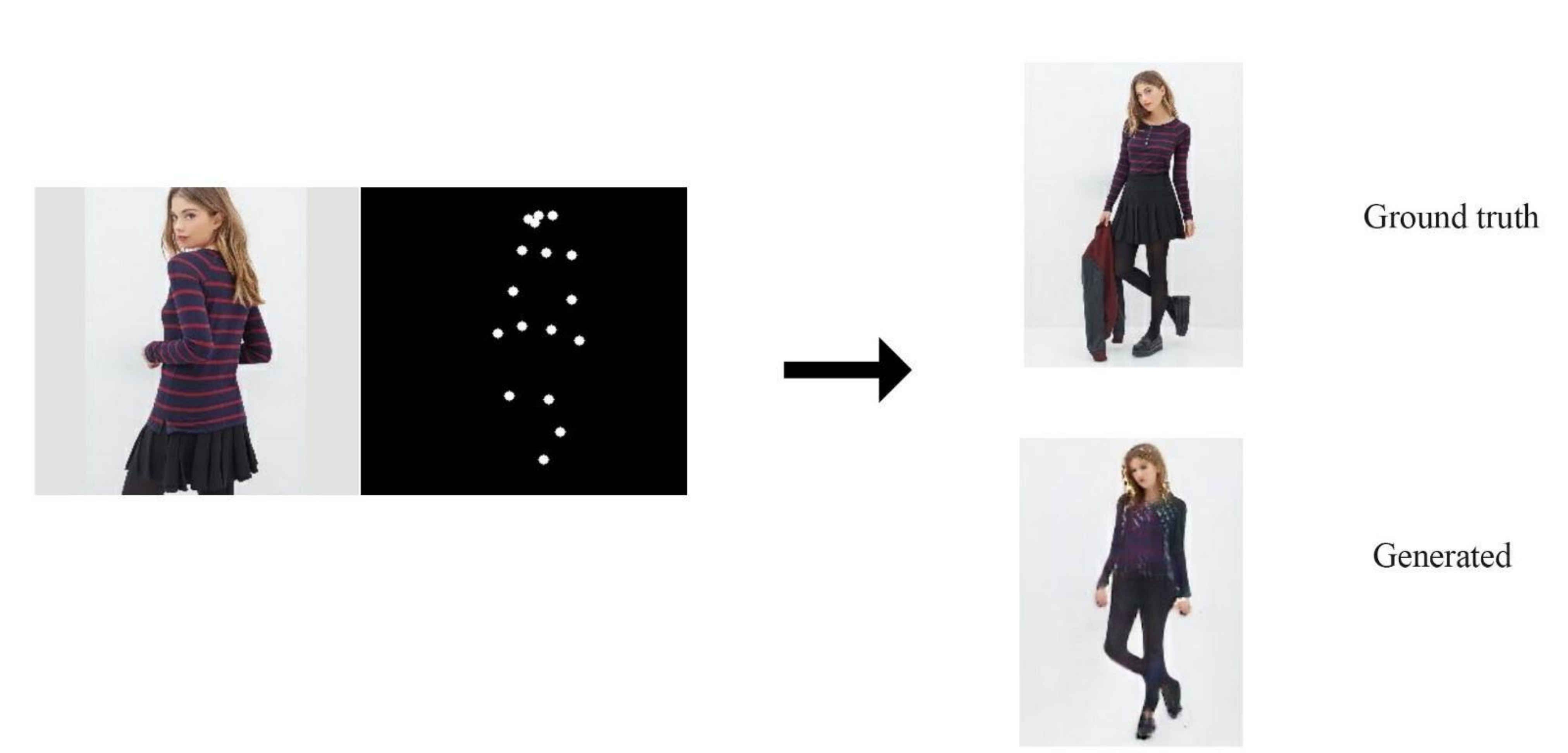
Generative models

Generating Data (is exciting)

Figure 7: Generated samples

https://arxiv.org/pdf/1708.05509.pdf

Generating Data (is exciting)



Generating Data (is exciting)

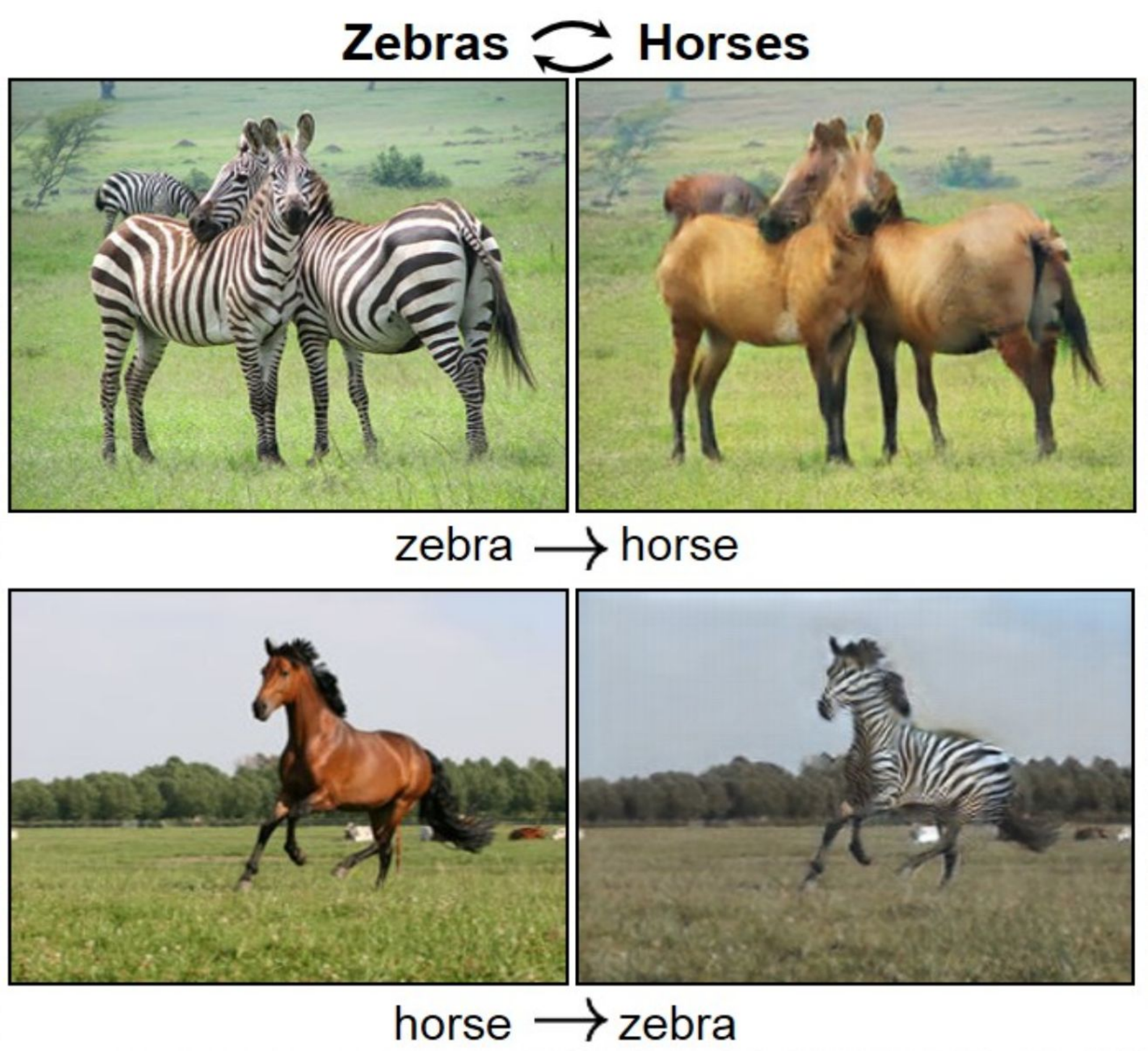


Figure 5: 1024 × 1024 images generated using the CELEBA-HQ dataset. See Appendix F for a larger set of results, and the accompanying video for latent space interpolations.

(a) MidiNet model 1

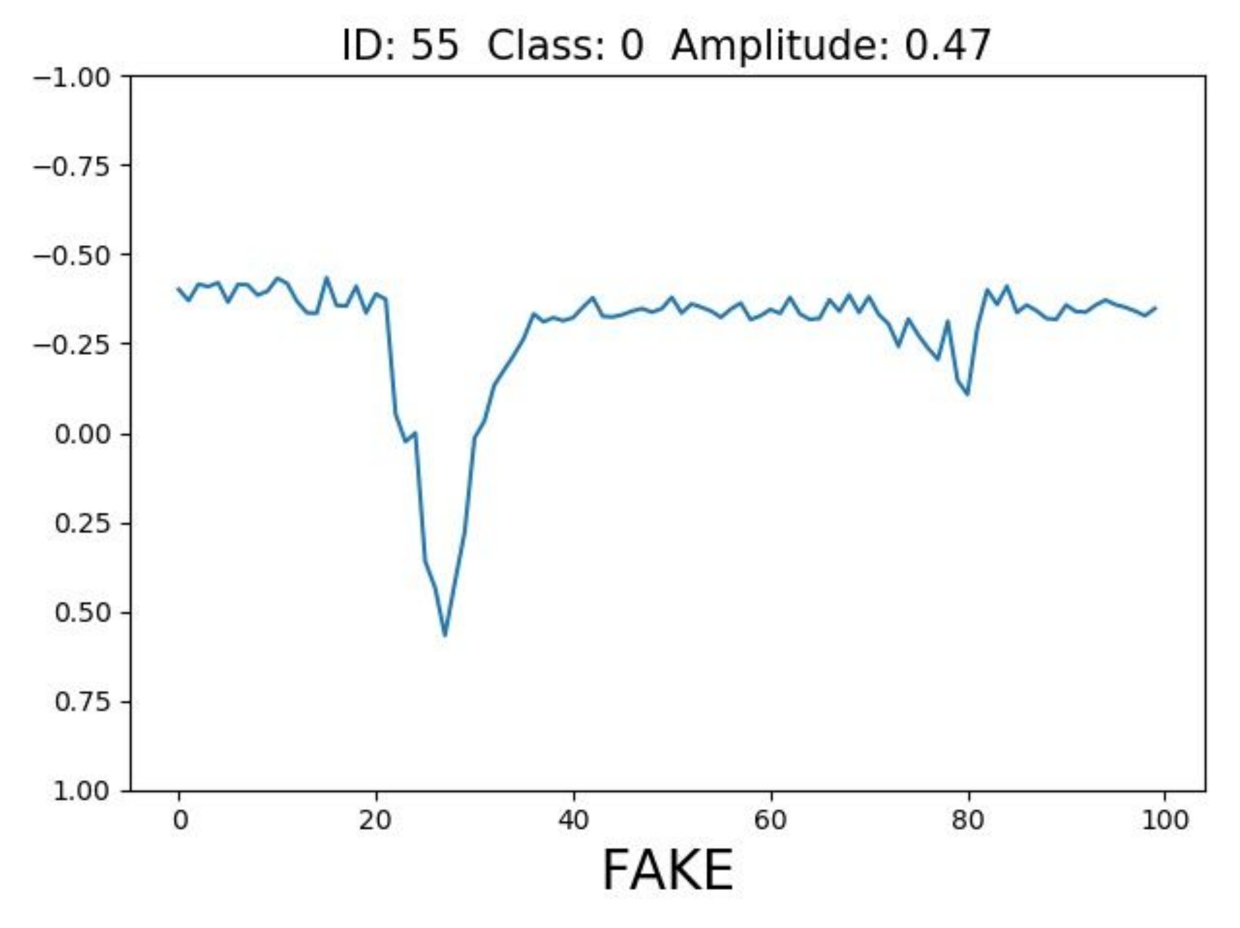
(b) MidiNet model 2

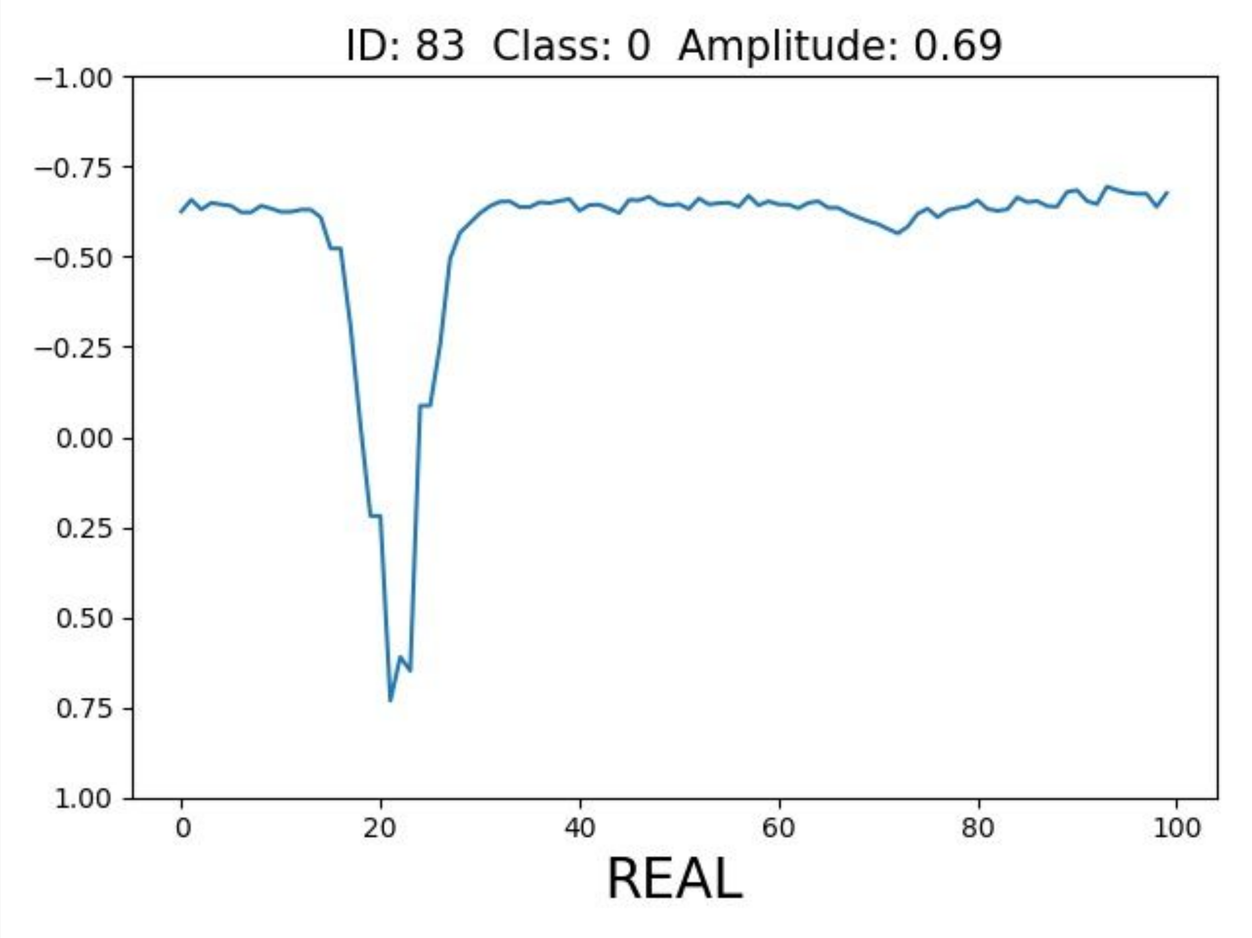
(c) MidiNet model 3

Figure 3. Example result of the melodies (of 8 bars) generated by different implementations of MidiNet.

Another use of generating new data is to give us ideas and options. Suppose we're planning a house. We can give the computer the space we have available, and its location. From this, the computer can can give us some ideas.

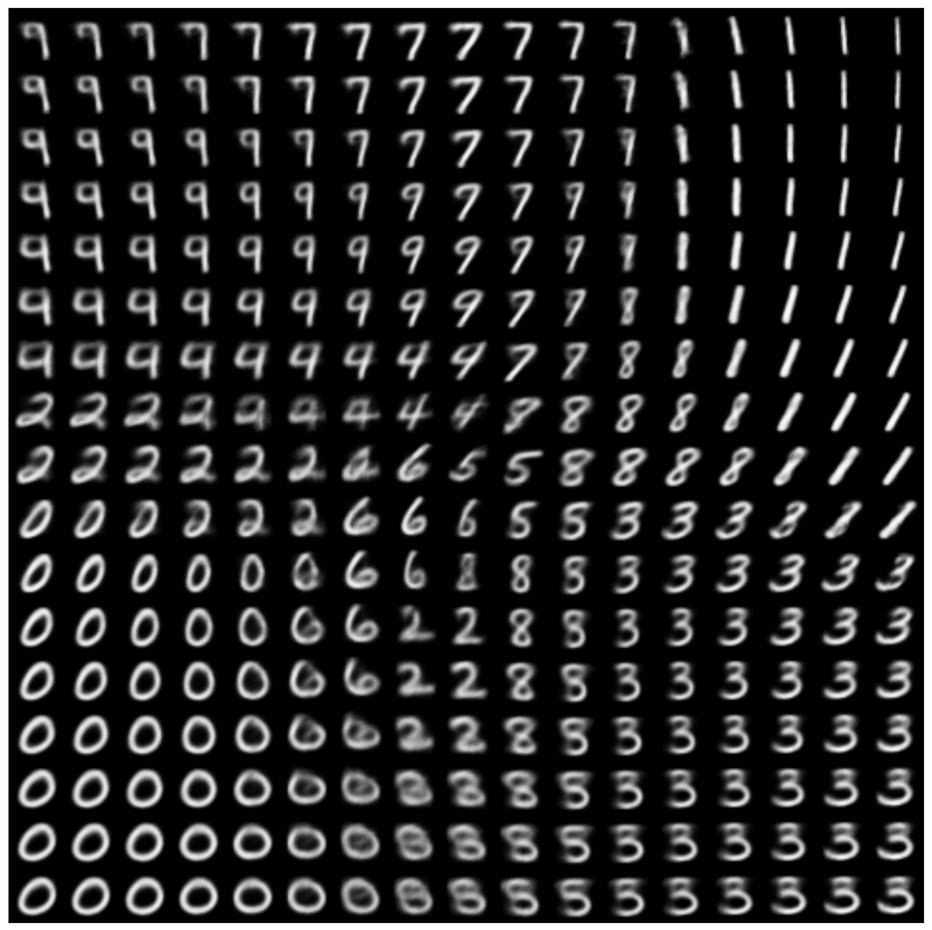
Big networks require big data, and getting high-quality, labeled data is difficult. If we're generating that data our selves, we can make as much of it as we like.





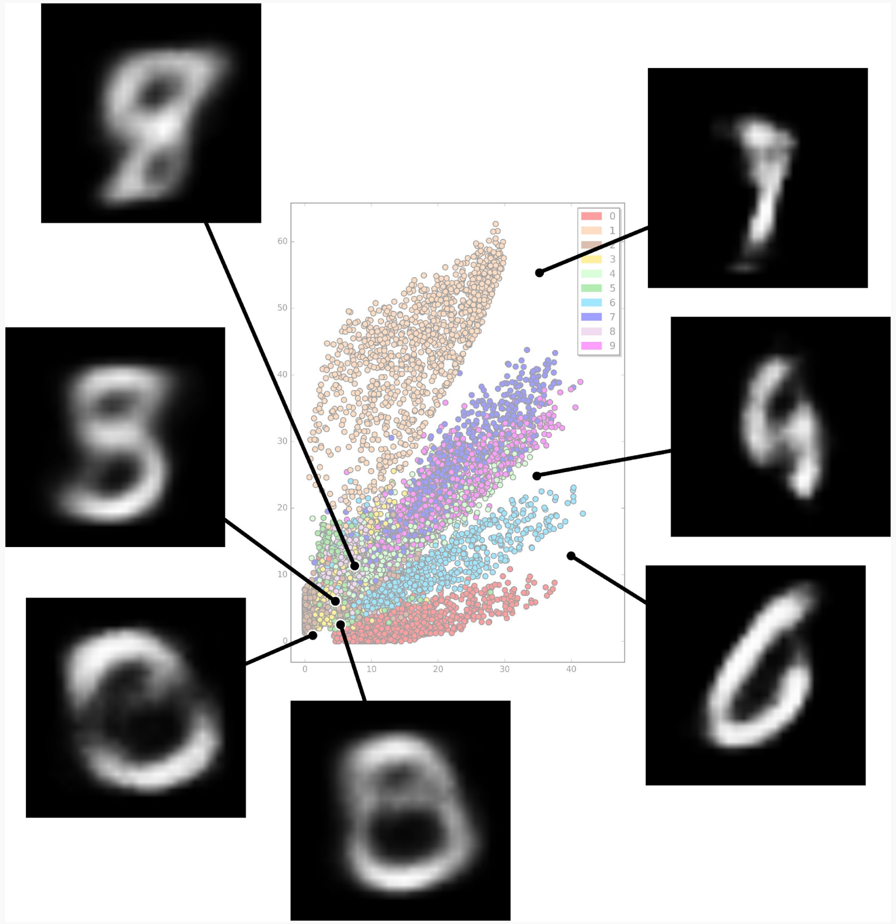
Generating Data

We saw how to generate new data with a AE in Lecture 12.



Problems with Autoencoders (from lecture 12)

- Gaps in the latent space
- Discrete latent space
- Separability in the latent space



Outline

Motivation for Variational Autoencoders (VAE)

Mechanics of VAE

Seperability of VAE

The math behind everything

Generative models

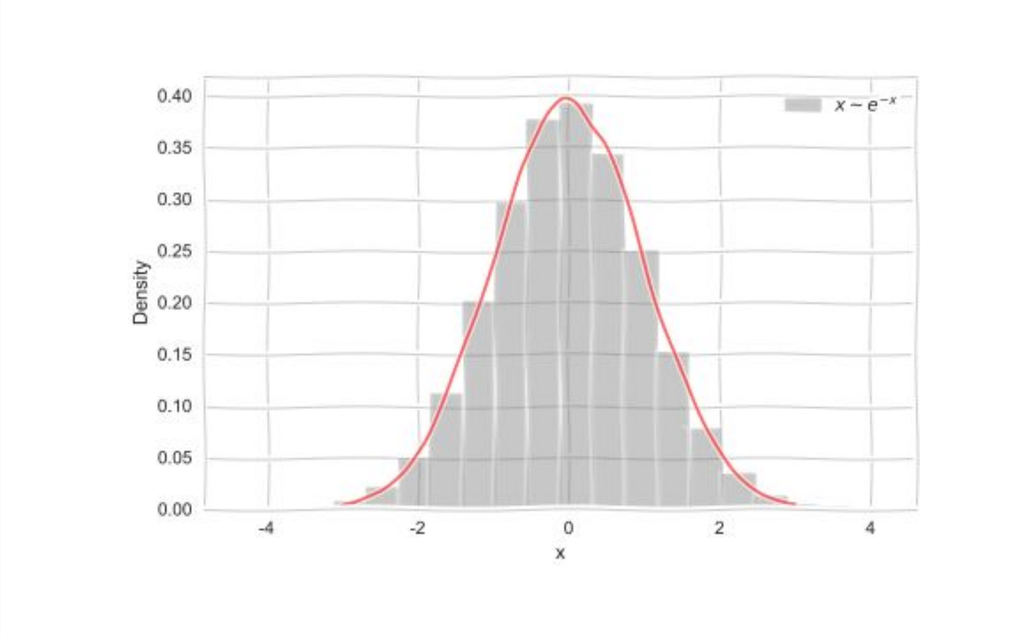
Generative models

Imagine we want to generate data from a distribution,

$$x \sim p(x)$$

e.g.

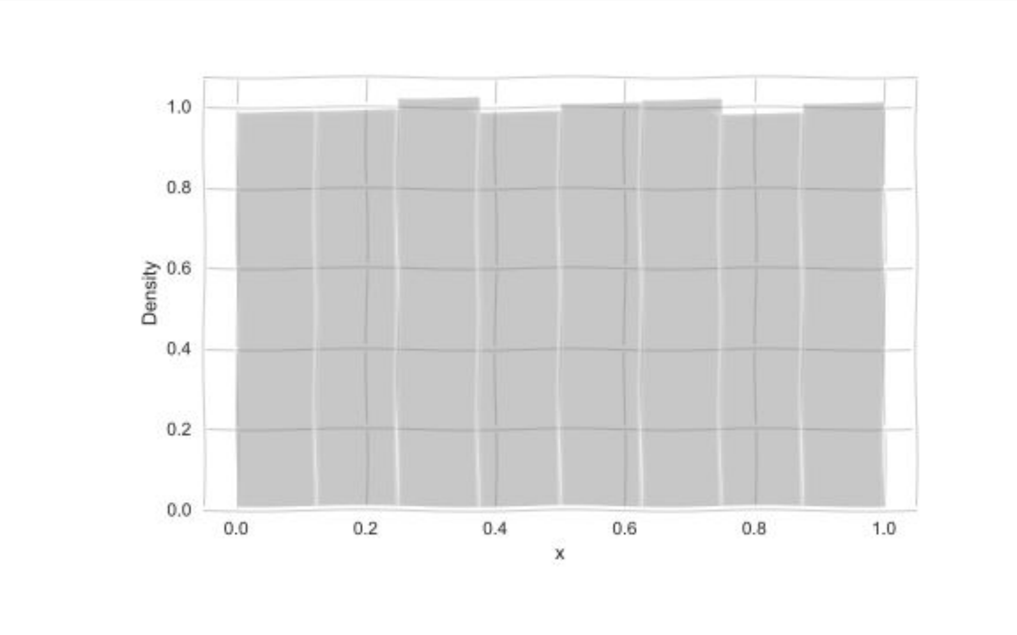
$$x \sim \mathcal{N}(\mu, \sigma)$$



Generative models

But how do we generate such samples?

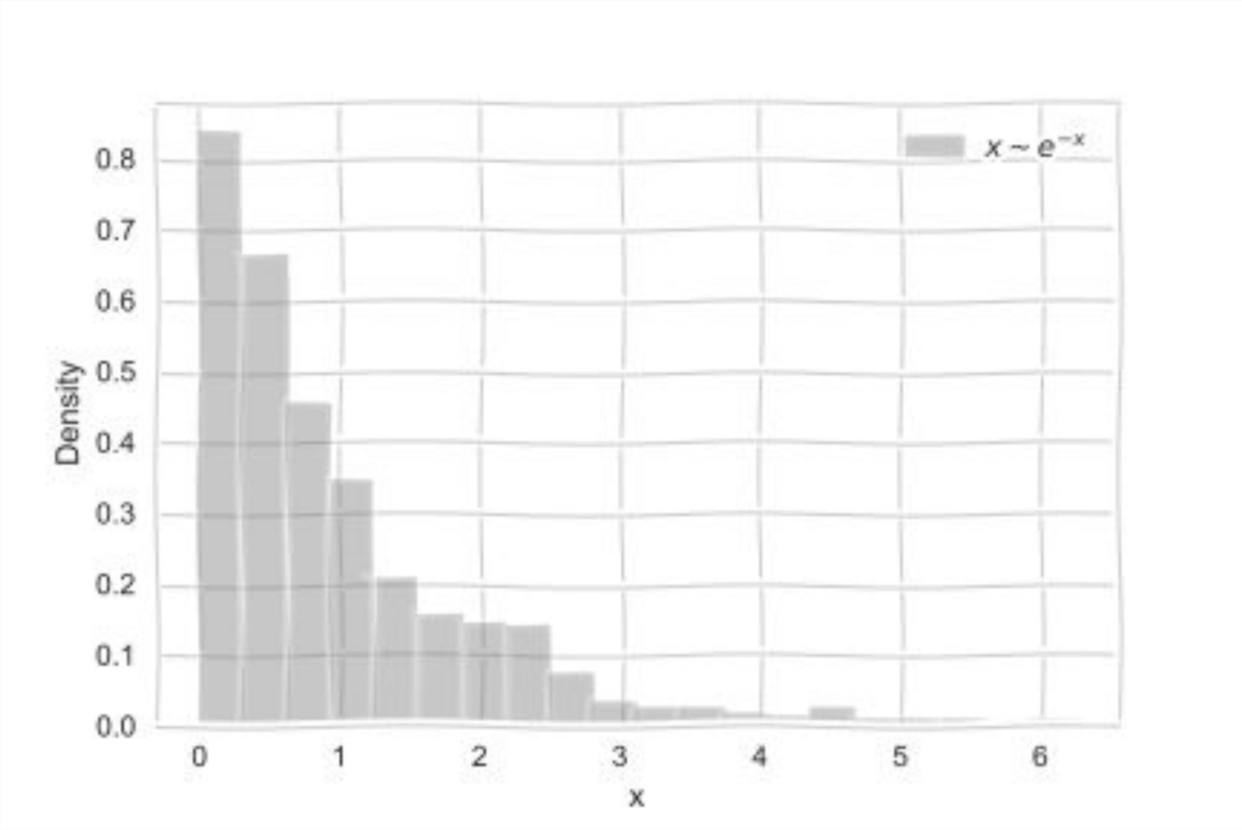
$$z \sim \mathrm{Unif}(0, 1)$$



Generative models

But how do we generate such samples?

$$z \sim \mathrm{Unif}(0,1) \quad \mathrm{x} = \ln \mathrm{z}$$



Generative models

In other words we can think that if we choose $\boldsymbol{z \sim Uniform}$ then there is a mapping:

$$x = f(z)$$

such as:

$$x \sim p(x)$$

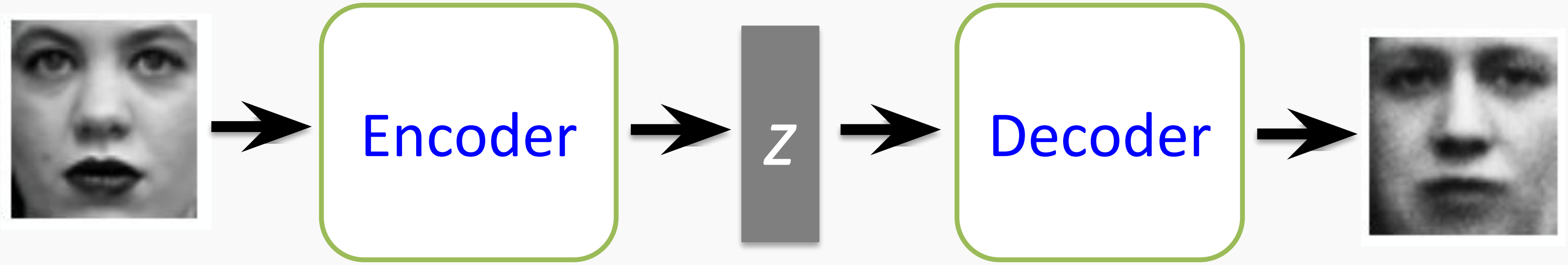
where in general f is some complicated function.

We already know that **Neural Networks are great in learning complex functions.**

$$\boxed{z \sim g(z)} \longrightarrow \boxed{x = f(z)} \longrightarrow \boxed{x \sim p(x)}$$

Traditional Autoencoders

In traditional autoencoders, we can thing of encoder and decoders as some function mapping.

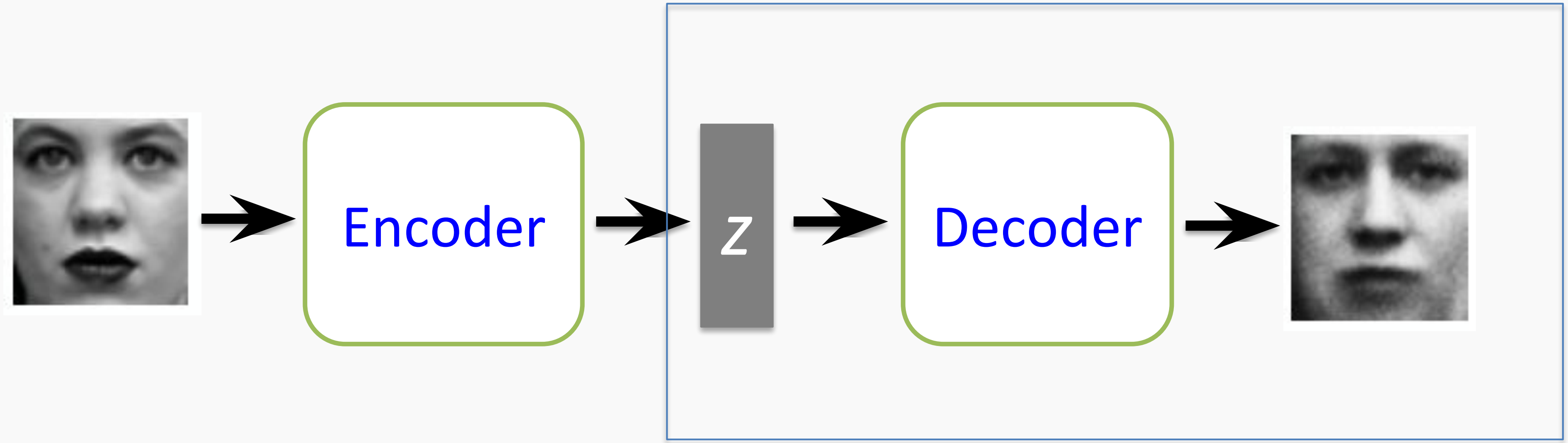


$$\boxed{z = h(x)}$$

$$\boxed{\hat{x} = f(z)}$$

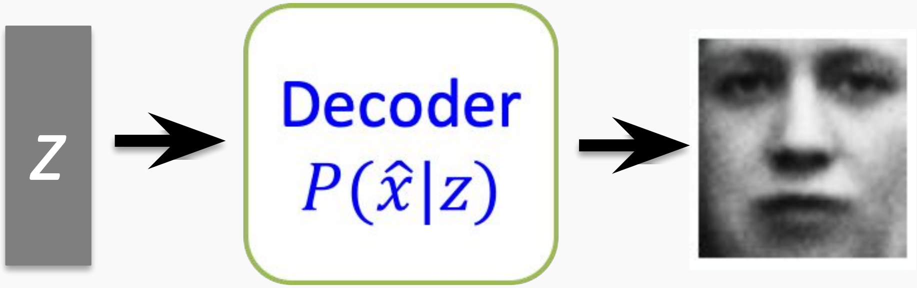
Variational Autoencoders

To go to variational autoencoders, we need to first add some stochasticity and think of it as a probabilistic modeling.



Variational Autoencoders

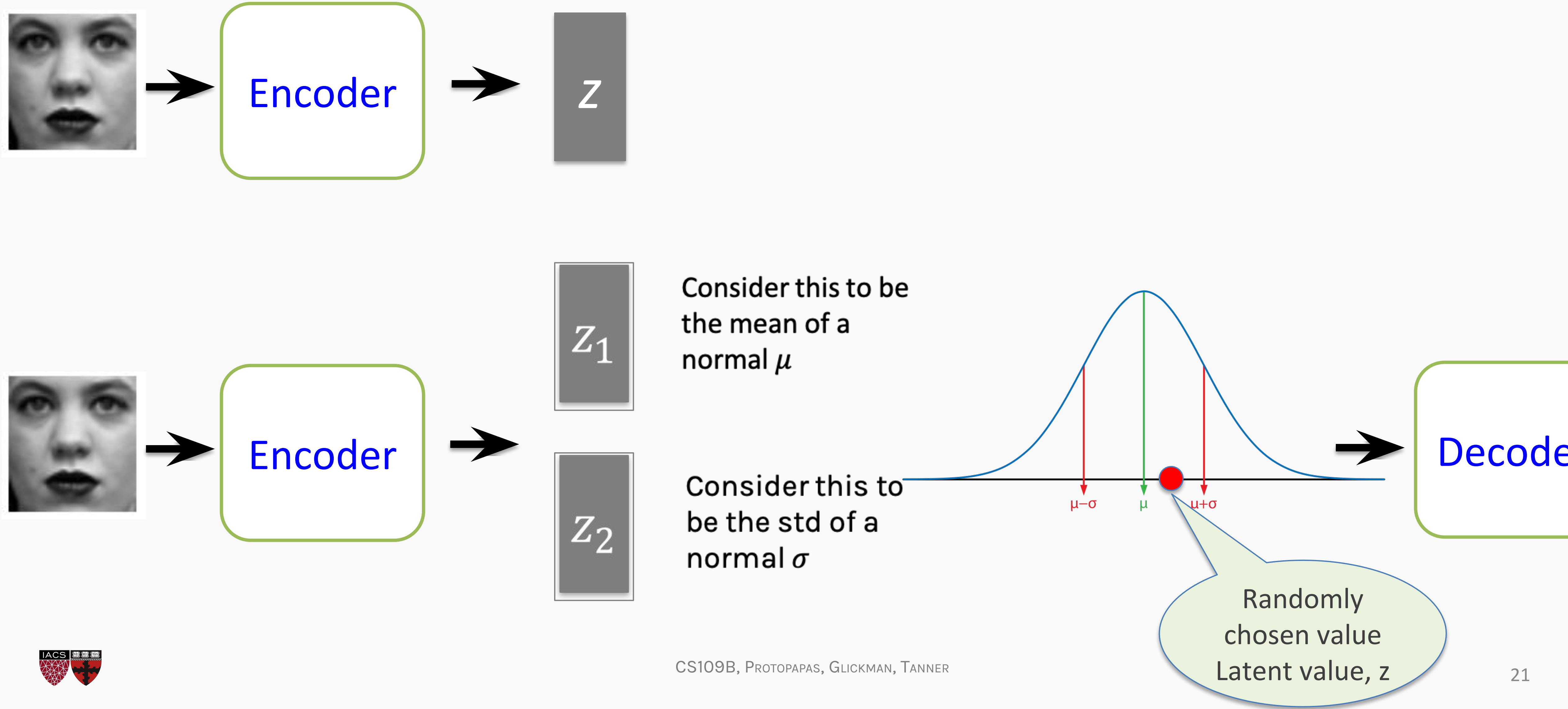
Sample from $g(z)$
e.g. Standard Gaussian



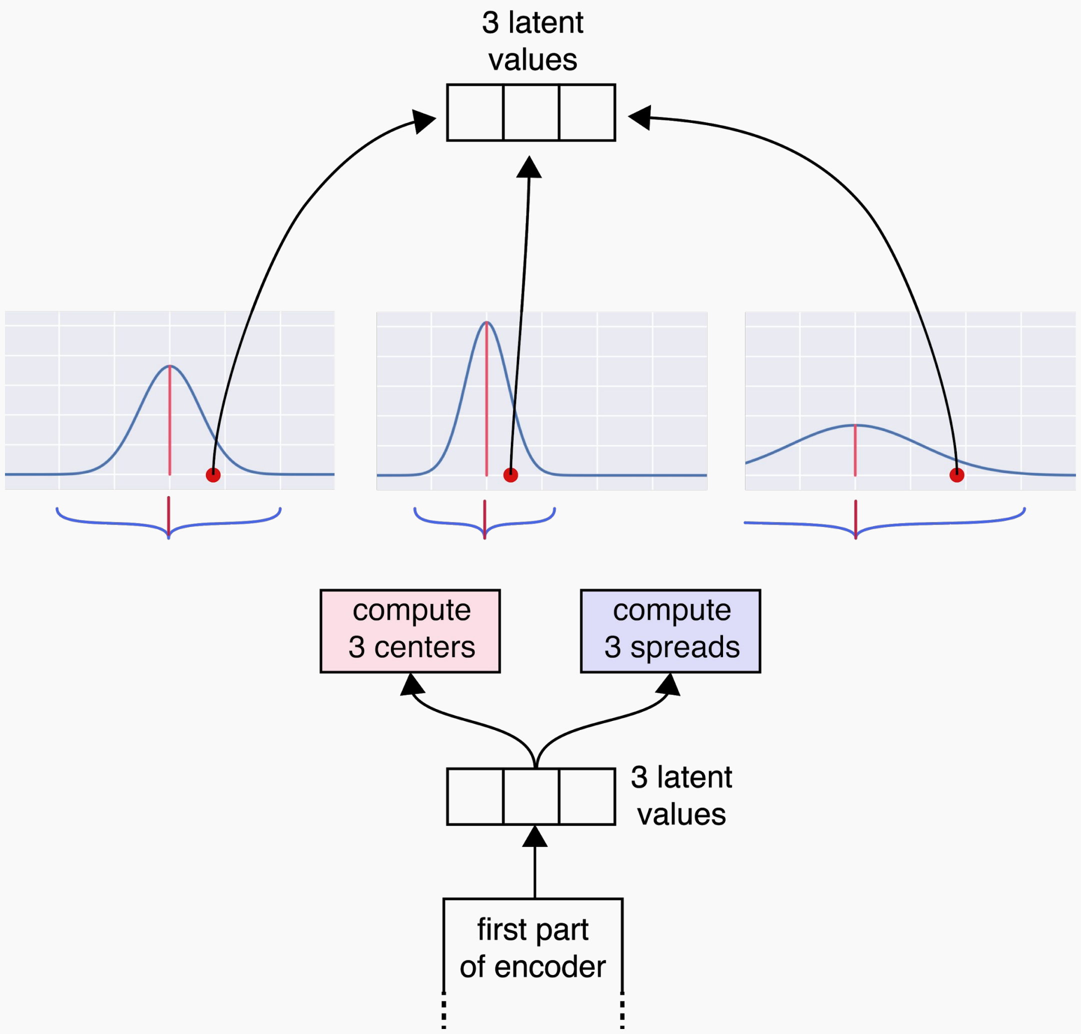
$z \sim g(z)$ $\hat{x} = f(z)$ $\hat{x} \sim P(x|z)$

Variational Autoencoders

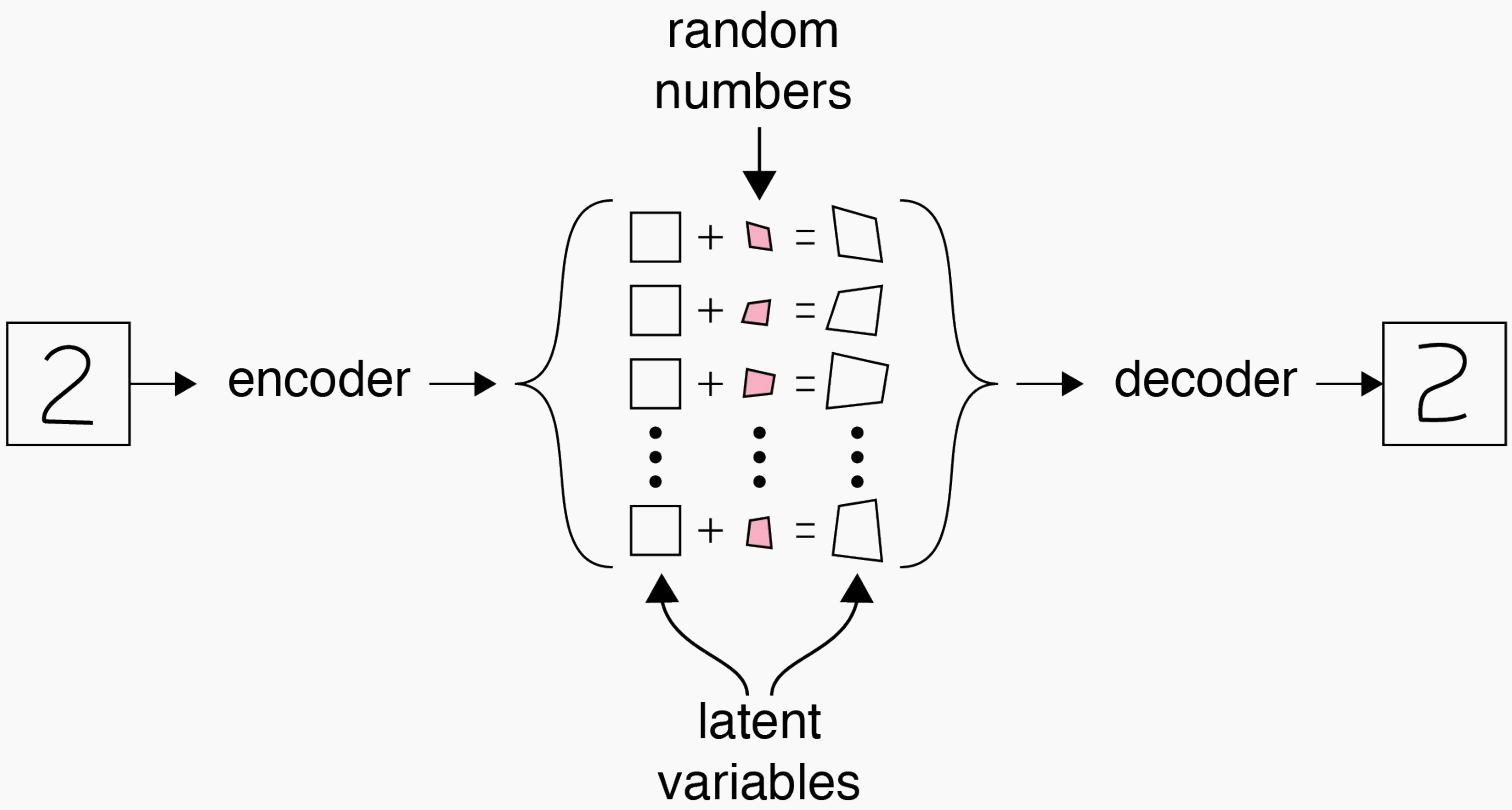
Traditional AE



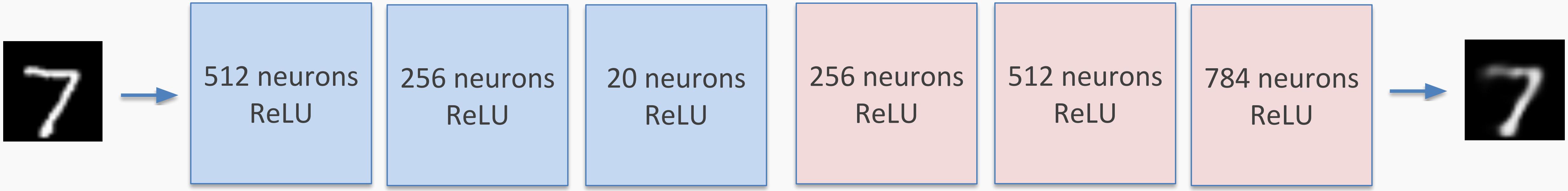
Variational Autoencoders

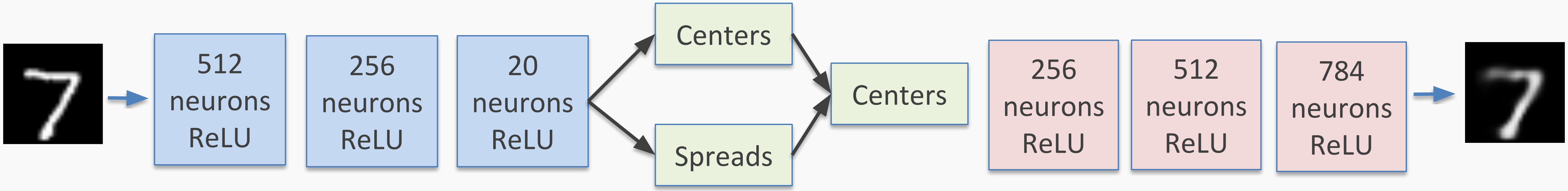


Variational Autoencoders



Variational Autoencoders





Outline

Motivation for Variational Autoencoders (VAE)

Mechanics of VAE

Separability of VAE

The math behind everything

Generative models

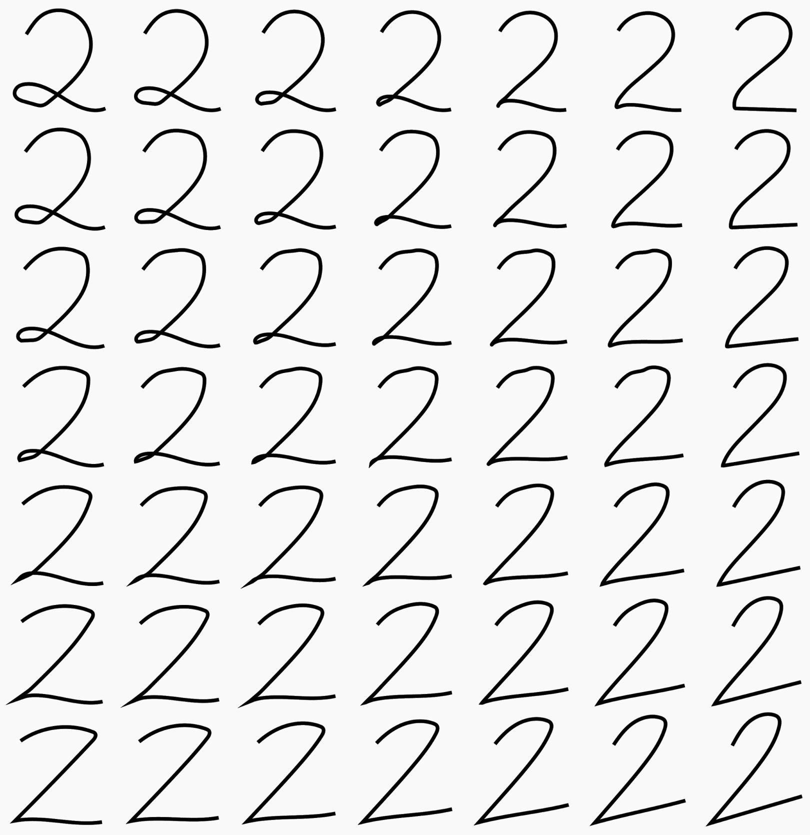
Separability in Variational Autoencoders

Separability is not only between classes but we also want similar items in the same class to be near each other.

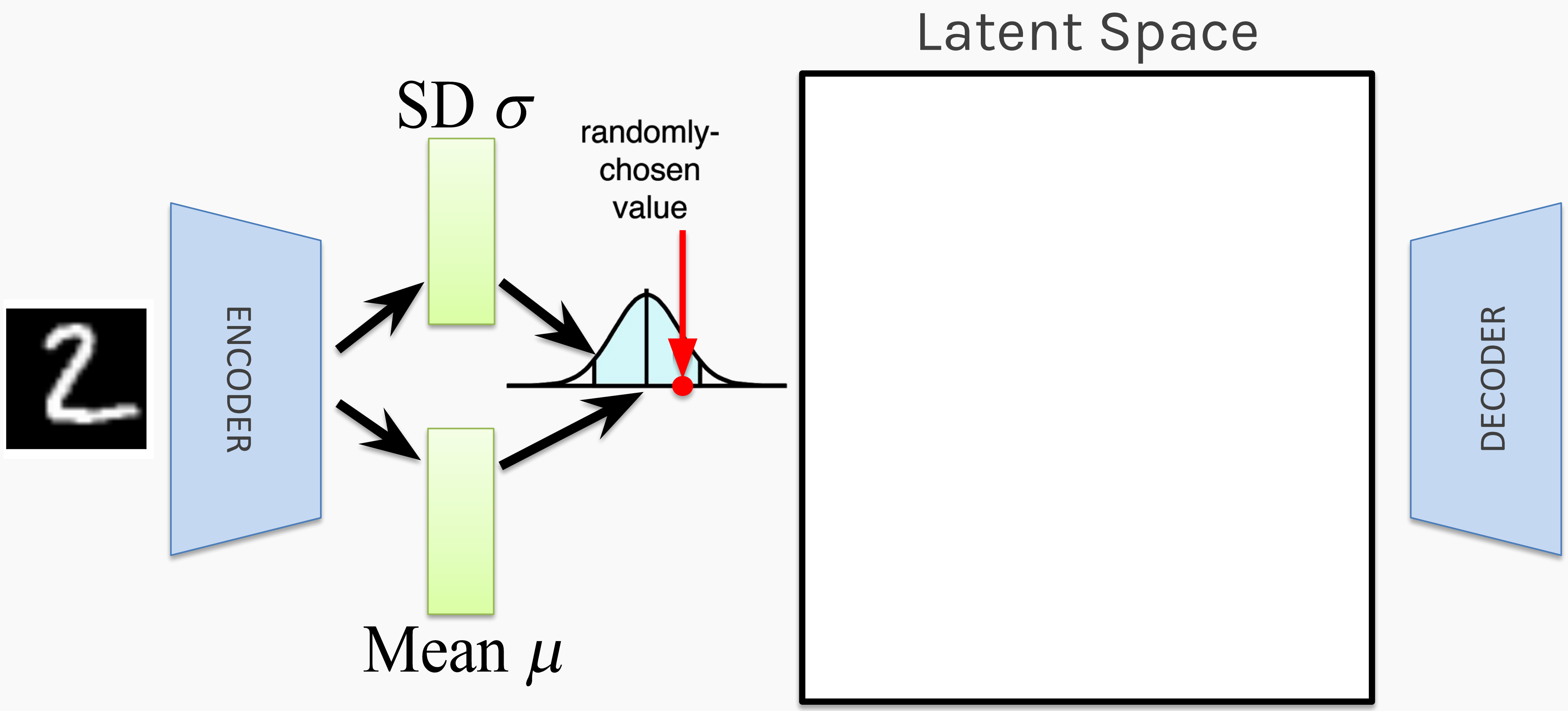
This is similar to word encoding we have talked in the previous lecture.

For example, there are different ways of writing "2", we want similar styles to end up near each other.

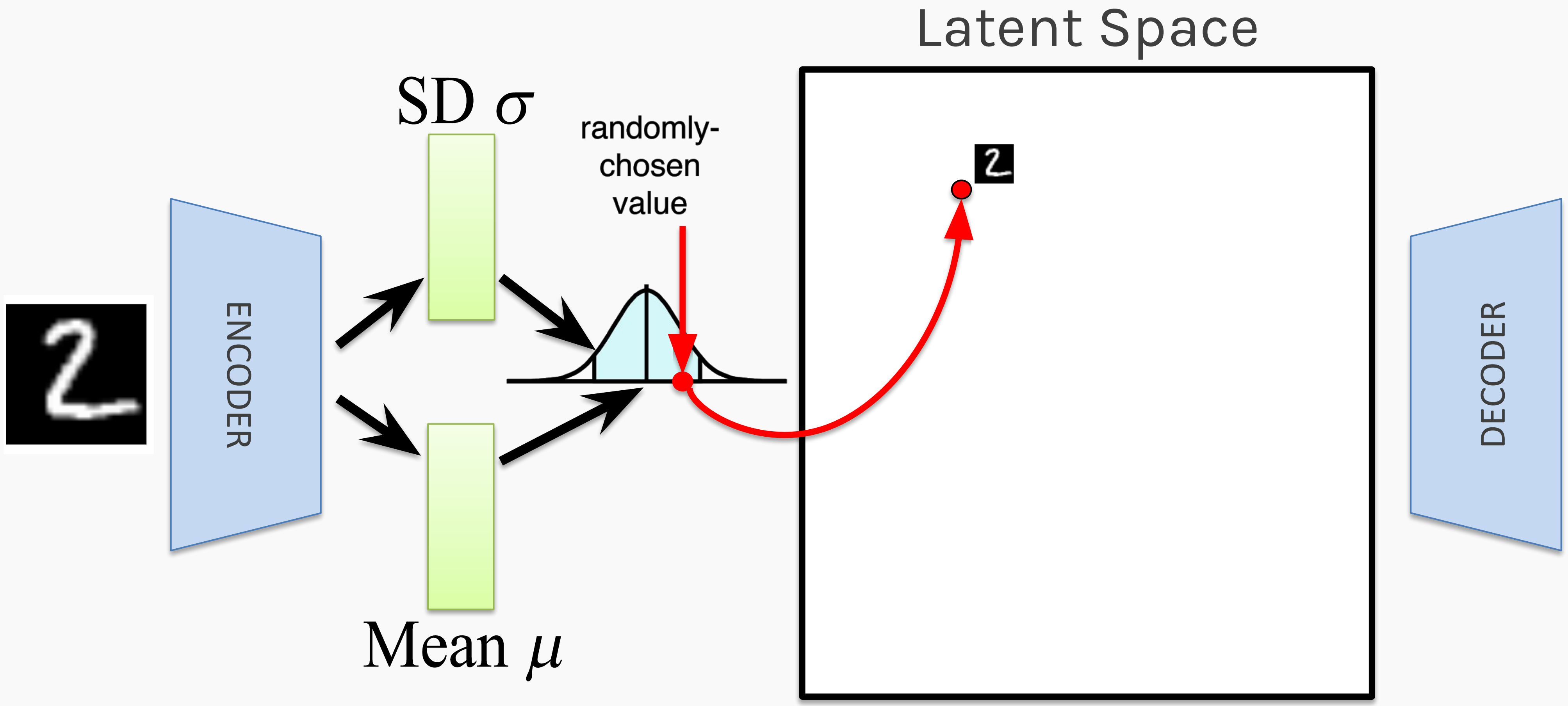
Let's examine VAE, there is something magic happening once we add stochasticity in the latent space.



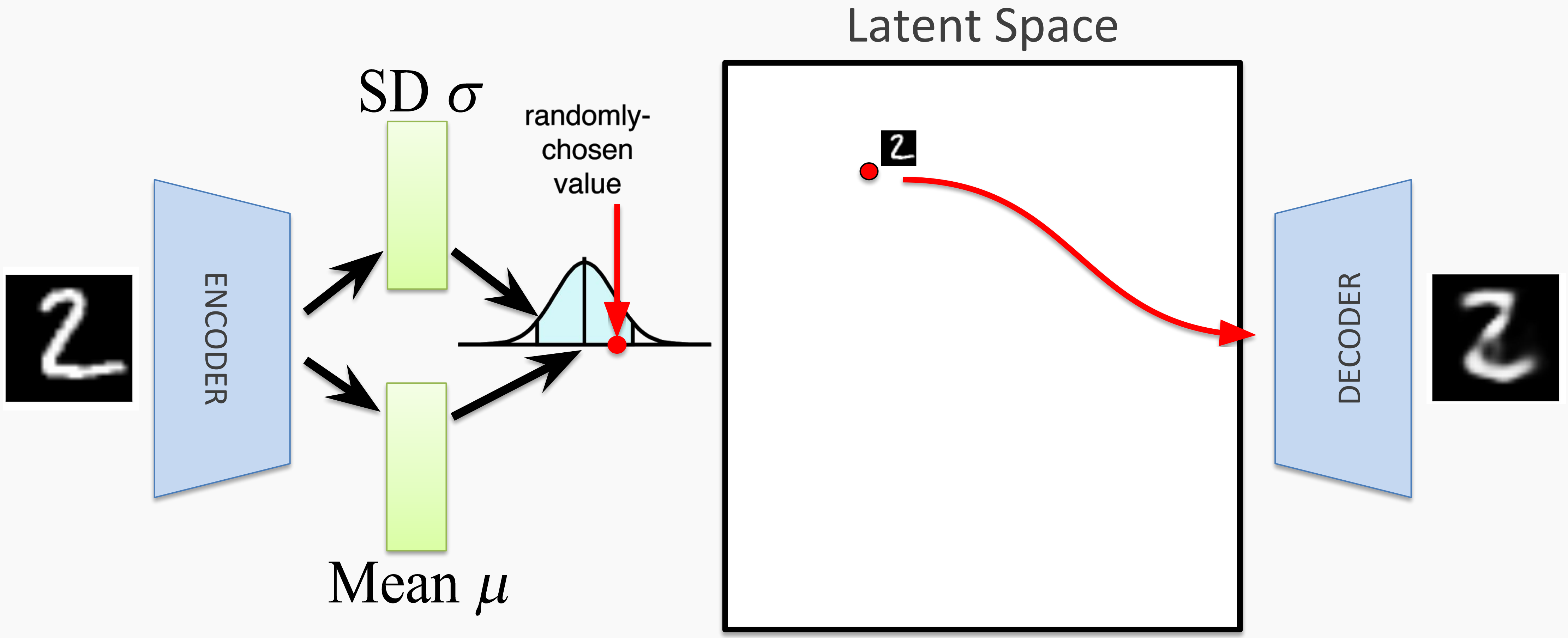
Separability in Variational Autoencoders



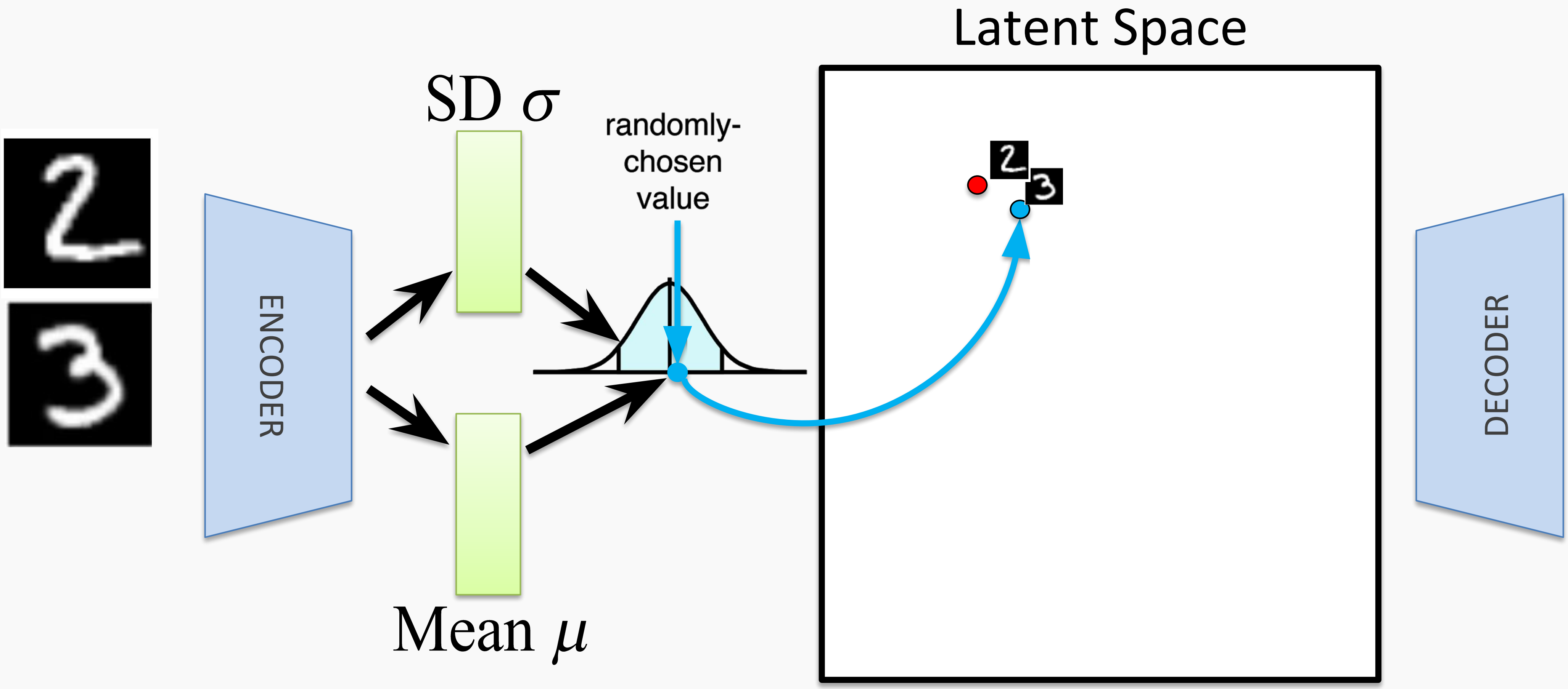
Separability in Variational Autoencoders



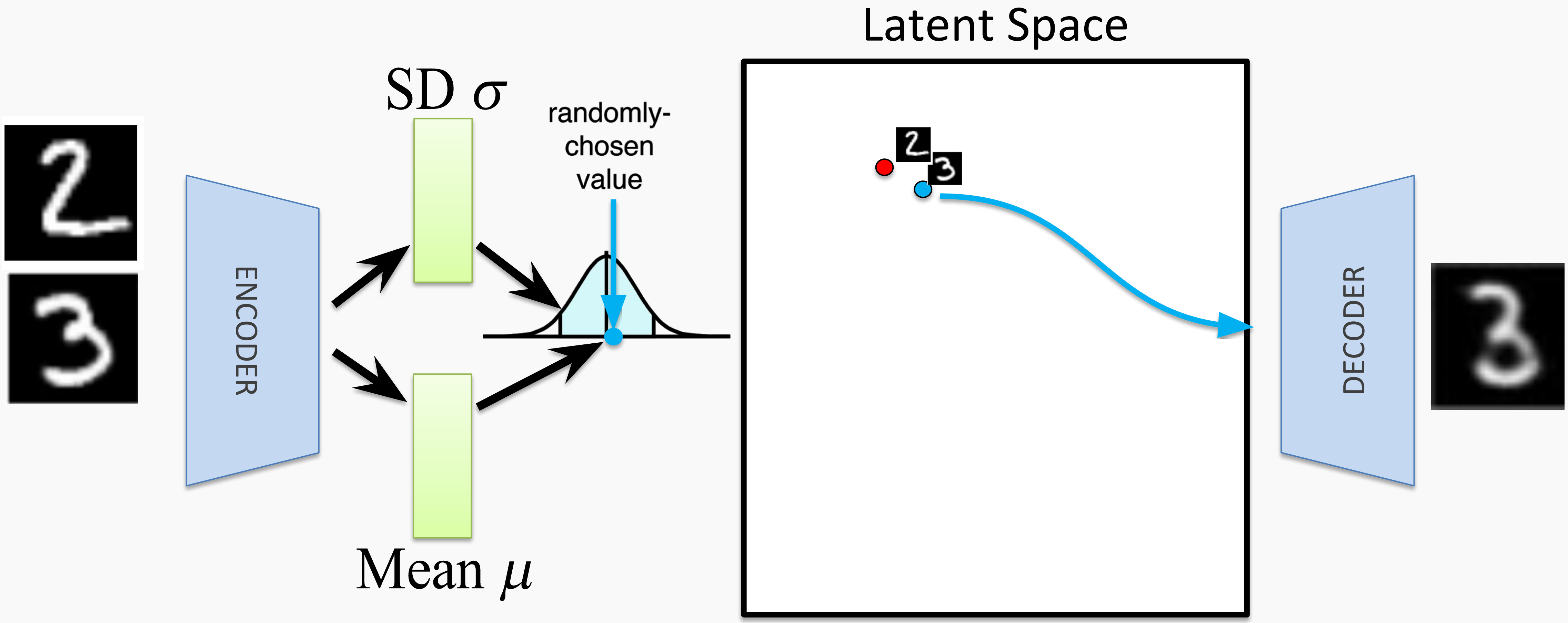
Blending Latent Variables



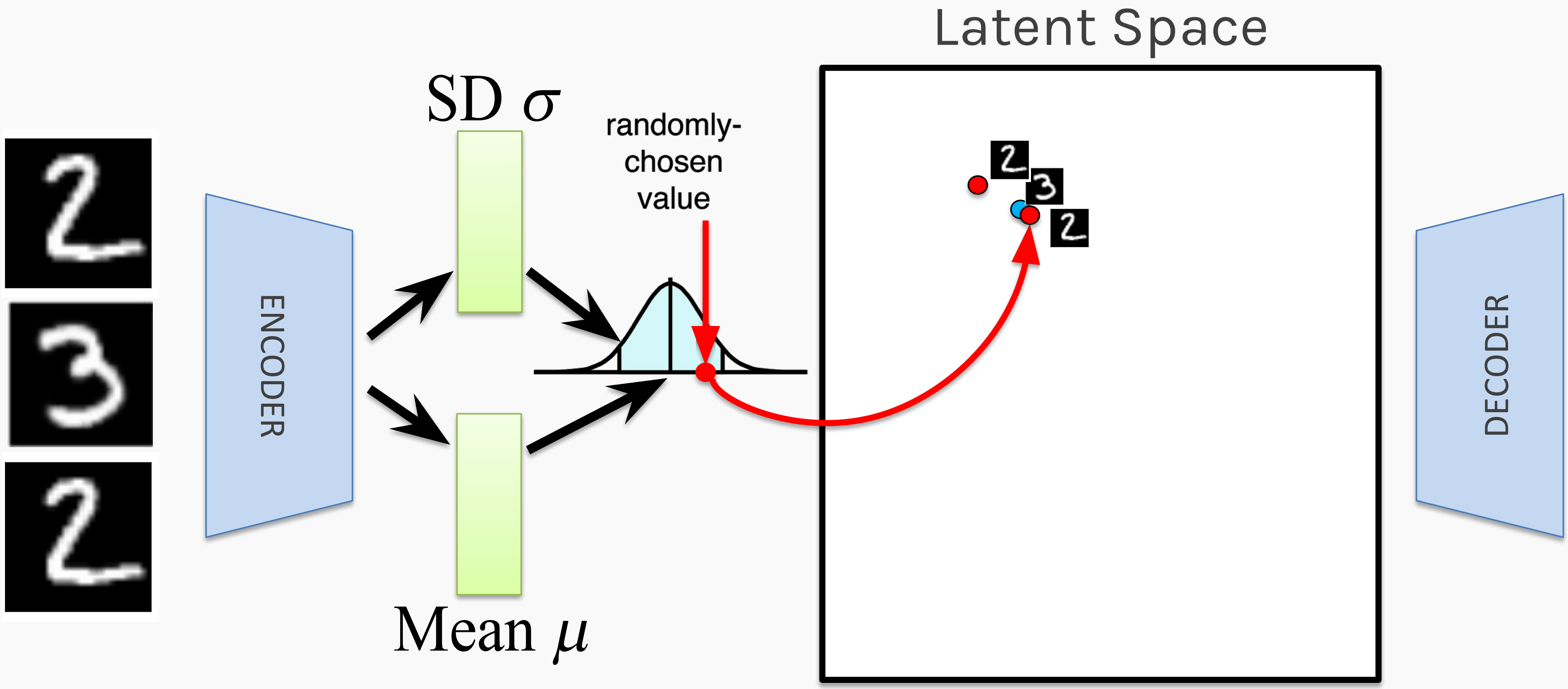
Separability in Variational Autoencoders



Separability in Variational Autoencoders

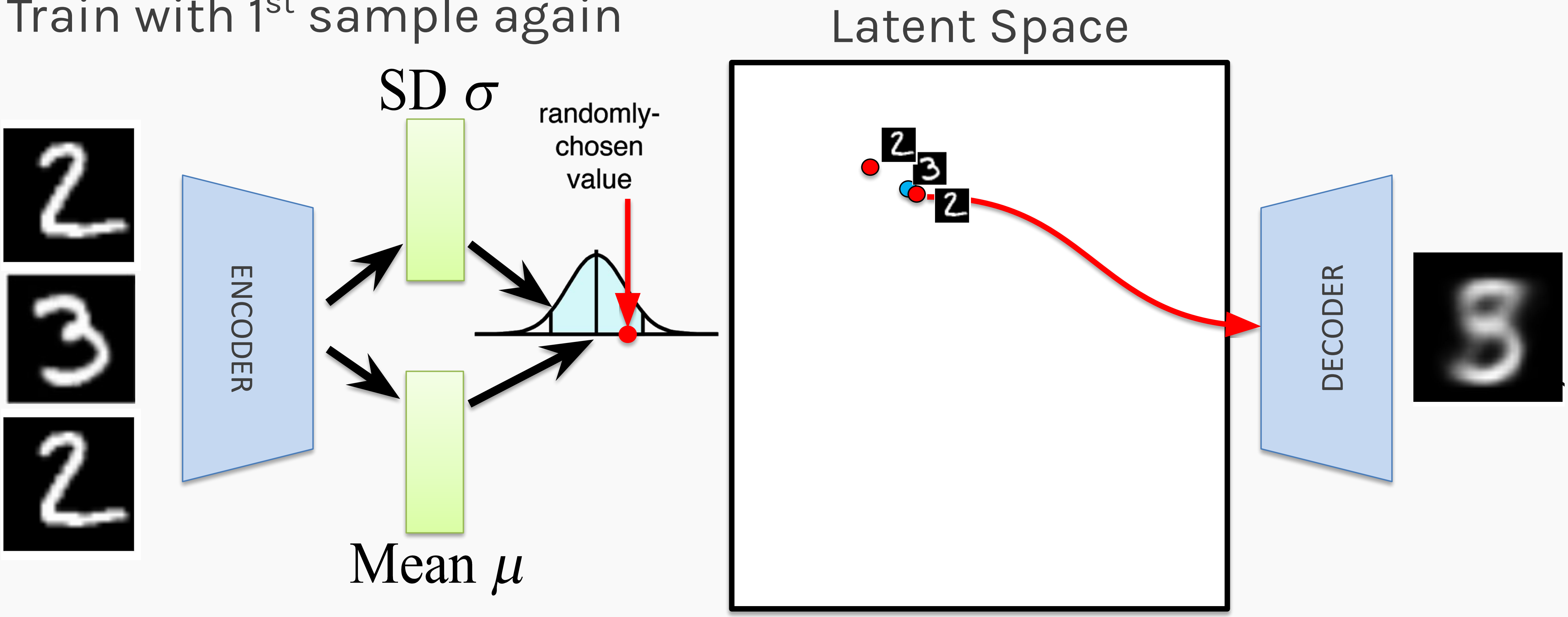


Separability in Variational Autoencoders

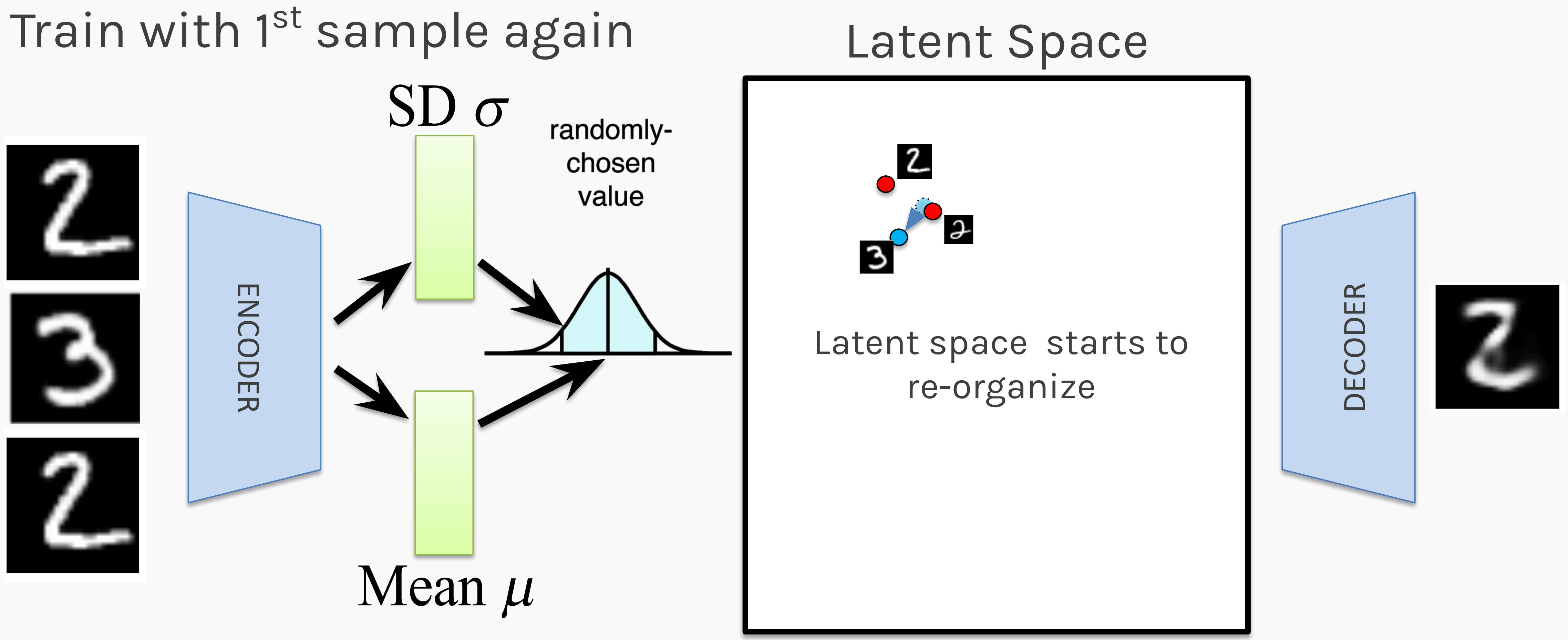


Separability in Variational Autoencoders

Train with 1st sample again

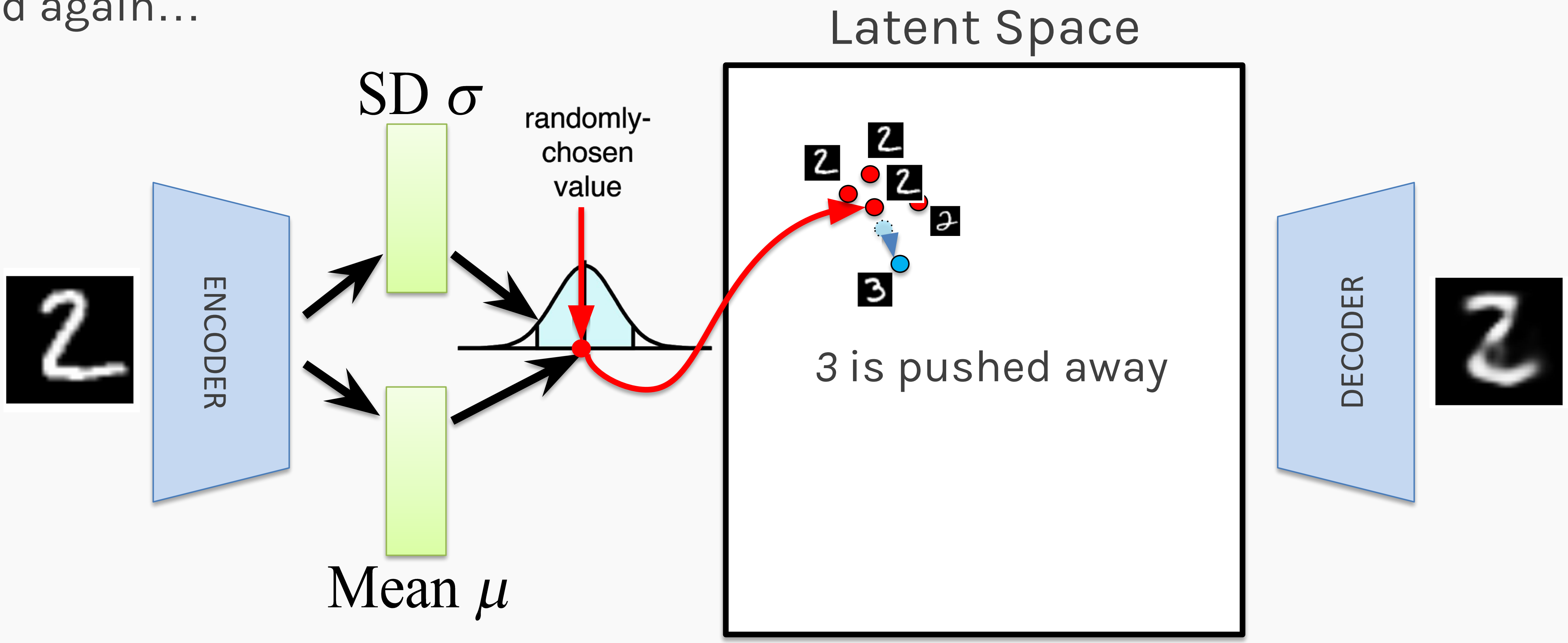


Separability in Variational Autoencoders



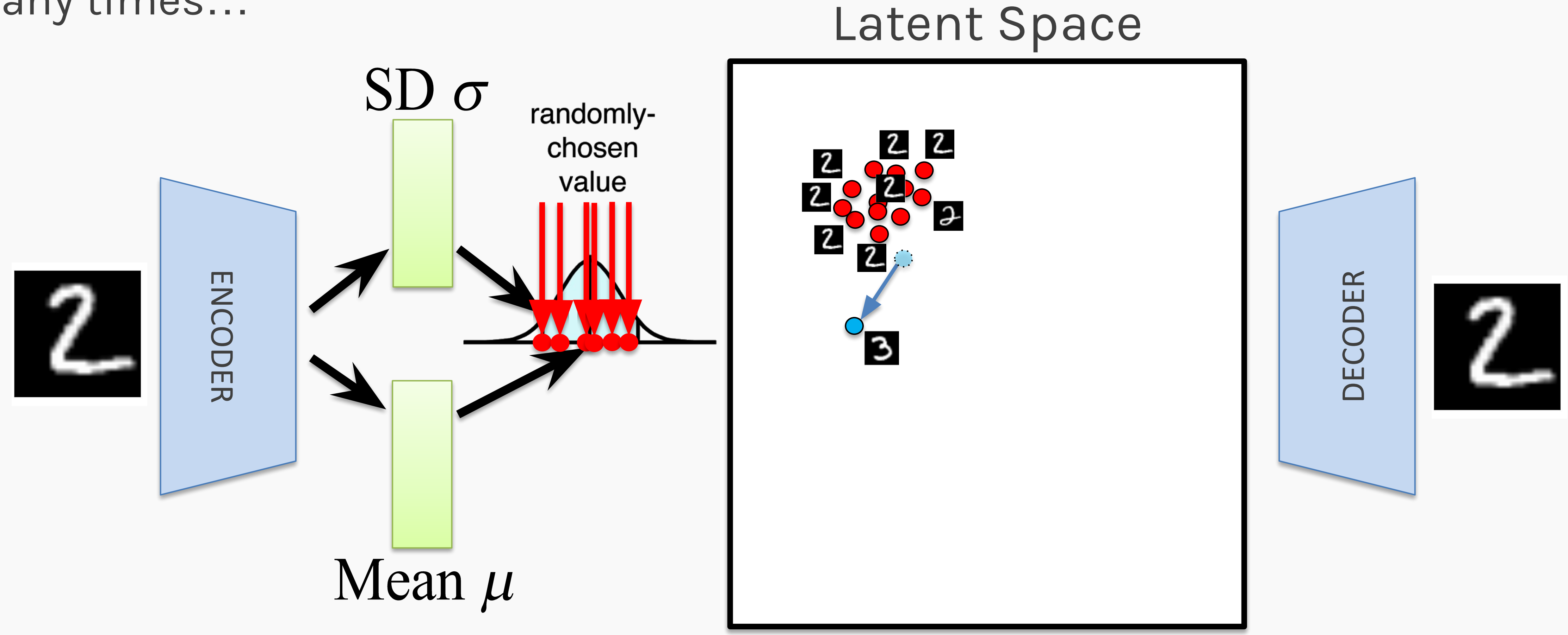
Separability in Variational Autoencoders

And again…



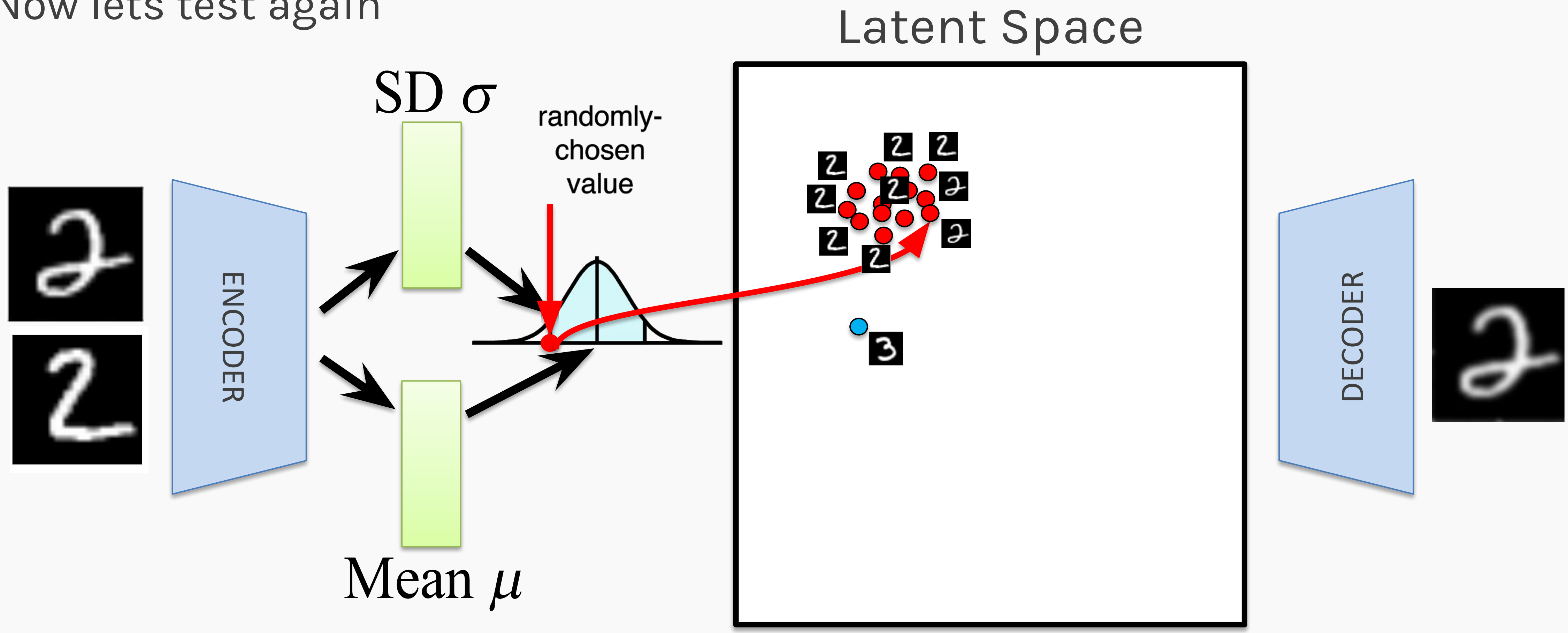
Separability in Variational Autoencoders

Many times…



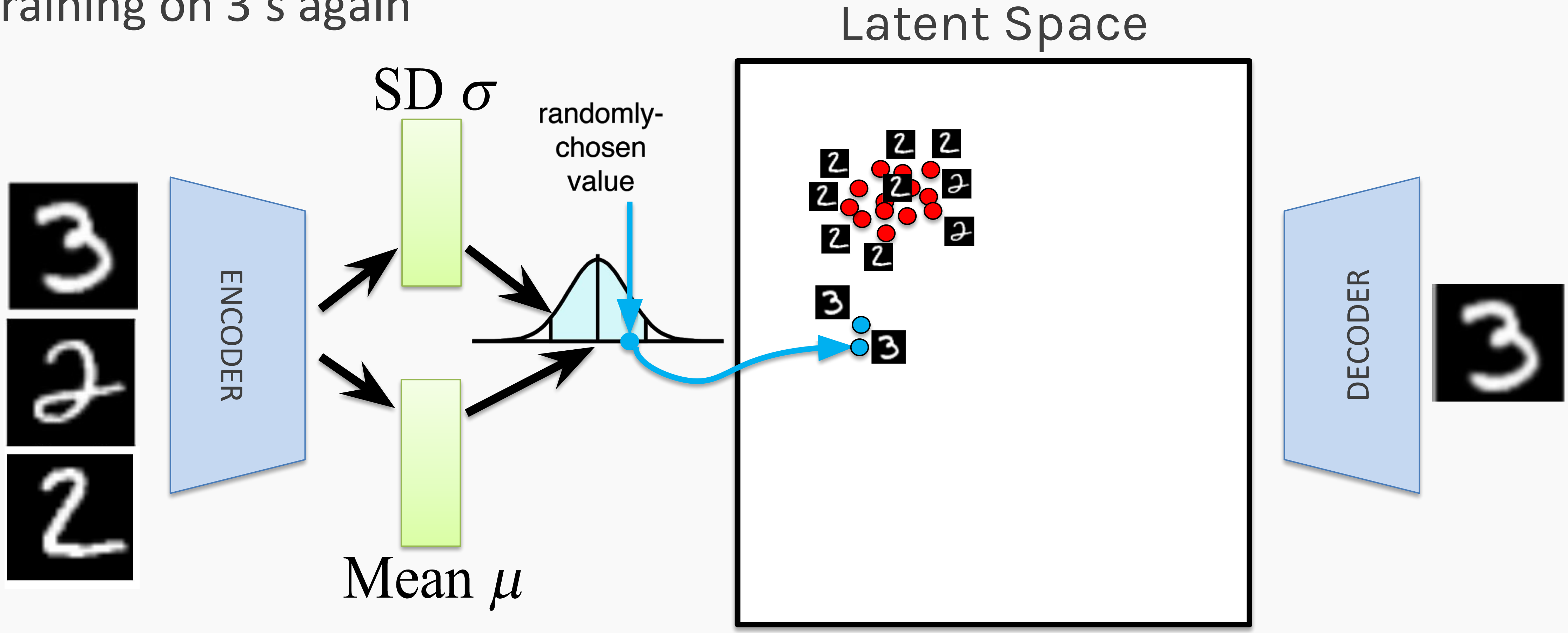
Separability in Variational Autoencoders

Now lets test again



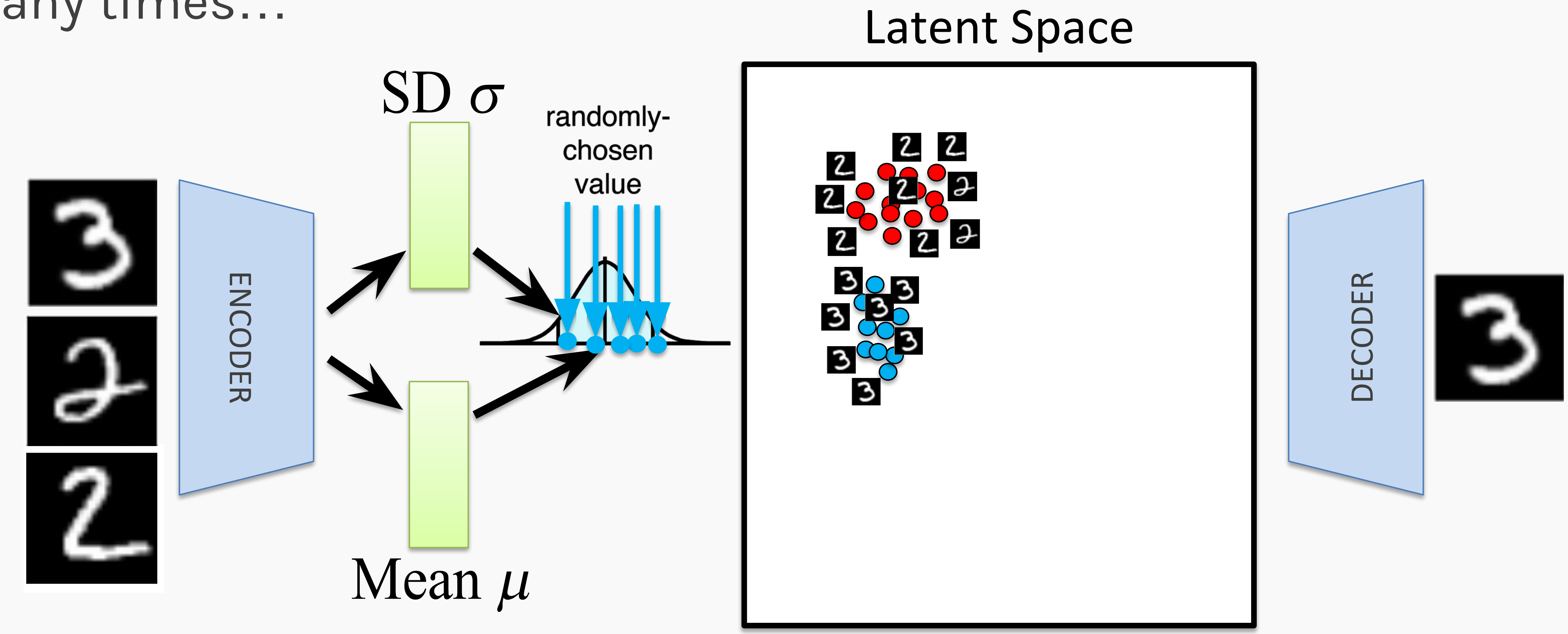
Separability in Variational Autoencoders

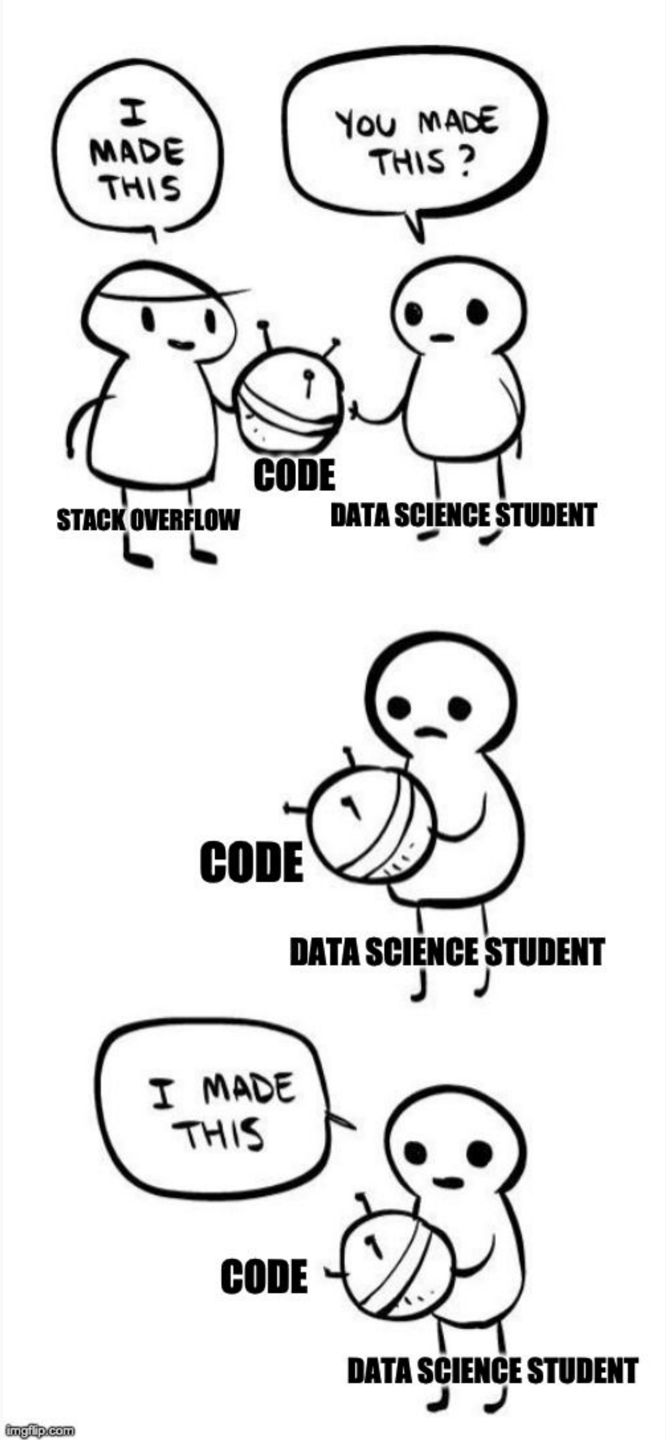
Training on 3's again



Separability in Variational Autoencoders

Many times...




I MADE THIS
YOU MADE THIS ?
CODE
STACK OVERFLOW
DATA SCIENCE STUDENT
CODE
DATA SCIENCE STUDENT
I MADE THIS
CODE
DATA SCIENCE STUDENT
imgflip.com

Outline

Motivation for Variational Autoencoders (VAE)

Mechanics of VAE

Seperability of VAE

The math behind everything

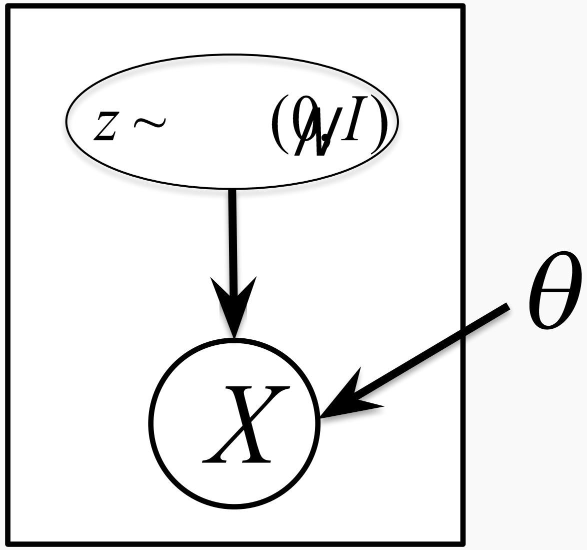
Generative models

VAE Likelihood

Neural network

$$p_\theta(x) = \int_z p_\theta(x|z)\boxed{p_\theta(z)}dz$$

Difficult to approximate in high dim through sampling



For most *z* values *p*(*x*|*z*) close to 0

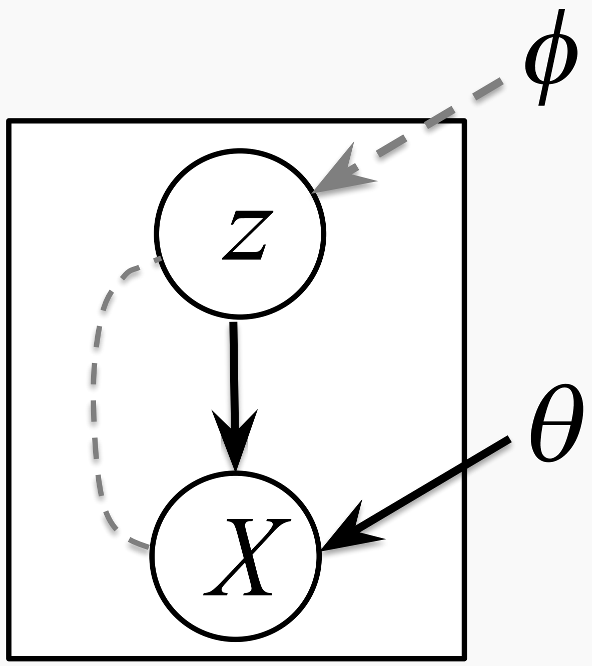
VAE Likelihood

Another neural net

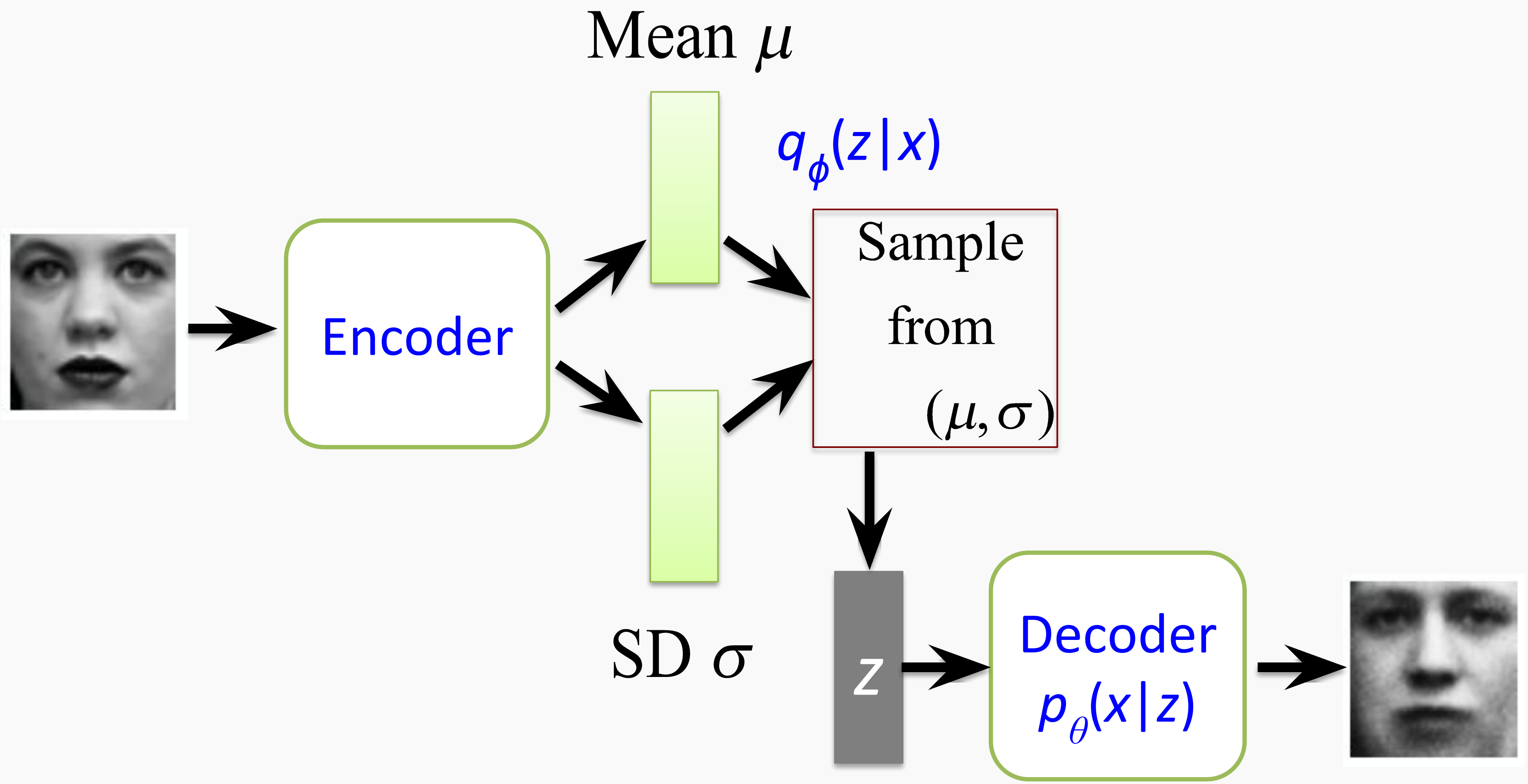
$$p_\theta(x) = \int_z p_\theta(x|z)\boxed{q_\phi(z|x)}dz$$

Proposal distribution:

Likely to produce values of *x* for which *p*(*x*|*z*) is non-zero



VAE Architecture



VAE Loss

Reconstruction Loss

$$-\mathbf{E}_{z\sim q_\phi(z|x)} \log\left(p_\theta\left(x|z\right)\right)$$

VAE Loss

Reconstruction Loss

Proposal distribution should resemble a Gaussian

$$-\mathbf{E}_{z\sim \boxed{q_\phi(z|x)}} \log\left(p_\theta\left(x|z\right)\right) + KL\left(\boxed{q_\phi\left(z|x\right)} \middle\| p_\theta(z)\right)$$

VAE Loss

Reconstruction Loss

Proposal distribution should resemble a Gaussian

$$-\mathbf{E}_{z\sim \boxed{q_\phi(z|x)}} \log\left(p_\theta\left(x|z\right)\right) + KL\left(\boxed{q_\phi\left(z|x\right)} \| p_\theta(z)\right)$$

$$\geq -\log p_\theta(x)$$

Variational upper bound on loss we care about!

Training VAE

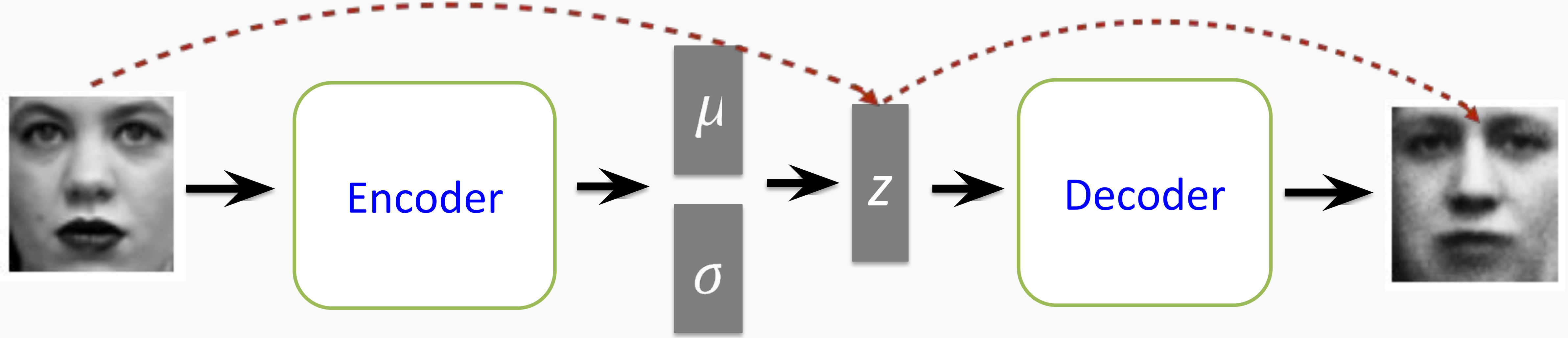
Apply stochastic gradient descent

Sampling step not differentiable

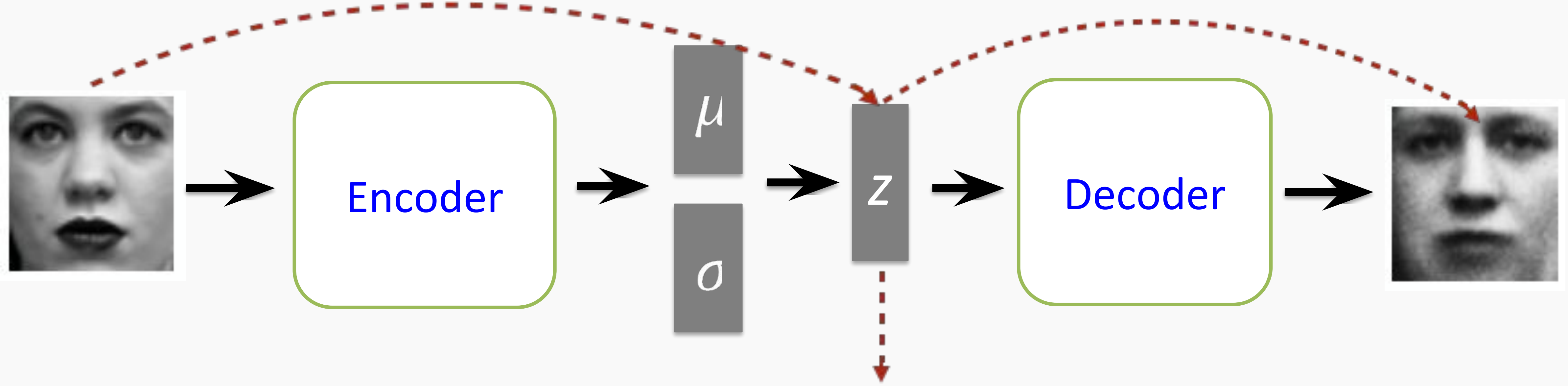
Use a re-parameterization trick

- Move sampling to input layer, so that the sampling step is independent of the model

Reparametrization Trick

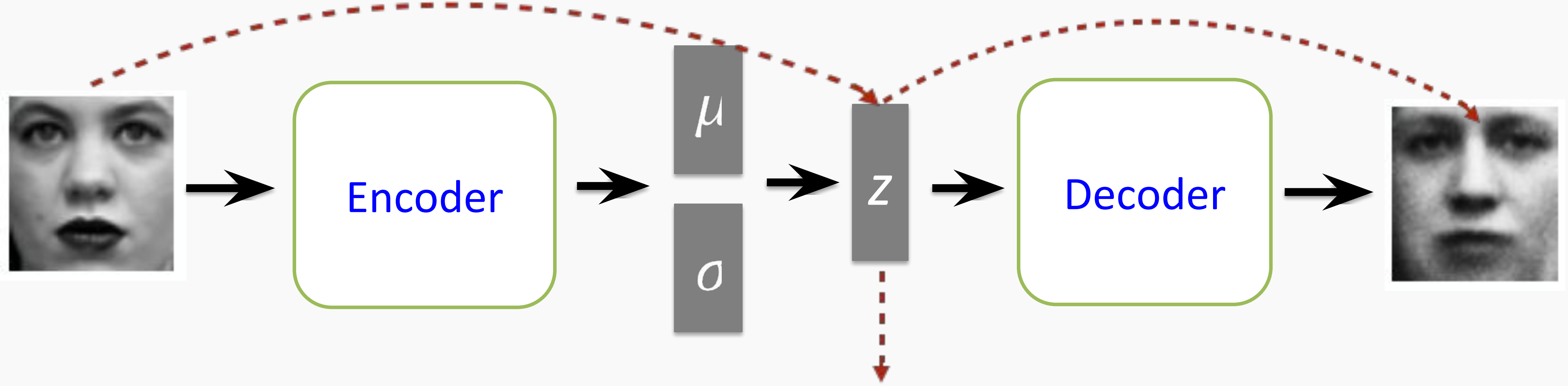


Reparametrization Trick



$$Z = \mu + \varepsilon \circ \sigma$$

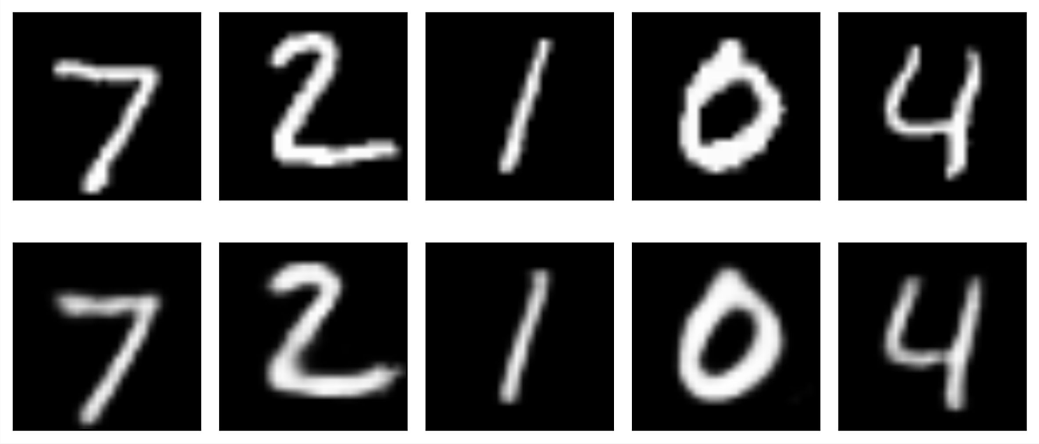
Reparametrization Trick

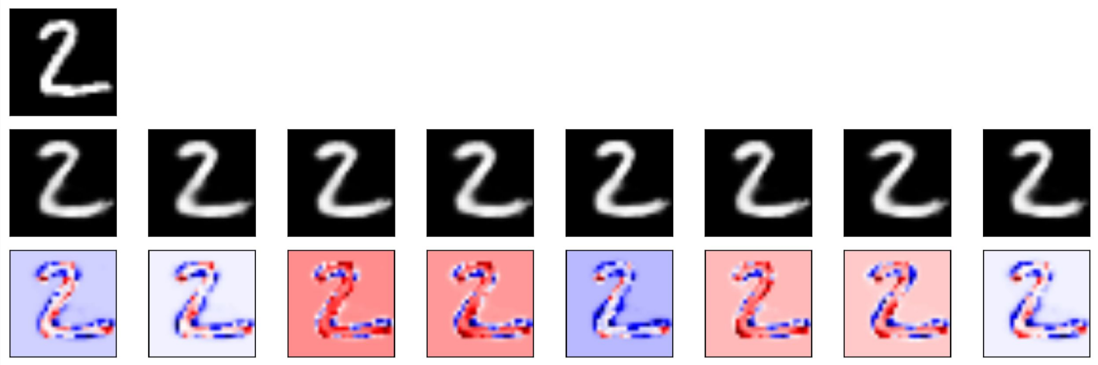


$$Z = \mu + \varepsilon \circ \sigma$$

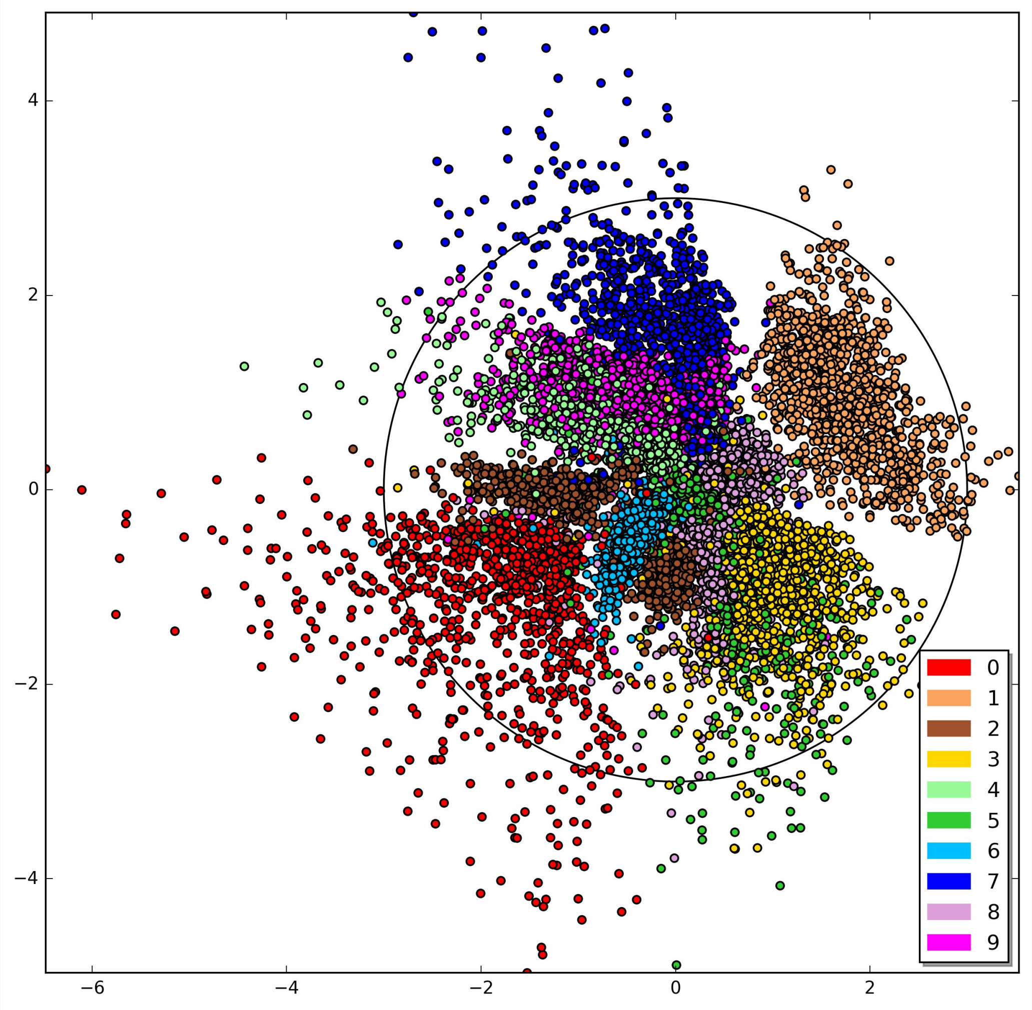
$$\varepsilon \sim N(0, I)$$

Training VAE

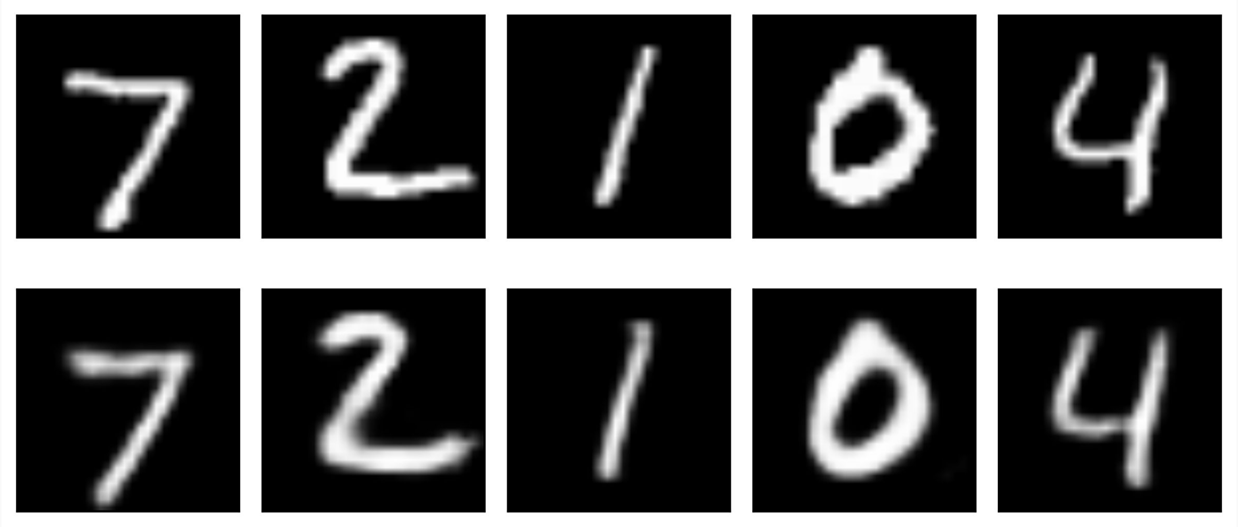




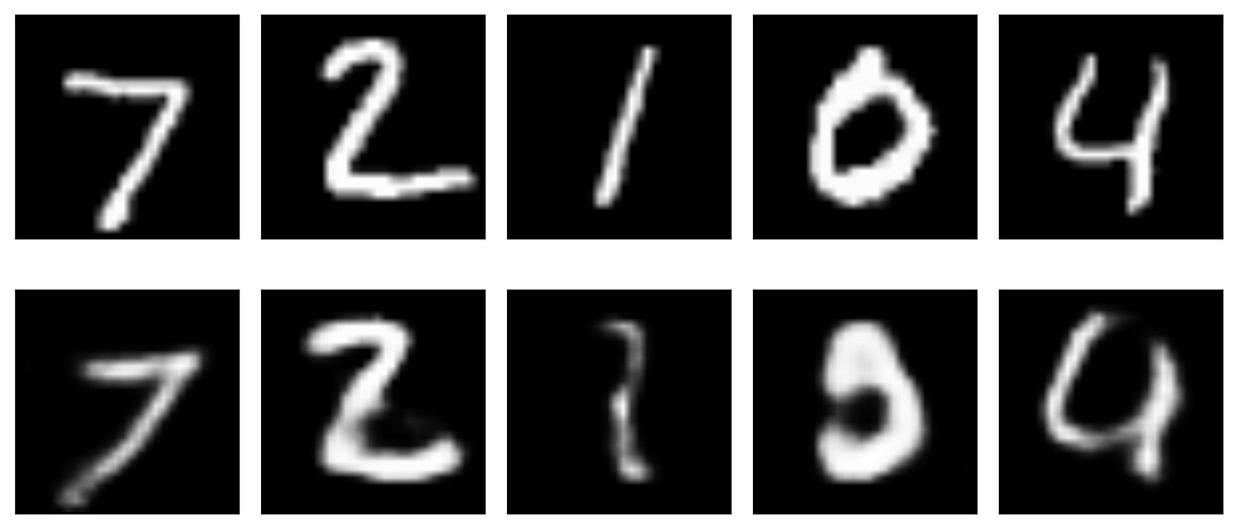
Parameter space VAE



Training VAE



10% error



30% error

Parameter space VAE