Trajectory Generation Using MPC For High Speed Overtaking

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ABSTRACT

In this study model predictive control is applied to a vehicle overtaking slower moving vehicles in a one way, two lane road. A risk map is defined considering the road boundaries, the center of the two lanes, and distance relative to other vehicles. The results of this study found that the vehicle was able to conduct safe lane changes while avoiding unsafe regions defined by the risk map. The video link for this study is as followed:
https://drive.google.com/file/d/1YX1ZyK55xWvT7UN3O9-mr-iWDYp2QAie/view?usp=sharing

1. Introduction

Autonomous driving has become one of the major goals for automotive companies, where companies like Tesla and Porche have demonstrated promising results in recent history [1]. One of the major goals functions that autonomous driving will need to accomplish is to perform lane changes, as lane changing accounts for at least 33 percent of all car accidents [2]. As such, automotive companies will need closely optimized control over vehicle motion and trajectory to allow for safe lane changes in many driving scenarios. In reality, lane change accidents are a result of many different driving scenarios, however a major common scenario is a faster vehicle hitting a slower moving target. In this paper, we use a robust model predictive control for a vehicle lane changing on a one way 2-lane road, where a fast moving vehicle overtakes a slower vehicle while avoiding collision. From a defined trajectory path, model predictive control will be used to guide the target vehicle on the trajectory path while subject to motion constraints.

2. Model Dynamics

For the model to define the dynamics of our system, we used the bicycle model discussed both in class and in another paper conducting a similar study [3].A nonlinear kinematic vehicle model assumes no slip between tire and road is found to be suitable for trajectory planning for highway driving. Furthermore, since normal driving on the highway involves small steering inputs, small angles approximation for the side-slip angle and steering angle are assumed. Under this assumption of small angles approximation the vehicle bicycle model is:

$$\dot{x} = v \tag{1}$$

$$\dot{y} = v\psi + \frac{l_r}{l_f + l_r} v\delta_f \tag{2}$$

$$\dot{\psi} = \frac{l}{l_f + l_r} v\delta_f \tag{3}$$

$$\dot{v} = a_x \tag{4}$$

where x and y are the longitudinal and lateral displacement of the centre of gravity in the I-frame, ψ is the inertial heading angle, v is the velocity of the vehicle, l_f is the distance of front axle from centre of gravity, and l_r is the distance of the rear axle from the centre of gravity. The control inputs are longitudinal acceleration a_x and front steering angle f. In this paper, the described model is used for computing the reachability sets of a vehicle to identify safe driving zones, while for the generation of the vehicle trajectory toward a target point. However the system dynamics faces nonlinearity of system. The model is rewritten as a linear time invariant (LTI) system subjected to an additive bounded disturbance. The system can be made as a linear parameter varying system. The resulting matrices for the system are shown below:

$$\dot{x}_a = A_c(v)x_a + B_c u \tag{4}$$

$$A_c(v) = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & v & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_c(v) = \begin{bmatrix} 0 & 0 \\ 0 & \dfrac{v \cdot l_f}{l_f + l_r} \\ 0 & \dfrac{v}{l_f + l_r} \\ 1 & 0 \end{bmatrix}$$

3. MPC Approach

In this project, a trajectory generation approach was deployed. In the first step the subject vehicle, lead vehicle 1, lead vehicle 2 and lead vehicle 3 are initialized. Afterwards, for each time step, the reachable set for the subject vehicle is calculated based on the cars bicycle model. To take the other lead vehicles and the surrounding into account for the MPC, a risk map is evaluated. The risk map is based on a road potential, lane potential and a car potential which will be discussed in detail in the next paragraphs. The risk map prevents the car from driving off the road or the lane and crashing into other vehicles by penalizing any actions leading to those events. At this point, an optimal target set with minimal cost is analyzed by combining the reachable set and the risk map. The target set combines the set where the risk in the risk map is the lowest and that is also reachable by the subject vehicle. In order to reach the end of the road the fastest, the target point for the CFTOC simulation is chosen as the furthest point in the target set along the x axis. The CFTOC calculates the predicted trajectory for our system with our chosen target point for each time step.

The MPC in this project has a simulation horizon of 165 steps and a prediction horizon of 10 steps. The cost function is defined as a quadratic function with Q, P and R as positive definite matrices. The cost is calculated as the error between the subject vehicle state and the target state. In order to keep a constant vehicle speed, the cost for the velocity difference is increased. The R matrix minimizes the input cost for the vehicle model. The terminal state has it's own cost matrix P. Additionally, X are the state constraints and U are the input constraints for the subject vehicle. However, The terminal constraint X_f in this MPC only considers the first two state variables namely the x position and y position. Therefore, the terminal constraint has the form $[1, 1, 1, 1]^T x_N = [x_{target}, y_{target}, noconstraint, noconstraint]^T$.

$$\begin{aligned}
\min_{x_0,\ldots,x_N,u_0,\ldots,u_{N-1}} \quad & (x_N - x_{target})^T P (x_N - x_{target}) + \\
& \sum_{i=0}^{N-1} (x_i - x_{target})^T Q (x_i - x_{target}) + u_i^T R u_i \\
\text{s.t.} \quad & x_{k+1} = A x_k + B u_k \qquad \forall k = 0, ..., N-1 \\
& x_{min} \leq x_k \leq x_{max} \qquad \forall k = 0, ..., N-1 \\
& u_{min} \leq u_k \leq u_{max} \qquad \forall k = 0, ..., N-1 \\
& x_0 = x(0) \\
& x_N = X_f
\end{aligned} \tag{5}$$

4. Risk Map

As mentioned in the introduction, the vehicle will be performing high speed lane changing on a one way, two lane road. The vehicle will need to have awareness of the road boundaries, the center of the 2 different lanes, and vehicle location relative to slower moving vehicles in the two lanes. As such, to define the risk map we define the road boundary potential, lane potential,car potentials, and the sum of all these potentials.

4.1. Road Boundary Potential

The road boundary potential is a potential function which prevents the subject vehicle from getting closer to the edges of the road. The general formula defining this potential is shown below:

$$U_{\text{road}} = \frac{1}{2} X_{\text{i}} \sum_{b=1}^{2} \left(\frac{1}{y_{\text{r}} - y_{\text{r},b}} \right)^2 \tag{6}$$

where X_{i} is a scaling factor to be defined, y_{r} is the target point, $y_{\text{r},b}$ is the y coordinate of the road edge at top or bottom of road. This potential function is designed to reach infinity near the boundaries, and approach zero at the center of the road.

4.2. Lane Potential

The lane potential function is a potential function which forces the subject vehicle to stay in the center of the road. The general form of this equation is defined below:

$$U_{\text{lane},i} = A_{\text{lane}} \exp \left(\frac{-\left(y_{\text{r}} - y_{\text{l},i}\right)^2}{2\sigma^2} \right) \tag{7}$$

where A_{lane} and σ are scaling factors, and $\eta_{\text{l},i}$ is the y coordinate of the ith lane.This potential creates a barrier between lanes to a have vehicle centered at lane center.

4.3. Vehicle Potential

The vehicle potential is the potential function which allows the subject vehicle to avoid lead vehicles in any of the two lanes. The general form of this potential equation is shown below:

$$U_{\text{car}} = A_{\text{car}} \left(\frac{e^{-\alpha K_{\text{d}}}}{K_{\text{d}}} \right) \tag{8}$$

where A_{car} and α are Yuwaka amplitude and scaling factors [4], and K_{d} is the distance from the nearest point to slower moving vehicles.

4.4. Combined Risk Map

Taking all of the potentials into consideration, a complete risk map for the subject vehicle can be compiled by summing the individual potentials as shown below:

$$U_{\text{r}} = U_{\text{road}} + U_{\text{lane}} + U_{\text{car}} \tag{9}$$

To generate a safe region for the trajectory, we assign a threshold limit to U_{r}, such that the sum of all potentials U_{r} should be less than or equal to the threshold limit U_{safe}.

4.5. Reachability Set

In this paper, the bicycle model is used for computing the reachability sets of the vehicle to identify safe driving zones. The safe regions of the road under the threshold limit U_{safe} surrounding the vehicle can be expressed using the set

$$G = \{p \in T_{\text{R}}^{\text{T}} : U(p) \leq U_{\text{safe}}\} \tag{10}$$

where p is any generic point on road and the MPC should plan the vehicle trajectories that keep it within this set. This set is dynamic and updates every time the nearby states of the vehicle change to provide an accurate representation of the environment. Yet, it does not consider the dynamics of the subject vehicle and hence some values of p in the set G may not be practically reachable.

4.6. Safe Intersection

The subject vehicle can theoretically reach any point p in the set G from its initial position p_0 without exceeding the maximum desired velocity v_{des}. The set of the points p on the road which are practically reachable by the subject vehicle is denoted by R such that $R \in \mathbb{R}^2$. The safe zones surrounding the subject vehicle which are reachable considering the vehicle dynamics and the risk map factors is

$$R_{\text{safe}} = G \cap R \tag{11}$$

5. Target Selection

To select a target point during the simulation, a point among the feasible set are checked, and are filtered for intersections between the feasible point set and points that satisfy the safe zone requirement, U_{r} less than U_{safe}. Of these filtered points, we select the target point as the point farthest in the x direction of the safe points. The farthest of the safe points is chosen to allow vehicle to travel more efficiently, as the vehicle will travel at it's largest displacement while being within the safe region.

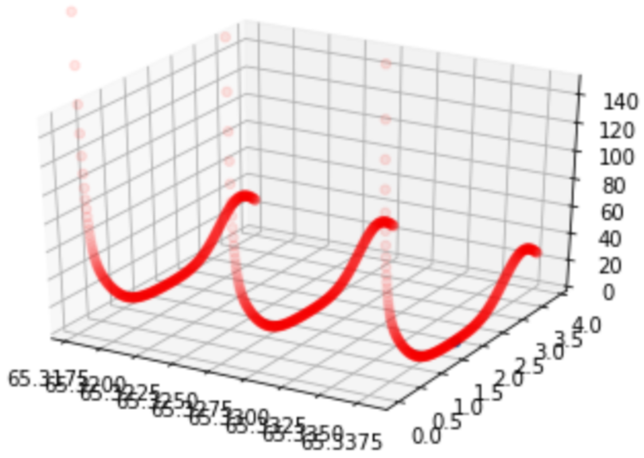

Fig. 1. Reachable set plot for system

6. Results

Using the aforementioned methods, risk maps were generated, targets were selected, and MPC was used to solve the control problem for each time step. The following table shows the values for constant parameters used in the risk map. Figure 2 shows the vehicle trajectory.

parameter	value
X_{i}	3
σ	0.5
α	0.16
A_{lane}	36
A_{car}	10
l_{f}	1.446
l_{r}	1.477
U_{safe}	10
v_{i}	32.67

From Figure 2, we observe that the vehicle behavior is as expected. The trajectory never exceeds the road boundaries, and the trajectories are centered in the lane when not undergoing lane change. Furthermore, the vehicle trajectory is able to avoid both current unsafe zones and future unsafe zones, as the trajectory does not intersect any of the green or red unsafe zones.

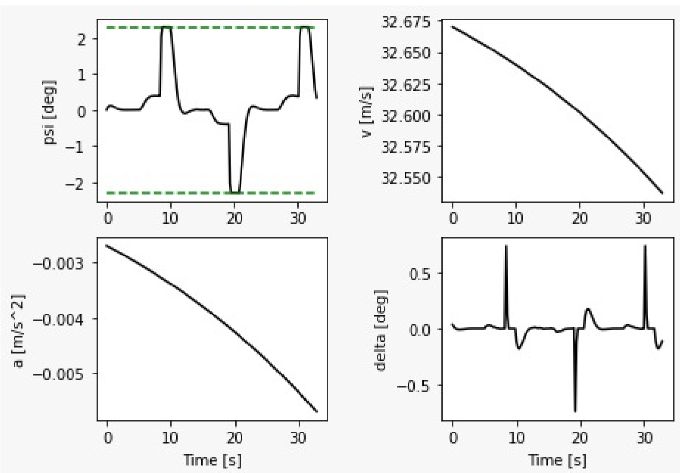

Fig. 3. System state and input over time.

Examining the inputs and and results of MPC, we see a slightly different picture. Looking at Figure 3, the car is slowly slowing down. This is an effect of the simulation and not having parameters perfectly tuned, but the difference is very small. Numerically, 0.125 m/s is equivalent to 0.45 kph or 0.28 mph, not something noticeable at highway speeds. Looking at steering angle delta, we see insignificant fluctuations except when changing lanes. When lane change occurs, the steering angle stays small as is desirable at highway speeds. This also has the effect of retaining accuracy of our linearization. There is slight oscillatory behavior of the system as the car returns gets adjusted to the new lane, but it is relatively minimal and the car always is able to stay in its lane. Looking at angle of the car relative to the lanes, the constraints are binding here. As the car changes lanes, in order to maintain safety, the angle was limited. Real cars can only turn so quickly and should not perform rapid movements when not necessary. In sum, the steering inputs and states are as expected. MPC is able to safely control the vehicle and the risk map and target point selection methods are shown to be effective in some real world scenarios.

7. Conclusion and Future work

This project proposes a control framework for autonomous overtaking, consisting of 1) generating a predicted risk map and a reachable set, 2) selecting a target point, and 3) trajectory generation using MPC. Obtaining the target point from the reachable set guarantees the optimization problem to be always feasible, and the risk map makes sure that the target point always lies in the safe zone. By tuning the prediction horizon and the area of the unsafe zones around the LVs, we made the SV not intrude into the current unsafe zones. Also, representing the unsafe zones with not state constraints but potential functions prevents the MPC from being a non-convex problem, which leads to good computational efficiency.

Future work for this control framework will focus on resolving its limit: the actual trajectory of the SV may intrude into the unsafe zones. While the target point is selected based on the risk map, the MPC we designed does not consider it, which may end up intruding into the unsafe zones. To resolve the limit while maintaining the problem to be non-convex, a novel approach for designing state constraints needs to be presented. For example, when the SV tries to change a lane, it should construct new state constraints based on the location of two LVs: 1) an LV on the same lane, in front of the SV and 2) the other one behind the SV, on the lane to which the SV plans to go. Once the new constraints were added, the target point may not be feasible anymore. Then, the control framework can extend its utility by adding a control module - which allows the SV to follow the LV ahead of it safely - when there is no feasible trajectory to change the lane.

References

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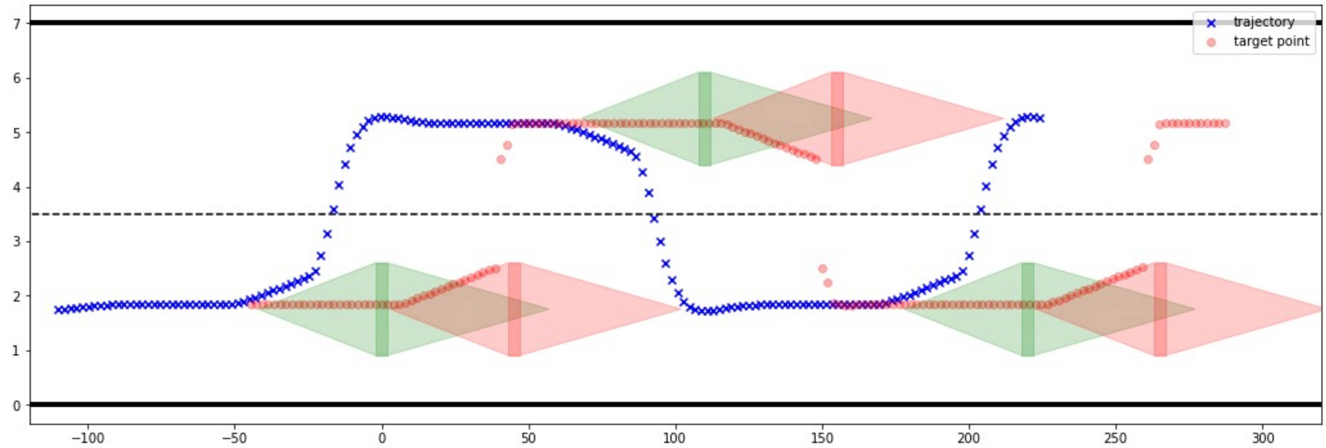

Fig. 2. Vehicle trajectory shown in blue with corresponding target points in red. Green triangles show current unsafe zones of leading vehicles and red triangles show expected unsafe zones of leading vehicles 2 seconds in the future. The trajectory and the other leading vehicles are plotted with respect to the first leading vehicle.