Digital Communication IC Design

Homework #4

(Due on 11/13)

Please upload your codes and reports (with simulation results and explanations) to the NTUCool.
Total points: 120 points

1. Let

$$\mathbf{H} = \frac{1}{\sqrt{L}} \sum_{l=1}^{L} \alpha_l \, \boldsymbol{a}^{\boldsymbol{ULA,r}}(\theta_l^r) \boldsymbol{a}^{\boldsymbol{ULA,t}}(\theta_l^t)^H$$

and

$$\boldsymbol{a}^{\boldsymbol{ULA}}(\theta) = [1 \quad e^{jkdsin(\theta)} \quad \ldots \quad e^{j(N-1)kdsin(\theta)}]^T.$$

Assume $kd = \pi$ and there are one transmitting antenna and 32 receiving antenna elements in the uniform linear antenna array. Given $L = 1$ and $\alpha_1 = 1$, please generate your own θ_1^r and θ_1^t such that $-\pi/2 \le \theta_1^r \le \pi/2$. Write them down. Please generate your channel matrix $\mathbf{H_1}$ with AoA θ_1^r and AoD θ_1^t, which is a matrix of dimension 32×1. Note that $\boldsymbol{a}^{\boldsymbol{ULA,r}}(\theta_l^r)$ is an 32×1 vector and $\boldsymbol{a}^{\boldsymbol{ULA,t}}(\theta_l^t)^H$ is a 1×1 vector.

(a) Now let's check the effect of a receiver performing beamforming. Given that

$$\boldsymbol{c}(m) = [1 \quad e^{j2\pi m/N} \quad \ldots \quad e^{j2\pi(N-1)m/N}]^T$$

is used as the phase shift for combiner at the receiver and $m = -16, -15, \ldots, 15$. First, use your last digit of student ID as m_1. Then, draw the beam pattern associated with m_1. Namely, fix $\boldsymbol{c}(m_1)$ and scan $|\boldsymbol{c}(m_1)^{\boldsymbol{H}}\mathbf{H}|$ for $-\pi/2 \le \theta_1^r \le \pi/2$ in polar coordinate. (10%)

(b) please check the magnitude of the combination result, $|\boldsymbol{c}(m)^{\boldsymbol{H}}\mathbf{H_1}|$ versus m and see if the best result will generate when $\varphi = sin^{-1}(2m/N)$ approaches θ_l^r. Draw the figure with x-axis indicating the value of m and y-axis is the magnitude of $|\boldsymbol{c}(m)^{\boldsymbol{H}}\mathbf{H_1}|$ for $m = -16, -15, \ldots, 15$. (10%)

(c) Given $L = 2$ and $\alpha_1 = 0.4 - 0.7j, \alpha_2 = 0.2 + 0.3j$, $\theta_2^r = \theta_1^r + 0.4$ (radian) , $\theta_2^t = \theta_1^t$. Please generate your channel matrix $\mathbf{H_2}$ with α_l, θ_l^r, θ_l^t for l=1,2. Repeats Step (b). (10%)

(d) In Q1(c), if you can set only one value of m (it means that you have one RF chain), what is your selection? If you can set two values of m (it means that you have two RF chains), what are your choices? Why? (10%)

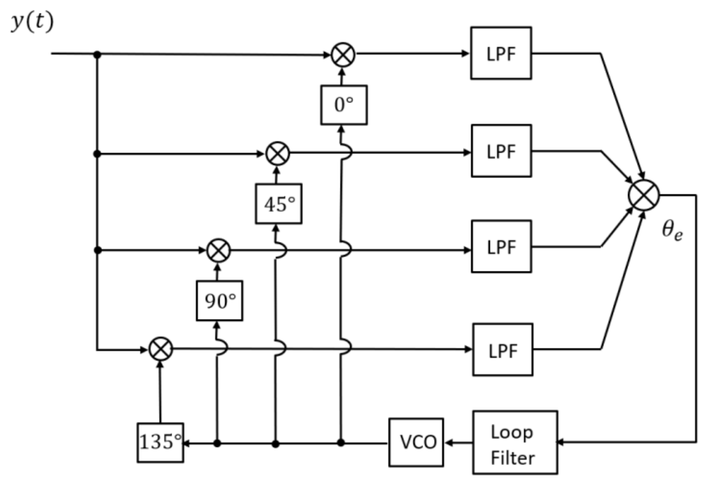
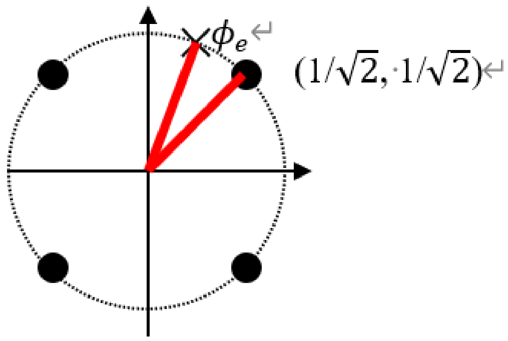


Fig. 1 QPSK Costas loop

2. Consider the QPSK Costas loop as in the above figure. The received signal is denoted as
$$y(t) = A\cos(2\pi f_c t + \theta_k + \varphi),$$
where θ_k is the k-th QPSK symbol from $\{\pi/4, 3\pi/4, -3\pi/4, -\pi/4\}$ and φ is the unknown phase offset. Assume the sinusoidal waves from VCO are given by $\cos(2\pi f_c t), \cos(2\pi f_c t + \frac{\pi}{4})$, $\cos(2\pi f_c t + \frac{\pi}{2})$, $\cos(2\pi f_c t + \frac{3\pi}{4})$. Please calculate error signal θ_e after the multiplier. (Note that you can assume the LPF keeps only the frequency component below f_c.) (10%)

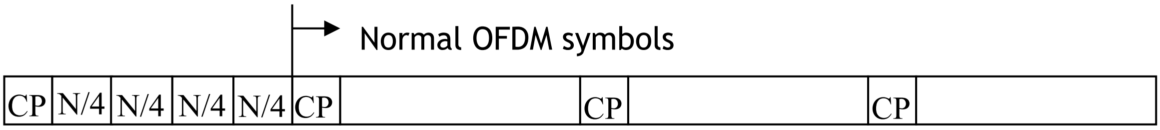
3. In the digital baseband decision-directed Costas loop, the error signal is calculated by $\theta_e(n) = s_{I,n}\hat{d}_{Q,n} - s_{Q,n}\hat{d}_{I,n}$ with $\hat{d}_{I,n} = sgn(s_{I,n})$ and $\hat{d}_{Q,n} = sgn(s_{Q,n})$, where $s_n = s_{I,n} + js_{Q,n}$, which is the received complex baseband signal, and $\hat{d}_n = \hat{d}_{I,n} + j\hat{d}_{Q,n}$, denoting the decision of the nth receive signal.
   (a) If a sign function is defined as
   $$sgn(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases},$$
   then an estimate θ_e is used for phase error ϕ_e. Show that
   $\theta_e(n) = Im\{s_n^* \hat{d}_n\} = s_{I,n}\hat{d}_{Q,n} - s_{Q,n}\hat{d}_{I,n}$.
   and use the following figure to explain why θ_e is a reasonable approximation of ϕ_e. (10%)



   (b) Please randomly generated 200 QPSK baseband symbols, which is denoted by p_n for $n = 0,1,\ldots,199$. Assume that a carrier frequency offset Δf exists and the system sampling period is equal to symbol period, $T_s = T$, namely one sample in one symbol. Given $\Delta f = 1/(500T_s)$, the received signal without noise is denoted by
   $$s_n = p_n e^{j2\pi(\Delta f)(nT_s)}$$
   please draw the true phase rotation versus time n due to the carrier frequency offset and your estimated results $\theta_e(n)$ versus time n (10%) (Please pay attention to the definition of the "sign" function in Matlab.)
   (c) In Q3(b), you will see the phenomenon called **phase ambiguity** when decision-directed Costas loop is used. Please explain it. (10%)

4. Please download "hw4.mat" from the NTUCool. Use "load hw4.mat" to retrieve the original data. It contains one variable "OFDMTx", which are the complex time-domain discrete signals generated by OFDM baseband transmitter. There are 32 samples in cyclic prefix ($N_g = 32$) and the FFT size

is 128 ($N = 128$). The first OFDM symbol is a preamble which has identical four repetitions as show in Fig. 3, and then three normal OFDM symbols follows the preamble. Assume that zero signals are received before and after the following waveform. Note that this is the best case for synchronization because channel fading and noise are not considered.



(a) Please draw magnitude of the delay and correlate result $\Phi_{DC}(m)$ by setting $R = 32$ and $L = 64$ where

$$\Phi_{DC}(m) = \left|\sum_{r=0}^{R-1} z_{m-r} z_{m-r-L}^{*}\right|. \ (10\%)$$

(Mark the X and Y value of the peak in the figure.)

(b) If a decision is made according to $\hat{m}_{DC} = \arg\max \Phi_{DC}(m)$, what's the result? (5%)

(c) Repeat (a) for $R = 96$, $L = 64$. (10%)

(Mark the X and Y value of the peak in the figure.)

(d) Following (c), if a decision is made according to $\hat{m}_{DC} = \arg\max \Phi_{DC}(m)$, what's the result? (5%)

(e) Please evaluate the results in Q4(b) and Q4(d). which one is better? Why? (10%)