Digital Communication IC Design

Homework #2

(Due on 10/16)

Please upload your codes and reports (with simulation results and explanations) to the NTUCool.
Total points: 110 points

1. The properties of **constant amplitude zero auto-correlation** (CAZAC) Zadoff-Chu sequence will be examined. Define

$$S(n,u) = e^{\frac{j\pi un(n+1)}{N}}$$

for $0 \le n \le N-1$.

(a) In the primary synchronization channel of the LTE system, $N = 63$, $u = 25, 29, 34$. Variable u corresponds to sector ID (0, 1, 2). For even-numbered students, please use u_1=25. For odd-number students, please use u_1=29. First, generate sequence $S(n,u_1)$ for $0 \le n \le N-1$. Now, examine its autocorrelation property

$$\Phi_s(l) = \frac{1}{N}\sum_{n=0}^{N-1} S(n,u_1)S^*(n-l,u_1)$$

Plot $|\Phi_s(l)|$ for $-N \le l \le N$. (10%)

(b) Now, examine its cross-correlation property with u_2=34

$$\Omega_s(l) = \frac{1}{N}\sum_{n=0}^{N-1} S(n,u_1)S^*(n-l,34)$$

Plot $|\Omega_s(l)|$ for $-N \le l \le N$ (10%)

(c) However, the mapping of Zadoff-Chu sequence to the data subcarriers in the LTE system is given as follows

$$X_k = \begin{cases} 0 & k = 0 \\ S(k+31, u_1) & -31 \le k \le 31, k \ne 0 \end{cases}$$

Please examine the properties of $|\Phi_s(l)|$ and $|\Omega_s(l)|$ of the primary synchronization signal X_k . Plot them for $-N \le l \le N$. (10%)

2. Walsh Hadamard code is also widely used for multiple access because of good cross-correlation. A Walsh Hadamard code matrix $\mathbf{W_N}$ of size $N \times N$ can be defined recursively as in the following.

$$\mathbf{W_1} = [\mathbf{1}] \quad \mathbf{W_{2N}} = \begin{bmatrix} \mathbf{W_N} & \mathbf{W_N} \\ \mathbf{W_N} & -\mathbf{W_N} \end{bmatrix} \quad .$$

(a) Denote your last digit of student ID as α. Please write a program to generate $\mathbf{W_{64}}$. Check for their cross correlation and plot the figure of the following equation

$$\Gamma_w(u) = \frac{1}{64}\sum_{j=1}^{64}(\mathbf{W_{64}}(j,\alpha+1)\mathbf{W_{64}}(j,u)) \text{ for } 1 \le u \le 64 \text{ (10\%)}$$

(b) Now, choose $\mathbf{c_1} = \mathbf{W_{64}}(:,\alpha+1)$ and $\mathbf{c_2} = \mathbf{W_{64}}(:,14)$ as two codes for user 1 and user 2. Randomly generate 8 symbols (d_0~d_7) from the set {+1,-1}. Spread the data by code 1 as $\mathbf{y} =$ $[d_0c_{1,1}\ d_0c_{2,1}\ \ldots d_0c_{64,1}\ \ d_1c_{1,1}\ \ldots\ d_1c_{64,1} \ldots d_7c_{1,1} \ldots d_7c_{64,1}\ 0\ 0\ 0\ 0]$. Use "stem" to plot the signals before spreading and after spreading. (10%)

(c) Assume at the Rx side, perfect synchronization is not achieved. Then, check the results if the receiver uses code 1 for dispreading, which includes multiplying the spreading code again and then summing them, i.e.

$$p(i) = \frac{1}{64}\sum_{j=1}^{64}\left(y_{64i+j+2}\right)c_{j,1}, \text{ where } y_r \text{ is the } r\text{th element of vector } \mathbf{y}.$$

Plot $p(i)$ using index i as the x-axis, for $i = 0,1,\ldots,7$. (10%)

(d) Assume at the Rx side, perfect synchronization is achieved. Then, check the results if the receiver uses code 2 for dispreading, which includes multiplying the spreading code again and then summing them, i.e.

$$p(i) = \frac{1}{64}\sum_{j=1}^{64}\left(y_{64i+j}\right)c_{j,2} = \frac{1}{64}\sum_{j=1}^{64}\left(d_ic_{j,1}\right)c_{j,2}, \text{ where } y_r \text{ is the } r\text{th}$$

element of vector $\mathbf{y}$. Plot $p(i)$ using index i as the x-axis. (10%)

(e) Please comment the results in (c) and (d). (5%) Which property should be examined for Walsh Hadamard code if we want to realize the effect of (c) in the design phase (5%)?

3. The octal representation for the coefficients of degree-9 primitive polynomial can be expressed by 1021_8 and 1423_8 with a_9 on the left to a_0 on the right. For even-numbered students, please use 1021_8. For odd-number students, please use 1423_8.

   (a) Draw the architecture of linear feedback shift register that you use. (10%)

   (b) Write a program to generate the ML sequence. If your last digit of student ID is s, then transform $s+1$ into binary representation and use it as the initial state. Print the first 20 binary elements, $(b_0, b_1, \ldots, b_{19})$ of your sequence (10%). Then, map the binary sequence by $d_n = (-1)^{b_n}$. Use d_n to calculate the results of periodic autocorrelation function for $-N \le l \le N$. (10%)

$$\Phi_s(l) = \frac{1}{N}\sum_{n=0}^{N-1} d_n d_{[n+l]_N}$$