Digital Communication IC Design

Homework #3

(Due on 2024/10/30)

Please upload your codes and reports (with simulation results and explanations) to the NTUCool.
Total points: 130 points

1. Consider one channel power delay profile listed in the following table.

Table 1: Channel power delay profile

Tap	Power	Delay (τ_i)
0	0 dB	0 ns
1	-3 dB	10 ns
2	-4 dB	40 ns
3	-8 dB	80 ns
4	-15 dB	100 ns

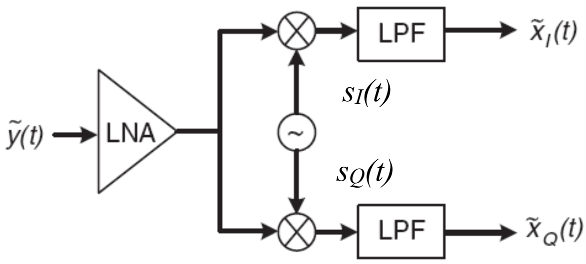
   (a) Please list the power ratio of 5 taps in linear scale. (5%)
   (b) Please calculate the mean excess delay. (5%)
   (c) Please calculate the RMS excess delay (τ_{RMS}). (5%)

2. Please generate complex Gaussian random variable (Z=X+jY) with zero mean and unit variance (i.e. the variance of both X and Y is 0.5) to obtain $\gamma_i,\ i = 0,1,\dots,4$.
   (a) Assume that both $h_i(t)$ and $\tau_i(t)$ are time-invariant so that $h_i(t) = h_i$ and $\tau_i(t) = \tau_i$. $g_i = \alpha_i * \gamma_i$, where α_i is the path **magnitude** gain (not power) from Question 1. Now normalize g_i so that $h_i = Kg_i$ and $\sum_{i=0}^{4} |h_i|^2 = 1$, which can keep the same signal energy under the multipath fading channel effect. Please write down your $h_i,\ i = 0,1,\dots,4$. (10%)
   (b) Given that the channel impulse response is written as $\sum_{i=0}^{4} h_i \delta(t - \tau_i)$. Now, use Fourier transform to derive its frequency domain channel response. Write down the equation. (10%)
   (c) Plot the magnitude and phase of channel frequency response in the range of -100 MHz and 100 MHz. (10%)
   (d) If the coherence bandwidth is defined as $B_c = 1/(5\tau_{RMS})$, where τ_{RMS} comes from Q1. Please calculate the coherence bandwidth. (5%) How many subcarriers should be allocated in a 100MHz bandwidth if we want to produce a flat-fading channel in each subcarrier of an OFDM systems? (5%)

3. If the transmitted RF signal is denoted as

$$\tilde{y}(t) = Re\{(x_I(t) + jx_Q(t))(C(t) + jS(t))\}$$

Ideally, $C(t) = \cos(\omega_c t)$ and $S(t) = \sin(\omega_c t)$ Assume at the **transmitter side,** IQ imbalance exists, and thus $C(t) = \left(1 + \frac{\varepsilon}{2}\right)\cos\left(\omega_c t + \frac{\emptyset}{2}\right)$ and $S(t) = \left(1 - \frac{\varepsilon}{2}\right)\sin\left(\omega_c t - \frac{\emptyset}{2}\right)$. Please demodulate the input signal $\tilde{y}(t)$ and express $\tilde{x}_I(t)$ and $\tilde{x}_Q(t)$ in terms of $x_I(t)$ and $x_Q(t)$ as well as the IQ imbalance parameters with the assumption of perfect demodulation, i.e. $s_I(t) = \cos(\omega_c t)$ and $s_Q(t) = -sin(\omega_c t)$. (15%).



4. Given the OFDM systems. The whole bandwidth is 100MHz and is partitioned into 64 subcarriers. The sampling frequency (f_s) is also 100MHz. Cyclic prefix (CP) of 16 samples is used in the guard interval.
   (a) Please randomly generate BPSK data X_k for each subcarrier k, $k = 0, 1, \dots, 63$.
   (b) Obtain the time-domain signal $x[n]$ for $n = 0, 1, \dots, 63$. Then, insert CP, which is indicated by $x[n]$ for $n = -16, -15, \dots, -1$. Draw the real part and imaginary part of the baseband transmitted waveform (10%)
   (c) Note that the discrete baseband equivalent channel in Q2(a) can be described by $h[n] = \sum_{i=0}^{4} h_i \delta[n - n_{\tau_i}]$, where $n_{\tau_i} = \frac{\tau_i}{T_s}$ denote the normalized delay from Table 1 and T_s is the sampling period. Assume the OFDM signal generated in Q4(b) passes through the channel. The received signal $y[n]$ is expressed as $y[n] = h[n] \otimes x[n]$. Draw the real part and imaginary part of the baseband received signal $y[n]$ for $n = -16, -15$, ..., 63, 64, ….. Note that you need to draw the entire output sequence of $y[n]$ considering the length of channel impulse response $h[n]$. (10%)
   (d) Now, remove CP and transform $y[n]$, for $n = 0, 1, \dots, 63$, into frequency domain to obtain the received frequency domain data Y_k. Calculate $H_k = \frac{Y_k}{X_k}$. Draw the magnitude and phase of H_k. (10%)

(e) Compare Q4(d) and Q2(c). How to interpret the results given that one of their X-axes is expressed by Hertz and other is expressed by index? (10%)

(f) Let's observe the results to see what will happen if the CP is not inserted. Now, generate $z[n] = h[n] \otimes x[n]$ for $n = 0,1 ,\ldots,63$. Transform $z[n]$ into frequency domain to obtain the received frequency domain data Z_k. Calculate $H'_k = \frac{Z_k}{X_k}$. Draw the magnitude and phase of H'_k. (10%)

(g) Compare Q4(d) and Q4(f). Please explain why H'_k can not acquire the proper channel frequency response. (10%)