高等數位訊號處理 hw1 姓名:葉冠宏 學號:R11943113

1.

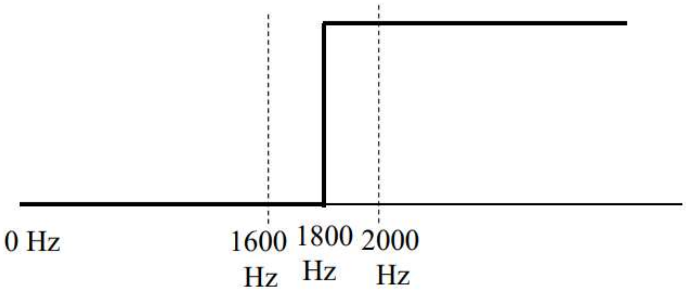
(1) Design a Mini-max **highpass** FIR filter such that (40 scores)

① Filter length = 21, ② Sampling frequency f_s = 8000Hz,

③ Pass Band 1800~4000Hz ④ Transition band: 1600~2000 Hz,

⑤ Weighting function: $W(F)$ = 1 for passband, $W(F)$ = 0.8 for stop band .

⑥ Set Δ = 0.0001 in Step 5.



※ The code should be handed out by NTUCool, too.

Show (a) the frequency response, (b) the impulse response $h[n]$, and (c) the maximal error for each iteration.

程式所跑的結果如下:

```
the frequency response
[0.06323356517190039, -0.06323356517190001, 0.06323356517190024, -0.06323356517190015, 0.06323356517190053,
0.9494131478624798, 1.0505868521375203, 0.9494131478624803, 1.0505868521375203, 0.9494131478624792,
1.0505868521375203, 0.9494131478624798]
the impulse response h[n]
[-0.03331262  0.00466773  0.03699781  0.02293908 -0.0316303  -0.03485657
  0.04898989  0.09451187 -0.04579777 -0.30943683  0.55456974 -0.30943683
 -0.04579777  0.09451187  0.04898989 -0.03485657 -0.0316303   0.02293908
  0.03699781  0.00466773 -0.03331262]
the maximal error for each iteration
[0.3238859363554261, 0.5843933291039262, 0.20283277902524663, 0.09151437385319072, 0.059844225911609455,
0.05148945277904467, 0.0505906409579595, 0.05058685213752079]
[Finished in 1.5s]
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2.

(2) (a) Which type of systems can be implemented by convolution?

(b) How do we convert convolution into an addition operation? (10 scores)

(a)

linear time-invariant (LTI) system。

(b)

如果原來是 y[n]=x[n] * h[n] convolution 的運算。我們可以對等號左右兩邊同時進行 Fourier transform 後變成 Y(f)=X(f)H(f)。最後我們對等號左右兩邊同時取 log 後變成 log (Y(f))= log(X(f)) + log(H(f))，這樣就有 addition operation 了。

3.

(3) (a) Describe three advantages of the FIR filter.

(b) How do we implement $y[n] = x[n] *(0.7^n u[n]+ 0.2^n u[n])$ using the recursive method where * means the convolution and $u[n]$ is the unit step function?

(10 scores)

(a)

第一點:輸出會有固定的長度。

第二點:通常需要較少的運算量。

第三點:FIR 比較穩定。

(b)

$y[n] = x[n] * (0.7^n u[n] + 0.2^n u[n])$

$= x[n] * 0.7^n + x[n] * 0.2^n$

假設 $H_1(z) = \sum_{n=0}^{\infty} 0.7^n z^{-n} = \sum_{n=0}^{\infty} (0.7z^{-1})^n = \frac{1}{1-0.7z^{-1}}$

同理 $H_2(z) = \sum_{n=0}^{\infty} 0.2^n z^{-n} = \frac{1}{1-0.2z^{-1}}$

$\Rightarrow Y(z) = X(z)H_1(z) + X(z)H_2(z) = X(z)\left[\frac{1}{1-0.7z^{-1}} + \frac{1}{1-0.2z^{-1}}\right]$

$\Rightarrow Y(z)\left[1-0.7z^{-1}\right]\left[1-0.2z^{-1}\right] = X(z)\left[1-0.2z^{-1} + 1-0.7z^{-1}\right]$

$\Rightarrow Y(z)\left[1-0.2z^{-1}-0.7z^{-1}+0.14z^{-2}\right] = X(z)\left[2-0.2z^{-1}-0.7z^{-1}\right]$

$\Rightarrow y[n] - 0.2y[n-1] - 0.7y[n-1] + 0.14y[n-2] = 2x[n] - 0.9x[n-1]$

$\Rightarrow y[n] - 0.9y[n-1] + 0.14y[n-2] = 2x[n] - 0.9x[n-1]$ #

4.

(4) What are the roles of (a) the transition band and (b) the weight function for minimax FIR filter design? (10 scores)

(a)

Transition band 是位於 passband 和 stopband 中間的地帶。

(b)

因為我們的目標是希望可以降地所設計的 filter 和理想 filter 在我們所關心的頻率區間的差距可以越小越好，因此我們希望可以藉由調整權重使演算法訓練的重點在所關注的頻率區間。當你側重其中一個區間時，相對的另一個區間的重要程度會降低，因而使其波動會和理想上的 filter 差距上升。也就是說，你只能側重某些頻率區間。權重可以控制哪些地方的誤差要大一點，哪些地方誤差要小一點。

5.

(5) Suppose that $x[n] = y(0.001n)$ and the length of $x[n]$ is 6000. If $X[m]$ is the FFT of $x[n]$, determine m such that $X[m]$ correspond to the frequencies of (a) 200Hz and (b) -100Hz. (10 scores)

(a) $x[n] = y(0.001n)$

$\Rightarrow \Delta t = 0.001,\ N = 6000$

$f = \frac{m}{N\Delta t} \Rightarrow m = f N \Delta t = 200 \times 6000 \times 0.001$

$= 1200$ #

(b) $\frac{1}{\Delta t} = 1000$, $X(f + f_s) = X(f)$, $X(-100 + 1000) = X(-100)$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \parallel$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad X(900)$

$\Rightarrow m = f N \Delta t = 900 \times 6000 \times 0.001$

$= 5400$ #

6.

(6) Use the <u>MSE method</u> to design the 7-point FIR filter that approximates the band filter of $H_d(F) = 1$ for $0.1 < |F| < 0.4$ and $H_d(F) = 0$ for $|F| < 0.1$ or $|F| > 0.4$.

(10 scores)

6、

$N=7 \Rightarrow N=2k+1,\ k=3$

If $0.1<|F|<0.4,\ Hd(F)=1$

If $|F|<0.1$ or $|F|>0.4,\ Hd(F)=0$

$$\Rightarrow S[0]=\int_{-1/2}^{1/2} Hd(F)\,dF=\int_{0.1}^{0.4} 1\ dF = F\Big|_{0.1}^{0.4}=0.3$$

$$S[n]=2\int_{-1/2}^{1/2}\cos(2\pi nF)\,Hd(F)\,dF=2\int_{0.1}^{0.4}\cos(2\pi nF)\,dF$$

$$=\frac{2}{2\pi n}\sin(2\pi nF)\Big|_{0.1}^{0.4}=\frac{1}{\pi n}\left[\sin(0.8\pi n)-\sin(0.2\pi n)\right]$$

$h[k]=h[3]=S[0]=0.3$

$$\begin{cases} h[k+n]=S[n]/2 \\ h[k-n]=S[n]/2 \end{cases} \Rightarrow h[2]=h[4]=S[1]/2=\frac{1}{2\pi}\Big[\sin(0.8\pi)-\underbrace{\sin(0.2\pi)}_{\sin(\pi-0.8\pi)}\Big]$$

$=0$

$$h[1]=h[5]=S[2]/2=\frac{1}{4\pi}\Big[\underbrace{\sin 1.6\pi}_{-\sin 0.4\pi}-\sin 0.4\pi\Big]$$

$$=\frac{1}{2\pi}\left[-\sin 0.4\pi\right]=-0.15136$$

$$h[0]=h[6]=S[3]/2=\frac{1}{3\pi}\Big[\underbrace{\sin 2.4\pi}_{\sin 0.4\pi}-\underbrace{\sin 0.6\pi}_{\sin(\pi-0.4\pi)}\Big]=0$$

$h[7]=0$

$\Rightarrow h[3]=0.3,\ h[2]=h[4]=0,\ h[1]=h[5]=-0.15136$

$h[0]=h[6]=0,\ h[7]=0$ 共

7.

(7) Estimate the length of the digital filter if both the passband ripple and the stopband ripple are smaller than 0.01, the sampling interval $\Delta_t = 0.0001$, and the transition band is from 3000Hz to 3300Hz. (10 scores)

2.

$\delta_1 = 0.01$, $\delta_2 = 0.01$

$\Delta F = (f_1 - f_2)/f_s = (3300 - 3000) / \frac{1}{0.0001}$

$= 300 \times 0.0001 = 0.03$

$N = \frac{2}{3} \frac{1}{\Delta F} \log_{10} \frac{1}{10\delta_1\delta_2} = \frac{2}{3} \times \frac{1}{0.03} \times \cancel{\log 10^3}\ 3$

$\doteqdot 66.67$

∴ 取一個近似的整數，$N = 67$ #

Extra: 為什麼 bilinear transform 可以完全避免 aliasing effect?

ANS: 因為在 Bilinear transform 裡面，actan 的定義域是負無限大到正無限大，投影到-pi/2 到 pi/2 之間，是一個 single conformal mapping，因此不會有重疊的現象，避免 aliasing effect。