高等數位訊號處理期末報告　姓名:葉冠宏　學號:R11943113

1.Wiener filter

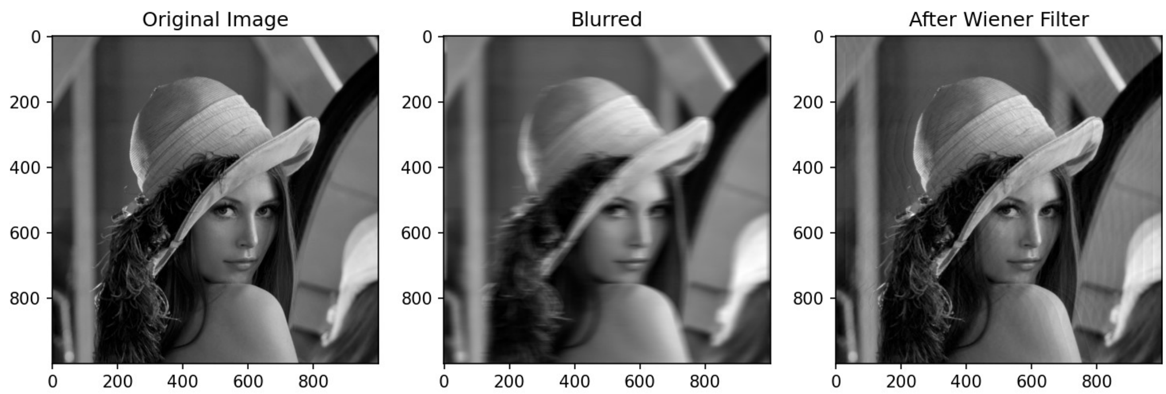
參考公式:

$$
\begin{aligned}
\hat{F}(u,v) &= \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)\left|H(u,v)\right|^2 + S_\eta(u,v)}\right]G(u,v) \\
&= \left[\frac{H^*(u,v)}{\left|H(u,v)\right|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v) \\
&= \left[\frac{1}{H(u,v)}\frac{\left|H(u,v)\right|^2}{\left|H(u,v)\right|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)
\end{aligned}
$$

程式執行:

python wiener.py

實作成果:



我們可以觀察到在我們實施完 wiener filter 後，原本模糊的影像變得清楚了。

2.Kalman filter

參考公式:

Predict [edit]

Predicted (*a priori*) state estimate	$\hat{\mathbf{x}}_{k\mid k-1} = \mathbf{F}_k \mathbf{x}_{k-1\mid k-1} + \mathbf{B}_k \mathbf{u}_k$
Predicted (*a priori*) estimate covariance	$\hat{\mathbf{P}}_{k\mid k-1} = \mathbf{F}_k \mathbf{P}_{k-1\mid k-1} \mathbf{F}_k^\mathsf{T} + \mathbf{Q}_k$

Update [edit]

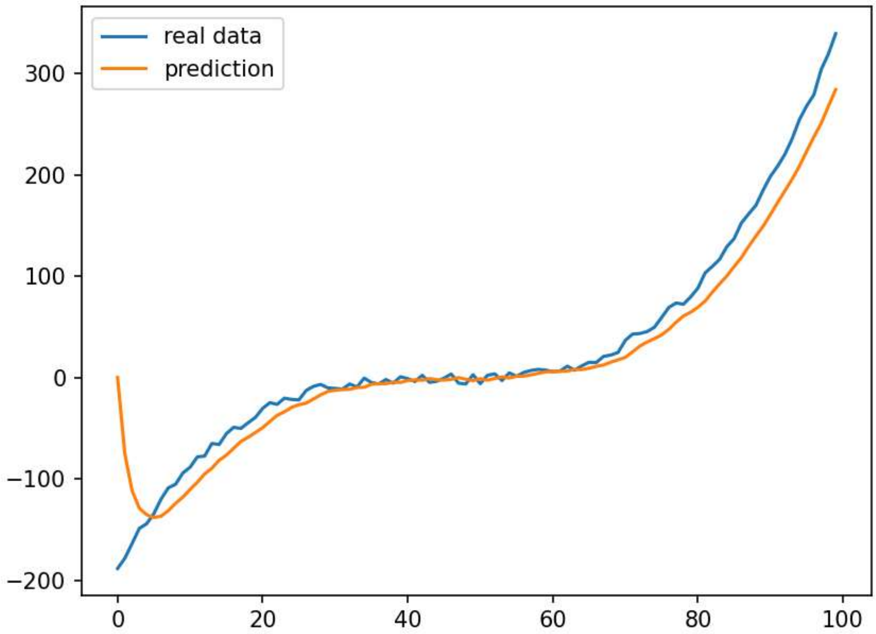
Innovation or measurement pre-fit residual	$\tilde{\mathbf{y}}_k = \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k\mid k-1}$
Innovation (or pre-fit residual) covariance	$\mathbf{S}_k = \mathbf{H}_k \hat{\mathbf{P}}_{k\mid k-1} \mathbf{H}_k^\mathsf{T} + \mathbf{R}_k$
Optimal Kalman gain	$\mathbf{K}_k = \hat{\mathbf{P}}_{k\mid k-1} \mathbf{H}_k^\mathsf{T} \mathbf{S}_k^{-1}$
Updated (*a posteriori*) state estimate	$\mathbf{x}_{k\mid k} = \hat{\mathbf{x}}_{k\mid k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k$
Updated (*a posteriori*) estimate covariance	$\mathbf{P}_{k\mid k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \hat{\mathbf{P}}_{k\mid k-1}$
Measurement post-fit residual	$\tilde{\mathbf{y}}_{k\mid k} = \mathbf{z}_k - \mathbf{H}_k \mathbf{x}_{k\mid k}$

$$\mathbf{P}_{k\mid k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k\mid k-1} (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)^\mathsf{T} + \mathbf{K}_k \mathbf{R}_k \mathbf{K}_k^\mathsf{T}$$

程式執行:

python kalman.py

實作成果:



我們可以觀察到在執行 Kalman filter 後，所預測的數據可以和真實數據很貼合。