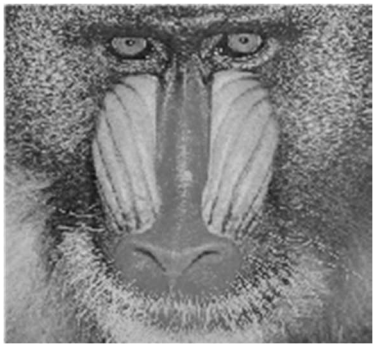
學號:R11943113 姓名:葉冠宏 高等數位訊號處理 hw4

1.

Img1:



Img2:

Img3:

結果:

```
ssim between img1 and img2 is 5.750117772167531e-05
ssim between img1 and img3 is 0.0007169946695234699
```

執行方式:

python test.py

2

(a) 假設現在有兩個複數，一組是 $a+bj$，另一組是 $c+dj$

則在正常的運算系法中，$(a+bj)(c+dj)=\underbrace{(ac-bd)}_{e}+\underbrace{(ad+bc)}_{f}j$

如果用矩陣系法表示

$$\begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} c & -d \\ d & c \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} c & c \\ c & c \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} 0 & -c-d \\ d-c & 0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

⇒ 在運算過程中，我們令 $g_1 = c(a+b)$，$h_1 = g_1$

$g_2 = (-c-d)b$，$h_2 = (d-c)a$

則實部 $= e = g_1 + g_2$，虛部 $= f = h_1 + h_2$

過程會有3個乘法 #

(b) 在正常的矩陣乘法中

$c_1 = a_1b_1 - a_2b_2 - a_3b_3 - a_4b_4$

$c_2 = b_2a_1 + b_1a_2 + b_4a_3 - b_3a_4$ ⇒ 需要16 MUL

$c_3 = b_3a_1 + b_4a_2 + b_1a_3 - b_2a_4$

$c_4 = -b_4a_1 + b_3a_2 + b_2a_3 + b_1a_4$

⇒ 如果可以用

$$\begin{bmatrix} c_1 \\ c_4 \end{bmatrix} = \underbrace{\begin{bmatrix} b_1 & -b_4 \\ -b_4 & b_1 \end{bmatrix}\begin{bmatrix} a_1 \\ a_4 \end{bmatrix}}_{\text{2 MUL}} + \underbrace{\begin{bmatrix} -b_2 & -b_3 \\ b_3 & b_2 \end{bmatrix}\begin{bmatrix} a_2 \\ a_3 \end{bmatrix}}_{\text{2 MUL}}$$

$$\begin{bmatrix} c_2 \\ c_3 \end{bmatrix} = \underbrace{\begin{bmatrix} b_2 & -b_3 \\ b_3 & -b_2 \end{bmatrix}\begin{bmatrix} a_1 \\ a_4 \end{bmatrix}}_{\text{2 MUL}} + \underbrace{\begin{bmatrix} b_1 & b_4 \\ b_4 & b_1 \end{bmatrix}\begin{bmatrix} a_2 \\ a_3 \end{bmatrix}}_{\text{2 MUL}}$$

⇒ 僅需8個MUL

#

3.

(a)

$MOL_{25} = 5\,MOL_5 + 5\,MOL_5 + 16\times 3 = 148$

$MOL_{125} = \cancel{25\times \underbrace{MOL_5}_{10}}^{250} + \cancel{5\times \underbrace{MOL_{25}}_{148}}^{740} + \cancel{3\times 144}^{288} = 1278$ #

$125 = 25\times 5$

$\frac{125}{12}$ 和 $\frac{125}{8}$ 都非整數

(b)

$147 = 7^2\times 3 = 49\times 3$

$\Rightarrow MOL_{147} = 49\times \underbrace{MOL_3}_{2} + 3\times \underbrace{MOL_{49}}_{332} = 1094$ #

$MOL_{49} = 7\,\underbrace{MOL_7}_{16} + 7\times MOL_7 + \underbrace{3\times 6\times 6}_{108} = 332$

(c) $385 = 5\times 7\times 11 = 35\times 11 \Rightarrow MOL_{385} = 35\times \underbrace{MOL_{11}}_{40} + 11\,\underbrace{MOL_{35}}_{150}$

$= 1400 + 1650 = 3050$ #

4.

所求 $= O(MNK\log_2 MNK)$ #

5.

detect \ fact	P	N	
P	TP 900	FP 100	1000
N	FN 0	TN	2000

$precision = \frac{TP}{TP+FP} = \frac{900}{1000} = 90\%$ #

$recall = \frac{TP}{TP+FN} = 100\%$ #

$\Rightarrow F\ score = 2\times \frac{precision\times recall}{precision + recall} = 2\times \frac{0.9}{1.9}$

$= 0.947$ #

6

(a) $N=1100$, $M=500$, $L_0=\frac{N}{L}\times 3(L+M-1)\left[\log_2(L+M-1)+1\right]$

① direct method : $3MN = 1650000$ #

② sectioned convolution :

經由程式的運算 $L_0=$ [illegible], $L_0+M-1=1599$

我們去搜尋 L_0 的點數 P，算 $L=P-M+1$，$S=\lceil \frac{N}{L} \rceil$

用 $2\times S\times MUL_P+3S\times P$ 去看數組的乘法量

最少，其中 $S=1,2,\ldots$

⇒ 經過程式運算，當 $P=1152$，即用 1152 點的 FFT

$L=653$，$S=1$ 時有最小乘法量 17632 #

③ non-sectioned

$P\geq N+M-1=1599$ ⇒ 選 $P=1680$，$MUL_{1680}=10420$

⇒ $2\times MUL_{1680}+3\times 1680=25880$ #

⇒ 最好的為 Sectioned Convolution #

(b) ① direct method: $3MN = 3 \times 40 \times 1100 = 132000$ #

② sectioned convolution: 經過計算, $L_0 = 208$

$L_0 + M - 1 = 287$

⇒ 經過計算, 最佳解是 $p = 336$, $L = 297$, $S = 3$

時有最小乘法量 11496 #

③ non-sectioned

$p \geq N + M - 1 = 1137$ ⇒ 選 $p = 1152$, $MUL_{1152} = 7088$

⇒ $2 \times MUL_{1152} + 3 \times 1152 = 17632$ #

⇒ 最好的是 sectioned convolution #

(c) ① direct method: $3MN = 3 \times 1100 \times 6 = 19800$ #

② sectional convolution: 經過計算, $L_0 = 19$

$L_0 + M - 1 = 24$

⇒ 經過計算, 最佳解是 $p = 24$, $L = 19$, $S = 5$ 時

有最小乘法量 7296 #

③ non-sectioned

$p \geq N + M - 1 = 1105$ ⇒ 選 $p = 1152$, $MUL_{1152} = 7088$

⇒ $2 \times MUL_{1152} + 3 \times 1152 = 17632$ #

⇒ 最好的是 sectioned convolution #

(d) ① direct method: $3MN = 3 \times 110 \times 2 = 660$ #

② sectioned convolution: 經過計算, $L_0 = 2$

$L_0 + M - 1 = 3$

⇒ 經過計算, 最佳解是 $P = 4$, $L = 3$, $S = 366$

有最小乘法量 4392 #

③ non-sectioned

$P \geq N + M - 1 = 1101$ ⇒ 選 $P = 1152$, $MUL_{1152} = 7168$

⇒ $2 \times MUL_{1152} + 3 \times 1152 = 17632$ #

⇒ 最好的是 sectioned convolution #

Extra: 問 MUL_{55}?

Ans: $= 11 \times MUL_5 + 5\,MUL_1 = 11 \times 10 + 5 \times 40 = 110 + 200$

$= 310$ #