3C5 - Gyroscopic Phenomena

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I'm guessing you're trying to say 'differences noticed' but also i don't think you need the word noticed

results were compared?

Abstract

Coriolis accelerations are hard to imagine as their occurrence day to day is very low. However the forces caused by these accelerations play a key role in applications involving gyroscopes. These applications can be anything from toy gyro's to specifying the whole motion of a body through space. By using a gyroscope setup in two ways these applications were investigated, and the results compared to theoretical calculations. There were slight differences notices between these, due to errors in measurement and physical quantities of the equipment.

unnecessary comma but kudos for trying

I. Introduction

pick a tense here and stick to it please

The objective of this lab was to observe gyroscopic motion and to relate this back to theory taught in the lecture series. The uses of gyroscopes in the real world were also to be explored.

II. Apparatus and Experimental Method

The experiments consisted of using a 10.8kg gyroscope mounted onto a stand with rotational freedom. The details of this arrangement as well as the experimental method can be located in the '3C5 Gyroscopic Phenomena' handout.

III. Results and Discussion

i. Preliminary Calculations

mutiple masses?

To estimate the moment of inertia 'C' about the axis of rotation, the mass was all assumed to reside 70mm from the rotation axis. The equation $J = \int r^2 dm = 4.2x0.07^2 = 0.0207kgm^2$, this results in a very close approximation to the value listed in the handout of $0.021kgm^2$.

where is the rest of this sentence

To calculate the moment of inertia about the two other axis use the perpendicular axis theorem which tells you they should be half the value of C as the other axis' are equal.

using 'you' in a lab report is a sin please restructure sentence perhaps something akin to '… perpendicular axis theorem was used, which states that ______'

The radius of gyration about pin 1 is calculated using $k = \sqrt{\frac{I_G}{m}} = \sqrt{\frac{0.084}{10.8}} = 0.088m$

In order to calculate the moment of inertia about pin 2 use $I = I_G + Mr^2 = 0.084 + 0.21^2 * 10.8 = 0.56kgm^2$

how about instead of telling me what to use, you tell me what you USED ok this is not an instruction manual

ii. Steady Precession

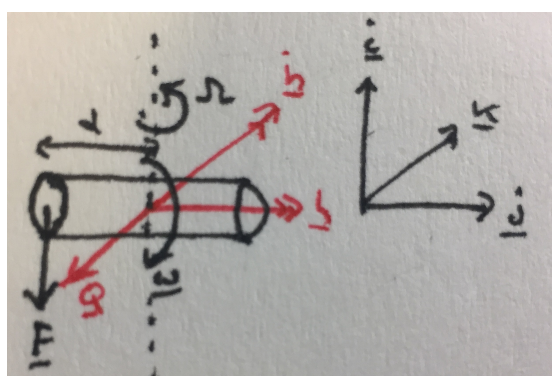

Figure 1: *Vector diagram of the steady procession set up*

In order to work out the direction of preces-

i'm just being anal but can this be on the next page please?

again stop telling me what to do bitch

sion during this experiment use figure 1 and equation 1, this gives you a clockwise rotation viewed from above.

$$
\begin{aligned}
\underline{Q} &= \dot{\underline{h}} \\
\underline{Q} &= -Fd\underline{k} \\
\dot{\underline{h}} &= \Omega \underline{i} x C\omega \underline{j} = C\omega\Omega\underline{k} \\
\Omega &= -\frac{Fd}{C\omega}\underline{k}
\end{aligned}
\tag{1}
$$

the force can be tuned into a force ok thanks Harvey

The 1kg mass is being held up by the reaction acting through the support. The force can be tuned into a force and a moment acting around the support as in figure 2. This weight is opposed by the reaction through the support, with the moment causing gyroscopic precession.

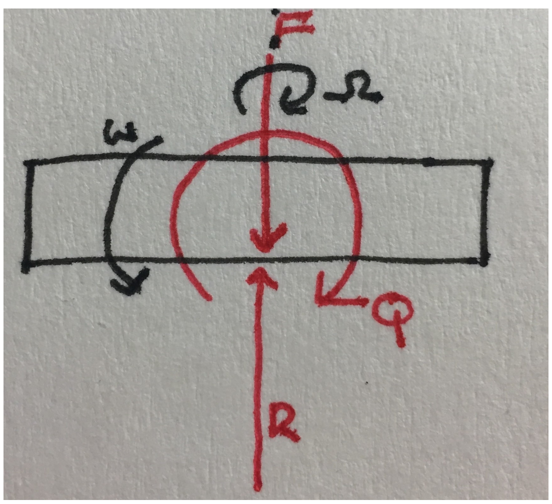

Figure 2: *Free body diagram of the gyroscope*

The disagreement between this and the experimental data in table 1 is likely due to IMPERFECT blablabla

Using equation 2 with the values listed in the lab handout and 5110rpm, a procession rate of $\dot{\phi} = 0.205\frac{rad}{s}$ or 34.27 seconds for one revolution. This is slightly disagreeing with our experimental data in table 1, this is likely due to non perfect inclination readings, and a moment of inertia different to what's recorded in the handout.

$$
\underline{Q} = C\omega\dot{\phi}sin\theta \tag{2}
$$

$\dot{\phi}$ is inversely proportional to $sin\theta$ therefore as

comma pls

you used therefore twice in the same sentence can U NOT

inclination angle changes, $sin\theta$ decreases and $\dot{\phi}$ increases therefore reducing the procession time. Table 1 shows the experimental data agreeing with this.

Table 1: *Shows the rate of steady precession varying with gyroscope inclination, this precession is caused by a 1kg hung mass.*

Inclination /degrees	Precession Time/s	Rotor Speed /rpm
90	36	5110
30	27	4950
45	33	5000
120	32	5160

doubled? idk context here

Using a mass of 2kg should half the procession time as the couple is double. In our experiment we recorded a time of 16 seconds.

iii. Nutation

Nutation frequency can be calculated using equation 3, where the first equation is inaccurate and doesn't include the whole assembly inertia. For all inclination this gives 20.8Hz for the uncorrected equation, and the values listed in table 2 once corrected. These calculated frequencies match the measured values closely at low angles.

w0T r u saying

$$
\begin{aligned}
p &= \frac{C\omega}{A} \\
p &= \frac{C\omega}{A} / \sqrt{1 + \frac{J_1}{A}Cot^2\theta + \frac{I_1}{A}cosec^2\theta}
\end{aligned}
\tag{3}
$$

Table 2: *Shows the nutation frequency varying with gyroscope inclination*

Inclination /degrees	Nutation Frequency/Hz	Theoretical Frequency/Hz
90	21.4	19.1
60	20.1	18.1
45	16.8	16.6
30	13.2	13.8

i'd personally use past tense in general but present tense is ok i guess

iv. The Rate Gyro

the couple acting on the rotor can be calculated using equation 2 with a rpm of 5000

By attaching springs to the assembly (see figure 3) a couple which varies with gyroscope inclination is created. Once again using equation 2 with a rpm of 5000 you can calculate the couple acting on the rotor. These are tabulated in table 3. The springs are approximately 60mm from the pins, therefore a force difference can be calculated between the springs that is creating this couple. Equation 4 is used here. Note only a force difference can be calculated as both springs were in tension not equilibrium at theta=90, and the length of spring wasn't recorded. The length difference between the springs is $0.12 cos\theta$ using a spring constant of 500N/m this gives forces differences of 42.4, 30 and 15.6N which is about 2.8 times smaller. This difference is likely due to a different spring constant and distance present in the apparatus.

are

what the fuck

full stop

…and 15.6N, which are about….

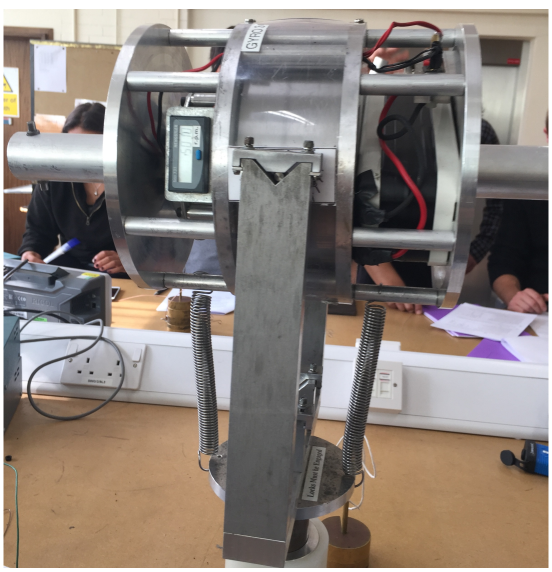

Figure 3: *Experimental set up of the rate gyro*

$$0.06(F_1 - F_2) = Q \tag{4}$$

This rate gyro can be used to measure rotation of the boat. The rotation will cause a moment to be generated in the gyro which can be measured to determine how the boat is turning.

Table 3: *Shows the procession rate created with two springs attached at various inclinations*

Inclination /degrees	Procession Time/s	Couple /Nm	$F_1 - F_2$ /N
135	6.9	7.1	118
120	10.8	5.5	92
75	30.7	2.2	36

If the gyro is impacted when precessing then nutation will still occur. Nutation is the transient response to an impact and will be present under any condition, the precessing is the steady state response caused by the input of a couple. The gyroscope is a linear system therefore these two separate responses can be added linearly and both occur simultaneously.

new sentence

comma

v. Gyroscopic Pendulum

The forces can once again be moved into a couple and force through support as in figure 2. The couple causes gyroscopic precession and with rate calculated using equation 1. The theoretical time is calculated with this equation and equation 5 with M=10.8kg and l=0.21m, 3.17s is calculated. This value agrees closely with the recorded data. In order to improve accuracy in measuring time ten rotations were recorded together.

forced?

comma

what are you even trying to say

$$Q = Mglsin\theta \tag{5}$$

Table 4: *Shows how procession time depends on inclination angle from vertical in set up 2*

Inclination /degrees	Procession Time/s
90	3.0
45	2.8
15	2.7

typo?

During this procession nutation can still occur as discussed earlier. The frequency of the oscillations was recorded using a slow motion camera and five wavelengths in order to increase accuracy. Using equation 3 you can once again get a theoretical prediction of this frequency , see table 5. This prediction agrees with the observed motion very well.

Table 5: *Shows how nutation frequency changes with inclination angle for set up 2*

Inclination /degrees	Nutation Frequency/Hz	Calculated Frequency/Hz
90	3.13	3.08
45	2.5	3.00

When you release the gyroscope from rest cuspidal motion is observed. The energy required to rotate the gyroscope is due to the drop in height of the gyroscope as precession occurs. For total energy to be conserved in equation 6 potential energy must be converted into kinetic energy for motion to occur.

$$TotalEnergy = KineticEnergy + PotentialEnergy \tag{6}$$

The motion is once again caused by the moment exerted on the supports and the change of moment of momentum. The damping throughout the system will causes these oscillations to diminish until the gyroscope is in steady state and precessing. This steady state occurs at an inclination less than 90 degrees. This end inclination was too small to measure in the experiment but can be calculated using equation 7, resulting in 0.106rad or 6 degrees.

$$\alpha = \frac{mgr(A + I_1)}{(C\omega)^2} \tag{7}$$

The damping of the system resulted in the observed motion differing to the theory in two regards. The sharp peak at the top of the motion appeared as more of a loop and was a lot smoother. The inclination loss at steady state appeared larger due to the energy losses.

IV. Conclusion

The theory and experimental observations matched up reasonably well throughout the tests. Differences between them can be accounted to various measurement errors. The inertias of the equipment would all vary from the values listed in the handout due to changes of battery etc. over the years. Measuring angles was inaccurate to to only have large divisions or no divisions to measure from.

5/10