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# 4M20 Robotics (2018) Coursework 1 - Note

**Q1**. (20%) Consider a two-legged robot shown in Figure 1 below. By using its three joint motors (shown by white circles), it can walk forward by swinging Foot 2 forward, then Foot 1 forward, for example. Assuming that Foot 1 is fixed on the ground at first, calculate the joint angles $\theta_1$, $\theta_2$, $\theta_3$ to achieve the end of Foot 2 [$X_5$, $Y_5$] to be at [0.15m, 0.05m] (with respect to [$X_1$, $Y_1$] = [0.0m, 0.0m]. Then what are those joint angles for Foot 2 landing jat [$X_5$, $Y_5$] =[0.12m, 0.0m] with the forth link perpendicular to the ground plane. $L_1$= $L_2$= $L_3$= $L_4$= 0.10m.

**A1.**

The answer to these questions can be obtained by deriving the forward and inverse kinematics of the Foot 2 [$X_5$, $Y_5$] with respect to the origin [$X_1$, $Y_1$] = [0.0m, 0.0m].
For the first part [$X_5$, $Y_5$] to be at [0.15m, 0.05m], there are not enough constraints to uniquely determine the answer, but some of the answers can be [$\theta_1$, $\theta_2$, $\theta_3$] = [33.8, 75.5, 70.7] (degrees), or [34.6, 72.7, 72.8] (degrees), for example. You can actually check the correct answer by using the forward kinematics.
In contrast, the second part of question has sufficient constraints, thus the answer should be [$\theta_1$, $\theta_2$, $\theta_3$] = [36.9, 106.2, 36.9] (degrees).

**Q2.** (10%) In the same two-legged robot, we consider each link has a point mass $M_n$ at the middle. Derive the Centre of Mass ($M_{COM}$) of the whole robot and its location as the function of joint angles $\theta_1$, $\theta_2$, $\theta_3$. Consider $M_1$= $M_2$= $M_3$= $M_4$= 0.10kg.

**A2.**

The center of mass of the whole robot can be obtained from
$[x_{com}, y_{com}] = \frac{1}{M}\sum x_i\, m_i, \frac{1}{M}\sum y_i\, m_i]$
= [l/8 *(5$s_1$ + 3 $s_{12}$ + $s_{123}$), l/8 *(7+5$c_1$ + 3 $c_{12}$ + $c_{123}$)]

**Q3.** (20%) When Foot 1 is not fixed on the ground, in order for this robot not to fall down, the projection of $M_{COM}$ on the ground ($X_{COM}$) needs to be within $0<X_{COM}<L_0$. Considering $L_1$= $L_2$= $L_3$= $L_4$= 0.10m, what is the minimum length of Foot 1 $L_0$ for this robot to achieve “static locomotion” all the time? And what is the maximum stride length (i.e. a possible longest step of Foot 2)? Discuss the drawback to achieve locomotion when $L_0$ is too large.

**A3.**

In order to achieve $0<X_{COM}<L_0$ for any joint angles of this robot, the largest $X_{COM}$ happens when the swing leg is stretched straight in parallel to the ground i.e. [$\theta_1$, $\theta_2$, $\theta_3$] = [90, 0, 0] (degrees). Under this assumption, the shortest $L_0$ should be 0.1125m.
If we restrict the joint angles, however, $L_0$ can be arbitrarily shorter. In theory, if the stride length is 0.0m, $L_0$ can be 0.0m, for example. Therefore, a trade-off exists the longer stride length (usually faster locomotion) requires longer feet, while longer feet are often troublesome to avoid collisions with ground and steps (e.g. for the terrain in Q4).
The longest stride length of this robot should be 0.2m, assuming that links and feet don’t collide to each other when they are passing through.

**Q4**. (50%) Given the design above, this robot needs to traverse the terrain with a step shown in Figure 2 below. Calculate the trajectories of $\theta_1$, $\theta_2$, $\theta_3$ such that the robot can reach the goal without falling down at the step. And estimate the minimum torque required for each of three motors, duration required to accomplish this task, and energy efficiency. Discuss whether motor trajectories would influence these performance metrics.

**A4.**

(a) Locomotion trajectory

Obviously there is no single answer to this question, but many different ways to achieve this task. The first important consideration is which of kinematic or dynamic motion control strategies we employ to design the trajectory. As we discussed in the lecture, an easier way is to consider kinematics only (even though dynamics can be faster and more efficient in locomotion). Here we consider only a kinematic motion control solution.

First, we need to identify the boundary conditions of kinematic motion control, i.e. (1) we consider only static locomotion (as in Q3); (2) links and foot don't collide with the step.

Second, we need two types of low-level controllers, Controller1 for Foot1 swinging (Foot 2 on the ground), and Controller2 for Foot 2 swinging (Foot 1 on the ground). Each of this controller needs as the input of where the swing foot should start and land, and the trajectories of joint angles need to check the two boundary conditions above).

Third, we need to design a planner, which determines where the footholds should be in order to accomplish the task. For the locomotion on the flat part of the terrain, this planner can set footholds for fastest and/or most efficient locomotion. For climbing up the step, this planner needs to consider whether the conditions of static locomotion can be actually possible.

(b) Motor torque

When designing a robot or its joint trajectories, it is important to estimate the maximum torque required (and often maximum speed, because speed and torque are usually trade-off relationship). Here we need two types of motor torque, i.e. torque for static moment (force x moment arm), and torque for dynamic motion (moment of inertia x angular acceleration). Unless you plan aggressive joint trajectories that requires considerably large joint angular acceleration, torque for dynamic motion in our model is a few orders of magnitude smaller than that of static torque. Therefore we can safely ignore the former.

Maximum motor torque is dependent on the joint trajectories. We could numerically calculate the values precisely by using simulation from the part (a) of this question, but a rough estimate can be obtained the worst case analysis such as when the links are stretched out furthest as in the foot length calculation in Q1. This estimate should give motor 1 and 3 to be (0.05m+0.15m+0.25m)*0.1kg*9.8m/s^2= 0.44 Nm, and motor 2 should be (0.05m+0.15m)*0.1kg*9.8m/s^2= 0.196 Nm.

(c) Energy efficiency

The total energy consumption of the system can be estimated by integrating, over time, the amount of torque multiplied by angular velocities of all joints during the locomotion. This energy consumption, therefore, is obviously dependent on the joint trajectories (both angles and angular velocities), the posture of the whole robot, how high/low each leg swings, and speed of locomotion. Energy efficiency of this locomotion task therefore can be calculated by: (Energy Efficiency) = (Energy Consumption)/((Robot Weight)+(Horizontal Locomotion Distance)). This is also known as "specific resistance" or "cost of transport".