1

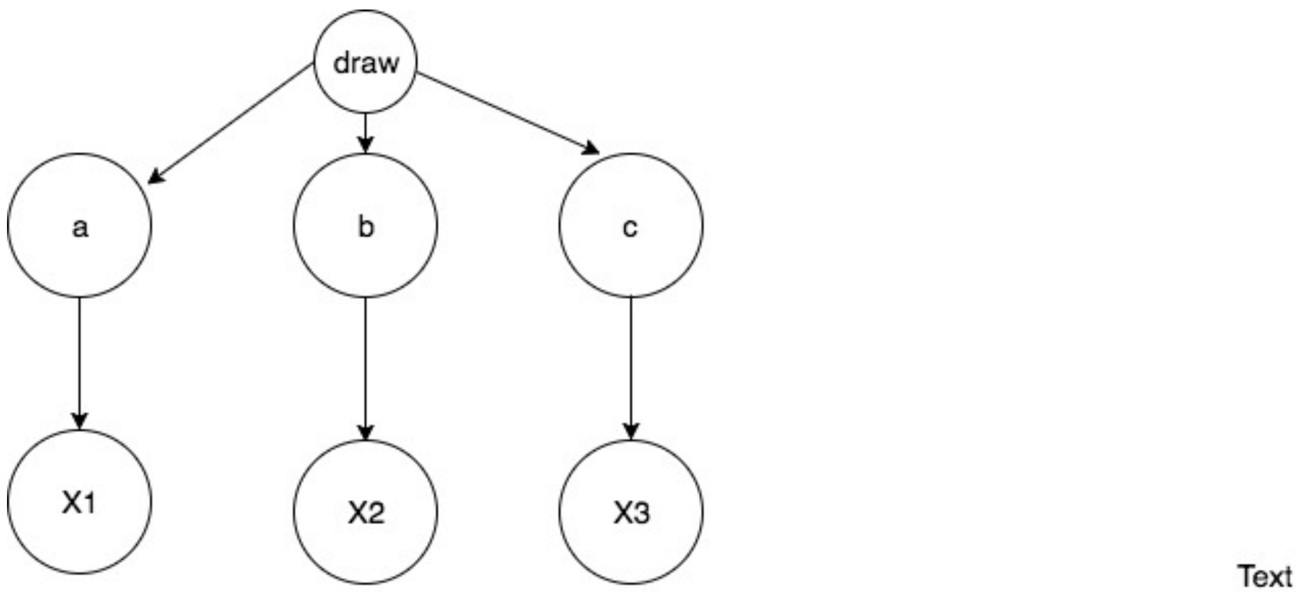
(1). Prove: $P(A,B|K) = P(A|B,K)P(B|K)$.

$$\begin{aligned} RHS &= P(A,B,K) / P(B,K) * P(B|K) \\ &= P(A,B,K) / [P(B|K)*P(K)] * P(B|K) \\ &= P(A,B,K)/P(K) \\ &= P(A,B|K) \\ &= LHS \end{aligned}$$

(2). Prove: $P(A|B,K) = P(B|A,K)*P(A|K)/P(B|K)$

$$\begin{aligned} LHS &= P(A,B,K) / P(B,K) \\ &= P(A,B,K) / [P(B|K)*P(K)] \\ &= P(A,B|K) / P(B|K) \\ &= P(B,A|K) / P(B|K) \\ &= P(B|A,K)*P(A|K) / P(B|K) \quad \text{(based on the conslusion from (1))} \\ &= RHS \end{aligned}$$

2.



There are two CPT tables:

Coin	Pr(Coin)
a	1/3
b	1/3
c	1/3

Coin	Outcome Xi	Pr(Xi|coin)
a	X1(heads)	0.2
b	X2(heads)	0.6
c	X3(heads)	0.8

3.

W	Black	Square	One	P
1	T	T	T	2/13
2	T	T	F	4/13
3	T	F	T	1/13
4	T	F	F	2/13
5	F	T	T	1/13
6	F	T	F	1/13
7	F	F	T	1/13
8	F	F	F	1/13

P(α1) = 2/13 + 4/13 + 1/13 + 2/13 = 9/13 (worlds 1, 2, 3 ,4)
P(α2) = 2/13 + 4/13 + 1/13+ 1/13 = 8/13 (worlds 1,2,5,6)
P(α3) = (2/13 + 4/13 + 1/13) / (1-1/13-1/13) = 7/11 (worlds 1,2,5)

Set1:
α: One
β: Square
γ: Black

Set2:
α: One
β: Square
γ: Not black

4(a). The assumptions are listed below:
I (A, NULL, BE)
I (B, NULL, AC)

I (C, A, BDE)
I (D, AB, CE)
I (E, B, ACDFG)
I (F, CD, ABE)
I (G, F, ABCDEH)
I (H, FE, ABCDG)

(b).
d_separated (A, BH, E):
False, in the path ACFHE, FHE is a converged value but H⊆BH, this path is not blocked.

d_seperated (G, D, E):
False, the path GFHE is not blocked.

d_seperated (AB, F, GH)
False, the path ADBEH is not blocked.

(c).
P (a, b, c, d, e, f, g, h) =
P(a|b,c,d,e,f,g,h)*P(b|c,d,e,f,g,h)*P(c|d,e,f,g,h)*P(d|e,f,g,h)*P(e|f,g,h)*
P(f|g,h)*P(g|h)*P(h)

(d).
A and B are independent, so
P (A=0, B=0) = P(A=0) * P(B=0)
= 0.8*0.3
= 0.24

Since A and E are independent,
P(E=1 | A=1) = P(E=1)
= P(E=1, B =1) + P(E=1, B=0)
= P(E=1| B=1) * P(B=1) + P(E=1| B=0) * P(B=0)
= 0.1*0.7 + 0.9*0.3
= 0.34