
CS161: FUNDAMENTALS OF ARTIFICIAL INTELLIGENCE

Fall 2017

Assignment 7. Due Sunday, November 26, 2017, 11:55pm

Please submit your solutions on CCLE. The submitted file should be plain text or a for-matted PDF file (no scans or pictures). Text files should have lines no longer than 100 characters and should be well-aligned when viewed with a monospace font.

1. Prove:

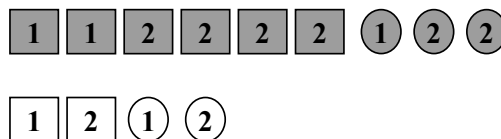
Generalized product rule: $\Pr(A, B \mid K) = \Pr(A \mid B, K)\Pr(B \mid K)$.

Generalized Bayes' rule: $\Pr(A \mid B, K) = \Pr(B \mid A, K)\Pr(A \mid K)/\Pr(B \mid K)$.

2. We have a bag of three biased coins a, b, and c with probabilities of coming up heads of 20%, 60%, and 80%, respectively. One coin is drawn randomly from the bag (with equal likelihood of drawing each of the three coins), and then the coin is flipped three times to generate the outcomes X_1 , X_2 , and X_3 .

Draw the Bayesian network corresponding to this setup and define the necessary CPTs (Conditional Probability Table).

3. (From Bayesian Networks by R. Neapolitan) Consider the set of objects below.



Mr. Y picked up an object at random from the above set. We want to compute the probabilities of the following events:

α_1 : the object is black;

α_2 : the object is square;

α_3 : if the object is one or black, then it is also square.

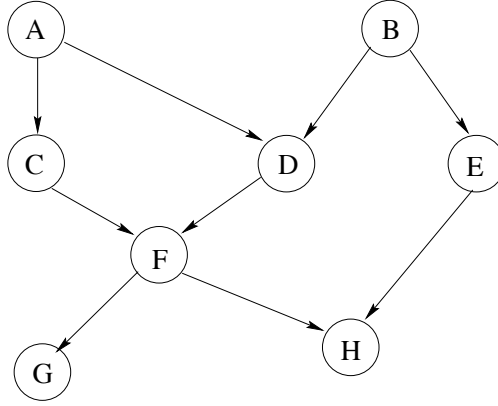


Figure 1: The DAG of a Bayesian network.

Construct the joint probability distribution of this problem. Use it to compute the above probabilities by explicitly identifying the worlds at which each α_i holds. Identify two sets of sentences α, β, γ such that α is independent of β given γ with respect to the constructed distribution.

4. Consider the DAG in Figure 1:

- (a) List the Markovian assumptions asserted by the DAG.
- (b) True or false? Why?
 - $d_separated(A, BH, E)$
 - $d_separated(G, D, E)$
 - $d_separated(AB, F, GH)$
- (c) Express $\Pr(a, b, c, d, e, f, g, h)$ in factored form using the chain rule for Bayesian networks.
- (d) Compute $\Pr(A = 0, B = 0)$ and $\Pr(E = 1 \mid A = 1)$. Justify your answers.

$\Pr(A = 0)$	$\Pr(A = 1)$
.8	.2

$\Pr(B = 0)$	$\Pr(B = 1)$
.3	.7

	$\Pr(E = 0 \mid B)$	$\Pr(E = 1 \mid B)$
$B = 0$.1	.9
$B = 1$.9	.1

	$\Pr(D = 0 \mid A, B)$	$\Pr(D = 1 \mid A, B)$
$A = 0, B = 0$.2	.8
$A = 0, B = 1$.9	.1
$A = 1, B = 0$.4	.6
$A = 1, B = 1$.5	.5