

$$1.(a). (P \Rightarrow \neg Q) \equiv (\neg P \vee \neg Q)$$

$$(Q \Rightarrow \neg P) \equiv (\neg Q \vee \neg P)$$

P	Q	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$	$\neg Q \vee \neg P$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

The last two columns are equal, thus those two pairs are equivalent.

$$(b). P \Leftrightarrow \neg Q \equiv (\neg P \vee \neg Q) \wedge (Q \vee P)$$

P	Q	$\neg P$	$\neg Q$	$P \Leftrightarrow \neg Q$	$(\neg P \wedge \neg Q) \vee (P \wedge Q)$
0	0	1	1	0	0
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

The last two columns are equal, thus these two pairs are equivalent.

$$2.(a). S \Rightarrow F \equiv \neg S \vee F$$

$$\neg S \Rightarrow \neg F \equiv S \vee \neg F$$

$$(\neg S \vee F) \Rightarrow (S \vee \neg F) \equiv (\neg(\neg S \vee F)) \vee (S \vee \neg F) \equiv (S \wedge \neg F) \vee (S \vee \neg F)$$

S	F	$\neg S$	$\neg F$	$S \wedge \neg F$	$S \vee \neg F$	$(S \wedge \neg F) \vee (S \vee \neg F)$
0	0	1	1	0	1	1
0	1	1	0	0	0	0
1	0	0	1	1	1	1
1	1	0	0	0	1	1

This is satisfiable (neither) because it contains both 1 and 0.

$$(b). S \Rightarrow F \equiv \neg S \vee F \quad (A)$$

$$(S \vee H) \Rightarrow F \vee E \equiv (\neg(S \vee H)) \vee F \equiv (\neg S \wedge \neg H) \vee F \quad (B)$$

$$A \Rightarrow B \equiv \neg A \vee B \equiv (S \wedge \neg F) \vee (\neg S \wedge \neg H) \vee F$$

S	F	H	$\neg S$	$\neg F$	$\neg H$	$S \wedge \neg F$	$\neg S \wedge \neg H \vee F$	$(S \wedge \neg F) \vee ((\neg S \wedge \neg H) \vee F)$
0	0	0	1	1	1	0	1	1
0	0	1	1	1	0	0	0	1
0	1	0	1	0	1	0	1	0
0	1	1	1	0	0	0	1	1
1	0	0	0	1	1	1	0	1
1	0	1	0	1	0	1	0	1
1	1	0	0	0	1	0	1	1
1	1	1	0	0	0	0	1	1

Since this expression contains both 1 and 0, thus it is satisfiable (neither) (c).

$$(S \wedge H) \Rightarrow F \equiv (\neg(S \wedge H)) \vee F \equiv \neg S \vee \neg H \vee \neg F \quad (A)$$

$$S \Rightarrow F \equiv \neg S \vee F$$

$$H \Rightarrow F \equiv \neg H \vee F$$

$$(\neg S \vee F) \vee (\neg H \vee F) \equiv \neg S \vee \neg H \vee F \quad (B)$$

$$A \Leftrightarrow B \equiv (\neg A \vee B) \wedge (\neg B \vee A)$$

S	H	F	$\neg S$	$\neg H$	$\neg F$	A	B	$\neg A \vee B$	$\neg B \vee A$	$(\neg A \vee B) \wedge (\neg B \vee A)$
0	0	0	1	1	1	1	1	1	1	1
0	0	1	1	1	0	1	1	1	1	1
0	1	0	1	0	1	1	1	1	1	1
0	1	1	1	0	0	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1	1
1	0	1	0	1	0	1	1	1	1	1
1	1	0	0	0	1	1	1	1	1	1
1	1	1	0	0	0	0	0	1	1	1

Since this expression always evaluates to 1, it's valid

3. a. My: mythical I: Immortal
Mg: magical H: Horned
Mo: mortal mammal

1. $My \Rightarrow I$

2. $\neg My \Rightarrow Mo$

3. $(I \vee Mo) \Rightarrow H$

4. $H \Rightarrow Mg$

$$b. \quad My \Rightarrow I : \quad \neg My \vee I$$

$$\neg My \Rightarrow Mo : \quad My \vee Mo$$

$$\begin{aligned} (I \vee Mo) \Rightarrow H : \quad (\neg(I \vee Mo)) \vee H &\equiv (\neg I \wedge \neg Mo) \vee H \\ &\equiv (\neg I \vee H) \wedge (\neg Mo \vee H) \end{aligned}$$

$$H \Rightarrow Mg : \quad \neg H \vee Mg$$

$$c. \quad 1. \quad \neg My \vee I$$

$$2. \quad My \vee Mo$$

$$3. \quad \neg I \vee H$$

$$4. \quad \neg Mo \vee H$$

$$5. \quad \neg H \vee Mg$$

$$6. \quad Mo \vee I \quad (1+2)$$

$$7. \quad I \vee H \quad (4+6)$$

$$8. \quad H \quad (3+7)$$

$$9. \quad Mg \quad (8+5)$$

We can only prove that the unicorn is magical and horned, can't prove that it is mythical.