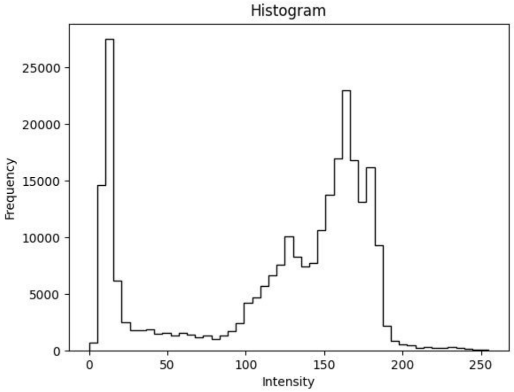
Histogram

Deepayan Bhowmik

Histogram: ?

The histogram function (or distribution) of an image is defined over all possible intensity levels.
For each intensity level, its value is equal to the number of the pixels with that intensity.



Histogram: Calculation

Simply count occurrences of each grayscale intensity value (or R,G,B colour value for colour images).

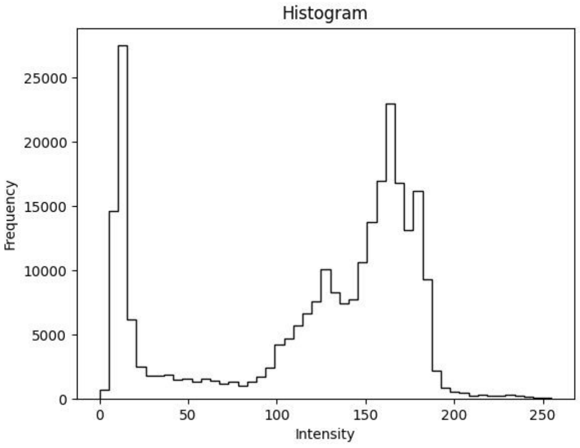
Algorithm pseudo-code:

```
initialise all histogram array entries to 0
for each pixel I(i,j) within the image I
 histogram(I(i,j)) = histogram(I(i,j))  + 1
end
```

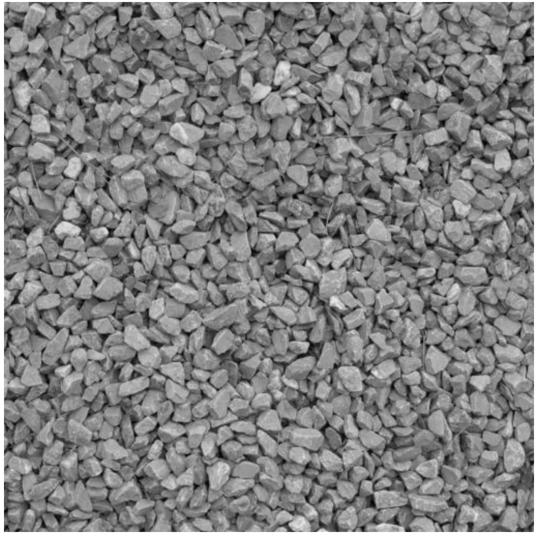
Histogram: Understanding

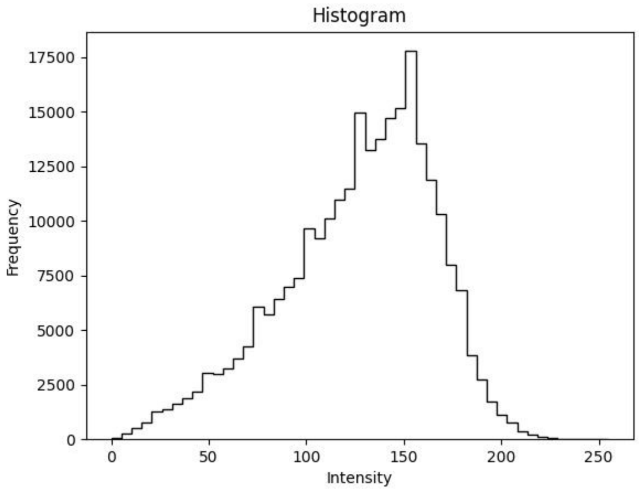
What is the max of the histogram x-axis ?
What is the max of the histogram y-axis ?
What part of this (simple) image does:

- The small peak in the histogram represent ?
- The large peak in the histogram represent ?



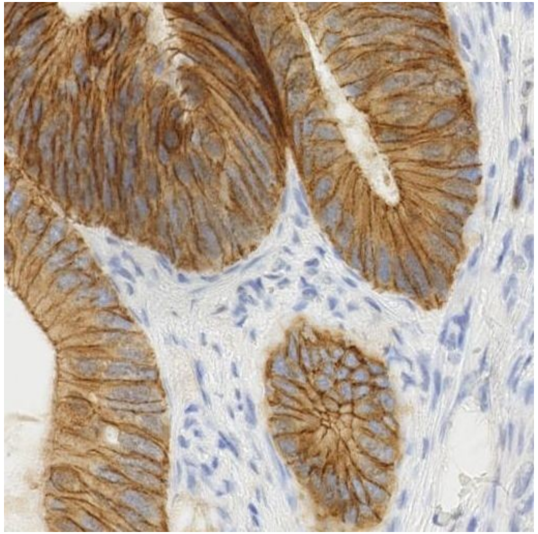
Histogram: More Complex ...

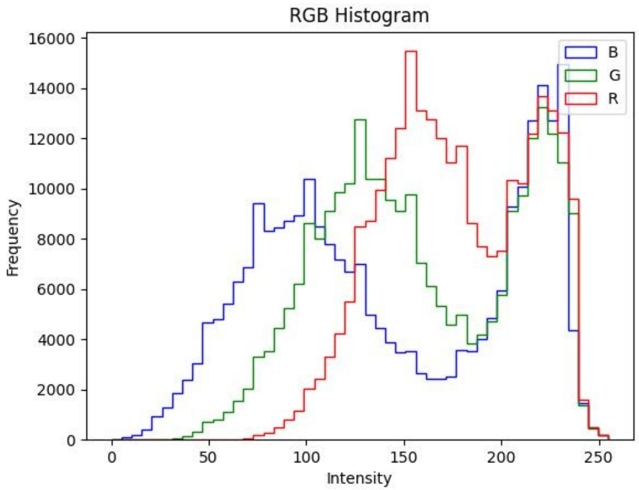




Histogram: Colour

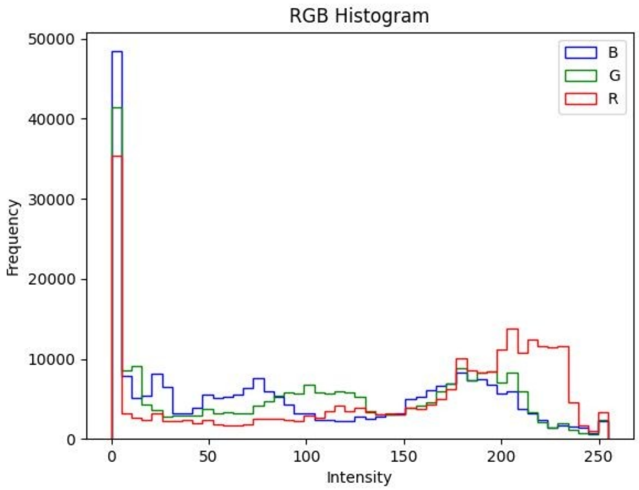
Extend to each R, G, B colour channel (separately).





Histogram: Colour More Complex

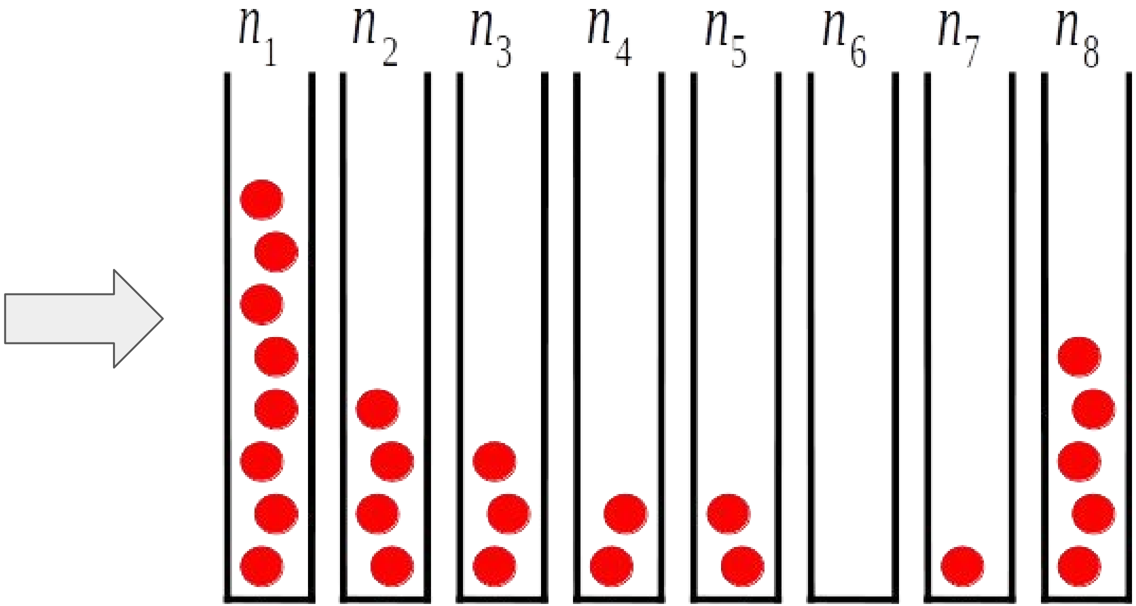
Extend to each R, G, B colour channel (separately).



Histogram: Construction

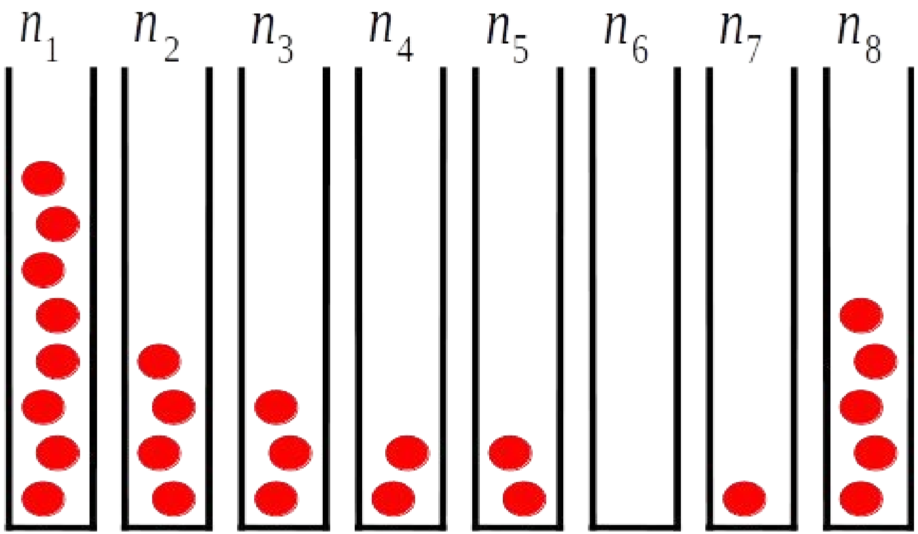
Consider a 5x5 image with integer intensities in the range between one and eight:

1	8	4	3	4
1	1	1	7	8
8	8	3	3	1
2	2	1	5	2
1	1	8	5	2



Histogram: Construction

Result: Intensity value v_k, has histogram function $h(v_k)$ value = n_k.



$$h(v_1)=8$$
$$h(v_2)=4$$
$$h(v_3)=3$$
$$h(v_4)=2$$
$$h(v_5)=2$$
$$h(v_6)=0$$
$$h(v_7)=1$$
$$h(v_8)=5$$
...

$$h(v_k)=n_k$$

Histogram: Normalization

The normalised histogram function is the histogram function divided by the total number of the pixels of the image:

$$p(v_k) = \frac{h(v_k)}{n} = \frac{n_k}{n}$$

The sum of the normalised histogram function over the range of all image pixel intensities is 1.0.

As such, it gives a measure of how likely is for a pixel to have a certain intensity.

That is, it gives the probability of occurrence the pixel intensity.

Histogram: Normalization

Our earlier 5 x 5 example image had 25 pixels in the image.

$$
\begin{aligned}
h(v_1) &= 8\\
h(v_2) &= 4\\
h(v_3) &= 3\\
h(v_4) &= 2\\
h(v_5) &= 2\\
h(v_6) &= 0\\
h(v_7) &= 1\\
h(v_8) &= 5\\
&\ldots
\end{aligned}
\qquad \Rightarrow \qquad
\begin{aligned}
p(v_1) &= 8/25 = 0.32\\
p(v_2) &= 4/25 = 0.16\\
p(v_3) &= 3/25 = 0.12\\
p(v_4) &= 3/25 = 0.08\\
p(v_5) &= 2/25 = 0.08\\
p(v_6) &= 0/25 = 0.00\\
p(v_7) &= 1/25 = 0.04\\
p(v_8) &= 5/25 = 0.20\\
&\ldots
\end{aligned}
$$

Histogram: Image Enhancement

Histograms are functions describing global information extracted from the image.

We can use this information to perform global image transforms.

We will look at three examples:

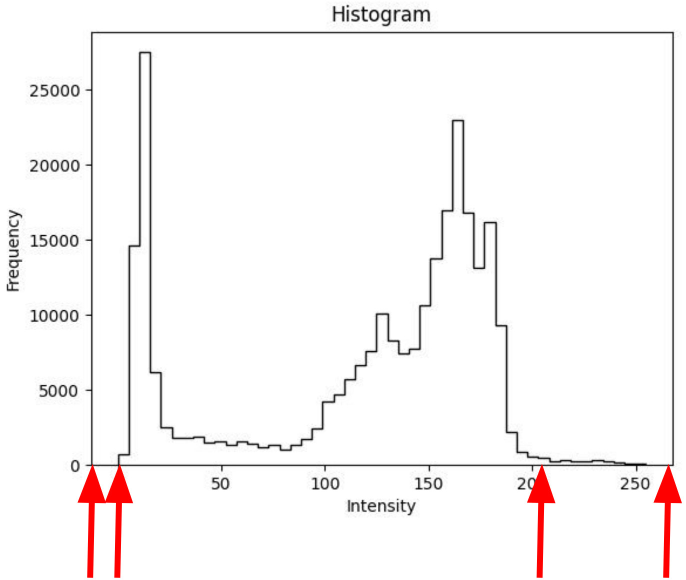
1. Contrast **Stretch**.
2. Histogram **Equalization**.
3. Histogram **Specification**.

Histogram: Contrast Stretch

Operation: stretches the pixel range over a larger dynamic range also known as normalisation

Approach: 4 measures from *h()*

- *a*: upper pixel quantisation limit.
- *b*: lower pixel quantisation limit.
- *c*: max. pixel value present.
- *d*: min. pixel value present.



$$I_{output}(i,j) = (I_{input}(i,j) - c)\left(\frac{a-b}{c-d}\right) + a$$

Histogram: Contrast Stretch

Histogram: Contrast Stretch Advanced

Use a robust method to select c and d.

First compute histogram of input

Use:

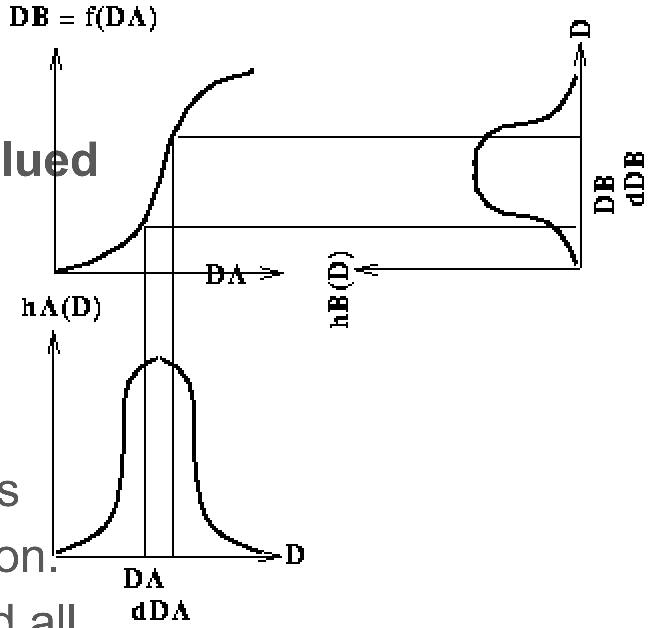
- Selection Method 1:
    - Select c and d at the 5th and 95th percentile points of distribution.
    - (i.e. 5% of pixels will be less than c and 5% greater than d).
- (or) Selection Method 2:
    - Find the most frequently image value (histogram peak).
    - Select a cut-off as a percentage of the peak.
    - Scan down from peak in either direction until last values above cut-off are reached. Select these as c and d.
    - N.B. method 2 is marginally weaker for complex, multi-peak histograms.

Histogram: Equalisation

Histogram modelling: modifying an image dynamic range so its histogram conforms to a given shape.

Histogram equalisation: use a monotonic, non-linear mapping so input pixels are mapped to an output image with a uniform histogram distribution.

Histogram: Equalisation



- **Map pixels** from image A to B using a **single-valued and monotonically increasing function *f()***.
- Transform the density DA to DB and similarly densities in the range DA->dDA to the range DB->dDB.
- ***f()* increases/decreases** the relative areas of this mapping (i.e. density) depending on its constitution.
- **Histogram equalisation**: output densities should all be an equal fraction of the maximum possible intensity value – i.e. all equalised to the same value.

Histogram: Equalisation

Reminder:

- Each histogram entry = density of a given intensity value.
- For a normalised histogram this is the probability density.
- A normalised histogram = probability distribution of image intensity.
- Histogram = statistical distribution of intensity levels (*h()*).

Thus construct cumulative histogram, *C()*, from *h()*:

$$C(i) = \sum_{v=0}^{v=i} h(v)$$

- Each entry in *C()* records the sum of the occurrence of each grey level upto and including the current entry.

Histogram: Equalisation

Thus .....

- *C(i)* records the frequency of occurrence all grey levels upto and including *h(i).*
- *C()* = single valued, monotonically increasing function:
    - In (very) simple language - an increasingly additive curve!
    - Gradient of *C()* ~= current equalisation of the image.
    - Think about it a constant gradient of 1 will be a perfectly equalised image (as increase at each step is constant).

Histogram: Equalisation

Idealised case: resulting equalised image will contain an equal number of pixels at each intensity level.

- Thus for L possible grey levels in an N pixel image the j^{th} entry of the cumulative histogram, $C(i)$ will have value jN/L
  - i.e. j times the equalised value.

General case: mapping between input pixel intensity value, i, and output pixel value, j, as follows:

- $C(i) = jN/L$

(for ideal case – as above) from which rearrangement gives:

- $j = (L/N)C(i)$

= mapping from input value i to output value j via the cumulative histogram of the input image.

Histogram: Equalisation

However max value of $j = L$ but range of grayscale values is strictly $0 \rightarrow (L-1)$ for a given image

- This is rectified by adding "-1" to the equation thus also requiring a check to ensure a value of $j=-1$ is not returned:

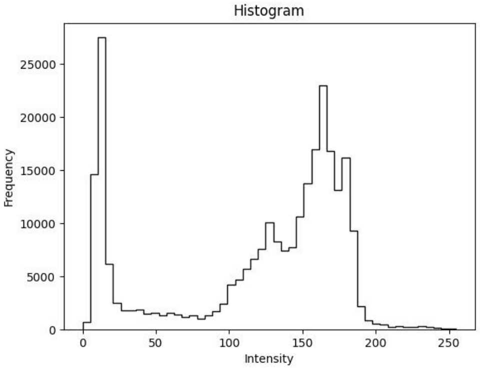
$$j = max(0, (L/N)C(i) -1)$$

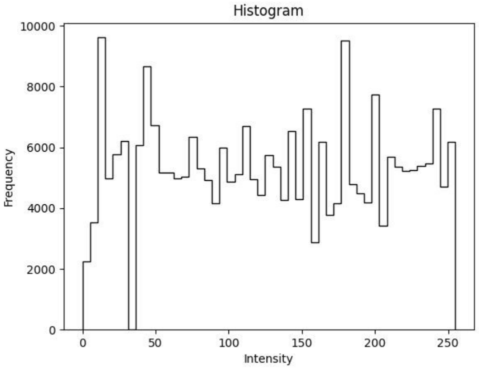
In our familiar notation, **Histogram Equalization** is:

$$I_{output}(x,y) = max(0, (L/N)C(I_{input}(x,y)) - 1)$$

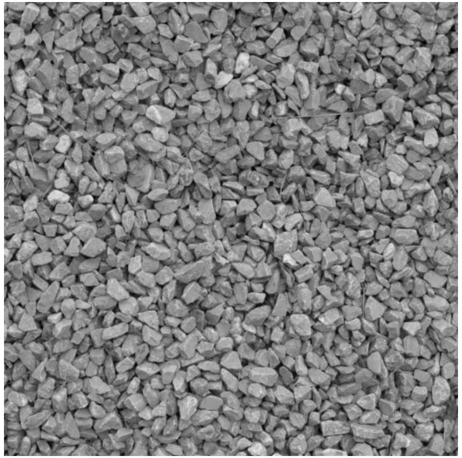
- $C()$ is the cumulative histogram for the input image, I_{input}.
- N is the number of pixels in the input image (CxR).
- L is the number of grey levels present.

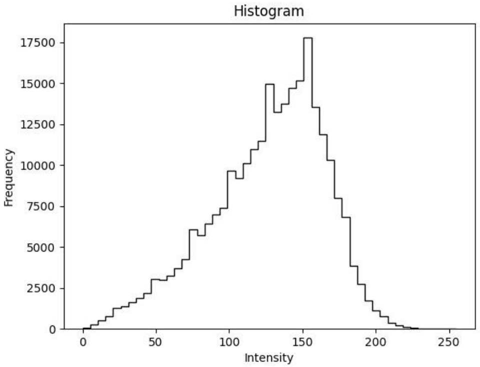
Histogram: Equalisation

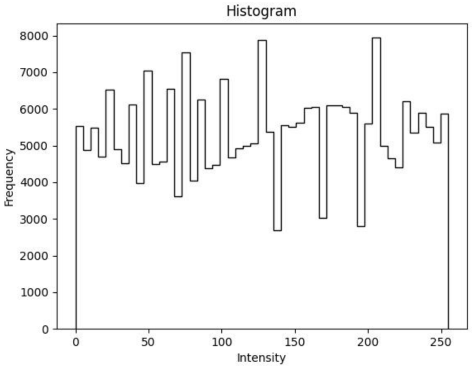




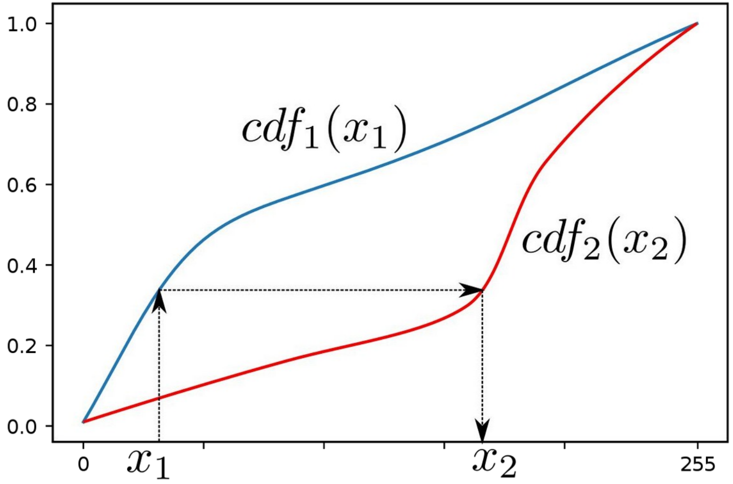
Histogram: Equalisation

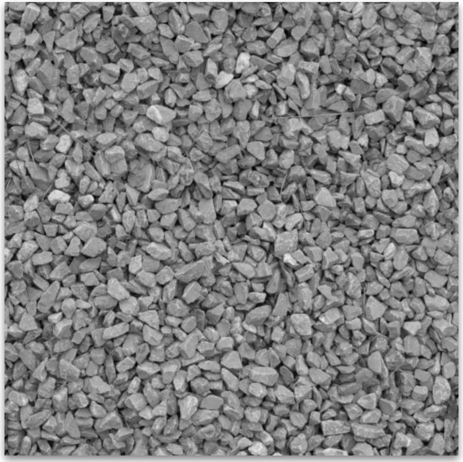


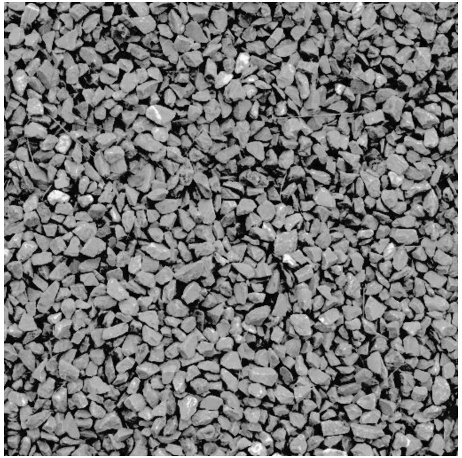




Histogram: Matching

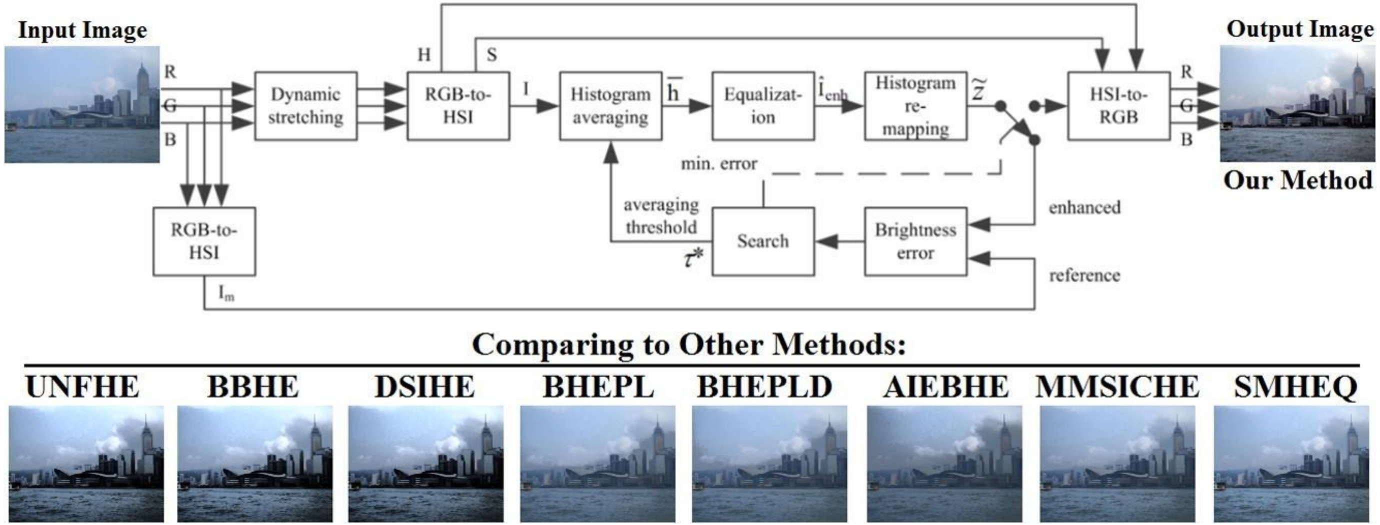






Applications

A pipelined approach including color channel stretching, histogram averaging and re-mapping is proposed. By using stretching, color information from a scene is restored.



SCF Lin, CY Wong, MA Rahman, G Jiang, S Liu, N Kwok, Image enhancement using the averaging histogram equalization (AVHEQ) approach for contrast improvement and brightness preservation Comput Electr Eng, 46 (2014), pp. 356-370.