Segmentation

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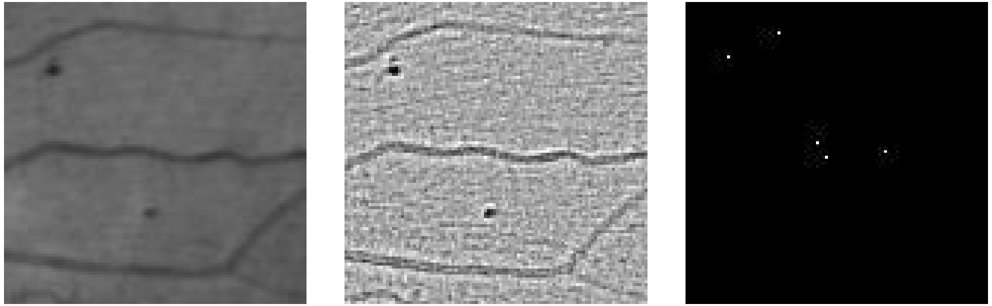
Image Segmentation

Image segmentation is the process of dividing an image into multiple parts. This is typically used to identify objects or other relevant information in digital images.

There are many different ways to perform image segmentation, including:

$$\text{Segmentation based on: } \begin{cases} \text{(1) Discontinuities} \begin{cases} \text{(1) Isolated points} \\ \text{(2) Lines} \\ \text{(3) Edges} \end{cases} \\ \text{(2) Similarity} \begin{cases} \text{(1) Thresholding} \\ \text{(2) Region growing} \\ \text{(3) Region splitting/merging} \end{cases} \end{cases}$$

Point Detection



−1	−1	−1
−1	8	−1
−1	−1	−1

Line Detection

−1	−1	−1
2	2	2
−1	−1	−1

Horizontal

−1	−1	2
−1	2	−1
2	−1	−1

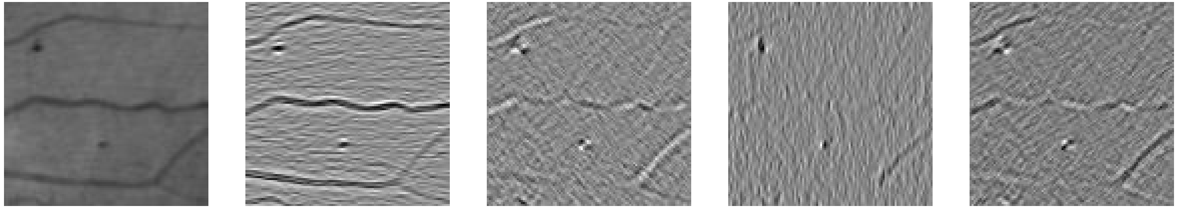
+45°

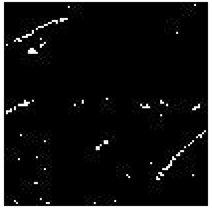
−1	2	−1
−1	2	−1
−1	2	−1

Vertical

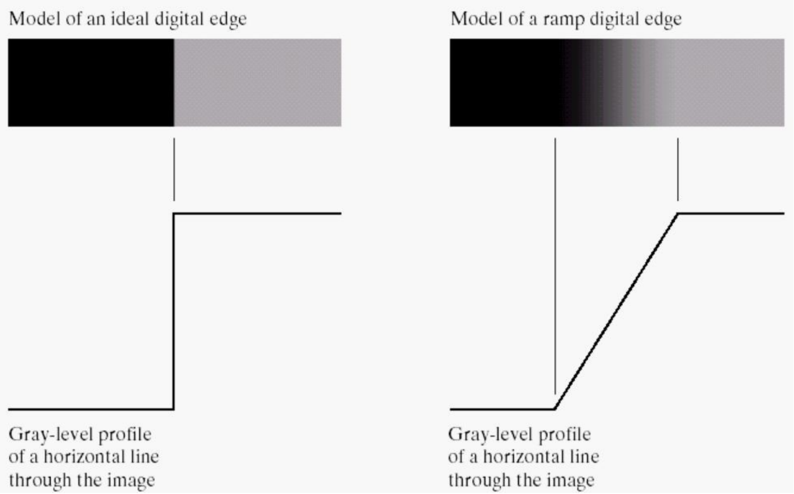
2	−1	−1
−1	2	−1
−1	−1	2

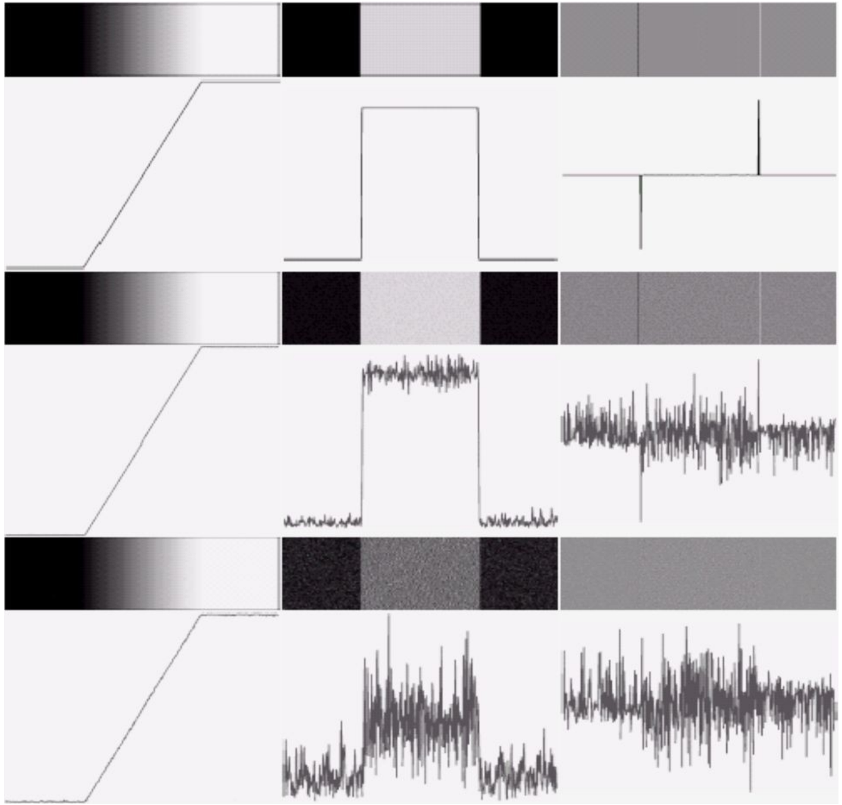
−45°





Edge Detection





Edge Detection: Gradient

Definition

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

Magnitude

$$\begin{aligned} \nabla f &= \mathrm{mag}(\nabla \mathbf{f}) \\ &= [G_x^2 + G_y^2]^{1/2} \\ &= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2} \end{aligned}$$

Direction

$$\alpha(x, y) = \tan^{-1}\left(\frac{G_y}{G_x}\right)$$

−1	0		0	−1
0	1		1	0

Roberts

−1	−1	−1		−1	0	1
0	0	0		−1	0	1
1	1	1		−1	0	1

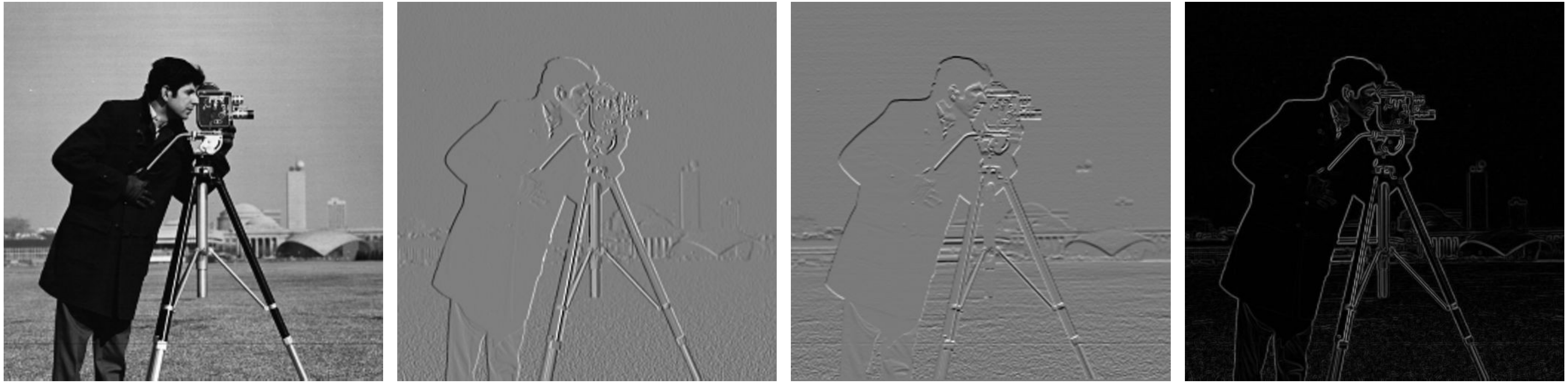
Prewitt

−1	−2	−1		−1	0	1
0	0	0		−2	0	2
1	2	1		−1	0	1

Sobel

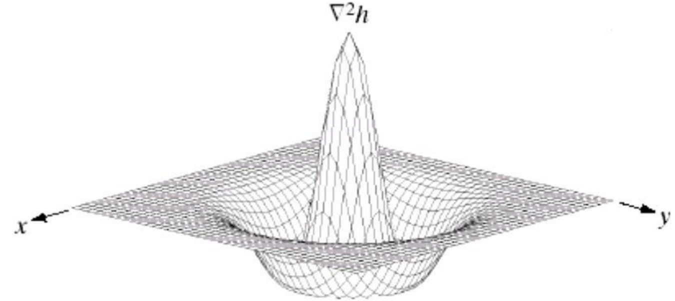
Edge Detection: Gradient

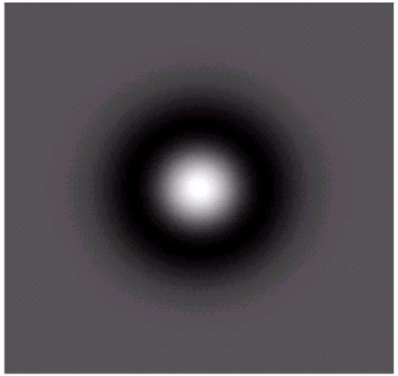
Robert

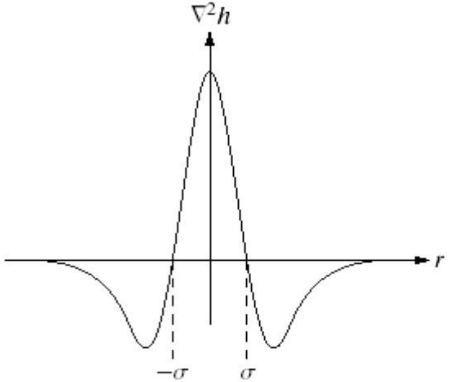


Sobel

Edge Detection: Laplacian







0	0	−1	0	0
0	−1	−2	−1	0
−1	−2	16	−2	−1
0	−1	−2	−1	0
0	0	−1	0	0

Edge Detection: Laplacian

Thresholding

A thresholded image $g(x, y)$ is defined as

$$g(x,y) = \begin{cases} 1, & f(x,y) > T \\ 0, & f(x,y) \leq T \end{cases},$$

where 1 is object and 0 is background

When $T = T[\, f(x,y)\,]$, threshold is **global**

When $T = T[\, p(x,y),\ f(x,y)\,]$, threshold is **local**

When $T = T[\, x,\ y,\ p(x,y),\ f(x,y)\,]$, threshold is **dynamic** or **adaptive**

Thresholding: Basic Global

Algorithm used to obtain T automatically

(1) Select an initial estimate for T

(2) Segment image using $T \longrightarrow$ Group G_1 (values $> T$)
Group G_2 (values $\leq T$)

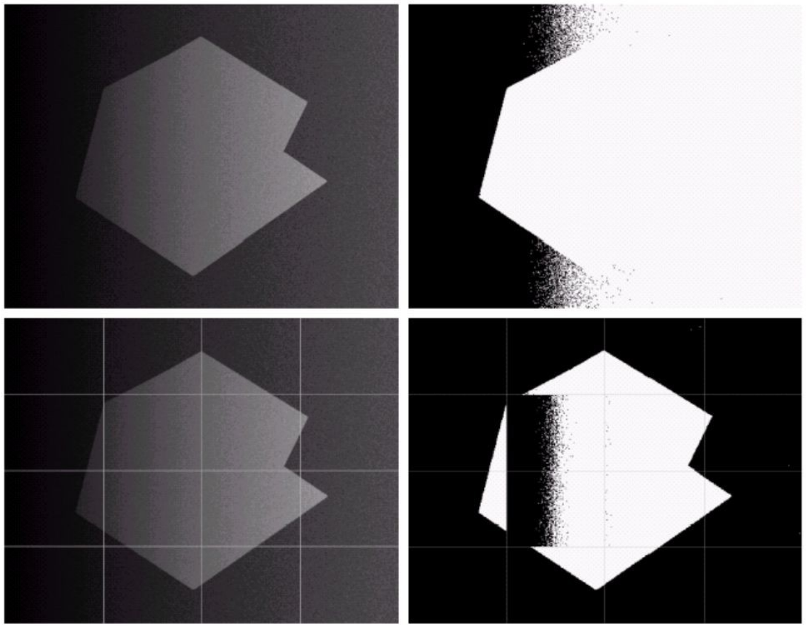
(3) Compute average gray level values for G_1, $G_2 \longrightarrow \mu_1$, μ_2

(4) Compute a new threshold value $T = \frac{1}{2}(\mu_1 + \mu_2)$

(5) Repeat (2) through (4) until the difference in T in successive iterations is smaller than T_0

101.98034790209195
91.79847700039974
88.59251602169817
87.96379671902227
87.76400903673864
87.76400903673864

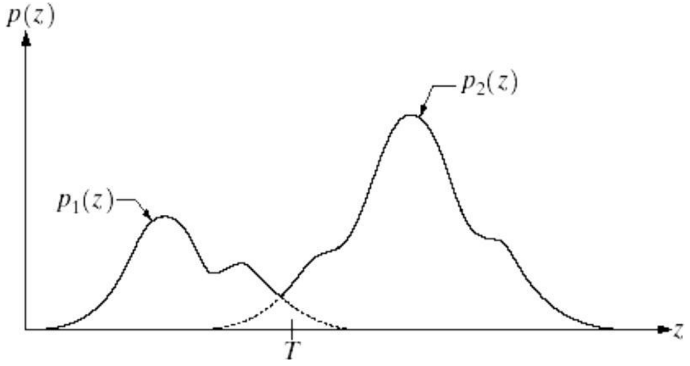
Thresholding: Basic Adaptive



1. Divide original image into subimages
2. Utilize a different threshold to segment each subimage
3. Difficulties: Subdivision and subsequent threshold estimation

Thresholding: Optimal Global and Adaptive

Assume two principal gray level regions...



Mixture PDF describing overall gray level variation...

$$p(z) = P_1\, p_1(z) + P_2\, p_2(z)$$

P_1: probability that pixel is object pixel
P_2: probability that pixel is background pixel

$$P_1 + P_2 = 1$$

Select T that minimizes average error in making decision

Probability in erroneously classifying background as object

$$E_1(T) = \int_{-\infty}^{T} p_2(z)\, dz$$

Probability in erroneously classifying object as background

$$E_2(T) = \int_{T}^{\infty} p_1(z)\, dz$$

Thresholding: Optimal Global and Adaptive

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Overall probability of error is

$$E(T) = P_2\, E_1(T) + P_1\, E_2(T)$$

Threshold value for which the error is minimal

$$P_1\, p_1(T) = P_2\, p_2(T) \quad \ldots \quad (**)$$

Approximate $p_1(z)$ and $p_2(z)$ with Gaussian densities...

$$p(z) = \frac{P_1}{\sqrt{2\pi}} e^{-(z-\mu_1)^2/2\sigma_1^2} + \frac{P_2}{\sqrt{2\pi}} e^{-(z-\mu_2)^2/2\sigma_2^2}$$

Using this equation in $(**)$ results in $AT^2 + BT + C = 0$, where

$$\begin{aligned} A &= \sigma_1^2 - \sigma_2^2 \\ B &= 2\,(\mu_1\,\sigma_2^2 - \mu_2\,\sigma_1^2) \\ C &= \sigma_1^2\,\mu_2^2 - \sigma_2^2\,\mu_1^2 + 2\,\sigma_1^2\,\sigma_2^2 \ln(\sigma_2\,P_1/\sigma_1\,P_2) \end{aligned}$$

Note that two threshold values are generally required

Also note that if $\sigma^2 = \sigma_1^2 = \sigma_2^2$, one threshold is sufficient:

$$T = \frac{\mu_1 + \mu_2}{2} + \frac{\sigma^2}{\mu_1 - \mu_2} \ln\left(\frac{P_2}{P_1}\right)$$

When $P_1 = P_2$ and/or $\sigma = 0$ the optimal threshold is the average of the means

Segmentation: Region Growing

- Start from a set of **seed points** and from these points grow the regions by appending to each seed those neighbouring pixels that have similar properties

- The selection of the seed points depends on the problem. When a priory information is not available, clustering techniques can be used: compute the above mentioned properties at every pixel and use the centroids of clusters

- The selection of **similarity criteria** depends on the problem under consideration and the type of image data that is available

- Descriptors must be used in conjunction with **connectivity** (adjacency) **information**

- Formulation of a **"stopping rule"**. Growing a region should stop when no more pixels satisfy the criteria for inclusion in that region.

- When a model of the expected results is partially available, the consideration of **additional criteria** like the size of the region, the likeliness between a candidate pixel and the pixels grown so far, and the shape of the region can improve the performance of the algorithm.

Segmentation: Region Splitting and Merging

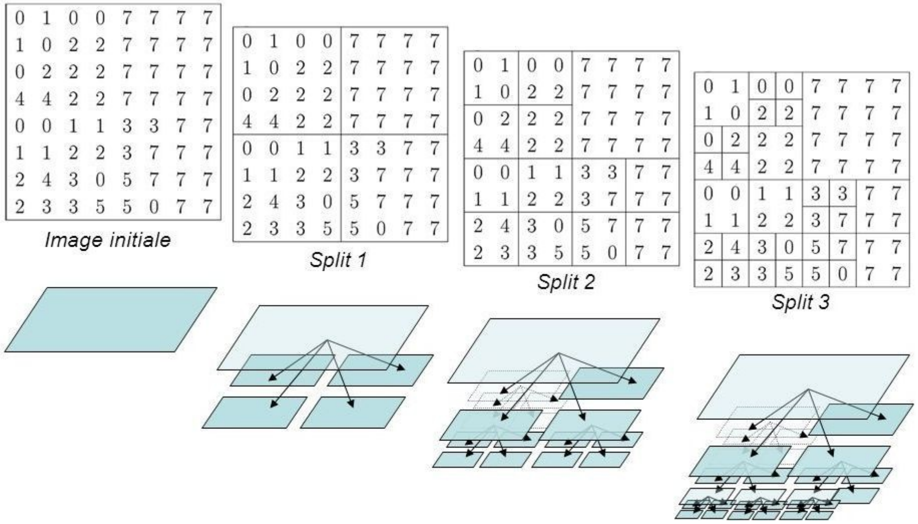
Subdivide an image initially into a set of arbitrary, disjoint regions and then merge and/or split the regions in an attempt to satisfy the necessary conditions

Let R represent entire image region and select a predicate P

(1) Split into four disjoint quadrants any region R_i for which $P(R_i) = \text{FALSE}$

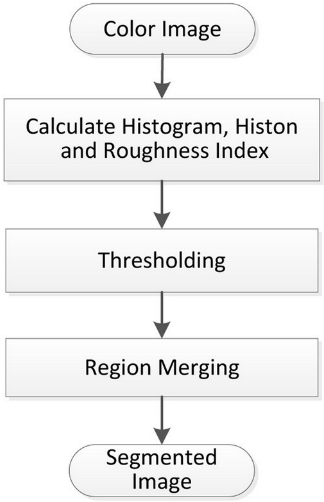
(2) Merge any adjacent regions R_j and R_k for which $P(R_j \cup R_k) = \text{TRUE}$

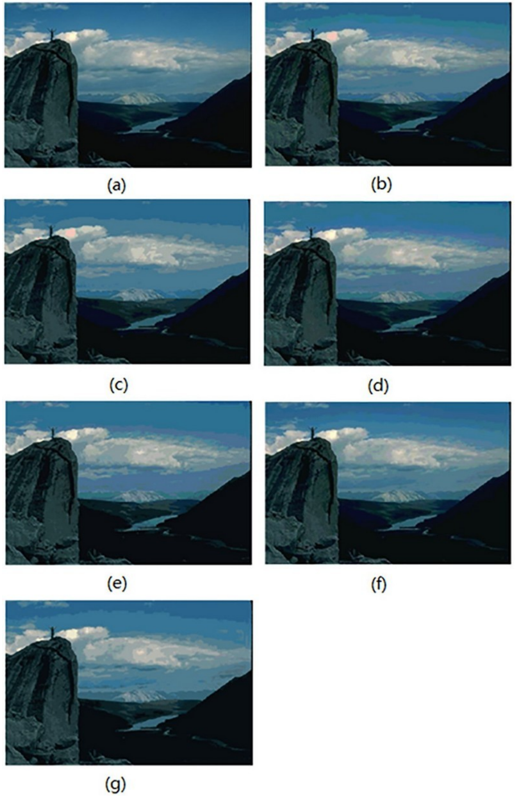
(3) Stop when no further merging or splitting is possible



Applications

Hierarchical-histogram-based thresholding - a multigranularity abstraction of the color image which can adaptively identify the thresholds from valleys.





Li, M., Wang, L., Deng, S., Zhou, C.: Color image segmentation using adaptive hierarchical-histogram thresholding. PLoS ONE 15(1), e0226345 (2020)

(a) original image, (b, c) initial segmented result and result after region merging based on the histon, (d, e) initial segmented result and result after region merging based on the roughness index, (f, g) initial segmented result and result after region merging based on AHHT.