Risk Aware Agent for Bidding in Second-Price Sealed-Bid Auctions and variants

This technical report introduces two forms of a risk aware agent for bidding in Simultaneous Second-Price Sealed-Bid Auctions but who's heuristics are general enough to be used in other auction settings. The main idea of the risk aware agent is the computation of a utility score that given predictions of prices over bundles of goods, considers the risk associated with the distribution of prices over each good contained in a given bundle. This utility is a generalization of the classic acquisition problem described in (Boyan & Greenwald, 2001) and used by every agent in (Yoon & Wellman, 2011) .

As a baseline, all agents from (Yoon & Wellman, 2011) were implemented to test the efficacy of the proposed risk-aware agent as well as parallel implementations of both point price predicting and distribution predicting algorithms in order to maximize the number of auction simulations executed in minimal time. The implementation language is Python and all code and documentation can be found in the github repository:

git://github.com/bam593/bmProjects.git

The contents can be viewed with a web browser at the address:

https://github.com/bam593/bmProjects/tree/master/courses/fall2011/csci2951

Motivation

Every strategy profile described in (Yoon & Wellman, 2011) assumes the bidding process can be decomposed into two independent modules:

1) Identify a set of goods that if won would maximize the agent's surplus, defined as valuation less cost (this step is known as the acquisition problem).
2) Compute bids to place on the goods in the bundle that solves that the acquisition problem.

That is we first define a bundle as a collection of goods that we can potentially procure at auction.

If there are m goods available, we define the set of all possible bundles, X, available for purchase as

$$X = \{x : x \in \mathbb{P}^m\}$$

where $\mathbb{P}^m$ represents the power set of m items. For example, lets' assume $m = 4$, we can enumerate every possible bundle using bit vectors of length 4, representing the set of all bundles as:

$$X = \{[0,0,0,0], [0,0,0,1], \dots, [1,1,1,0], [1,1,1,1]\}$$

If the j^{th} entry is 1 in the i^{th} bundle bit vector, this indicates the j^{th} good is purchased in bundle i, otherwise the j^{th} good is not included in the bundle.

The agents' valuation function is then a function that assigns a scalar to each bundle in the powerset and is known to each agent apriori

$$v : \mathbb{P}^m \rightarrow \mathbb{R}.$$

The function used in this report are described in (Sodomka & Greenwald).

Given a vector of prices representing the price paid for each good in a bundle, $p \in \mathbb{R}^m$ we can calculate the cost , c, associated with each bundle as the dot product of the i^{th} bundle and the price vector p and thus compute the surplus σ for each bundle.

$$c = x_i \cdot p$$

$$\begin{aligned}\sigma_i(X,p) &= v - c \\ &= v - x_i \cdot p\end{aligned}$$

Therefore the acquisition problem of step (1) is a matter of solving (Boyan & Greenwald, 2001):

$$X^* = ACQ_i(p) = \arg\max_{X \subseteq \mathcal{X}} \sigma_i(X,p)$$

The non-additive valuation function described in (Sodomka & Greenwald) and (Yoon & Wellman, 2011) is used to model complementary goods. Goods A and B are complementary if:

$$v(A\ U\ B) > v(A) + v(B)$$

That is if we are able to purchase only part of the bundle that solves the acquisition problem, the valuation of the bundle may be zero exposing the agent to a possible negative surplus as he will still be obligated to pay for the items he wins. Therefore, given X^*, (Yoon & Wellman, 2011) describe different strategy profiles that bid differently on this bundle, using techniques based on marginal value calculations shade the bids up to make sure the agent obtains the necessary goods in the target bundle. They also describe a profile bidEvaluator, that uses multiple base strategy profiles to generate candidate bids then evaluate the confidence in each bid against a price distribution and choose the bid by evaluating the candidate bids against the probability distribution over prices of goods.

There are two versions of the risk aware strategy profile, riskAware1 and riskAware2.

<u>riskAware1</u>

riskAware1 attempts to solve a more general form of the acquisition problem, thus modifying the first step above. Given this alternative target bundle, we can use any strategy described in (Yoon & Wellman, 2011) to produce a bid.

From economics, mean-variance utility function for returns on investments is defined as (Bodi, Kane, & Marcus, 2010):

$$U = \mathbb{E}_f[r] - \frac{1}{2} * A * var[r]$$

The free parameter A defines how risk-adverse a particular investor is, $\mathbb{E}[r]$ defines the expected rate of return of an investment portfolio given a probability distribution f and $var[r]$ defines the variance associated with the rate of return of the investment and serves as a measure of risk.

Though this function is used in the context of identifying optimal investments for an investor given the investors' level of risk aversion, we can modify this function to conform to our problem.

Define:

$$U = \mathbb{E}_f[\sigma_i] - \frac{1}{2}\, A\, var[\sigma_i]$$

Using the following property of variance:

$$var[\sigma_i] = var\left[v_i + \sum_{j=1}^{m} x_{ij} * p_j \right]$$

$$= var\left[\sum_{j=1}^{m} x_{ij} * p_j \right]$$

we write the mean-variance utility as:

$$U = \mathbb{E}_f[\sigma_i] - \frac{1}{2}\, A\, var\left[\sum_{j=1}^{m} x_{ij}\, p_j \right]$$

Let us assume as in (Yoon & Wellman, 2011) that prices are independent over goods j,

$$U = \mathbb{E}_f[\sigma_i] - \frac{1}{2} A \sum_{j=1}^{m} x_{ij}^2 var[p_j]$$

$$= \mathbb{E}_f[\sigma_i] - \frac{1}{2} A \sum_{j=1}^{m} \delta(j \in X_i) var[p_j]$$

Given the function v and a distribution over prices, we easily calculate $\mathbb{E}_f[\sigma_i]$ and $var[p_j]$. The intuition behind this heuristic is that we still want to pick the bundle that maximizes the expected surplus for the agent, however, there may be "safer" bundles to bid on quantifying the risk associate with each bundle by the sum of the variance of the individual goods in the bundle.

We can adapt the variance term to better quantify risks present in auctions. Given an arbitrary distribution over the price of a good, we can see that the risk posed to the agent are the realization of closing prices that are greater than what the agent will bid.

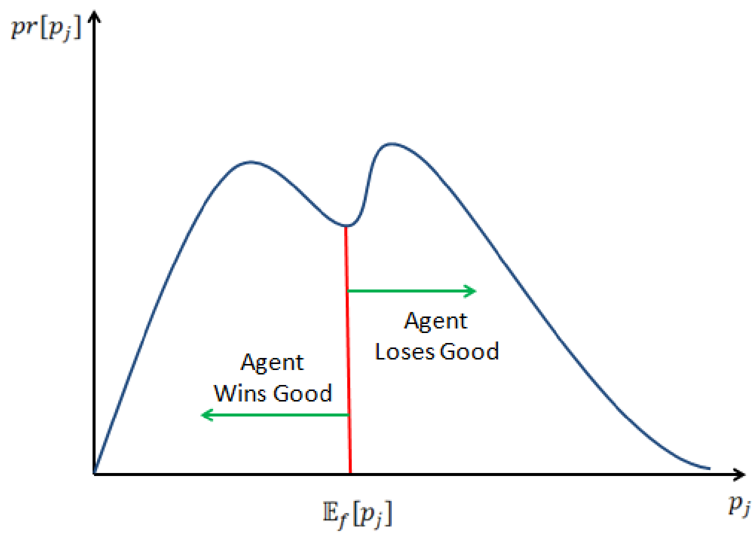

Figure 1: Agent Risk

For an auction with one item for sale, the agent risks losing the item to another agent if the closing price is greater than his bid. Therefore, quantifying risk associated with the expected surplus of a bundle by the sum of marginal variances overestimates the risk associated with each good. Instead we wish to quantify only the variance with respect to closing prices that are above the expected value.

We can do this by calculating an upper-partial variance (Bodi, Kane, & Marcus, 2010):

$$\widehat{UPV_j} = \sum_{t=1}^{T} \delta(p_{jt} \geq \mathbb{E}_f(p_{jt})(\mathbb{E}_f[p_j] - p_{jt})^2$$

This estimator is assuming a discrete (histogram) distribution over prices with bins indexed by t. We add a price only to the variance (risk) calculation when computing the expected surplus utility if the realized price is greater than the expected value.

Thus the final form of our utility function is:

$$U = \mathbb{E}_f[\sigma_i] - \frac{1}{2} A \sum_{j=1}^{m} \delta(j \in X_i)\widehat{UPV_j}$$

We can then define the acquisition problem as:

$$X^* = \arg\max_{X \subseteq \mathcal{X}} U(X, p)$$

Given the optimal bundle X^* we can then calculate a bid based on any strategy outlined in (Yoon & Wellman, 2011), e.g. bidding target prices or marginal values etc.

Note that A is a free parameter of our system and can be learned or chosen experimentally. There are some intuitive explanations for different values of A:

- $A > 0$
    - Agent is risk-adverse
    - The agent's utility is reduced proportionally to the quantified risk
- $A < 0$
    - Agent is a risk-lover or gambler
    - The agent gains utility not only through increased expected surplus but by the action of taking more risky positions
- $A = 0$
    - The agent is risk-neutral
    - The agent does not factor risk into the overall utility of a bundle
    - This reduces to the classical acquisition formulation and shows that the proposed utility is a more general form of the original acquisition problem in (Boyan & Greenwald, 2001)

riskAware2

In riskAware2 we assumed we could produce a strategy profile by dividing the bidding process into two modules, one identifying an optimal bundle on which to bid and another defining what to bid given an optimal bundle.

It may be advantageous to consider the bidding problem as one global problem.

Let us define a utility as in riskAware1 with some modifications.

Let X^{final} indicate the bundle of goods that are purchased after placing a bid, that is the bundle of goods the agent has successfully obtained.

Let us make the surplus dependent on X^{final} rather than a specific bundle X_i,

$$\sigma(p) = v - X^{final} \cdot p$$

$$\mathbb{E}_f\big(\sigma(p)\big) = \mathbb{E}[v - X^{final} \cdot p]$$

Recall, f is a distribution over closing prices, not p itself. So if we view p as a non-random quantity, the only random variable in this equation is X^{final} which is dependent on bid, p and the probability that the bid with be greater than the closing price p_c.

$$\mathbb{E}_f\big(\sigma(p)\big) = v - \mathbb{E}_f\big[X^{final}\big|p\big] \cdot p$$

assuming that the distributions over closing prices are independent:

$$\mathbb{E}_f\big[X^{final}\big|p\big] = \sum_{j=1}^{m}(P[p_i \geq p_{ic}] \cdot 1 + P[p_i < p_{ic}] \cdot 0)$$

$P[p_i > p_{ic}]$ indicates the good i was won and so the i^{th} good is included in the bundle ($X_i^{final} = 1$). Where $P[p_i < p_{ic}]$ indicates the good i was not obtained ($X_i^{final} = 0$). This equation then reduces to:

$$\begin{aligned}\mathbb{E}_f\left[X^{final} \middle| p\ \right] &= \sum_{j=1}^{m} P[p_i \geq p_{ic}] \\ &= \sum_{j=1}^{m} F(p_i)\end{aligned}$$

so returning to the expected surplus given a bid:

$$\begin{aligned}\mathbb{E}_f[\sigma(p)] &= v - \sum_{j=1}^{m} P[p_i \geq p_{ic}]p_i \\ &= v - \sum_{j=1}^{m} F(p_i)p_i\end{aligned}$$

We are now ready to express the a utility function explicitly dependent on a bid p.

$$\begin{aligned}U(p) &= v - X^{final} \cdot p - \frac{1}{2}A\sum_{j=1}^{m} \delta(j \in X^{final})\widehat{UPV_j} \\ &= v - \sum_{j=1}^{m} F(p_i)p_i - \frac{1}{2}A\sum_{j=1}^{m} \delta(j \in X^{final})\sum_{t=1}^{T} \delta(p_{jt} \geq \mathbb{E}_f(p_{jt})\left(\mathbb{E}_f[p_j] - p_{jt}\right)^2\end{aligned}$$

It may be possible to repose the bidding problem as a single optimization of utility over the price vector. That is can we search the utility space by modifying the values of p in order to optimize the utility then take the optimizing p to be our bid?

We can then take the expected value of this utility:

$$\mathbb{E}_f[U(p)] = \mathbb{E}_f[\sigma_i(p)] - \frac{1}{2}A\sum_{j=1}^{m} \mathbb{E}_f[\delta(j \in X_i)]\sum_{t=1}^{T} \mathbb{E}_f[\delta(p_{jt} \geq \mathbb{E}_f(p_{jt})]\left(\mathbb{E}_f[p_j] - p_{jt}\right)^2$$

Note that the CDF, $F(x)$ of a distribution $f(x)$ can be written as:

$$F(x) = \mathbb{E}_f[\delta(X \leq x)]$$

hence:

$$1 - F(x) = \mathbb{E}_f[\delta(X > x)]$$

Also note that $\delta(j \in X_i)$ indicates that the j^{th} good is contained in the bundle X_i