RBS

JEE-MAIN 2017

Test Booklet Code

D

PAPER - 1 : MATHEMATICS, PHYSICS & CHEMISTRY

Do not open this Test Booklet until you are asked to do so.

Read carefully the Instructions on the Back Cover of this Test Booklet.

Important Instructions:

1. Immediately fill in the particulars on this page of the Test Booklet with ***only Black Ball Point Pen*** provided in the examination hall.
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of **90** questions. The maximum marks are **360.**
5. There are ***three*** parts in the question paper A, B, C consisting of, **Mathematics, Physics and Chemistry** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
6. *Candidates will be awarded marks as stated above in instruction No. 5 for correct response of each question. 1/4 (one fourth) marks of the total marks allotted to the question (i.e., 1 mark) will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
7. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. For writing particulars/ marking responses on **Side-1** and **Side-2** of the Answer Sheet ***use only Black Ball Point Pen*** *provided in the examination hall.*
9. No candidates is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc., except the Admit Card inside the examination room/hall.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in **4** pages (Pages **20–23**) at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room / Hall. ***However, the candidates are allowed to take away this Test Booklet with them.***
12. The CODE for this Booklet is **D**. Make sure that the CODE printed on **Side–2** of the Answer Sheet and also tally the serial number of the Test Booklet and Answer Sheet are the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the invigilator for replacement of both the Test Booklet and the Answer Sheet.
13. **Do not fold or make any stray marks on the Answer Sheet.**

Name of the Candidate (in Capital letters) : ____________________

Roll Number : in figures | | | | | | | | |

: in words ____________________

Examination Centre Number : | | | | | | |

Name of Examination Centre (in Capital letters) : ____________________

Candidate's Signature : ____________________

1. Invigilator's Signature : ____________________

2. Invigilator's Signature : ____________________

Read the following instructions carefully :

1. The candidates should fill in the required particulars on the Test Booklet and Answer Sheet (***Side-1***) with ***Black Ball Point Pen.***
2. For writing/marking particulars on ***Side-2*** of the Answer Sheet, use ***Black Ball Point Pen only*****.**
3. The candidates should not write their Roll Numbers anywhere else (except in the specified space) on the Test Booklet/Answer Sheet.
4. Out of the four options given for each question, only one option is the correct answer.
5. For each ***incorrect response, ¼ (one-fourth)*** of the total marks allotted to the question (i.e., 1 mark) will be deducted from the total score. ***No deduction*** from the total score, however, will be made ***if no response*** is indicated for an item in the Answer Sheet.
6. Handle the Test Booklet and Answer Sheet with care, *as under no circumstances (except for discrepancy in Test Booklet Code and Answer Sheet Code), another set will be provided.*
7. The candidates are not allowed to do any rough work or writing work on the Answer Sheet. All calculations/writing works are to be done in the space provided for this purpose in the Test Booklet itself, marked 'Space for Rough Work'. This space is given at the bottom of each page and in 4 pages (Pages 20 – 23) at the end of the booklet.
8. On completion of the test, the candidates must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. **However, the candidates are allowed to take away this Test Booklet with them.**
9. Each candidate must show on demand his/her Admit Card to the Invigilator.
10. No candidate, without special permission of the Superintendent or Invigilator, should leave his/her seat.
11. The candidates should not leave the Examination Hall without handing over their Answer Sheet to the Invigilator on duty and sign the Attendance Sheet again. Cases where a candidate has not signed the Attendance Sheet second time will be deemed not to have handed over the Answer Sheet and dealt with as an unfair means case. **The candidates are also required to put their left hand THUMB impression in the space provided in the Attendance Sheet.**
12. Use of Electronic/Manual Calculator and any Electronic Item like mobile phone, pager etc. is prohibited.
13. The candidates are governed by all Rules and Regulations of the Examination body with regard to their conduct in the Examination Hall. All cases of unfair means will be dealt with as per Rules and Regulations of the Examination body.
14. No part of the Test Booklet and Answer Sheet shall be detached under any circumstances.
15. **Candidates are not allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, electronic device or any other material except the Admit Card inside the examination room/hall.**

Questions and Solutions

PART- A : MATHEMATICS

1. If S is the set of distinct values of 'b' for which the following system of linear equations $x + y + z = 1$, $x + ay + z = 1$, $ax + by + z = 0$ has no solution, then S is :

(1) an empty set
(2) an infinite set
(3) a finite set containing two or more elements
(4) a singleton

1. (4)

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = -(a-1)^2$$

$\Delta_x = (a - 1)$
$\Delta_y = 0$
$\Delta_z = -a(a - 1)$

If $a = 1$ $\Rightarrow \Delta = \Delta_x = \Delta_y = \Delta_z = 0$
$\Rightarrow$ Infinite solution

For $b = 1$ & $a = 1$ $\Rightarrow x + y + z = 1$
$x + y + z = 0$
$\Rightarrow$ no solution

Only one value of b.

2. The following statement $(p \to q) \to [(\sim p \to q) \to q]$ is :

(1) a tautology
(2) equivalent to $\sim p \to q$
(3) equivalent to $p \to \sim q$
(4) a fallacy

2. (1)

$(p \to q) \to [(\sim p \to q) \to q]$ is

1	2	3	4	5	6	7
p	q	~p	$p \to q$	$\sim p \to q$	$[(\sim p \to q) \to q]$	$4 \to 6$
T	T	F	T	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	F	T	T	F	T	T

3. If $5\left(\tan^2 x - \cos^2 x\right) = 2\cos 2x + 9$, then the value of $\cos 4x$ is :

(1) $-\frac{3}{5}$
(2) $\frac{1}{3}$
(3) $\frac{2}{9}$
(4) $-\frac{7}{9}$

3. (4)

$$5\left(\tan^2 x - \cos^2 x\right) = 2\cos 2x + 9$$

$$5\left(\sec^2 x - 1 - \cos^2 x\right) = 2\,(2\cos^2 x - 1) + 9$$

$$5\left(\frac{1}{\cos^2 x} - 1 - \cos^2 x\right) = 4\cos^2 x - 2 + 9$$

$$5\left(\frac{1-\cos^2 x-\cos^4 x}{\cos^2 x}\right) = 4\cos^2 x + 7$$

$5 - 5\cos^2 x - 5\cos^4 x = 4\cos^4 x + 7\cos^2 x$

$\therefore\ 9\cos^4 x + 12\cos^2 x - 5 = 0$

Put, $\cos^2 x = m$

$\therefore\ 9m^2 + 12m - 5 = 0$

$\therefore\ 9m^2 + 15m - 3m - 5 = 0$

$3m(3m+5) - 1(3m+5) = 0$

$\therefore\ (3m+5)(3m-1) = 0$

$\therefore\ m = \frac{1}{3}$ or $m = -\frac{5}{3}$

$\therefore\ \cos^2 x = \frac{1}{3}$ or $\cos^2 x = -\frac{5}{3}$

But, $\cos^2 x \neq -\frac{5}{3}$

$\therefore\ \cos^2 x = \frac{1}{3}$

Now, $\cos 4x = 2\cos^2 2x - 1$

$= 2(\cos 2x)^2 - 1$

$= 2(2\cos^2 x - 1)^2 - 1$

$= 2\left[2\times\frac{1}{3}-1\right]^2 - 1 = 2\left[\frac{2}{3}-1\right]^2 - 1 = 2\left[-\frac{1}{3}\right]^2 - 1$

$= \frac{2}{9} - 1 = \frac{2-9}{9} = -\frac{7}{9}$

4. For three events A, B and C, P (Exactly one of A or B occurs) = P (Exactly one of B or C occurs) = P (Exactly one of C or A occurs) $= \frac{1}{4}$ and P (All the three events occur simultaneously $= \frac{1}{16}$.
Then the probability that at least one of the events occurs is :

(1) $\frac{7}{32}$ (2) $\frac{7}{16}$ (3) $\frac{7}{64}$ (4) $\frac{3}{16}$

4. (2)

P(exactly one of A or B occurs) $= \frac{1}{4} \Rightarrow P(A) + P(B) - 2P(A\cap B) = \frac{1}{4}$

P(exactly one of B & C occurs) $= \frac{1}{4} \Rightarrow P(B) + P(C) - 2P(B\cap C) = \frac{1}{4}$

P(exactly one of C or A occurs) $= \frac{1}{4} \Rightarrow P(C) + P(A) - 2P(A\cap C) = \frac{1}{4}$

$P(A\cap B\cap C) = \frac{1}{16}$

Ad. $[P(A) + P(B) + (C) - P(A\cap B) - P(B\cap C) - P(C\cap A)] = \frac{3}{8}$ …. (1)

Probability of at least one event

$= P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$

$= \frac{3}{8} + \frac{1}{16} = \frac{6+1}{16} = \frac{7}{16}$

5. Let ω be a complex number such that 2ω + 1 = z, where z = $\sqrt{-3}$. If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$,

then k is equal to :

(1) −z (2) z (3) −1 (4) 1

5. (1)

$w = \frac{\sqrt{3}i-1}{2}$ $w = \frac{-1+\sqrt{3}i}{2}$, $w^2 = \frac{-1-\sqrt{3}i}{2}$

$1 + w + w^2 = 0$, $w^3 = 1$

$$\therefore\ 3k = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -w^2-1 & w^2 \\ 1 & w^2 & w^7 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w \end{vmatrix}$$

$= 1(w^2 - w^4) - 1(w - w^2) + 1(w^2 - w)$

$= w^2 - w^4 - w + w^2 + w^2 - w$

$= 3w^2 - 2w - w^4$

$= 3w^2 - 2w - w = 3(w^2 - w)$

$$\therefore\ k = w^2 - w = \frac{-1-\sqrt{3}i}{2} - \frac{-1+\sqrt{3}i}{2}$$

$$= \frac{-1-\sqrt{3}i+1-\sqrt{3}i}{2} = \frac{-2\sqrt{3}i}{2} = -\sqrt{3}i = -z$$

($\because$ 2w + 1 = z = $\sqrt{3}$ i)

6. Let k be an integer such that the triangle with vertices (k, −3k), (5, k) and (−k, 2) has area 28 sq. units. Then the orthocenter of this triangle is at the point :

(1) $\left(2, -\frac{1}{2}\right)$ (2) $\left(1, \frac{3}{4}\right)$ (3) $\left(1, -\frac{3}{4}\right)$ (4) $\left(2, \frac{1}{2}\right)$

6. (4)

$$\Delta = \frac{1}{2}\left|(k^2+10+3k^2) - (-15k - k^2 + 2k)\right| = 28$$

$$\left|5k^2 + 13k + 10\right| = 56$$

$5k^2 + 13k + 10 = \pm 56$

$5k^2 + 13k + 10 = 56$

$5k^2 + 13k - 46 = 0$

$5k^2 + 23k - 10k - 46 = 0$

$k(5k + 23) - 2(5k + 23) = 0$

$k = 2, -\frac{23}{5}$

k = 2 as integer

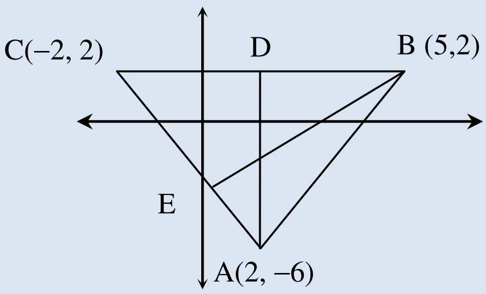
A ≡(2, −6), B ≡ (5, 2) , C ≡ (−2, 2).

As BC perpendicular to x−axis

⇒ eqn. of altitude AD is x = 2

For BE, $y - 2 = \frac{1}{2}(x - 5)$

Solving orthocentre $\left(2, \frac{1}{2}\right)$



7. Twenty meters of wire is available for fencing off a flower–bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower–bed is :

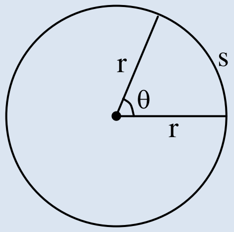
(1) 12.5 (2) 10 (3) 25 (4) 30

7. (3)

$2r + s = 20$ (length of wire)

Now $s = r\theta$

$\therefore \; 2r + r\theta = 20 \qquad \therefore \; \theta = \dfrac{20-2r}{r}$



$\therefore \; A = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}r^2\left(\dfrac{20-2r}{r}\right)$

$A = 10\,r - r^2$

$\therefore \; \dfrac{dA}{dr} = 10 - 2r = 0 \Rightarrow r = 5$

$\dfrac{d^2A}{dr^2} = -2 < 0 \Rightarrow$ Area is maximum

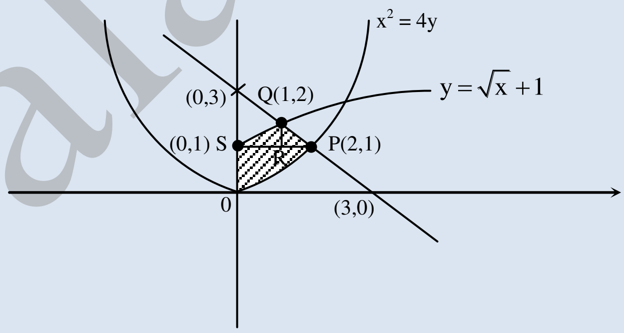
$\therefore$ Maximum Area $= \dfrac{1}{2}r^2\theta = \dfrac{1}{2}\times 25\times\dfrac{20-2r}{r}$

$= \dfrac{1}{2}\times 25\times\dfrac{20-10}{5} \; = \dfrac{1}{2}\times 5\times 10 = 25$ sq.m.

8. The area (in sq. units) of the region $\left\{(x, y): x \ge 0, x + y \le 3, x^2 \le 4y \text{ and } y \le 1+\sqrt{x}\right\}$ is :

(1) $\dfrac{59}{12}$ (2) $\dfrac{3}{2}$ (3) $\dfrac{7}{3}$ (4) $\dfrac{5}{2}$

8. (4)



$$\text{Area} = \int_0^1 x\,dy + \int_0^1 y\,dx + \Delta PQR$$

$$= \int_0^1 2\sqrt{y}\,dy + \int_0^1 \sqrt{x}\,dx + \frac{1}{2}1\times 1$$

$$= 2\times\frac{2}{3} + \frac{2}{3} + \frac{1}{2} = 2 + \frac{1}{2} = \frac{5}{2}$$

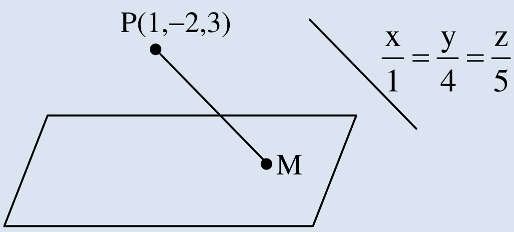
9. If the image of the point P (1, –2, 3) in the plane, $2x + 3y - 4z + 22 = 0$, measured parallel to the line, $\dfrac{x}{1} = \dfrac{y}{4} = \dfrac{z}{5}$ is Q, then PQ is equal to :

(1) $3\sqrt{5}$ (2) $2\sqrt{42}$ (3) $\sqrt{42}$ (4) $6\sqrt{5}$

9. (2)

Line PM (Parallel to given line) is

$$\frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{5} = r(\text{Let})$$



$M \equiv (r + 1, 4r - 2, 5r + 3)$

M satisfy plane, $2x + 3y - 4z + 22 = 0$

$\Rightarrow 2r + 2 + 12r - 6 - 20r - 12 + 22 = 0$

$-6r + 6 = 0$

$r = 1$

So, $M = (2, 2, 8)$

$$PM = \sqrt{(2-1)^2 + (2+2)^2 + (8-3)^2} = \sqrt{1+16+25} = \sqrt{42}$$

So, $PQ = 2\sqrt{42}$

10. If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}\,.\,g(x)$, then g(x) equals :

(1) $\frac{9}{1+9x^3}$ (2) $\frac{3x\sqrt{x}}{1-9x^3}$ (3) $\frac{3x}{1-9x^3}$ (4) $\frac{3}{1+9x^3}$

10. (1)

$$y = \tan^{-1}\left[\frac{2\cdot\left(3x\sqrt{x}\right)}{1-\left(3x\sqrt{x}\right)^2}\right]. \quad \text{Let } 3x\sqrt{x} = \tan\theta$$

$$= \tan^{-1}(\tan 2\theta) = 2\theta = 2\tan^{-1}\left(3x\sqrt{x}\right)$$

$$\frac{dy}{dx} = 2\cdot\frac{1}{1+9x^3}\; 3\cdot\frac{3}{2}\; x^{\frac{1}{2}} = \frac{9\sqrt{x}}{1+9x^3} = \sqrt{x}\cdot\frac{9}{1+9x^3}$$

$$\Rightarrow g(x) = \frac{9}{1+9x^3}$$

11. If $(2+\sin x)\frac{dy}{dx} + (y+1)\cos x = 0$ and y(0) = 1, then $y\left(\frac{\pi}{2}\right)$ is equal to :

(1) $\frac{1}{3}$ (2) $-\frac{2}{3}$ (3) $-\frac{1}{3}$ (4) $\frac{4}{3}$

11. (1)

$$\frac{dy}{dx} + \frac{y\cos x}{2+\sin x} + \frac{\cos x}{2+\sin x} = 0$$

$$\frac{dy}{dx} + \frac{\cos x}{2+\sin x}y = -\frac{\cos x}{2+\sin x}$$

$$\text{I.F.} = e^{\int\frac{\cos x}{2+\sin x}dx} = 2+\sin x$$

$$y\,.\,(2+\sin x) = c + \int(-\cos x)dx$$

y (2 + sin x) = c − sin x

Given y (0) = 1

1(2 + 0) = c − 0

c = 2

Soln. y(2 − sin x) = 2 − sin x

$$y(\pi/2) = \frac{2-1}{2+1} = \frac{1}{3}$$

12. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If ∠BPC = β, then tan β is equal to :

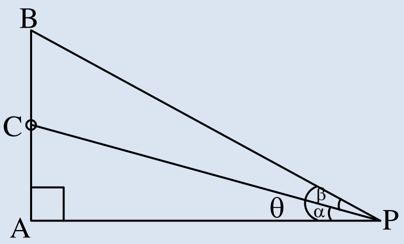
(1) $\frac{6}{7}$ (2) $\frac{1}{4}$ (3) $\frac{2}{9}$ (4) $\frac{4}{9}$

12. (3)

AP = 2AB

$$\tan\theta = \frac{AB}{AP} = \frac{1}{2}$$

$$\Rightarrow \tan\alpha = \frac{AC}{AP} = \frac{\frac{AB}{2}}{2AB} = \frac{1}{4}$$



$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$$

$$\frac{1}{2} = \frac{\frac{1}{4}+\tan\beta}{1-\frac{1}{4}.\tan\beta}$$

$$\frac{1}{2} = \frac{1+4\tan\beta}{4-\tan\beta} \quad \Rightarrow \quad 2 + 8\tan\beta = 4 - \tan\beta$$

$$9\tan\beta = 2$$

$$\tan\beta = \frac{2}{9}$$

13. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to :

(1) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$ (2) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ (3) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$ (4) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

13. (2)

$$A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$$

$$3A^2 + 12A = \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix} + \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix} = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

$$\text{Adj. } (3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$$

14. For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then :

(1) b, c and a are in G.P. (2) b, c and a are in A.P.

(3) a, b and c are in A.P. (4) a, b and c are in G.P.

14. (2)

$$225a^2 + 9b^2 + 25c^2 - 75ac - 45ab - 15bc = 0$$

$$\Rightarrow 450a^2 + 18b^2 + 50c^2 - 150ac - 90ab - 30bc = 0$$

$$\Rightarrow (15a - 5c)^2 + (15a - 3b)^2 + (3b - 5c)^2 = 0$$

$$\Rightarrow 15a = 5c \quad 15a = 3b \quad 3b = 5c$$

$$\therefore 15a = 3b = 5c$$

$$\Rightarrow \frac{a}{1} = \frac{b}{5} = \frac{c}{3}$$

$\Rightarrow$ b, c, a in A.P.

15. The distance of the point (1, 3, –7) from the plane passing through the point (1, –1, –1) having normal perpendicular to both the lines $\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{3}$ and $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1}$ is :

(1) $\frac{20}{\sqrt{74}}$ (2) $\frac{10}{\sqrt{83}}$ (3) $\frac{5}{\sqrt{83}}$ (4) $\frac{10}{\sqrt{74}}$

15. (2)

Plane passing (1, –1,–1) is

$$a(x - 1) + b(y + 1) + c(z + 1) = 0$$

Normal of line containing both the line is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 5\hat{i} + 7\hat{j} + 3\hat{k}$$

From given condition normal of plane parallel to $5\hat{i} + 7\hat{j} + 3\hat{k}$

$\Rightarrow$ Plane is $5(x - 1) + 7(y + 1) + 3(z + 1) = 0$

$5x + 7y + 3z + 5 = 0$

Distance = $\left| \dfrac{5 \times 1 + 7 \times 3 + 3 \times (-7) + 5}{\sqrt{5^2 + 7^2 + 3^2}} \right| = \dfrac{10}{\sqrt{83}}$

16. Let $I_n = \int \tan^n x \, dx, (n > 1)$. If $I_4 + I_6 = a \tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to :

(1) $\left(-\frac{1}{5}, 1\right)$ (2) $\left(\frac{1}{5}, 0\right)$ (3) $\left(\frac{1}{5}, -1\right)$ (4) $\left(-\frac{1}{5}, 0\right)$

16. (2)

$I_n = \int \tan^{n-2} x . (\sec^2 x - 1) \, dx$

$= \int \tan^{n-2} x . \sec^2 x \, dx - \int \tan^{n-2} x \, dx$

$I_n = \dfrac{\tan^{n-1} x}{n-1} - I_{n-2}$

$I_n + I_{n-2} = \dfrac{\tan^{n-1} x}{n-1} + C$

$n = 6$

$I_6 + I_4 = \dfrac{\tan^5 x}{5} + C$

Equate $a = \dfrac{1}{5}$, $b = 0$

$(a, b) = \left(\dfrac{1}{5}, 0\right)$

17. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is $x = -4$, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is :

(1) $2y - x = 2$ (2) $4x - 2y = 1$ (3) $4x + 2y = 7$ (4) $x + 2y = 4$

17. (2)

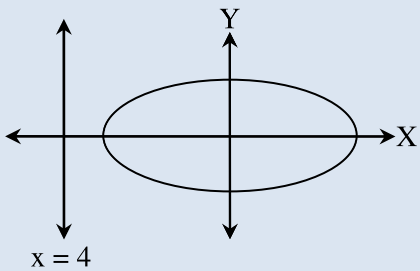
$e = \dfrac{1}{2}$

$x = -\dfrac{a}{e}$

$-4 = -\dfrac{a}{\frac{1}{2}}$

$4 = 2a$

$a = 2$



$e^2 = 1 - \dfrac{b^2}{a^2}$

$\dfrac{1}{4} = 1 - \dfrac{b^2}{4}$

$\dfrac{b^2}{4} = 1 - \dfrac{1}{4}$

$b^2 = 3$

ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{3} = 1$

$\dfrac{2x}{4} + \dfrac{2y}{3}\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = -\dfrac{2x}{4} \times \dfrac{3}{2y} = -\dfrac{3x}{4y}$

$m_{tangent} = -\dfrac{3 \times 1}{4 \times 3} = -\dfrac{1}{2}$

Normal at $\left(1, \dfrac{3}{2}\right)$ is

$y - \dfrac{3}{2} = 2(x - 1)$

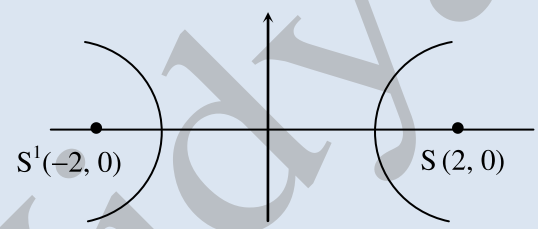
$2y - 3 = 4x - 4$

$4x - 2y - 1 = 0$

18. A hyperbola passes through the point $P\left(\sqrt{2}, \sqrt{3}\right)$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point :

(1) $\left(3\sqrt{2}, 2\sqrt{3}\right)$ (2) $\left(2\sqrt{2}, 3\sqrt{3}\right)$ (3) $\left(\sqrt{3}, \sqrt{2}\right)$ (4) $\left(-\sqrt{2}, -\sqrt{3}\right)$

18. (2)



foci $S \equiv (\pm ae, 0)$

$2ae = 4$

$a^2e^2 = 4$

$a^2\left(1 + \dfrac{b^2}{a^2}\right) = 4$

$a^2 + b^2 = 4$

$a^2 = 4 - b^2$

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

It passes P $\left(\sqrt{2}, \sqrt{3}\right)$

$\dfrac{2}{a^2} = \dfrac{3}{b^2} = 1$

$\dfrac{2}{4 - b^2} - \dfrac{3}{b^2} = 1$

$2b^2 - 12 + 3b^2 = b^2(4 - b^2)$

$b^4 + b^2 - 12 = 0$

$(b^2 + 4)(b^2 - 3) = 0$

$b^2 = 3$

$\Rightarrow a^2 = 1$

Hyperbola $\dfrac{x^2}{1} - \dfrac{y^2}{3} = 1$

Tangent at $\left(\sqrt{2}, \sqrt{3}\right)$ is $x\sqrt{2} - \dfrac{y\sqrt{3}}{3} = 1$

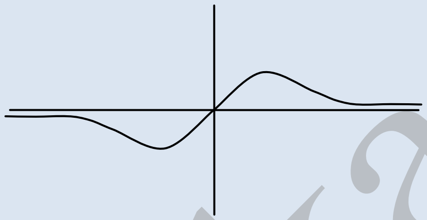
Obviously $\left(2\sqrt{2}, 3\sqrt{3}\right)$ satisfy

19. The function $f : R \to \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$ defined as $f(x) = \dfrac{x}{1+x^2}$, is :

(1) invertible
(2) injective but not surjective
(3) surjective but not injective
(4) neither injective nor surjective

19. (3)

Obviously from graph not one–one.



$$f'(x) = \frac{1-x^2}{(1+x^2)^2} < 0 \qquad \forall\, x \in R$$

$\Rightarrow$ f(x) $\downarrow$

Obviously range $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$.

20. $\displaystyle\lim_{x\to\frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals :

(1) $\dfrac{1}{24}$ (2) $\dfrac{1}{16}$ (3) $\dfrac{1}{8}$ (4) $\dfrac{1}{4}$

20. (2)

Let $L = \displaystyle\lim_{x\to\frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$

Put, $\pi - 2x = \theta \;\Rightarrow\; \pi - \theta = 2x \;\Rightarrow\; x = \dfrac{\pi}{2} - \dfrac{\theta}{2}$

As $x \to \dfrac{\pi}{2}, \theta \to 0$

$$\therefore\; L = \lim_{\theta\to 0} \frac{\cot\left(\frac{\pi}{2} - \frac{\theta}{2}\right) - \cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right)}{\theta^3}$$

$$= \lim_{\theta\to 0} \frac{\tan\frac{\theta}{2} - \sin\frac{\theta}{2}}{\theta^3} = \lim_{\theta\to 0} \frac{\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} - \sin\frac{\theta}{2}}{\theta^3} = \lim_{\theta\to 0} \frac{\sin\frac{\theta}{2} - \sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{\theta^3 \cos\frac{\theta}{2}}$$

$$= \lim_{\theta\to 0} \frac{\sin\frac{\theta}{2}\left(1 - \cos\frac{\theta}{2}\right)}{\cos\frac{\theta}{2}\,\theta^3} = \lim_{\theta\to 0} \frac{\sin\frac{\theta}{2}}{\frac{\theta}{2}} \times \frac{1}{2} \times \lim_{\theta\to 0} \frac{2\sin^2\frac{\theta}{4}}{\frac{\theta^2}{16}} \times \frac{1}{16} \times \lim_{\theta\to 0} \frac{1}{\cos\frac{\theta}{2}}$$

$$= 1 \times \frac{1}{2} \times 2 \times \frac{1}{16} \times 1 = \frac{1}{16}$$

21. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$, $\left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right| = 3$ and the angle between $\vec{c}$ and $\vec{a} \times \vec{b}$ be 30°. Then $\vec{a} . \vec{c}$ is equal to :

(1) $\frac{25}{8}$ (2) 2 (3) 5 (4) $\frac{1}{8}$

21. (2)

$\vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a} \times \vec{b}| = 3$

Given : $|(\vec{a} \times \vec{b}) \times \vec{c}| = 3$

$|\vec{a} \times \vec{b}|\,|\vec{c}| \sin 30° = 3$

$3 \times |\vec{c}| \cdot \frac{1}{2} = 3$

$|\vec{c}| = 2$

$|\vec{c} - \vec{a}| = 3$

$(\vec{c} - \vec{a}) \cdot (\vec{c} - \vec{a}) = 9$

$|\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a} \cdot \vec{c} = 9$

$4 + 9 - 2\vec{a} \cdot \vec{c} = 9$

$\vec{a} \cdot \vec{c} = 2$

22. The normal to the curve y (x − 2) (x − 3) = x + 6 at the point where the curve intersects the y−axis passes through the point :

(1) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ (2) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (3) $\left(\frac{1}{2}, -\frac{1}{3}\right)$ (4) $\left(\frac{1}{2}, -\frac{1}{3}\right)$

22. (2)

y−axis at (0, 1)

$y\,(x^2 - 5x + 6) = x + 6$

$$y = \frac{x + 6}{x^2 - 5x + 6}$$

$$\frac{dy}{dx} = \frac{\left(x^2 - 5x + 6\right) \cdot 1 - (x + 6)(2x - 5)}{\left(x^2 - 5x + 6\right)^2}$$

$$\left.\frac{dy}{dx}\right|_{(0,1)} = \frac{6 - 6(-5)}{6^2} = 1$$

Normal at (0, 1) is $y - 1 = -1\,(x - 0)$

$y + x = 1$

It passes $\left(\frac{1}{2}, \frac{1}{2}\right)$

23. If two different numbers are taken from the set {0, 1, 2, 3, …, 10}; then the probability that their sum as well as absolute difference are both multiple of 4, is :

(1) $\frac{6}{55}$ (2) $\frac{12}{55}$ (3) $\frac{14}{45}$ (4) $\frac{7}{55}$

23. (1)

{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

$$n(s) = {}^{11}C_2 = \frac{11 \times 10}{1 \times 2} = 55$$

Let A be the event that the sum as well as absolute difference of two different numbers taken from given get are both multiple of 4

$2 + 6 = 8$ divisible by 4

$|6 - 2| = 4$ divisible by 4

$2 + 10 = 12$ divisible by 4

$|10-2|=8$ divisible by 4

$4 + 8 = 12$ divisible by 4

$|8 - 4| = 4$ divisible by 4

$$P(A)=\frac{n(A)}{n(S)}=\frac{6}{55}.$$

24. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is :

(1) 485 (2) 468 (3) 469 (4) 484

24. (1)

Man $\rightarrow$ (7 friends) Wife $\rightarrow$ (7 friends)

4 Ladies 3 Men 3 Ladies 4 Men

3 friends

Party Invites $\rightarrow$ 3 Ladies and 3 Men

(G_1) 4 Ladies + 3 Men 3 Ladies + 4 Men (G_2)

(3) (3)

3 Ladies and 3 Men

G_1	G_2	
3 L	3 M	${}^4C_3 \times {}^4C_3$
3 M	3 L	${}^3C_3 \times {}^3C_3$
1 L and 2 M	2 L and 1 M	${}^4C_1 \times {}^3C_2 \times {}^3C_2 \times {}^4C_1$
2 L and 1 M	1 L and 2 M	${}^4C_2 \times {}^3C_1 \times {}^3C_1 \times {}^4C_2$

$$\text{Req. No.} = \left({}^4C_3 \times {}^4C_3\right)+\left({}^3C_3 \times {}^3C_3\right)+\left({}^4C_1 \times {}^3C_2 \times {}^3C_2 \times {}^4C_1\right)+\left({}^4C_2 \times {}^3C_1 \times {}^3C_1 \times {}^4C_2\right)$$

$$= (4\times4)+(1\times1)+(4\times3\times3\times4)+\left(\frac{4\times3}{2}\times3\times3\times\frac{4\times3}{2}\right)$$

$$= 16 + 1 + 144 + 324 = 485$$

25. The value of

$\left({}^{21}C_1-{}^{10}C_1\right)+\left({}^{21}C_2-{}^{10}C_2\right)+\left({}^{21}C_3-{}^{10}C_3\right)+\left({}^{21}C_4-{}^{10}C_4\right)+\ldots+\left({}^{21}C_{10}-{}^{10}C_{10}\right)$ is :

(1) $2^{21}-2^{11}$ (2) $2^{21}-2^{10}$ (3) $2^{20}-2^{9}$ (4) $2^{20}-2^{10}$

25. (4)

$$\left({}^{21}C_1 + {}^{21}C_2 + \ldots\ldots {}^{21}C_{10}\right) - \left({}^{10}C_1 + {}^{10}C_2 + \ldots\ldots + {}^{10}C_{10}\right)$$

$$= (2^{20} - 1) - (2^{10} - 1)$$

$$= 2^{20} - 2^{10}$$

As $${}^{21}C_0 + \left({}^{21}C_1 + {}^{21}C_2 + \ldots. + {}^{21}C_{10}\right) + \left({}^{21}C_{11} + \ldots.. + {}^{21}C_{20}\right) + {}^{21}C_{21} = 2^{21}$$

$$2\left({}^{21}C_1 + {}^{21}C_2 + \ldots.. + {}^{21}C_{10}\right) = 2^{21} - 2$$

$${}^{21}C_1 + {}^{21}C_2 + \ldots.. + {}^{21}C_{10} = 2^{20} - 1$$

26. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one−by−one, with replacement, then the variance of the number of green balls drawn is :

(1) $\frac{12}{5}$ (2) 6 (3) 4 (4) $\frac{6}{25}$

26. (1)

G - 15, Y - 10

P(Green ball in each step) = $\frac{15}{25} = \frac{3}{5}$

P(Yellow ball in each step) = $\frac{2}{5}$

Variance = npq

$= 10 \times \frac{3}{5} \times \frac{2}{5} = \frac{12}{5}$

27. Let a, b, c $\in$ R. If f(x) = $ax^2 + bx + c$ is such that a + b + c = 3 and f (x + y) = f(x) + f(y) + xy, $\forall$ x, y $\in$ R, then $\sum_{n=1}^{10} f(n)$ is equal to :

(1) 330 (2) 165 (3) 190 (4) 255

27. (1)

$f(x) = ax^2 + bx + c$

$f(1) = a + b + c = 3$

$f(1 + 1) = 2\,f(1) + 1$

$f(2) = 2 \times 3 + 1 = 7$

$f(2 + 1) = f(2) + f(1) + 2$

$f(3) = 7 + 3 + 2 = 12$

$f(3 + 1) = f(3) + f(1) + 3$

$= 12 + 3 + 3 = 18$

$S_n = 3 + 7 + 12 + 18 + \ldots\ldots\ldots + t_n$

$- S_n = - 3 \pm 7 \pm 12 + \ldots\ldots\ldots \pm t_{n-1} \pm t_n$

$0 = (3 + 4 + 5 + 6 + \ldots\ldots\ldots\ldots + \text{n terms}) - t_n$

$$t_n = \frac{n}{2}[6 + (n-1)] = \frac{n}{2}[n+5] = \frac{n^2 + 5n}{2}$$

$$\sum_{n=1}^{10} f(n) = \frac{1}{2}\left[\sum_{n=1}^{10} n^2 + 5\sum_{n=1}^{10} n\right] = \frac{1}{2}\left[\frac{10\,(11)\,(21)}{6} + 5 \times \frac{10\,(11)}{2}\right] = \frac{1}{2}[55 \times 7 + 55 \times 5]$$

$$= \frac{55 \times 12}{2} = 330$$

28. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is :

(1) $2\left(\sqrt{2}+1\right)$ (2) $2\left(\sqrt{2}-1\right)$ (3) $4\left(\sqrt{2}-1\right)$ (4) $4\left(\sqrt{2}+1\right)$

28. (3)

Let centre of circle (0, k)

As touches $x - y = 0$ line

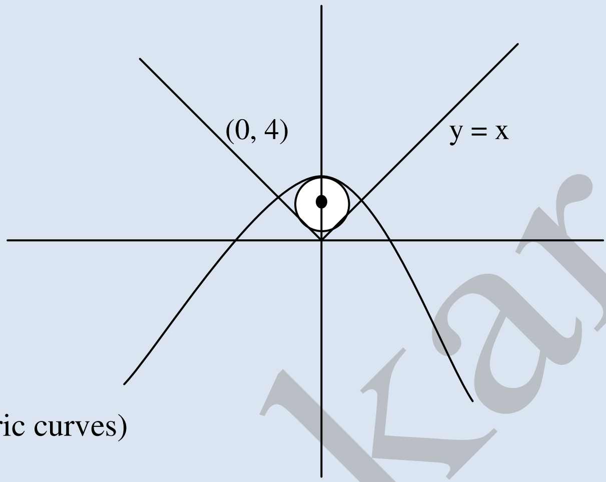
$\Rightarrow \left|\frac{0-k}{\sqrt{2}}\right| = r$ … (radius)

$r^2 = \frac{k^2}{2}$

Circle is $(x - 0)^2 + (y - k)^2 = \frac{k^2}{2}$

As touches $y = 4 - x^2$

$\Rightarrow$ It passes (0, 4) … (as symmetric curves)



$\Rightarrow 0 + (4 - k)^2 = \frac{k^2}{2}$

$\Rightarrow k^2 - 16k + 32 = 0$

$k = 8 \pm 4\sqrt{2}$

Radius for minimum radius

$= 4-\left(8-4\sqrt{2}\right) = 4\sqrt{2}-4 = 4\left(\sqrt{2}-1\right)$

29. If for a positive integer n, the quadratic equation, $x(x+1)+(x+1)(x+2)+...+(x+\overline{n-1})(x+n)=10n$ has two consecutive integral solutions, then n is equal to :

(1) 12 (2) 9 (3) 10 (4) 11

29. (4)

Let the roots be α, $\alpha + 1$

$\alpha(\alpha + 1) + (\alpha + 1)(\alpha + 2) \ldots + (\alpha + (n - 1)(\alpha + n) = 10n$

and $(\alpha + 1)(\alpha + 2) + (\alpha + 2)(\alpha + 3) + \ldots + (\alpha + n)(\alpha + n + 1) = 10n$

Subtract, $\alpha(\alpha + 1) - (\alpha + n)(\alpha + n + 1) = 0$

$\alpha^2 + \alpha - \alpha^2 - (2n + 1)\alpha - n(n + 1) = 0$

$-2n\alpha - n(n + 1) = 0$

$2\alpha = -(n + 1)$

$\alpha = -\frac{n+1}{2}$

So ‘n’ must be odd

$n = 9, \ \alpha = -4$

$n = 11, \alpha = -5$

Put, $\alpha = -4$, $(-4)(-3) + (-3)(-2) + (-2)(-1) + (-1)0 + 0 \times 1 + 1 \times 2 + \ldots + 4 \times 5$

$= 12 + 6 + 2 + 2 + 6 + 12 + 20$

$= 60 \neq 10n$

Put, $\alpha = -5$, $(-5)(-4) + (-4)(-3) + \ldots + 5 \times 6$

$= 20 + 12 + 6 + 2 + 2 + 6 + 12 + 20 + 30$

$= 110 = 10n$

$\therefore$ $n = 11$

30. The integral $\int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x}$ is equal to:

(1) – 2 (2) 2 (3) 4 (4) – 1

30. (2)

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x} \quad \ldots (1)$$

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1-\cos x} \quad \ldots (2)$$

By adding, … (1) + (2)

$$2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{2}{1-\cos^2 x}dx$$

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \operatorname{cosec}^2 x\, dx = -\cot x\Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = -\left[\cot\frac{3\pi}{4} - \cot\frac{\pi}{4}\right] = -[-1 - 1] = 2.$$

PART- B : PHYSICS

31. A radioactive nucleus A with half time T, decays into a nucleus B. At t = 0, there is no nucleus B. At sometimes t, the ratio of the number of B to that of A is 0.3. Then t is given by :

(1) $t = \frac{T}{\log(1.3)}$ (2) $t = \frac{T}{2}\frac{\log 2}{\log 1.3}$ (3) $t = T\frac{\log 1.3}{\log 2}$ (4) $t = T\log(1.3)$

31. (3)

$$A \xrightarrow{T} B$$

$t = 0$ N_0 0

$t =$ $N_A = N_0 \cdot e^{-\lambda t}$

$$N_B = N_0\left(1-e^{-\lambda t}\right)$$

$$\frac{N_B}{N_A} = \frac{1-e^{-\lambda t}}{e^{-\lambda t}} = 0.3 \Rightarrow e^{-\lambda t} = \frac{1}{1.3}$$

$$\lambda t = \ell n\, 1.3$$

$$\boxed{t = \frac{T\ell n 1.3}{\ell n 2}}$$

32. The following, observations were taken for determining surface tension t of water by capillary method :
diameter of capillary, D = 1.25×10^{-2} m
rise of water, h = 1.45×10^{-2} m
Using g = 9.80 m/s^2 and the simplified relation $T = \frac{r\,h\,g}{2} \times 10^3$ N/m, the possible error in surface tension is closest to :

(1) 10 % (2) 0.15% (3) 1.5 % (4) 2.4 %

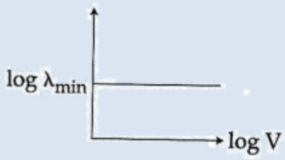
32. (3)

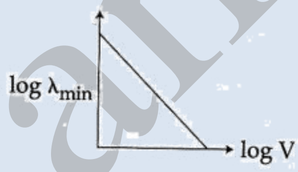
$$T = \frac{rgh}{2} \times 10^3$$

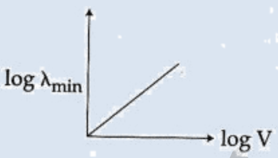
$$\frac{dT}{T} = \frac{dr}{r} + \frac{dg}{g} + \frac{dh}{h}$$

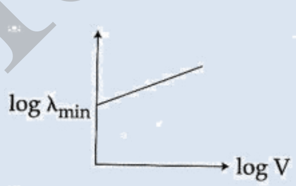
$$\%\ \frac{dT}{T} = \left(\frac{0.01}{1.25} + \frac{0.01}{9.80} + \frac{0.01}{1.45}\right) \times 100 = 0.8 + 0.1 + 0.7 = 1.6\%$$

33. An electron beam is accelerated by a potential different V to hit a metallic target to produce X–rays. It produces continuous as well as characteristic X–rays. If λ_{min} is the smallest possible wavelength of X–ray in the spectrum, the variation of log λ_{min} with log V correctly represented in:

(1) 

(2) 

(3) 

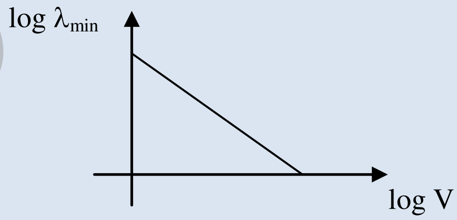
(4) 

33. (2)

$$KE_{max} = ev = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} \propto \frac{1}{V}$$

log λ_{min} = log C – log V
graph is a straight line with slope = –1
intercept = +ve



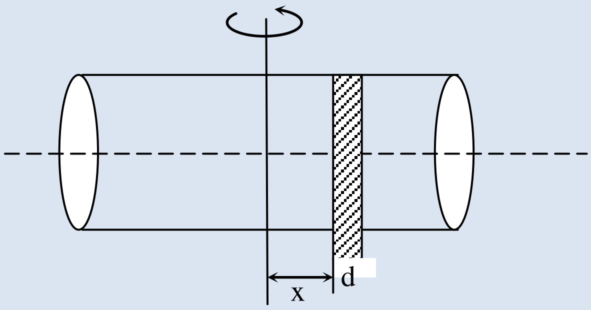
34. The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. What is the ratio of l/R such that the moment of inertia is minimum?

(1) $\frac{3}{\sqrt{2}}$ (2) $\sqrt{\frac{3}{2}}$ (3) $\frac{\sqrt{3}}{2}$ (4) 1

34. (2)

$$dm = \frac{M}{L}dx$$

$$dI = \frac{dm \cdot R^2}{4} + dm \cdot x^2$$



$$I = \frac{MR^2}{4L}\int_{-L/2}^{L/2} dx + \frac{M}{L}\int_{-L/2}^{L/2} x^2\,dx$$

$$I = \frac{MR^2}{4L}\times L + \frac{M}{3L}\left[\left(\frac{L}{2}\right)^3 + \left(\frac{L}{2}\right)^3\right]$$

$$I = \frac{M}{12}\left(3R^2 + L^2\right)$$

For I to be minimum $(3R^2 + L^2)$ should be minimum

$f = 3R^2 + L^2$

$$\frac{df}{dL} = 6R\frac{dR}{dL} + 2L = 0$$

$$\frac{dR}{dL} = -\frac{L}{3R} \qquad \ldots(1)$$

Again volume $V = \pi R^2 L$ = constant

$\therefore\ 2\pi R\, dR\cdot L + \pi R^2\cdot dL = 0$

$2L\cdot dR + R\cdot dL = 0$

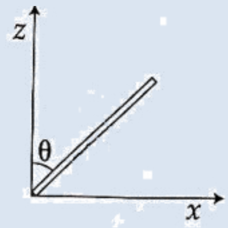
$$\frac{dR}{dL} = -\frac{R}{2L} \qquad \ldots(2)$$

From (1) and (2)

$$\frac{L}{3R} = \frac{R}{2L}$$

$$\therefore\ 2L^2 = 3R^2 \qquad \therefore\ \frac{L}{R} = \sqrt{\frac{3}{2}}$$

35. A slender uniform rod of mass M and length ℓ is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes at angle θ with the vertical is :



(1) $\frac{2g}{3\ell}\cos\theta$ (2) $\frac{3g}{2\ell}\sin\theta$

(3) $\frac{2g}{3\ell}\sin\theta$ (4) $\frac{3g}{2\ell}\cos\theta$

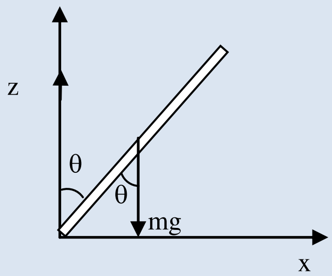
35. (2)

Torque about origin

$\tau = I\alpha$

$$mg\times\frac{L}{2}\sin\theta = \frac{ML^2}{3}\times\alpha$$

$$\alpha = \frac{3g\sin\theta}{2L}$$



36. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that,

$C_p - C_v = a$ for hydrogen gas

$C_p - C_v = b$ for nitrogen gas

The correct relation between a and b is :

(1) a = 28 b (2) $a = \frac{1}{14}b$ (3) a = b (4) a = 14 b

36. (4)

Molar specific heat $C' = \dfrac{\Delta Q}{n \cdot \Delta T}$

Specific heat $C = \dfrac{\Delta Q}{m \cdot \Delta T}$

$\dfrac{C'}{C} = \dfrac{m}{n} = M$ (Molecular weight)

$C = \dfrac{C'}{M}$

$C_P^{'} - C_V^{'} = R$

$C_P - C_V = \dfrac{R}{M}$

for Hydrogen M = 2

$a = C_P - C_V = \dfrac{R}{2}$

for nitrogen gas M = 28

$b = C_P - C_V = \dfrac{R}{28}$

$\dfrac{a}{b} = 14$

37. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by : (Given : room temperature = 30°C, specific heat of copper = 0.1 cal/gm°C)

(1) 825°C (2) 800°C (3) 885°C (4) 1250°C

37. (3)

Heat released by ball = Heat absorbed by 'Calorimeter + water' system

$100 \times 0.1 \times (T - 75) = 170 \times 1 \times (75 - 30) + 100 \times 0.1 \times (75 - 30)$

$(T - 75) = 17 \times 45 + 45$

$T - 75 = 810$

$T = 885°C$

38. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth ($\Delta\omega_m$) of the signal is such that $\Delta\omega_m << \Delta\omega_c$. Which of the following frequencies is not contained in the modulated wave?

(1) $\omega_c - \omega_m$ (2) ω_m (3) ω_c (4) $\omega_m + \omega_c$

38. (2)

39. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to the sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be :

(1) -2.5×10^{25} (2) -1.61×10^{23} (3) 1.38×10^{23} (4) 2.5×10^{25}

39. (1)

$n_i = \dfrac{P_0 V_0}{RT_1}$

$n_f = \dfrac{P_0 V_0}{RT_2}$

$$n_f - n_i = \frac{P_0V_0}{R}\left[\frac{1}{T_2} - \frac{1}{T_1}\right] = \frac{10^5 \times 30}{8.31}\left[\frac{1}{300} - \frac{1}{290}\right] = -41.5$$

Number of molecules = $-\ 41.5 \times 6.023 \times 10^{23} = -\ 2.5 \times 10^{25}$

40. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is :

(1) 15.6 mm (2) 1.56 mm (3) 7.8 mm (4) 9.75 mm

40. (3)

d = 0.5 mm

D = 150 cm

$\lambda_1 = 650$ nm and $\lambda_2 = 520$ nm

Fringe width $\beta = \dfrac{\lambda D}{d}$

$$\beta_1 = \frac{650 \times 10^{-9} \times 1.5}{0.5 \times 10^{-3}} = 1950\ \mu m = 1.95\ mm$$

$$\beta_2 = \frac{520 \times 10^{-9} \times 1.5}{0.5 \times 10^{-3}} = 1560\ \mu m = 1.56\ mm$$

where both waves coincide

y = L.C.M. of β_1 and β_2

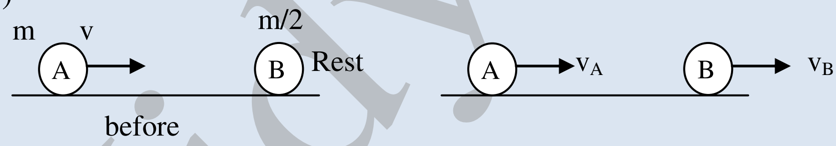
y = 7.8 mm

$y = 4\beta_1 = 5\beta_2 = 7.8$ mm

41. A particle A of mass m and initial velocity v collides with a particle B of mass $\dfrac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de–Broglie wavelengths λ_A and λ_B after the collision is :

(1) $\dfrac{\lambda_A}{\lambda_B} = \dfrac{1}{2}$ (2) $\dfrac{\lambda_A}{\lambda_B} = \dfrac{1}{3}$ (3) $\dfrac{\lambda_A}{\lambda_B} = 2$ (4) $\dfrac{\lambda_A}{\lambda_B} = \dfrac{2}{3}$

41. (3)



From conservation of linear momentum

$mv = mv_A + \dfrac{m}{2}v_B$ $\therefore$ $v = v_A + \dfrac{v_B}{2}$

also, $v = v_B - v_A$

$2v = \dfrac{3v_B}{2}$ (add)

$\therefore$ $v_B = \dfrac{4v}{3}$ and $v_A = \dfrac{v}{3}$

$$\frac{\lambda_A}{\lambda_B} = \frac{\dfrac{h}{mv_A}}{\dfrac{h}{\dfrac{m}{2}v_B}} = \frac{\dfrac{1}{\dfrac{v}{3}}}{\dfrac{1}{\dfrac{1}{2} \times \dfrac{4v}{3}}} = 2$$

42. A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillation is :

(1) 8.76 s (2) 6.65 s (3) 8.89 s (4) 6.98 s

42. (2)

Time period is given by

$$T = 2\pi\sqrt{\frac{I}{MB}} = 2\pi\sqrt{\frac{7.5\times10^{-6}}{6.7\times10^{-2}\times0.01}} = 6.28\times1.00\times0.1$$

$\therefore$ for 10 oscillation = 6.28 × 1.06 × 0.1 × 10 = 6.65 S

43. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x−axis. When subjected to an electric field $\overrightarrow{E_1} = E\hat{i}$, it experiences a torque $\overrightarrow{T_1} = \tau\,\hat{k}$. When subjected to another electric field $\overrightarrow{E_2} = \sqrt{3}\,E_1\hat{j}$ it experiences a torque $\overrightarrow{T_2} = -\overrightarrow{T_1}$. The angle θ is :

(1) 90° (2) 30° (3) 45° (4) 60°

43. (4)

$$\vec{T}_1 = \vec{p}\times\vec{E}_1 = \left(p\cos\theta\,\hat{i} + p\sin\theta\hat{j}\right)\times E\hat{i}$$

$$\vec{T}_1 = \left(-p\sin\theta E\right)\hat{k}$$

In the second situation

$$\vec{T}_2 = \vec{p}\times\vec{E}_2 = \left(p\cos\theta\hat{i} + p\sin\theta\hat{j}\right)\times\sqrt{3}\,E\hat{j} = \left(\sqrt{3}p\cos\theta\,E\right)\hat{k}$$

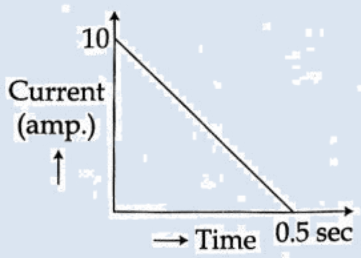
as $\vec{T}_2 = -\vec{T}_1$

$\therefore\ \sqrt{3}p\cos\theta\,E = p\sin\theta$

$\tan\theta = \sqrt{3}$

$\therefore\ \theta = 60°$

44. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is :



(1) 275 Wb (2) 200 Wb

(3) 225 Wb (4) 250 Wb

44. (4)

Equation of current in the coil

$$i = -\frac{10}{0.5}t + 10$$

$i = -20\,t + 10$

for the coil

$$-\frac{d\phi}{dt} = iR$$

$d\phi = -R\,i\,dt = -R\,(-\ 20t + 10)\,dt$

$$\therefore\ \Delta\phi = -R\int_0^{0.5}\left(-20t+10\right)dt - 100\left[-10t^2+10t\right]_0^{0.5}$$

$$= -1000\left[-t^2+t\right]_0^{0.5} = -1000\left[-(0.25-0)+(0.5-0)\right] = -1000\times0.25 = -250$$

$\therefore$ Magnitude of change in flux = 250 Wb

45. A time dependent force F = 6t acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be :

(1) 18 J (2) 4.5 J (3) 22 J (4) 9 J

45. (2)

$$a = \frac{F}{m} = \frac{6t}{1} = 6t$$

$$\frac{dv}{dt} = 6t$$

$$dv = 6t\, dt$$

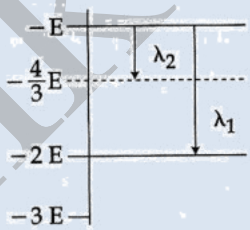
$$v = 6\int_0^t t\, dt$$

$$\therefore\ \frac{ds}{dt} = 3t^2$$

$$\therefore\ ds = 3t^2\, dt$$

$$w = \int F\cdot ds = \int_0^i 6t\cdot 3t^2 dt = 18\int_0^1 t^3 dt = \frac{18}{4}t^4\Big|_0^1$$

46. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths, $r = \lambda_1/\lambda_2$, is given by :



(1) $r = \frac{1}{3}$ (2) $r = \frac{4}{3}$

(3) $r = \frac{2}{3}$ (4) $r = \frac{3}{4}$

46. (1)

$$\lambda = \frac{hc}{\Delta E}$$

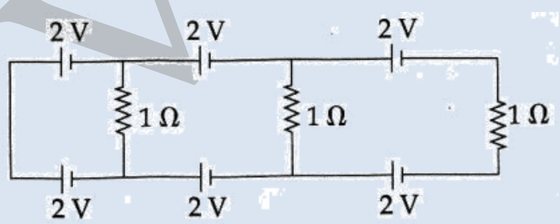
$$\therefore\ \lambda_1 = \frac{hc}{-E-(-2E)} = \frac{hc}{E} \quad \ldots\ldots\ldots\ldots\ (1)$$

$$\lambda_2 = \frac{hc}{-E-\left(-\frac{4}{3}E\right)} = \frac{3hc}{E} \quad \ldots\ldots\ldots\ldots\ (2)$$

(1) / (2) gives

$$r = \frac{\lambda_1}{\lambda_2} = \frac{1}{3}$$

47.



In the above circuit the current in each resistance is :

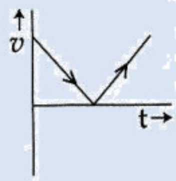
(1) 0 A (2) 1 A (3) 0.25 A (4) 0.5 A

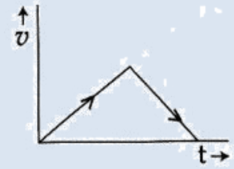
47. (1)

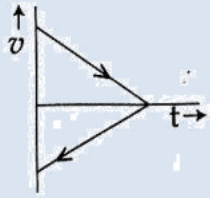
Points at two ends of 1 Ω resistors are at the same potential.

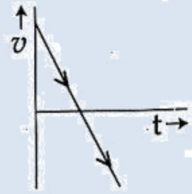
$\therefore$ Current through them = 0.

48. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity *vs* time?

(1)


(2)


(3)


(4)


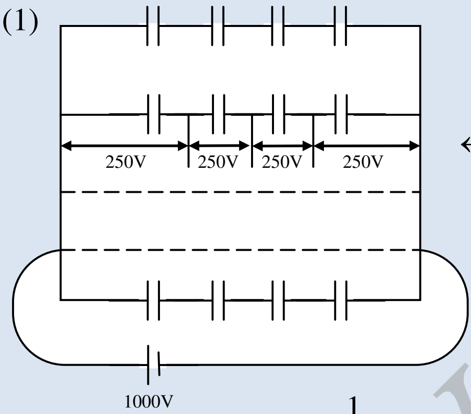
48. (4)

At the highest point direction or sign of velocity changes so only correct answer can be (4).

49. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is :

(1) 32 (2) 2 (3) 16 (4) 24

49. (1)

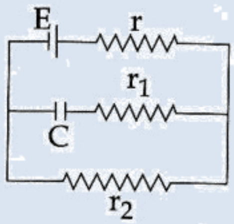


← 8 such rows and 4 capacitors in each row.

Capacitance of one row = $\frac{1}{4}\mu F$

$\therefore$ Total capacitance = $2\mu F$ and number of capacitance = $\frac{8}{\frac{1}{4}} = 32$.

50. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be :



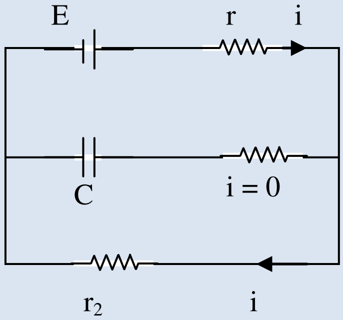
(1) $CE\frac{r_1}{(r_1+r)}$ (2) CE

(3) $CE\frac{r_1}{(r_2+r)}$ (4) $CE\frac{r_2}{(r+r_2)}$

50. (4)

In steady state

$$i = \frac{E}{r+r_2}$$



$\therefore$ Potential difference across the capacitor = $\frac{E\,r_2}{r+r_2}$

and charge on the capacitor = $\frac{CE\,r_2}{(r+r_2)}$.

51. In a common emitter amplifier circuit using an n–p–n transistor, the phase difference between the input and the output voltages will be :

(1) 180° (2) 45° (3) 90° (4) 135°

51. (1)

The input and output voltage and 180° apart in phase.

52. Which of the following statements is false?

(1) Kirchhoff's second law represents energy conservation.

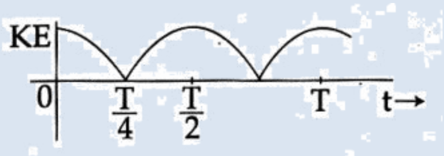
(2) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude.

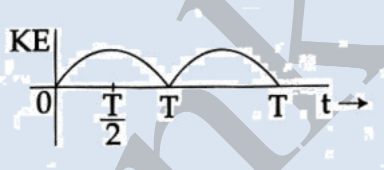
(3) In a balanced Wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.

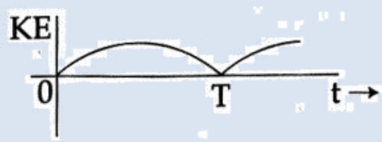
(4) A rheostat can be used as a potential divider.

52. (3)

53. A particle is executing simple harmonic motion with a time period T. At time t = 0, it is at its position of equilibrium. The kinetic energy–time graph of the particle will look like :

(1) 

(2) 

(3) 

(4) 

53. (1)

The correct graph is (1). as K.E. and P.E. both oscillate with double the frequency of SHM.

54. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light = 3×10^8 ms^{-1})

(1) 15.3 GHz (2) 10.1 GHz (3) 12.1 GHz (4) 17.3 GHz

54. (4)

Doppler's effect in light

If observer is moving towards source

$$f = \sqrt{\frac{1+\beta}{1-\beta}} \cdot f_0, \quad \text{where, } \beta = \frac{v}{c} = \frac{1}{2}$$

$$f = \sqrt{\frac{1+\frac{1}{2}}{1-\frac{1}{2}}} \cdot f_0 = \sqrt{3}\, f_0 = 17.3\,\text{GHz}$$

55. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of :

(1) $\frac{1}{81}$ (2) 9 (3) $\frac{1}{9}$ (4) 81

55. (2)

density = constant

volume = constant

Area × length = constant

$$\frac{dA}{A}+\frac{dL}{L}=0$$

$$\frac{dA}{A}=-\frac{dL}{L}=-9$$

Area decreases by a factor of 9.

$$\text{Stress} = \frac{Mg}{A}\propto\frac{1}{A}$$

Stress increase by a factor of 9.

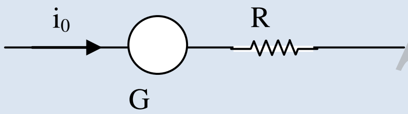
56. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0 – 10 V is :

(1) $4.005 \times 10^3\ \Omega$ (2) $1.985 \times 10^3\ \Omega$ (3) $2.045 \times 10^3\ \Omega$ (4) $2.535 \times 10^3\ \Omega$

56. (2)

$i_0 = 5mA$

$R_g = 15\Omega$

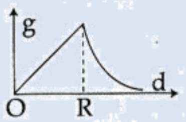


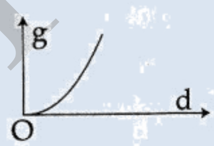
$v_0 = i_0\left(R_g + R\right)$

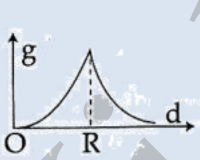
$$R = \frac{v_0}{i_0} - R_g = \frac{10}{5\times10^{-3}} - 15$$

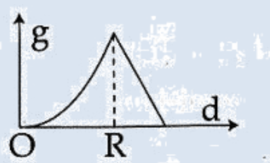
$R = 2 \times 10^3 - 15 = 2000 - 15 = 1985\ \Omega$

57. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius) :

(1) 

(2) 

(3) 

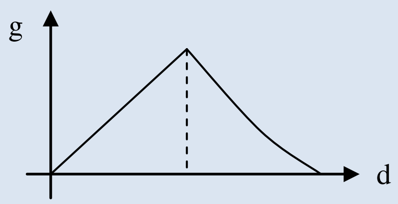
(4) 

57. (1)

According due to gravity is same as gravitation field due to earth.

(I) for $d \le R$

$$E = \frac{GMd}{R^3}\propto d$$



(II) for $d \ge R$

$$E = \frac{GM}{d^2}\propto\frac{1}{d^2}$$

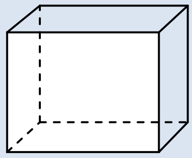
58. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by :

(1) $3PK\alpha$ (2) $\frac{P}{3\alpha K}$ (3) $\frac{P}{\alpha K}$ (4) $\frac{3\alpha}{PK}$

58. (2)

Volume strain due to excess pressure

$$\frac{\Delta V}{V} = \frac{1}{K} \times \Delta P \qquad \ldots\ldots\ldots\ldots\ (1)$$



Increase in volume due to increase in temperature.

$$\frac{\Delta V}{V} = \lambda \Delta T = 3\alpha \cdot \Delta T \qquad \ldots\ldots\ldots\ldots\ (2)$$

from equation (1) & (2)

$$3\alpha \cdot \Delta T = \frac{P}{K}$$

Increase in temperature $\boxed{\Delta T = \frac{P}{3K\alpha}}$

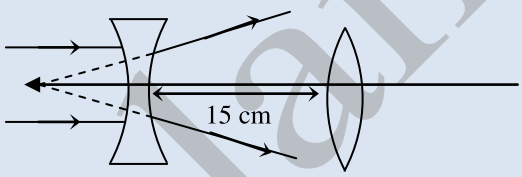
59. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is :

(1) real and at a distance of 6 cm from the convergent lens.
(2) real and at a distance of 40 cm from convergent lens.
(3) virtual and at a distance of 40 cm from convergent lens.
(4) real and at a distance of 40 cm from the divergent lens.

59. (2)

for converging lens

$u = -(15 + 25) = -40$ cm.



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40}$$

$v = +40$ cm

60. A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -k\upsilon^2$. Its initial speed is $\upsilon_0 = 10\ ms^{-1}$. If, after 10 s, its energy is $\frac{1}{8}m\upsilon_0^2$, the value of k will be :

(1) 10^{-1} kg m^{-1} s^{-1} (2) 10^{-3} kg m^{-1} (3) 10^{-3} kg s^{-1} (4) 10^{-4} kg m^{-1}

60. (4)

$m = 10^{-2}$ kg, $fr = -kv^2$, $v_0 = 10$ m/s, after t = 10 sec

$$KE = \frac{1}{8}mv_0^2$$

$$\text{final velocity} = \frac{v_0}{2} = 5\ m/s$$

$$fr = -kv^2$$

$$\text{Acceleration } \frac{dV}{dt} = -\frac{kv^2}{m}$$

$$-\int_{v_0}^{v_0/2} \frac{dv}{v^2} = \frac{k}{m}\int_0^{10} dt \quad \Rightarrow \quad \left[+\frac{1}{v}\right]_{v_0}^{v_0/2} = \frac{10k}{m}$$

$$\frac{2}{v_0} - \frac{1}{v_0} = \frac{10k}{m}$$

$$k = \frac{m}{10} \times \frac{1}{v_0} = \frac{10^{-2}}{10 \times 10} = 10^{-4}$$

PART- C : CHEMISTRY

61. 1 gram of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01186 mole of CO_2. The molar mass of M_2CO_3 in g mol^{-1} is :
(1) 84.3 (2) 118.6 (3) 11.86 (4) 1186

61. (1)

$$\underset{\frac{1}{M}\,mol}{M_2CO_3} + 2HCl \longrightarrow 2MCl + H_2O + \underset{\frac{1}{M} = 0.01186}{CO_2}$$

$$M = \frac{1}{0.01186} = 84.3\ g/mol$$

62. Given,
$C_{(graphite)} + O_2(g) \rightarrow CO_2(g)$;
$\Delta_r H^o = -\ 393.5$ kJ mol^{-1}
$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l)$;
$\Delta_r H^o = -285.8$ kJ mol^{-1}
$CO_2(g) + 2H_2O(l) \rightarrow CH_4(g) + 2O_2(g)$;
$\Delta_r H^o = +\ 890.3$ kJ mol^{-1}
Based on the above thermochemical equations, the value of $\Delta_r H^o$ at 298 K for the reaction $C_{(graphite)} + 2H_2(g) \rightarrow CH_4(g)$ will be :
(1) + 144.0 kJ mol^{-1} (2) −74.8 kJ mol^{-1} (3) −144.0 kJ mol^{-1} (4) + 74.8 kJ mol^{-1}

62. (2)

$C_{(graphite)} + O_2(g) \rightarrow CO_2(g)$; $\Delta_r H_1^o = -\ 393.5$ kJ mol

$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l)$; $\Delta H_2^o = -285.8$ kJ mol

$CO_2(g) + 2H_2O(l) \rightarrow CH_4(g) + 2O_2(g)$; $\Delta H_3^o = +\ 890.3$ kJ mol

$C_{(graphite)} + 2H_2(g) \rightarrow CH_4(g)$ $\Delta H = ?$

$$\Delta H^o = \Delta H_1^o + 2\Delta H_2^o + \Delta H_3^o = -74.8\ kJ/mol$$

63. The freezing point of benzene decreases by 0.45°C when 0.2 g of acetic acid is added to 20 g of benzene. If acetic acid associates to form a dimer in benzene, percentage association of acetic acid in benzene will be :
(K_f for benzene = 5.12 K kg mol^{-1})
(1) 80.4% (2) 74.6% (3) 94.6% (4) 64.6%

63. (3)

	$2\,CH_3COOH$	$\rightarrow$	$(CH_3COOH)_2$
t = 0	1		0
t = t	$1-\alpha$		$\alpha/2$

$$i = 1 - \alpha + \frac{\alpha}{2} = 1 - \frac{\alpha}{2}$$

$$\Delta T_f = i\,K_f\,m$$

$$0.45 = i(5.12)\left(\frac{0.2/60}{20\times10^{-3}}\right)$$

$$i = 0.52 = 1 - \alpha/2$$

$$\Rightarrow \alpha = 0.946 \text{ i.e. } 94.6\ \%$$

64. The most abundant elements by mass in the body of a healthy human adult are : Oxygen (61.4%); Carbon (22.9%), Hydrogen (10.0%); and Nitrogen (2.6%). The weight which a 75 kg person would gain if all ^{1}H atoms are replaced by ^{2}H atoms is :

(1) 37.5 kg (2) 7.5 kg (3) 10 kg (4) 15 kg

64. (2)

Weight of hydrogen atoms = $\frac{10}{100} \times 75 = 7.5$ kg

If all ^{1}H atoms are replaced by ^{2}H i.e. increases in weight by 7.5 kg.

65. ΔU is equal to :

(1) Isobaric work (2) Adiabatic work (3) Isothermal work (4) Isochoric work

65. (2)

Using Ist law of thermodynamics

$\Delta U = q + w$

For adiabatic process, q = 0

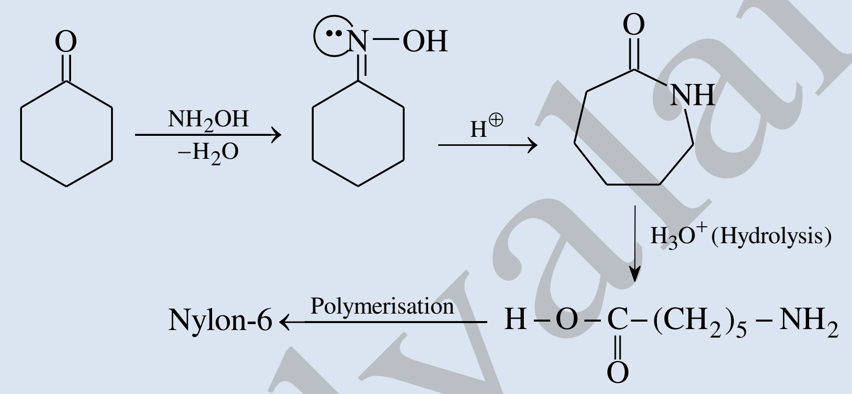
$\Rightarrow \Delta U = w$

$\therefore$ ΔU is equal to adiabatic work.

66. The formation of which of the following polymers involves hydrolysis reaction?

(1) Bakelite (2) Nylon 6, 6 (3) Terylene (4) Nylon 6

66. (4)



67. Given $E^{\circ}_{Cl_2/Cl^-} = 1.36V, E^{\circ}_{Cr^{3+}/Cr} = -0.74V$

$E^{\circ}_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33\,V,\ E^{\circ}_{MnO_4^-/Mn^{2+}} = 1.51V.$

Among the following, the strongest reducing agent is :

(1) Mn^{2+} (2) Cr^{3+} (3) Cl^- (4) Cr

67. (4)

Strongest reducing agent i.e. least value of reduction potential

$\Rightarrow E^{o}_{Cr^{+3}/Cr} = -0.74$ V

68. The Tyndall effect is observed only when following conditions are satisfied:

(a) The diameter of the dispersed particles is much smaller than the wavelength of the light used.

(b) The diameter of the dispersed particle is not much smaller than the wavelength of the light used.

(c) The refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude.

(d) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude.

(1) (b) and (d) (2) (a) and (c) (3) (b) and (c) (4) (a) and (d)

68. (1)

Tyndall effect is observed only when the following two conditions are satisfied.

(i) The diameter of the dispersed particles is not much smaller than the wavelength of the light used ; and

(ii) The refractive indices of the dispersed phase and the dispersion medium differ greatly in magnitude.

69. In the following reactions, ZnO is respectively acting as a/an :

(a) $ZnO + Na_2O \rightarrow Na_2ZnO_2$

(b) $ZnO + CO_2 \rightarrow ZnCO_3$

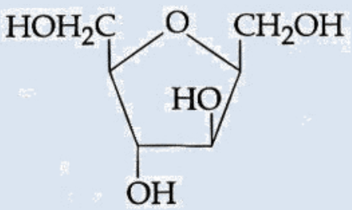
(1) base and base (2) acid and acid (3) acid and base (4) base and acid.

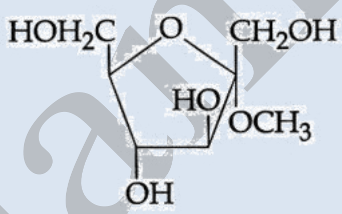
69. (3)

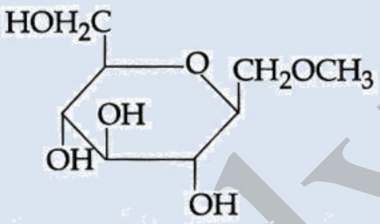
$$\underset{\text{Acid}}{ZnO} + \underset{\text{Base}}{Na_2O} \longrightarrow Na_2ZnO_2$$

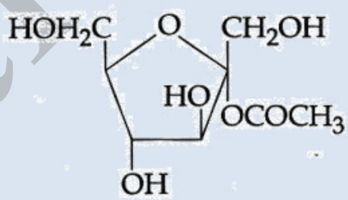
$$\underset{\text{Base}}{ZnO} + \underset{\text{Acid}}{CO_2} \longrightarrow ZnCO_3$$

70. Which of the following compounds will behave as a reducing sugar in an aqueous KOH solution?

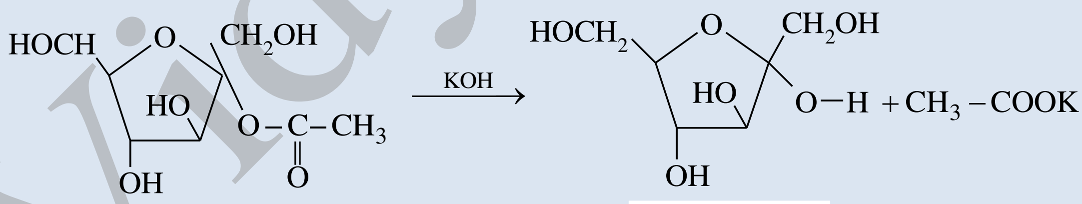
(1) 

(2) 

(3) 

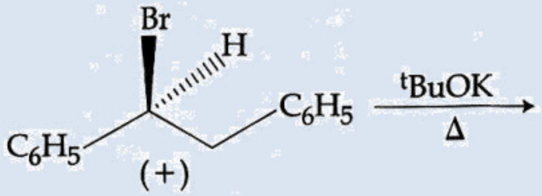
(4) 

70. (4)



Hemi-Acetal is reducing sugar

71. The major product obtained in the following reaction is :



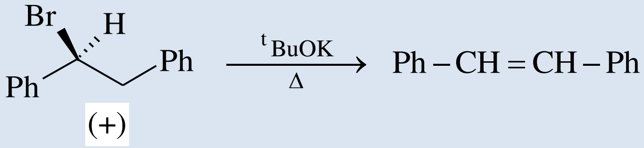
(1) $C_6H_5CH = CHC_6H_5$

(2) $(+)C_6H_5CH(O^tBu)CH_2C_6H_5$

(3) $(-)C_6H_5CH(O^tBu)CH_2C_6H_5$

(4) $(\pm)C_6H_5CH(O^tBu)CH_2C_6H_5$

71. (1)



72. Which of the following species is not paramagnetic?
(1) CO (2) O_2 (3) B_2 (4) NO

72. (1)
CO is diamagnetic
O_2, B_2 and NO are paramagnetic

73. On treatment of 100 mL of 0.1 M solution of $CoCl_3.6H_2O$ with excess $AgNO_3$; 1.2×10^{22} ions are precipitated. The complex is :
(1) $[Co(H_2O)_3Cl_3].3H_2O$ (2) $[Co(H_2O)_6]Cl_3$
(3) $[Co(H_2O)_5Cl]Cl_2.H_2O$ (4) $[Co(H_2O)_4Cl_2]Cl.2H_2O$

73. (3)
No of moles of AgCl formed = $\frac{1.2\times10^{22}}{6\times10^{23}}$ = 0.02 mol
No of moles of $CoCl_3{\cdot}6H_2O$ = 0.1 × 100 × 10^{-3} = 0.01 mol
No of chlorine atoms outside the co-ordination sphere = $\frac{0.02}{0.01}$ = 2
i.e. $[Co(H_2O)_5Cl]\ Cl_2{\cdot}H_2O$

74. pK_a of a weak acid (HA) and pK_b of a weak base (BOH) are 3.2 and 3.4, respectively. The pH of their salt (AB) solution is :
(1) 6.9 (2) 7.0 (3) 1.0 (4) 7.2

74. (1)
$$pH = \frac{1}{2}[pK_w + pK_a - pK_b] = \frac{1}{2}[14+3.2-3.4] = 6.9$$

75. The increasing order of the reactivity of the following halides for the S_N1 reaction is :

$CH_3CHCH_2CH_3$ with Cl on the second carbon $\quad$ $CH_3CH_2CH_2Cl$
(I) $\quad$ (II)
$p-H_3CO-C_6H_4-CH_2Cl$
(III)

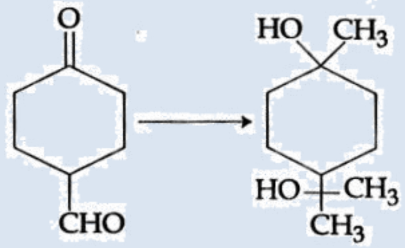
(1) (II) < (I) < (III) (2) (I) < (III) < (II) (3) (II) < (III) < (I) (4) (III) < (II) < (I)

75. (1)
Rate of S_N1 Reaction ∝ Stability of Carbocation
⇒ II < I < III

76. Both lithium and magnesium display several similar properties due to the diagonal relationship; however, the one which is incorrect, is :
(1) both form soluble bicarbonates
(2) both form nitrides
(3) nitrates of both Li and Mg yield NO_2 and O_2 on heating
(4) both form basic carbonates

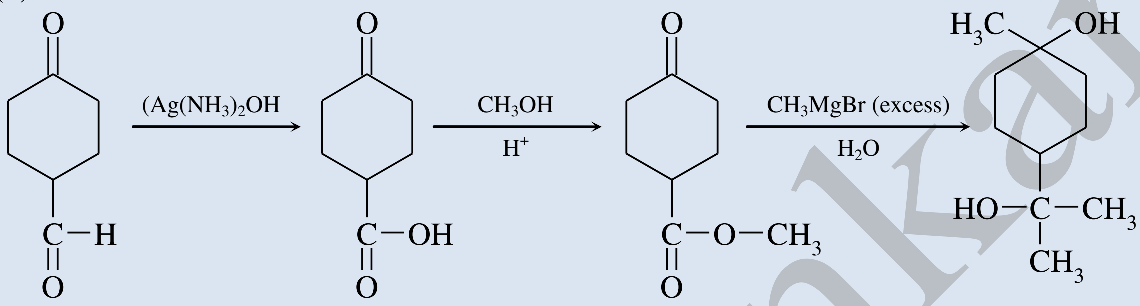
76. (4)

77. The correct sequence of reagents for the following conversion will be :



(1) CH_3MgBr, H^+/CH_3OH, $[Ag(NH_3)_2]^+OH^-$
(2) CH_3MgBr, $[Ag(NH_3)_2]^+OH^-$, H^+/CH_3OH
(3) $[Ag(NH_3)_2]^+OH^-$, CH_3MgBr, H^+/CH_3OH
(4) $[Ag(NH_3)_2]^+OH^-$, H^+/CH_3OH, CH_3MgBr

77. (4)



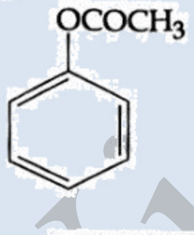
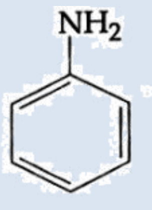
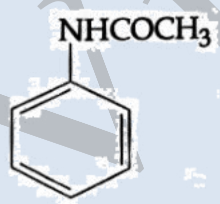
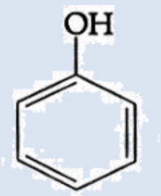
78. The products obtained when chlorine gas reacts with cold and dilute aqueous NaOH are :

(1) ClO_2^- and ClO_3^- (2) Cl^- and ClO^-
(3) Cl^- and ClO_2^- (4) ClO^- and ClO_3^-

78. (2)

$Cl_2 + 2NaOH_{(\text{Cold and dilute})} \rightarrow NaCl + NaOCl + H_2O$

79. Which of the following compounds will form significant amount of meta product during mono–nitration reaction?

(1) 
(2) 
(3) 
(4) 

79. (2)

Aniline gives significant amount of meta product.

80. 3-Methyl–pent-2-ene on reaction with HBr in presence of peroxide forms an addition product. The number of possible stereoisomers for the product is :

(1) Zero (2) Two (3) Four (4) Six

80. (3)

$$H_3C-CH{=}\underset{}{\overset{CH_3}{\overset{|}{C}}}-CH_2-CH_3 \xrightarrow[\text{Peroxide}]{\text{HBr}} H_3C-\underset{\underset{Br}{|}}{\overset{\times}{C}H}-\overset{CH_3}{\overset{|}{\underset{\times}{C}H}}-CH_2-CH_3$$

$$2^2 = 4$$

81. Two reactions R_1 and R_2 have identical pre–exponential factors. Activation energy of R_1 exceeds that of R_2 by 10 kJ mol^{-1}. If k_1 and k_2 are rate constants for reactions R_1 and R_2 respectively at 300 K, then $\ln(k_2/k_1)$ is equal to :
(R = 8.314 J $mol^{-1}K^{-1}$)
(1) 12 (2) 6 (3) 4 (4) 8

81. (3)

$$K_1 = A\,e^{-E_{a_1}/RT}$$

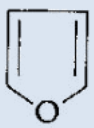
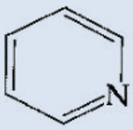
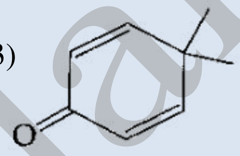
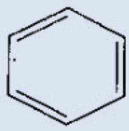
$$K_2 = A\,e^{-E_{a_2}/RT}$$

$$(E_a)_1 = (E_{a_2})_2 + 10\,kJ/mol$$

$$\frac{k_2}{k_1} = \frac{A\,e^{-E_{a_2}/RT}}{A\,e^{-E_{a_1}/RT}} = e^{(E_{a_1}-E_{a_2})} = e^{\frac{10\times10^3}{8.314\times300}} = e^4$$

$$\ell n\left(\frac{K_2}{K_1}\right) = 4$$

82. Which of the following molecules is least resonance stabilized?

(1)  (2)  (3)  (4) 

82. (3)
(1), (2) & (4) are aromatic

83. The group having isoelectronic species is :

(1) O^-, F^-, Na, Mg^+ (2) O^{2-}, F^-, Na, Mg^{2+}
(3) O^-, F^-, Na^+, Mg^{2+} (4) O^{2-}, F^-, Na^+, Mg^{2+}

83. (4)

O^{2-}, F^-, Na^+ and Mg^{+2} are Isoelectronic species contains 10 e^-.

84. The radius of the second Bohr orbit for hydrogen atom is :
(Planck's Const. h = 6.6262 × 10^{-34} Js; mass of electron = 9.1091 × 10^{-31} kg; charge of electron e = 1.60210 × 10^{-19} C; permittivity of vacuum $\in_0$ = 8.854185 × 10^{-12} kg^{-1} m^{-3} A^2)

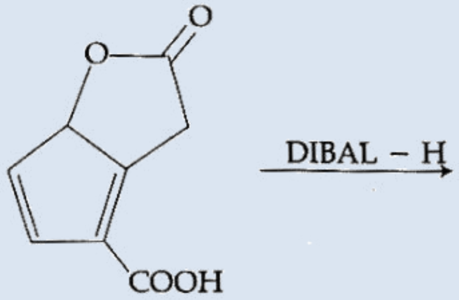
(1) 4.76 Å (2) 0.529 Å (3) 2.12 Å (4) 1.65 Å

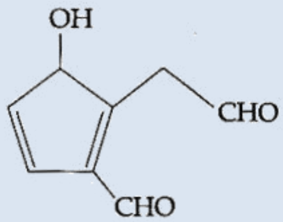
84. (3)

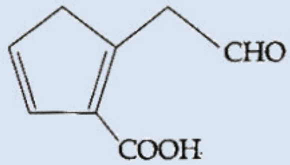
$$r_n = \frac{0.529\,n^2}{z}$$

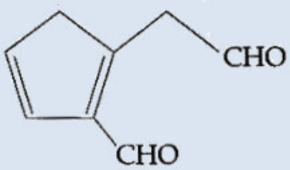
$$\text{Å} = \frac{(0.529\times4)}{1}\,\text{Å} = \frac{(0.529\times4)}{1}\,\text{Å} = 2.12\,\text{Å}$$

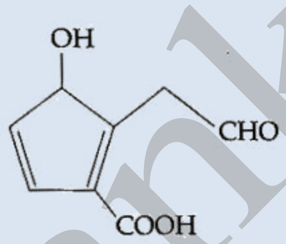
85. The major product obtained in the following reaction is :



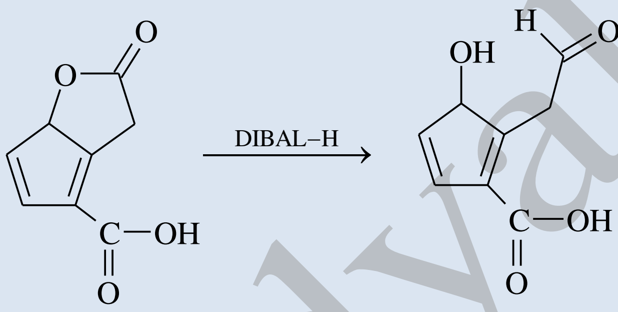
(1) 

(2) 

(3) 

(4) 

85. (4)



86. Which of the following reactions is an example of a redox reaction?

(1) $XeF_2 + PF_5 \rightarrow [XeF]^+ PF_6^-$
(2) $XeF_6 + H_2O \rightarrow XeOF_4 + 2HF$
(3) $XeF_6 + 2H_2O \rightarrow XeO_2F_2 + 4HF$
(4) $XeF_4 + O_2F_2 \rightarrow XeF_6 + O_2$

86. (4)

$XeF_4 + O_2F_2 \rightarrow XeF_6 + O_2$

Xenon is oxidized from +4 to +6 whereas oxygen is reduced from +1 to 0.

87. A metal crystallises in a face centred cubic structure. If the edge length of its unit cell is ‘a’, the closest approach between two atoms in metallic crystal will be:

(1) $2\sqrt{2}\,a$
(2) $\sqrt{2}\,a$
(3) $\frac{a}{\sqrt{2}}$
(4) 2a

87. (3)

For FCC, $4r = \sqrt{2}\,a$

The closest approach between two atoms in metallic crystal $= 2r$

$$= \frac{\sqrt{2}\,a}{2} = \frac{a}{\sqrt{2}}$$

88. Sodium salt of an organic acid 'X' produces effervescence with conc. H_2SO_4. 'X' reacts with the acidified aqueous $CaCl_2$ solution to give a white precipitate which decolourises acidic solution of $KMnO_4$. 'X' is :

(1) $HCOONa$ (2) CH_3COONa (3) $Na_2C_2O_4$ (4) C_6H_5COONa

88. (3)

$$Na_2C_2O_4 + \text{conc } H_2SO_4 \longrightarrow Na_2SO_4 + H_2O + CO + CO_2 \uparrow$$

$$Na_2C_2O_4 + CaCl_2 \longrightarrow \underset{\text{white ppt}}{CaC_2O_4} \downarrow + 2NaCl$$

89. A water sample has ppm level concentration of following anions

$F^- = 10$; $SO_4^{2-} = 100$; $NO_3^- = 50$

The anion/anions that make/makes the water sample unsuitable for drinking is/are :

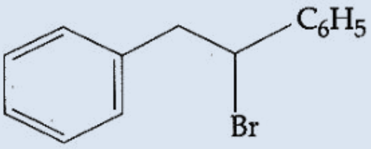
(1) both SO_4^{2-} and NO_3^- (2) only F^-

(3) only SO_4^{2-} (4) only NO_3^-

89. (2)

90. Which of the following upon treatment with tert-BuONa followed by addition of bromine water, fails to decolourize the colour of bromine?

(1)



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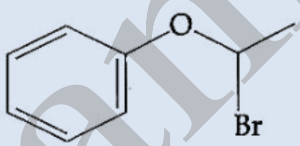

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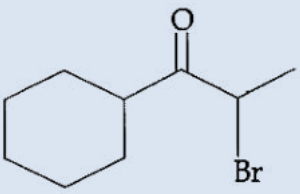

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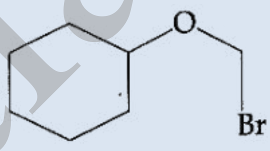

90. (4)

Due to absence of Beta-hydrogen in compound-4, no alkene is formed.
Alkene and alkyne de-colourises Br_2-water.

Previous Year's Official Cutoff Trends of JEE Main						
S. No.	**Category**	**2017***	**2016**	**2015**	**2014**	**2013**
1	General	95–100	100	105	115	113
2	OBC-NCL	65–70	70	70	74	70
3	SC	45–50	52	50	53	50
4	ST	40–45	48	44	47	45
* Expected Cutoff						