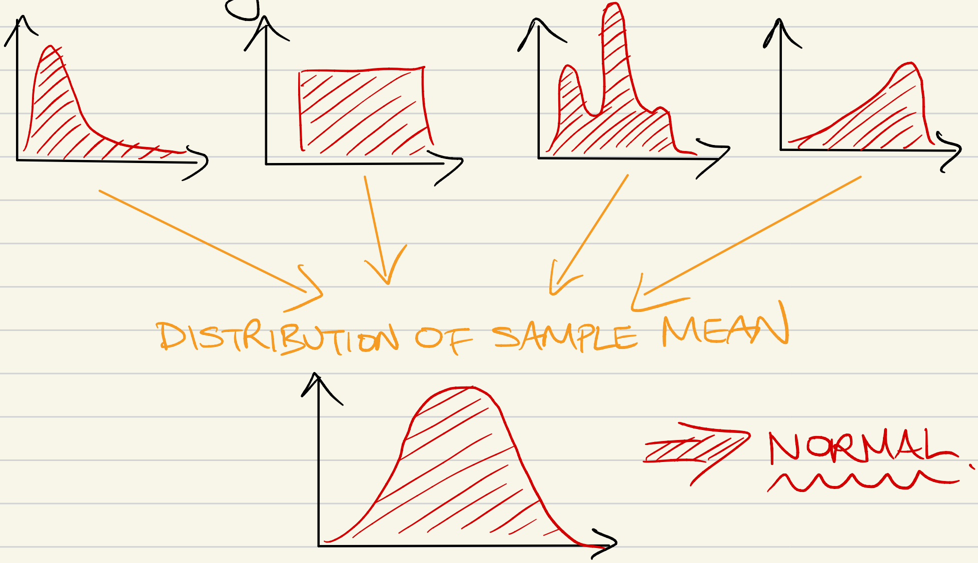
Lecture 4

- Central Limit Theorem
  - Pick N random numbers from any $h(x)$ [tails must fall off faster than $1/x^2$].
  - Mean of those N samples will converge to a Gaussian as $N \longmapsto$.



Amazing! Underlies the reason why repeat observations improve measurements as $1/\sqrt{N}$.

- Multivariate pdfs
  - Characterizing pdfs in more than one dimension or parameter.
  - Multivariate Gaussians

$$p(x|\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

$$\Downarrow$$

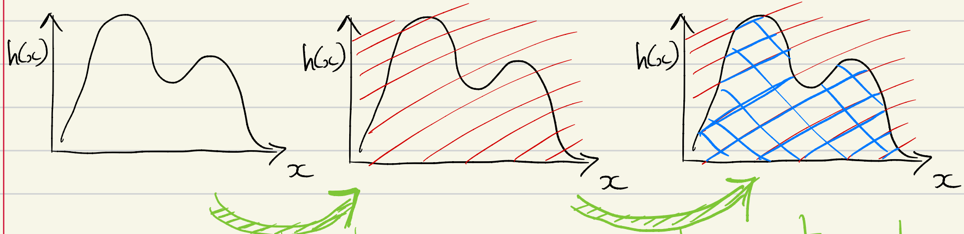
$$p(\vec{x}|\vec{\mu},C) = \frac{1}{\sqrt{\det(2\pi C)}} e^{-\frac{1}{2}(\vec{x}-\vec{\mu})^T C^{-1}(\vec{x}-\vec{\mu})}$$

$$\ldots\; C = \begin{pmatrix} \sigma_1^2 & & 0 \\ & \sigma_2^2 & \\ 0 & & \sigma_3^2 \; \ldots \end{pmatrix}$$

- Correlation coefficients
  - Parametric $\Rightarrow$ Pearson's ... $\underbrace{r = \frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum(x_i-\bar{x})^2\sum(y_i-\bar{y})^2}}}_{\text{SUSCEPTIBLE TO OUTLIERS}}$
  - Non-parametric $\Rightarrow$ SPEARMAN'S ... as Pearson's but with sample ranks
    $\Rightarrow$ KENDALL'S ... compares the # samples with same + different sign

- Sampling from arbitrary distributions

① Rejection sampling



generate random samples from a proposal, where proposal pdf > h(x)

only accept samples that fall below h(x)

⇒ what if we didn't have to waste samples?

② Inverse transform sampling

numerically find distribution CDF

sample from $U[0,1]$ then map to "x" with $H^{-1}(x)$

⇒ easy, no waste!

↳ only really works in 1D though...