Singular and Non-Singular

The square matrix A is said to singular, if determinant of A is zero, otherwise it is non-singular.

Reverse (Inverse of a matrix):

$$A^{-1} = \frac{1}{A} = \frac{1}{|A|}(\text{adj} A)$$

If A is a non-singular matrix then $A^{-1} = \frac{1}{|A|}(\text{adj} A)$

1. Symmetric Matrix:

A square matrix is said to be symmetric, if its equal to its transpose i.e $A = A^T$

2. Skew symmetric matrix:

A square matrix is said to be skew symmetric, if it is equal to negative of its transpose.

i.e $A = -A^T$

Hint: All Diagonal Elements should be zero to skew symmetric.

Eg: (1)

$A = \begin{pmatrix} 1 & a \\ a & 2 \end{pmatrix}_{2\times 2}$ $\quad A = \begin{pmatrix} 1 & 4 & 5 \\ 4 & 2 & 6 \\ 5 & 6 & 3 \end{pmatrix}_{3\times 3}$

$A^T = \begin{pmatrix} 1 & a \\ a & 2 \end{pmatrix}$ $\quad A^T = \begin{pmatrix} 1 & 4 & 5 \\ 4 & 2 & 6 \\ 5 & 6 & 3 \end{pmatrix}_{3\times 3}$

(2)

$A = \begin{pmatrix} 0 & -4 \\ 4 & 0 \end{pmatrix}_{2\times 2}$ $\quad A = \begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{pmatrix}_{3\times 3}$

$A^T = \begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix}$ $\quad A^T = \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{pmatrix}$

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Theorem:

Every square matrix can be expressed as the sum of symmetric and skew symmetric matrix.

Let 'A' $= \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

$$A = P + Q$$

To show that P is symmetric and Q is skew symmetric

$$P = \frac{1}{2}(A + A^T)$$

$$P^T = \left[\frac{1}{2}(A+A^T)\right]^T$$

$$= \frac{1}{2}(A+A^T)^T$$

$$= \frac{1}{2}\left[A^T + (A^T)^T\right]$$

$$= \frac{1}{2}(A^T + A)$$

$$= \frac{1}{2}(A + A^T)$$

$$P^T = P$$

$\therefore$ P is symmetric

$$Q = \frac{1}{2}(A - A^T)$$

$$Q^T = \left[\frac{1}{2}(A - A^T)\right]^T$$

$$= \frac{1}{2}(A - A^T)^T$$

$$= \frac{1}{2}\left[A^T - (A^T)^T\right]$$

$$= \frac{1}{2}\left[A^T - A\right]$$

$$Q^T = -\frac{1}{2}\left[A - A^T\right]$$

$$Q^T = -Q$$

$\therefore$ Q is skew symmetric

Express $\begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix}$ as the sum of Symmetric and skew-Symmetric.

let $A = \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix}$

$A^T = \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix}$

$$\frac{1}{2}(A + A^T) = \frac{1}{2}\left(\begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 12 & 12 & 14 \\ 12 & 4 & 10 \\ 14 & 10 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 6 & 7 \\ 6 & 2 & 5 \\ 7 & 5 & 1 \end{bmatrix}$$

$$\frac{1}{2}(A - A^T) = \frac{1}{2}\left(\begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 4 & -4 \\ -4 & 0 & -4 \\ 4 & 4 & 0 \end{bmatrix}$$

$$\therefore = \begin{bmatrix} 0 & 2 & -2 \\ -2 & 0 & -2 \\ 2 & 2 & 0 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 6 & 6 & 7 \\ 6 & 2 & 5 \\ 7 & 5 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 & -2 \\ -2 & 0 & -2 \\ 2 & 2 & 0 \end{bmatrix}$$

Conjugate of a Matrix:

A Matrix Obtained by replacing its elements by the corresponding conjugate complex number. It is denoted by $\bar{A}$.

Eg: $A = \begin{bmatrix} 1+i & 2-i \\ 3+i & 4-i \end{bmatrix}$

$$\bar{A} = \begin{bmatrix} 1-i & 2+i \\ 3-i & 4+i \end{bmatrix}$$

Properties:

(i) $(\overline{A+B}) = \bar{A} + \bar{B}$

(ii) $\overline{KA} = \bar{K}\bar{A}$

(iii) $(\overline{A \cdot B}) = \bar{A} \cdot \bar{B}$

Hermitian and Skew-hermitian:

A Square matrix A,

$A = \bar{A}^T \rightarrow$ Hermitian,

$A = -\bar{A}^T \rightarrow$ Skew hermitian.

Hint:

- For hermitian matrix, all the diagonals elements are real.
- For Skew hermitian, all the diagonal elements are purely imaginary or zero

Identify A is Skew hermitian:

$$A = \begin{bmatrix} -2i & 4-7i \\ -4-7i & 0 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 2i & 4+7i \\ -4+7i & 0 \end{bmatrix}$$

$$\bar{A}^T = \begin{bmatrix} 2i & -4+7i \\ 4+7i & 0 \end{bmatrix}$$

$$-\bar{A}^T = \begin{bmatrix} -2i & 4-7i \\ -4-7i & 0 \end{bmatrix}$$

$$-\bar{A}^T = A$$

$$\therefore A = -\bar{A}^T$$

Identify whether it is hermitian or skew hermitian:

$$A = \begin{bmatrix} 4 & 6+4i & 3-7i \\ 6-4i & 7 & 2+5i \\ 3+7i & 2-5i & 0 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 4 & 6-4i & 3+7i \\ 6+4i & 7 & 2-5i \\ 3-7i & 2+5i & 0 \end{bmatrix}$$

$$\bar{A}^T = \begin{bmatrix} 4 & 6+4i & 3-7i \\ 6-4i & 7 & 2+5i \\ 3+7i & 2-5i & 0 \end{bmatrix}$$

$\therefore A = \bar{A}^T$

$\therefore$ It is hermitian

Theorem:

If A and B are hermitian

Show that AB+BA is hermitian and AB-BA is skew hermitian.

Given:

A and B are hermitian

i.e $A = \bar{A}^T$, $B = B^*$

$A = A^*$

To Prove:

$(AB+BA) = (AB+BA)^*$

$(AB-BA) = -(AB-BA)^*$

Proof:

Consider $(AB+BA)^* = (AB)^* + (BA)^*$

$= B^*A^* + A^*B^*$

$= BA + AB$

$= AB + BA$

$\therefore$ AB+BA is hermitian.

$-(AB-BA)^* = -[(AB)^* - (BA)^*]$

$= -[B^*A^* - A^*B^*]$

$= -[BA - AB]$

$= AB - BA$

$\therefore$ AB-BA is skew hermitian.

Theorem:

If A is a Square Matrix. Show that $A+A^*$, A^*A are hermitian, $A-A^*$ is skew hermitian.

Given:

A is a Square matrix.

To Prove: (i) To Prove $A+A^*$ is hermitian

(i.e) $A+A^* = (A+A^*)^*$

(ii) To Prove A^*A is hermitian

(i.e) $A^*A = (A^*A)^*$

(iii) To Prove $A-A^*$ is skew hermitian, (i.e) $A-A^* = -(A-A^*)^*$

Proof:

(i) $(A+A^*)^* = A^* + (A^*)^*$

$= A^* + A$

$\therefore A+A^* = (A+A^*)^*$

$A+A^*$ is hermitian.

(ii) $(A^*A)^* = (A^*)^* (A)^*$

$= A^* A$

$\therefore (A^*A)^* = A^*A$

A^*A is hermitian

(iii) $-(A-A^*)^* = -[A^* - (A^*)^*]$

$= -[A^* - A]$

$= -A^* + A$

$\therefore -(A-A^*)^* = A - A^*$

$\therefore A-A^*$ is skew hermitian.

Every Square matrix can be Expressed as Sum of hermitian and skew hermitian. Here, the A can be written as

$$A = \frac{1}{2}(A+A^*) + \frac{1}{2i}(A-A^*)$$

Show that B^*AB is hermitian or skew hermitian according as A is hermitian or skew hermitian.

Given: B^*AB

Let A be hermitian, $A = A^*$

$(B^*AB)^* = B^*A^*(B^*)^*$

$= B^*A(B)$

$= B^*AB$.

$\therefore B^*AB$ is hermitian.

Let A be skew hermitian,

$A = -A^*$

$(B^*AB)^* = B^* A^* (B^*)^*$

$= B^* (-A) B$

$= -(B^*AB)$

$\therefore B^*AB$ is skew hermitian

H.W

1. Show that the matrix A^2 is symmetric if either A is symmetric or skew symmetric

2. Express $\begin{bmatrix} 2 & 3 & 5 \\ 3 & 4 & 7 \\ 6 & 2 & 1 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrix

3. Show that $\begin{bmatrix} i & 1+i & 2 \\ -1+i & 3i & i \\ -2 & i & 0 \end{bmatrix}$ is skew hermitian

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Orthogonal and Unitary matrices

If A is orthogonal, $AA^T = A^TA = I$

If A is Unitary, $AA^* = A^*A = I$

Theorem:

If A and B are orthogonal matrices of the same order, then AB and BA are also orthogonal.

Given: A and B are orthogonal matrices, i.e $AA^T = A^TA = I$

$B.B^T = B^T.B = I$

(i) T.P.T: AB is orthogonal

(i.e) $(AB).(AB)^T = (AB)^T.AB = I$

Proof:

$AB.(AB)^T = AB.(B^T.A^T)$

$= A.(B.B^T).A^T$

$= A(I).A^T$

$= A.A^T$

$= I$

$\therefore AB$ is orthogonal.

(ii) BA is orthogonal

(i.e) $BA(BA)^T = (BA)^T.BA = I$

$BA.(BA)^T = BA.(A^T.B^T)$

$= B.(A.A^T).B^T$

$= B(I)B^T$

$= BB^T$

$BA(BA)^T = I$

$\therefore (BA)^T . BA = I$

$\therefore$ BA is orthogonal.

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H.W

2. Solution:

Given: Let $A = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 4 & 7 \\ 6 & 2 & 1 \end{bmatrix}$

To Prove: $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

Proof:

$$A^T = \begin{bmatrix} 2 & 3 & 6 \\ 3 & 4 & 2 \\ 5 & 7 & 1 \end{bmatrix}$$

$$\frac{1}{2}(A + A^T) = \frac{1}{2}\left(\begin{bmatrix} 2 & 3 & 6 \\ 3 & 4 & 2 \\ 5 & 7 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 5 \\ 3 & 4 & 7 \\ 6 & 2 & 1 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 4 & 6 & 11 \\ 6 & 8 & 9 \\ 11 & 9 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 11/2 \\ 3 & 4 & 9/2 \\ 11/2 & 9/2 & 1 \end{bmatrix}$$

$$\frac{1}{2}(A - A^T) = \frac{1}{2}\left(\begin{bmatrix} 2 & 3 & 5 \\ 3 & 4 & 7 \\ 6 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 6 \\ 3 & 4 & 2 \\ 5 & 7 & 1 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$$

$$\frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & 0 & -1/2 \\ 0 & 0 & 5/2 \\ 1/2 & -5/2 & 0 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 2 & 3 & 11/2 \\ 3 & 4 & 9/2 \\ 11/2 & 9/2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -1/2 \\ 0 & 0 & 5/2 \\ 1/2 & -5/2 & 0 \end{bmatrix}$$

3. Solution:

Given: Let $A = \begin{bmatrix} i & 1+i & 2 \\ -1-i & 3i & i \\ -2 & -i & 0 \end{bmatrix}$

To Prove: $A = -A^*$

$A = -\bar{A}^T$

Proof:

$$\bar{A} = \begin{bmatrix} -i & 1-i & 2 \\ -1+i & -3i & +i \\ -2 & -i & 0 \end{bmatrix}$$

$$\bar{A}^T = \begin{bmatrix} -i & -1+i & -2 \\ 1-i & -3i & +i \\ 2 & +i & 0 \end{bmatrix}$$

$$-\bar{A}^T = \begin{bmatrix} i & 1+i & 2 \\ -1-i & 3i & i \\ -2 & -i & 0 \end{bmatrix}$$

$-\bar{A}^T = A$

$\therefore$ Hence the given matrix is skew hermitian.

1. Solution:

Given: $A = A^T$

(i) To Prove: $A^2 = (A \cdot A)^T = (A^2)^T$

Proof: $(A^2)^T = (A \cdot A)^T$

~~$A^2 = (A \cdot A)^T$~~

$(A^2)^T = A^T \cdot A^T$

$= A \cdot A$

$(A^2)^T = A^2$

$\therefore A^2$ is symmetric

(ii) Prove: $A^2 = +(A^2)^T$, Given: $A = -A^T$

$\therefore A^T = -A$

Proof:

$(A^2)^T = (A \cdot A)^T$

~~$= (A \cdot A)^T$~~

$= A^T \cdot A^T$

$= (-A) \cdot (-A)$

$(A^2)^T = A^2$

$A^2 = (A^2)^T$

$\therefore A^2$ is symmetric.

28.06.2018

Show that the matrix $\frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ is Orthogonal.

Solution:

Given: Let $A = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$

To Prove: $AA^T = A^T A = I$

Proof:

$$A^T = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

$$AA^T = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 1+4+4 & -2-2+4 & -2+4-2 \\ -2-2+4 & 4+1+4 & 4-2-2 \\ -2+4-2 & 4-2-2 & 4+4+1 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$A^T A = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = I$$

$\therefore\ AA^T = A^T A = I$

Hence Proved

$\therefore$ Given matrix is Orthogonal

Show that $\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ is Orthogonal

Solution:

Given: let $A = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

To Prove: $AA^T = A^T A = I$

Proof:

$$A^T = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$AA^T = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3}+\frac{1}{6}+\frac{1}{2} & \frac{1}{3}-\frac{2}{6}+0 & \frac{1}{3}+\frac{1}{6}-\frac{1}{2} \\ \frac{1}{3}+\frac{(-2)}{6}+0 & \frac{1}{3}+\frac{4}{6}+0 & \frac{1}{3}-\frac{2}{6}+0 \\ \frac{1}{3}+\frac{1}{6}-\frac{1}{2} & \frac{1}{3}-\frac{2}{6}+0 & \frac{1}{3}+\frac{1}{6}+\frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2+1+3}{6} & \frac{2-3+0}{6} & \frac{}{6} \\ \frac{}{6} & \frac{}{6} & \frac{}{6} \\ \frac{}{6} & \frac{}{6} & \frac{}{6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2+1+3}{6} & \frac{1}{3}-\frac{1}{3}+0 & \frac{2+1-3}{6} \\ \frac{1}{3}-\frac{1}{3} & \frac{3}{3} & \frac{1}{3}-\frac{1}{3} \\ \frac{2+1-3}{6} & \frac{1}{3}-\frac{1}{3} & \frac{2+1+3}{6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{6}{6} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{6}{6} \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I$$

$$A^T A = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3}+\frac{1}{3}+\frac{1}{3} & \frac{2-2}{\sqrt{18}} & -\frac{1}{\sqrt{6}}+0+\frac{1}{\sqrt{6}} \\ \frac{2-2}{\sqrt{18}} & \frac{1}{6}+\frac{4}{6}+\frac{1}{6} & \frac{-1}{\sqrt{12}}+0+\frac{1}{\sqrt{12}} \\ -\frac{1}{\sqrt{6}}+\frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{12}}+0+\frac{1}{\sqrt{12}} & \frac{1}{2}+0+\frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{3} & 0 & 0 \\ 0 & \frac{6}{6} & 0 \\ 0 & 0 & \frac{2}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A^T A = I$

$\therefore \; AA^T = A^T A = I$

Hence Proved

$\therefore$ The given matrix is Orthogonal

Show that $A = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$ is Unitary

Solution:

Given: $A = \begin{bmatrix} \frac{1+i}{2} & -\frac{1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$

$$= \frac{1}{2} \begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$$

To Prove: $A\bar{A}^T = \bar{A}^T A = I$

$$\bar{A} = \frac{1}{2} \begin{bmatrix} 1-i & -1-i \\ 1-i & 1+i \end{bmatrix}$$

$$\bar{A}^T = \frac{1}{2} \begin{bmatrix} 1-i & 1-i \\ -1-i & 1+i \end{bmatrix}$$

$$A\bar{A}^T = \frac{1}{2} \begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1-i & 1-i \\ -1-i & 1+i \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 2+2 & 2-1-\cancel{i}+\cancel{i}-1 \\ 2-1-\cancel{i}+\cancel{i}+1 & 2+2 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$A\bar{A}^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\bar{A}^T A = \frac{1}{2}\begin{bmatrix} 1-i & 1-i \\ -1-i & 1+i \end{bmatrix} \frac{1}{2}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 2+2 & -1+i+i+1+1-i-i-1 \\ -1-i-i+1+1-i+1-1 & 2+2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore A\bar{A}^T = \bar{A}^T A = I$

Hence Proved.

A is Unitary matrix

H.W

1. S.T $\begin{bmatrix} \frac{1+i}{\sqrt{7}} & \frac{2+i}{\sqrt{7}} \\ \frac{2-i}{\sqrt{7}} & \frac{-1+i}{\sqrt{7}} \end{bmatrix}$ is Unitary.

Solution:

Given: let $A = \begin{bmatrix} \frac{1+i}{\sqrt{7}} & \frac{2+i}{\sqrt{7}} \\ \frac{2-i}{\sqrt{7}} & \frac{-1+i}{\sqrt{7}} \end{bmatrix}$

$$= \frac{1}{\sqrt{7}}\begin{bmatrix} 1+i & 2+i \\ 2-i & -1+i \end{bmatrix}$$

To Prove: $A\bar{A}^T = \bar{A}^T A = I$

Proof:

$$\bar{A} = \frac{1}{\sqrt{7}}\begin{bmatrix} 1-i & 2-i \\ 2+i & -1-i \end{bmatrix}$$

$$\bar{A}^T = \frac{1}{\sqrt{7}}\begin{bmatrix} 1-i & 2+i \\ 2-i & -1-i \end{bmatrix}$$

$$A\bar{A}^T = \frac{1}{\sqrt{7}}\begin{bmatrix} 1+i & 2+i \\ 2-i & -1+i \end{bmatrix} \frac{1}{\sqrt{7}}\begin{bmatrix} 1-i & 2+i \\ 2-i & -1-i \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 2+5 & (1+i)(2+i)+(2+i)(-1-i) \\ (2-i)(1-i)+(-1+i)(2-i) & 5+2 \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 7 & 3+3i-3-3i \\ 3-3-3i+3i & 7 \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$A\bar{A}^T = I$

$$\bar{A}^T A = \frac{1}{\sqrt{7}}\begin{bmatrix} 1-i & 2+i \\ 2-i & -1-i \end{bmatrix} \frac{1}{\sqrt{7}}\begin{bmatrix} 1+i & 2+i \\ 2-i & -1+i \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 2+5 & (1-i)(2+i)+(2+i)(-1+i) \\ (2-i)(1+i)+(-1-i)(2-i) & 5+2 \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\bar{A}^T A = I$$

$$\therefore A\bar{A}^T = \bar{A}^T A = I$$

Hence, the given matrix is Unitary.

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2. P.T $\begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ is orthogonal.

Solution:

Given: let $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

To Prove: $AA^T = A^T A = I$

Proof:

$$A^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+\frac{1}{4}+\frac{\sqrt{3}\sqrt{3}}{4} & 0+\frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4} \\ 0+0+0 & 0+\frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4} & 0+\frac{3}{4}+\frac{1}{4} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{4}{4} & 0 \\ 0 & 0 & \frac{4}{4} \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0+\frac{1}{4}+\frac{3}{4} & 0-\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4} \\ 0 & 0-\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4} & 0+\frac{3}{4}+\frac{1}{4} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{4}{4} & 0 \\ 0 & 0 & \frac{4}{4} \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore AA^T = A^T A = I$$

Hence Proved.

29.06.2018 PROBLEMS:

1. Find x, y, z and w if

$$3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$$

Sol:

$$\begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}$$

Equating the corresponding elements,

$\Rightarrow$ $3x = x+4$

$3x - x = 4$

$2x = 4^2$

$\boxed{x=2}$

$3y = 6+x+y$

$2y = x+6$

$y = \frac{2+6}{2}$

$y = 8/2$

$\boxed{y=4}$

$3w = 2w+3$

$\boxed{w=3}$

$3z = -1+z+w$

$2z = -1+3$

$2z = 2$

$\boxed{z=1}$

2. Find x, y & z if $\begin{bmatrix} x+3 & 3x-2y \\ -3x-z & x+y+z \end{bmatrix} = \begin{bmatrix} 2 & -7+2y \\ y+4 & 2x \end{bmatrix}$

Solution:

$$\begin{bmatrix} x+3 & 3x-2y \\ -3x-z & x+y+z \end{bmatrix} = \begin{bmatrix} 2 & -7+2y \\ y+4 & 2x \end{bmatrix}$$

Equating the corresponding elements,

$\Rightarrow$ $x+3 = 2$

$\boxed{x=-1}$

$3x-2y = -7+2y$

$3(-1) = -7+4y$

$-3 = -7+4y$

$4 = 4y$

$\boxed{y=1}$

$x+y+z = 2x$

$-1+1+z = 2(-1)$

$\boxed{z=-2}$

3. If $u = \begin{bmatrix} 2 & -3 & 4 \end{bmatrix}_{1\times3}$, $x = \begin{bmatrix} 0 & 2 & 3 \end{bmatrix}_{1\times3}$

$v = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}_{3\times1}$, $y = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}_{3\times1}$ find i) $uv+xy$

ii) $4uv+5[x(3v-y)]$

Solution:

i) $uv+xy$.

$uv = \begin{bmatrix} 2 & -3 & 4 \end{bmatrix}_{1\times3} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}_{3\times1}$

$= \begin{bmatrix} 6-6+4 \end{bmatrix}$

$uv = \begin{bmatrix} 4 \end{bmatrix}$

$xy = \begin{bmatrix} 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$

$= \begin{bmatrix} 0+4+12 \end{bmatrix}$

$xy = \begin{bmatrix} 16 \end{bmatrix}$

$\therefore$ $uv+xy = 4+16$

$= 20$

(ii) $4UV + 5\ [x(3V - y)]$

$4UV = 4(4)$

$= 16$

$5[x(3V - y)]$.

$$3V - y = \begin{bmatrix} 9 \\ 6 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix}$$

$$5[x(3V-y)] = 5\left(\begin{bmatrix} 0 & 2 & 3 \end{bmatrix}_{1\times 3} \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix}_{3\times 1}\right)$$

$$= 5\ [0 + 8 - 3]$$

$$= 5(5)$$

$$= 25$$

$\therefore 4UV + 5\ [x(3V - y)] = 16 + 25$

$= 41$.

H.W

1. Find ABC if $A = [x\ \ y\ \ z]_{1\times 3}$,

$$B = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}_{3\times 3},\quad C = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times 1}$$

2. $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ P.T. $A^2 - 4A + 3I = 0$

3. Solve x if $\begin{bmatrix} 4 & -2 \\ 7 & 1 \end{bmatrix} + 3x = \begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}$

Problems:

Find 'x' & 'y'. $2x - 3y = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix}$

$3x + 2y = \begin{bmatrix} 7 & 1 \\ 4 & 5 \end{bmatrix}$.

Solution:

$\times 3 \Rightarrow \quad 6x - 9y = \begin{bmatrix} 6 & 15 \\ 9 & 3 \end{bmatrix} \rightarrow (1)$

$\times 2 \Rightarrow \quad 6x + 4y = \begin{bmatrix} 14 & 2 \\ 8 & 10 \end{bmatrix} \rightarrow (2)$

$(1) - (2) \Rightarrow$

$$\cancel{6x} - 9y = \begin{bmatrix} 6 & 15 \\ 9 & 3 \end{bmatrix}$$

$$\underset{(-)}{\cancel{6x}} + \underset{(-)}{4y} = -\begin{bmatrix} 14 & 2 \\ 8 & 10 \end{bmatrix}$$

$$-13y = \begin{bmatrix} 6 & 15 \\ 9 & 3 \end{bmatrix} + \begin{bmatrix} -14 & -2 \\ -8 & -10 \end{bmatrix}$$

$$-13y = \begin{bmatrix} -8 & 13 \\ 1 & -7 \end{bmatrix}$$

$$y = \begin{bmatrix} 8/13 & -1 \\ -\frac{1}{13} & 7/13 \end{bmatrix}$$

$$2x - 3y = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix}$$

$$2x - 3\begin{bmatrix} \frac{8}{13} & -1 \\ -\frac{1}{13} & \frac{7}{13} \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix}$$

$$2x + \begin{bmatrix} -24/13 & 3 \\ 3/13 & -\frac{21}{13} \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix}$$

$$2x = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} -24/13 & 3 \\ 3/13 & -21/13 \end{bmatrix}$$

$$2x = \begin{bmatrix} 2 + \frac{24}{13} & 5 - 3 \\ 3 - \frac{3}{13} & 1 + \frac{21}{13} \end{bmatrix}$$

$$2x = \begin{bmatrix} \frac{50}{13} & 2 \\ 36/13 & 34/13 \end{bmatrix}$$

$$2x = \begin{bmatrix} \frac{50}{13} & 2 \\ 36/13 & 34/13 \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{25}{13} & 2/2 \\ 18/13 & 17/13 \end{bmatrix}$$

$$\therefore \; x = \begin{bmatrix} 25/13 & 1 \\ 18/13 & 17/13 \end{bmatrix}, \quad y = \begin{bmatrix} 8/13 & -1 \\ -1/13 & 7/13 \end{bmatrix}$$

H.W

Answers:

1. Solution:

$$ABC = \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$BC = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}_{3\times3} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times1}$$

$$BC = \begin{bmatrix} ax + hy + gz \\ hx + by + fz \\ gx + fy + cz \end{bmatrix}_{3\times1}$$

$$ABC = \begin{bmatrix} x & y & z \end{bmatrix}_{1\times3} \begin{bmatrix} ax + hy + gz \\ hx + by + fz \\ gx + fy + cz \end{bmatrix}_{3\times1}$$

$$= x(ax + hy + gz) + y(hx + by + fz) + z(gx + fy + cz)$$

$$= ax^2 + hyx + gxz + hxy + by^2 + fyz + gxz + fyz + cz^2$$

$$= ax^2 + by^2 + cz^2 + 2hxy + 2fyz + 2gxz$$

Q. Solution:

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \qquad \text{P.T. } A^2 - 4A + 3I = 0$$

$$A^2 = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1 & -2-2 \\ -2-2 & 1+4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$$

$$-4A = \begin{bmatrix} -8 & 4 \\ 4 & -8 \end{bmatrix}$$

$$A^2 - 4A + 3I = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} + \begin{bmatrix} -8 & 4 \\ 4 & -8 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore A^2 - 4A + 3I = 0$$

Hence Proved

Q. Solution:

$$\begin{bmatrix} 4 & -2 \\ 7 & 1 \end{bmatrix} + 3x = \begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}$$

Find x:

$$3x = \begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 7 & 1 \end{bmatrix}$$

$$3x = \begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} -4 & 2 \\ -7 & -1 \end{bmatrix}$$

$$3x = \begin{bmatrix} -3 & -1 \\ -5 & 4 \end{bmatrix}$$

$$\therefore x = \begin{bmatrix} -1 & -1/3 \\ -5/3 & 4/3 \end{bmatrix}$$

2/7/18

02/07/18 Show that 3x3 matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ Satisfies the Eqn. $A^3 - 6A^2 + 9A - 4I = 0$ & Hence deduce A^{-1}.

Solution:

$$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = A^2 \times A$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 22 & -21 & 22 \end{bmatrix}$$

$$-6A^2 = -6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -36 & 30 & -30 \\ 30 & -36 & 30 \\ -30 & 30 & -36 \end{bmatrix}$$

$$9A = 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix}$$

$$-4I = -4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$A^3 - 6A^2 + 9A - 4I = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 22 & -21 & 22 \end{bmatrix} + \begin{bmatrix} -36 & 30 & -30 \\ 30 & -36 & 30 \\ -30 & 30 & -36 \end{bmatrix}$$

$$+ \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} + \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^3 - 6A^2 + 9A - 4I = 0$$

Hence Proved

Deduction:

$$\therefore \boxed{A^{-1} = \frac{1}{4}\left[A^2 - 6A + 9I\right]}$$

$$\Rightarrow \frac{1}{4}$$

$$A^3 - 6A^2 + 9A - 4I = 0$$

x by A^{-1} on both sides

$$A^{-1}(A^3 - 6A^2 + 9A - 4I) = A^{-1} \cdot 0$$

$A^2-6A+9A\cdot A^{-1}-4IA^{-1}=0$

$A^2-6A+9I-4A^{-1}=0$

$A^2-6A+9I=4A^{-1}$

$A^{-1}=\frac{1}{4}\left[A^2-6A+9I\right]$

$$\therefore A^{-1}=\frac{1}{4}\left(\begin{bmatrix}6&-5&5\\-5&6&-5\\5&-5&6\end{bmatrix}-6\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}+9\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\right)$$

$$=\frac{1}{4}\left(\begin{bmatrix}6&-5&5\\-5&6&-5\\5&-5&6\end{bmatrix}+\begin{bmatrix}-12&6&-6\\6&-12&6\\-6&6&-12\end{bmatrix}+\begin{bmatrix}9&0&0\\0&9&0\\0&0&9\end{bmatrix}\right)$$

$$=\frac{1}{4}\begin{bmatrix}6-12+9&-5+6+0&-6+5+0\\-5+6+0&6-12+9&-5+6+0\\5-6+0&-5+6+0&6-12+9\end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{4}\begin{bmatrix}3&1&-1\\1&3&1\\-1&1&3\end{bmatrix}$$

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1. A Company is considering which of the three methods of production. It should use to produce three goods A, B & C. The amount of each good produced by each method is shown in the matrix.

$$\begin{array}{c}\\ \text{Method 1}\\ \text{Method 2}\\ \text{Method 3}\end{array}\begin{array}{c}\begin{array}{ccc}A&B&C\end{array}\\ \begin{bmatrix}4&8&2\\5&7&1\\5&3&9\end{bmatrix}\end{array}$$

The row matrix $(10\quad 4\quad 6)$ represents the profit per unit for the goods A, B & C in order, using matrix multiplication. Find which method maximises the total profit.

Solution:

Y Representing the profit for goods ~~lot~~ per unit.

Let $X=\begin{bmatrix}4&8&2\\5&7&1\\5&3&9\end{bmatrix}_{3\times3}$, $Y=\begin{bmatrix}10\\4\\6\end{bmatrix}_{3\times1}$

$$XY=\begin{bmatrix}40+32+12\\50+28+6\\50+12+54\end{bmatrix}$$

$$XY=\begin{bmatrix}84\\84\\116\end{bmatrix}$$

Method 3 maximises the profit.

2. A man buys 8 dozens of mangoes, 10 dozens of apples and 4 dozens of bannanas. Mangoes cost ₹ 18 / dozen, Apples ₹ 9 / dozen and bannanas ₹ 6 / dozen. Represent the quantity bought by a row matrix and prices by the column matrix and hence obtain the total cost.

Solution:

Let X represent the quantities and Y represent the price.

Row matrix, $X = \begin{bmatrix} 8 & 10 & 4 \end{bmatrix}$

Column matrix, $Y = \begin{bmatrix} 18 \\ 9 \\ 6 \end{bmatrix}$

Total cost, $XY = \begin{bmatrix} 8 & 10 & 4 \end{bmatrix}_{1\times 3} \begin{bmatrix} 18 \\ 9 \\ 6 \end{bmatrix}_{3\times 1}$

$= \begin{bmatrix} 144 + 90 + 24 \end{bmatrix}$

$= 258$

∴ The total cost is ₹ 258

04/07/2018

If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ & $(A+B)^2 = A^2 + B^2$. Find 'a' and 'b' using the values of a and b, verify whether $AB = BA$

Solution:

$A+B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$

$(A+B) = \begin{bmatrix} a+1 & 0 \\ b+2 & -2 \end{bmatrix}$

$(A+B)^2 = \begin{bmatrix} (a+1) & 0 \\ (b+2) & (-2) \end{bmatrix} \begin{bmatrix} a+1 & 0 \\ b+2 & -2 \end{bmatrix}$

$= \begin{bmatrix} (a+1)^2 + \cancel{(a+1)(b+2)}^{0} & 0+0 \\ (a+1)(b+2)+(-2)(b+2) & 0+4 \end{bmatrix}$

$= \begin{bmatrix} (a+1)^2 + \cancel{ab+2a+b+2}^{0} & 0 \\ ab+2a+b+2-2b-4 & 4 \end{bmatrix}$

$(A+B)^2 = \begin{bmatrix} a^2+2a+1 & 0 \\ ab+2a-b-2 & 4 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & -1+1 \\ 2-2 & -2+1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b & a-1 \\ ab-b & b+1 \end{bmatrix}$$

$$A^2+B^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} a^2+b & a-1 \\ ab-b & b+1 \end{bmatrix}$$

$$= \cancel{\begin{bmatrix} -a^2-b+0 & -a+1+0 \\ 0-ab+b & 0-b+0 \end{bmatrix}} = \begin{bmatrix} a^2+b-1 & a-1 \\ ab-b & b \end{bmatrix}$$

$$= \cancel{\begin{bmatrix} -a^2-b & -a+1 \\ -ab+b & -b \end{bmatrix}} = \begin{bmatrix} a^2+b-1 & a-1 \\ ab-b & b \end{bmatrix}$$

$(A+B)^2 = A^2+B^2$

Equating the Co-ordinates,

$0 = +a-1$, $\cancel{b+1=4}$

$\boxed{a=1}$ $\boxed{b=4}$

$\therefore \quad A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix}$

Verification: $AB = BA$

$$AB = \begin{bmatrix} 1-4 & 1+1 \\ 2-4 & 2+1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 \\ -2 & 3 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1+2 & -1-1 \\ 4-2 & -4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 \\ 2 & -3 \end{bmatrix}$$

$$= -\begin{bmatrix} -3 & 2 \\ -2 & 3 \end{bmatrix}$$

$AB = -BA$

$\therefore \quad AB \neq -BA$

$$A = \begin{bmatrix} 5 & 4 & -2 \\ 4 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix}, \qquad (A-10I)(A-I)=0$$

& find A^3.

$(A-10I)(A-I) \Rightarrow$

$$\left(\begin{bmatrix} 5 & 4 & -2 \\ 4 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & -10 \end{bmatrix}\right)\left(\begin{bmatrix} 5 & 4 & -2 \\ 4 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} -5 & 4 & -2 \\ 4 & -5 & -2 \\ -2 & -2 & -8 \end{bmatrix} \begin{bmatrix} +4 & 4 & -2 \\ 4 & 4 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 20+16+4 & -20+16+4 & 10-8-2 \\ +16-20+4 & 16-20+4 & -8+10-2 \\ -8-8+16 & -8-8+16 & 4+4-8 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 0$ $\quad$ ∴ Hence Proved

To find A^3:

Given:

$(A-10I)(A-I) = 0$

$A^2 - AI - 10I \cdot A + 10I^2 = 0$

$A^2 - A - 10A + 10I = 0$

$A^2 - 11A + 10I = 0$, $\quad \boxed{A^2 = 11A - 10I}$ → ①

× by A on both sides,

$A^3 - 11A^2 + 10I \cdot A = 0 \cdot A$

$A^3 - 11A^2 + 10A = 0$

$A^3 = 11A^2 - 10A$ $\quad$ (∵ By eqn ①)

$= 11(11A - 10I) - 10A$

$= 121A - 110I - 10A$

$A^3 = 111A - 110I$

$$A^3 \Rightarrow 111\begin{bmatrix} 5 & 4 & -2 \\ 4 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix} - 110\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 555 & 444 & -222 \\ 444 & 555 & -222 \\ -222 & -222 & 222 \end{bmatrix} + \begin{bmatrix} -110 & 0 & 0 \\ 0 & -110 & 0 \\ 0 & 0 & -110 \end{bmatrix}$$

$$\therefore A^3 = \begin{bmatrix} 445 & 444 & -222 \\ 444 & 445 & -222 \\ -222 & -222 & 112 \end{bmatrix}$$

H.W

(1) $A = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$ S.T. $A(A-I)(A+2I)=0$

* Problems Under Adjoint:

Find the adjoint of $\begin{bmatrix} 3 & 1 & 2 \\ 2 & 2 & 5 \\ 4 & 1 & 0 \end{bmatrix}$.

Solution:

let $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 2 & 5 \\ 4 & 1 & 0 \end{bmatrix}$

$$\text{Co-factor of } A = \begin{pmatrix} \begin{vmatrix} 2 & 5 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 2 & 5 \\ 4 & 0 \end{vmatrix} & \begin{vmatrix} 2 & 2 \\ 4 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 4 & 0 \end{vmatrix} & -\begin{vmatrix} 3 & 1 \\ 4 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ 2 & 5 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} \end{pmatrix}$$

$$= \begin{bmatrix} (0-5) & -(0-20) & (2-8) \\ -(0-2) & (0-8) & -(3-4) \\ (5-4) & -(15-4) & (6-2) \end{bmatrix}$$

$$\text{adj}A = \begin{bmatrix} -5 & 20 & -6 \\ 2 & -8 & 1 \\ 1 & -11 & 4 \end{bmatrix}^T$$

$$\text{adj}A = \begin{bmatrix} -5 & 2 & 1 \\ 20 & -8 & -11 \\ -6 & 1 & 4 \end{bmatrix}$$

2. Find Adjoint. A for, $A = \begin{bmatrix} 3 & 3 & 4 \\ 3 & 2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$

Solution:

$$\text{Co-factor of } A = \begin{bmatrix} \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} & -\begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} & \begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} & -\begin{vmatrix} 3 & 3 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} 3 & 4 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 3 & 4 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 3 \\ 3 & 2 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} (-4-1) & -(-6-1) & (3-2) \\ -(-6-4) & (-6-4) & -(3-3) \\ (3-8) & -(3-12) & (6-9) \end{bmatrix}$$

$$\text{adj}A = \begin{bmatrix} -5 & 7 & 1 \\ 10 & -10 & 0 \\ -5 & 9 & -3 \end{bmatrix}^T$$

$$= \begin{bmatrix} -5 & 10 & -5 \\ 7 & -10 & 9 \\ 1 & 0 & -3 \end{bmatrix}$$

Find the inverse of a matrix, A,

$$\begin{bmatrix} 2 & +1 & 3 \\ 3 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

Solution:

$$A^{-1} = \frac{1}{|A|}(\operatorname{adj} A)$$

$$|A| = 2(1-4) - 1(3-6) + 3(6-3)$$

$$= 2(-3) - 1(-3) + 3(3)$$

$$= -6 + 3 + 9$$

$$|A| = 6$$

$\therefore A^{-1}$ exists.

$$\text{Co-factor of } A = \begin{bmatrix} \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 3 & 2 \end{vmatrix} \\ -\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} \\ \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} \end{bmatrix}.$$

$$= \begin{bmatrix} (1-4) & -(3-6) & (6-3) \\ -(1-6) & (2-9) & -(4-3) \\ (2-3) & -(4-9) & (2-3) \end{bmatrix}$$

$$\operatorname{adj} A = \begin{bmatrix} -3 & 3 & 3 \\ 5 & -7 & -1 \\ -1 & 5 & -1 \end{bmatrix}^T$$

$$\therefore \operatorname{adj} A = \begin{bmatrix} -3 & 5 & -1 \\ 3 & -7 & 5 \\ 3 & -1 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{6}\begin{bmatrix} -3 & 5 & -1 \\ 3 & -7 & 5 \\ 3 & -1 & -1 \end{bmatrix}$$

Find the inverse of $\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$.

Let $A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$

$$|A| = -2 - 12$$

$$= -14$$

$$\operatorname{adj} A = \begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{-1}{14}\begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}$$

Solve $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} X = \begin{bmatrix} 3 & 5 \\ 5 & 9 \end{bmatrix}$

Solution:

$$AX = B$$

$$X = A^{-1}B$$

To find A^{-1}:

$$|A| = 4-6$$
$$= -2$$

$$(\text{adj } A) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= -\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 5 & 9 \end{bmatrix}$$

$$= -\frac{1}{2}\begin{bmatrix} 12-10 & 20-18 \\ -9+5 & -15+9 \end{bmatrix}$$

$$= -\frac{1}{2}\begin{bmatrix} 2 & 2 \\ -4 & -6 \end{bmatrix}$$

$$= \frac{1}{1}\begin{bmatrix} -2 & -2 \\ 4 & 6 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} -1 & -1 \\ 2 & 3 \end{bmatrix}$$

H.W

① $A = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$ S.T $A(A-I)(A+2I) = 0$

Solution:

$A(A-I)$ ~~(A+2I)~~ $\Rightarrow$

$$\begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}\left(\begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}\begin{bmatrix} 1 & -3 & 1 \\ 3 & 0 & 3 \\ -5 & 2 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 2-9-5 & -6+0+2 & 2-9-5 \\ 3+3-15 & -9+0+6 & 3+3-15 \\ -5+6+20 & 15+0-8 & -5+6+20 \end{bmatrix}$$

$$A(A-I) = \begin{bmatrix} -12 & -4 & -12 \\ -9 & -3 & -9 \\ 21 & 7 & 21 \end{bmatrix}$$

$$(A+2I) = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 & 1 \\ 3 & 3 & 3 \\ -5 & 2 & -2 \end{bmatrix}$$

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$A(A-I).(A+2I) \Rightarrow$

$$\begin{bmatrix} -12 & -4 & -12 \\ -9 & -3 & -9 \\ 21 & 7 & 21 \end{bmatrix} \begin{bmatrix} 4 & -3 & 1 \\ 3 & 3 & 3 \\ -5 & 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -48-12+60 & 36-12-24 & -12-12+24 \\ -36-9+45 & 27-9-18 & -9-9+18 \\ 84+21-105 & -63+21+42 & 21+21-42 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 0$

$\therefore A(A-I)(A+2I) = 0$

Hence Proved.

06/07/2018

Method of Reduction

Consider the matrix A/I

Step (i) Divide the first row of matrix by the element a_{11} to make it unity. Then make other elements in the

Step (ii) Make the element in the a_{22} unity, then make other elements in the second column zero.

Step (iii) Make the element in the a_{33} unity, then make other elements in the third column zero.

PROBLEMS:

1. Find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ by the method of reduction.

Solution:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$A/I = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 3 & -2 & 1 & 0 & 1 & 0 \\ 4 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -8 & -8 & -3 & 1 & 0 \\ 0 & -6 & -11 & -4 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R_2 - 3R_1 \\ R_3 - 4R_1 \end{array}$$

$-3 \quad -6 \quad -9 \quad -3$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 3/8 & -1/8 & 0 \\ 0 & -6 & -11 & -4 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R_2/-8 \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 1 & 1 & \frac{3}{8} & -\frac{1}{8} & 0 \\ 0 & 0 & -5 & -\frac{1}{4} & -\frac{3}{4} & 1 \end{array}\right] \begin{array}{l} R_1 - 2R_2 \\ \\ R_3 + 6R_2 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 1 & 1 & \frac{3}{8} & -\frac{1}{8} & 0 \\ 0 & 0 & 1 & \frac{7}{20} & \frac{3}{20} & -\frac{1}{5} \end{array}\right] R_3/-5$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{2}{20} & \frac{2}{20} & \frac{1}{5} \\ 0 & 1 & 0 & \frac{1}{40} & -\frac{11}{40} & \frac{1}{5} \\ 0 & 0 & 1 & \frac{7}{20} & \frac{3}{20} & -\frac{1}{5} \end{array}\right] \begin{array}{l} R_1 - R_3 \\ R_2 - R_3 \\ \\ \end{array}$$

$$\therefore A^{-1} = \begin{bmatrix} -\frac{1}{10} & \frac{1}{10} & \frac{1}{5} \\ \frac{1}{40} & -\frac{11}{40} & \frac{1}{5} \\ \frac{7}{20} & \frac{3}{20} & -\frac{1}{5} \end{bmatrix}$$

09.07.18

1. Solve $x+y+z=6$, $x+2y+3z=14$, $-x+y-z=-2$ using matrices.

The eqns. can be written in the form

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \\ -2 \end{bmatrix}$$

$$A \quad X = B$$

$$X = A^{-1}B$$

To find A^{-1}:

$$|A| = 1(-2-3) - 1(-1+3) + 1(1+2)$$

~~Cofactor of A=~~

$$= 1(-5) - 1(2) + 1(3)$$

$$= -5 - 2 + 3$$

$$= -4$$

$$\neq 0$$

$$\text{Co-factor of } A = \begin{bmatrix} \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ -1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} (-2-3) & -(-1+3) & (1+2) \\ -(-1-1) & (-1+1) & -(1+1) \\ (3-2) & -(3-1) & (2-1) \end{bmatrix}$$

$$\text{adj}A = \begin{bmatrix} -5 & -2 & 3 \\ 2 & 0 & -2 \\ 1 & -2 & 1 \end{bmatrix}^T$$

$$\text{adj}A = \begin{bmatrix} -5 & 2 & 1 \\ -2 & 0 & -2 \\ 3 & -2 & 1 \end{bmatrix}$$

$$A^{-1} = -\frac{1}{4} \begin{bmatrix} -5 & 2 & 1 \\ -2 & 0 & -2 \\ 3 & -2 & \end{bmatrix}$$

$X = A^{-1}B$

$$= -\frac{1}{4}\begin{bmatrix} -5 & 2 & 1 \\ -2 & 0 & -2 \\ 3 & -2 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 14 \\ -2 \end{bmatrix}$$

$$= -\frac{1}{4}\begin{bmatrix} -30+28-2 \\ -12+0+4 \\ 18-28-2 \end{bmatrix}$$

$$= -\frac{1}{4}\begin{bmatrix} -4 \\ -8 \\ -12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore$ $x = 1$, $y = 2$, $z = 3$

2. $3x+4y+5z=18$, $2x-y+8z=13$, $5x-2y+7z=20$

Solution:

The matrix eqn. is

$$\begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 18 \\ 13 \\ 20 \end{bmatrix}$$

A X B

$X = A^{-1}B$

To find A^{-1}:

$$|A| = 3(-7+16) - 4(14-40) + 5(-4+5)$$
$$= 3(9) - 4(-26) + 5(1)$$
$$= 27 + 104 + 5$$
$$= 136 \neq 0$$

$$\text{Co-factor of } A = \begin{bmatrix} \begin{vmatrix} -1 & 8 \\ -2 & 7 \end{vmatrix} & -\begin{vmatrix} 2 & 8 \\ 5 & 7 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} \\ -\begin{vmatrix} 4 & 5 \\ -2 & 7 \end{vmatrix} & \begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix} & -\begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} \\ \begin{vmatrix} 4 & 5 \\ -1 & 8 \end{vmatrix} & -\begin{vmatrix} 3 & 5 \\ 2 & 6 \end{vmatrix} & \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} (-7+16) & -(14-40) & (-4+5) \\ -(28+10) & (21-25) & -(-6-20) \\ (32+5) & -(24-10) & (-3-8) \end{bmatrix}$$

$$\text{adj}A = \begin{bmatrix} 9 & 26 & 1 \\ -38 & -4 & 26 \\ 37 & -14 & -11 \end{bmatrix}^T$$

$$\text{adj}A = \begin{bmatrix} 9 & -38 & 37 \\ 26 & -4 & -14 \\ 1 & 26 & -11 \end{bmatrix}$$

$x = A^{-1}B$

$$= \frac{1}{136}\begin{bmatrix} 9 & -38 & 37 \\ 26 & -4 & -14 \\ 1 & 26 & -11 \end{bmatrix}\begin{bmatrix} 18 \\ 13 \\ 20 \end{bmatrix}$$

$$= \frac{1}{136}\begin{bmatrix} 162 - 494 + 740 \\ 468 - 52 - 280 \\ 18 + 338 - 220 \end{bmatrix}$$

$$= \frac{1}{136}\begin{bmatrix} 408 \\ 136 \\ 136 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore x = 3, \ y = 1, \ z = 1$

Rank of a Matrix:

No. of non zero rows $= \rho(A)$

① Find the rank of $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & -4 \\ -3 & 1 & -2 \end{bmatrix}$

Sol:

let $A = \begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & -4 \\ -3 & 1 & -2 \end{bmatrix}$

$$\sim \begin{bmatrix} 3 & -1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_2/2 + R_1 \\ R_3 + R_1 \end{matrix}$$

$\therefore \rho(A) = 1$

② let $A = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 2 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & 2 & 5 \\ 3 & 4 & 5 & 2 \end{bmatrix} \quad R_2 \to R_2 - R_1$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & 1 & 2 & 5 \end{bmatrix} \quad \begin{matrix} R_3 \to R_3 - 3R_1 \\ -3 \quad -3 \quad -3 \quad 3 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$\rho(A) = 2$

H.W.

1. Solve: $2x + 4y + z = 5, \ x + y + z = 6, \ 2x + 3y + z = 6$.
2. Find the ~~inverse~~ rank of the matrix.

$$\begin{bmatrix} 1 & -7 & 3 & -3 \\ -1 & 20 & -2 & 25 \\ 5 & -2 & 4 & 7 \end{bmatrix}$$

H.W Answers

1. Solution:

The Matrix Eqn. is

$$\underbrace{\begin{bmatrix} 2 & 4 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 5 \\ 6 \\ 6 \end{bmatrix}}_{B}$$

$$X = A^{-1}B$$

To find A^{-1}:

$$|A| = 2(1-3) - 4(1-2) + 1(3-2)$$
$$= 2(-2) - 4(-1) + 1(1)$$
$$= -\cancel{4} + \cancel{4} + 1$$
$$= 1$$

$$\text{Co-factor of } A = \begin{bmatrix} \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} \\ -\begin{vmatrix} 4 & 1 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 4 \\ 2 & 3 \end{vmatrix} \\ \begin{vmatrix} 4 & 1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} (1-3) & -(1-2) & (3-2) \\ -(4-3) & (2-2) & -(6-8) \\ (4-1) & -(2-1) & (2-4) \end{bmatrix}$$

$$\text{adj}A = \begin{bmatrix} -2 & 1 & 1 \\ -1 & 0 & 2 \\ 3 & -1 & -2 \end{bmatrix}^T$$

$$\text{adj}A = \begin{bmatrix} -2 & -1 & 3 \\ 1 & 0 & -1 \\ 1 & 2 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} -2 & -1 & 3 \\ 1 & 0 & -1 \\ 1 & 2 & -2 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \begin{bmatrix} -2 & -1 & 3 \\ 1 & 0 & -1 \\ 1 & 2 & -2 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} -10-6+18 \\ 5+0-6 \\ 5+\cancel{12}-\cancel{12} \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$$

$$\therefore \boxed{x = 2, \quad y = -1, \quad z = 5}$$

2. Solution:

let $A = \begin{bmatrix} 1 & -7 & 3 & -3 \\ 7 & 20 & -2 & 25 \\ 5 & -2 & 4 & 7 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & -7 & 3 & -3 \\ 0 & 69 & -23 & 46 \\ 5 & -2 & 4 & 7 \end{bmatrix}$ $R_2 \to R_2 - 7R_1$

-7 49 -21 21

$\sim \begin{bmatrix} 1 & -7 & 3 & -3 \\ 0 & 69 & -23 & 46 \\ 0 & 33 & -11 & 22 \end{bmatrix}$ $R_3 \to R_3 - 5R_1$

-5 35 -15 15

$\sim \begin{bmatrix} 1 & -7 & 3 & -3 \\ 0 & 69 & -23 & 46 \\ 0 & 3 & -1 & 2 \end{bmatrix}$ $R_3/11$

$\sim \begin{bmatrix} 1 & -7 & 3 & -3 \\ 0 & 69 & -2 & 25 \\ 0 & 0 & 0 & 25 \end{bmatrix}$ $\cancel{R_3 \to 7R_3 + 3R_1}$

0 21 -7 14

3 -21 9 -9

$\cancel{\rho(A) = 3}$

$\sim \begin{bmatrix} 1 & -7 & 3 & -3 \\ 0 & 69 & -23 & 46 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \to 23R_3 - R_2$

$\boxed{\rho(A) = 2}$

10.07.18

1. Find the rank of the matrix,

$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -2 & -1 & 6 & 7 \end{bmatrix}$

Solution:

$\sim \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 5 & -3 \\ -2 & -1 & 6 & 7 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$

-2 -4 -2 -8

$\sim \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 5 & -3 \\ 0 & 3 & 4 & 15 \end{bmatrix}$ $R_3 \to R_3 + 2R_1$

$\sim \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 2 & 1 \end{bmatrix}$ $\cancel{R_3 \to 2R_3 - 3R_1}$

-3 -6 -3 -12

$\rho(A) = 3$

Test for Consistency of Linear Eqns.

Consider the Eqns. $a_{11}x_1 + a_{12}x_2 + \dots a_{m}x_n = b_1$

$\vdots$

$a_{m1}x_1 + a_{m2}x_2 + \dots a_{mn}x_n = b_m$

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_n \end{bmatrix}$$

$$[A, B] = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ \vdots & & & & \\ a_{m1} & a_{m2} & \dots & a_{mn} & b_n \end{bmatrix}$$

If $\rho(A) = \rho(A, B) = r$ System is Consistent & has solutions

$\rho(A) \neq \rho(A, B)$ System is inconsistent & no solution

If $r = n$, Unique solution

$r < n$, infinitely many solution

PROBLEMS:

1. ~~Solve~~ S.T the System of Equations are consistent. $3x - 4y = 2$, $5x + 2y = 12$, $-x + 3y = 1$.

The Matrix Eqn. is

$$A = \begin{bmatrix} 3 & -4 \\ 5 & 2 \\ -1 & 3 \end{bmatrix}$$

$$[A, B] = \begin{bmatrix} 3 & -4 & 2 \\ 5 & 2 & 12 \\ -1 & 3 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} -1 & 3 & 1 \\ 5 & 2 & 12 \\ 3 & -4 & 2 \end{bmatrix} \quad R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & -3 & -1 \\ 5 & 2 & 12 \\ 3 & -4 & 2 \end{bmatrix} \quad \times(-1) R_1$$

$$\sim \begin{bmatrix} 1 & -3 & -1 \\ 0 & 17 & 17 \\ 3 & -4 & 2 \end{bmatrix} \quad R_2 \to R_2 - 5R_1$$

$-5 \quad 15 \quad 5$

$$\sim \begin{bmatrix} 1 & -3 & -1 \\ 0 & 17 & 17 \\ 0 & 5 & 5 \end{bmatrix} \quad R_3 \to R_3 - 3R_1$$

$-3 \quad 9 \quad -3$

$$\sim \begin{bmatrix} 1 & 3 & -1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad \begin{matrix} R_2/7 \\ R_3/5 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 3 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore \rho(A,B) = 2 \quad , \rho(A) = 2$

$\therefore \rho(A) = \rho(A,B)$

$\therefore$ System is consistent and has infinitely many ~~Unique~~ solution.

11.07.18

1. ST the system of Eqns, $x-3y-8z=-10$, $3x+4y-4z=0$, $2x+5y+6z=13$ are consistent and solve them.

Solution

The Matrix eqn is

$$\begin{bmatrix} 1 & -3 & -8 \\ 3 & 4 & -4 \\ 2 & 5 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \\ 13 \end{bmatrix}$$

A   X   B

$$[A,B] = \begin{bmatrix} 1 & -3 & -8 & -10 \\ 3 & 4 & -4 & 0 \\ 2 & 5 & 6 & 13 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -3 & -8 & -10 \\ 0 & 13 & 20 & 30 \\ 0 & 11 & 22 & 33 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 3R_1 & -3 \ 9 \ 24 \ 30 \\ R_3 \to R_3 - 2R_1 & -2 \ 6 \ 16 \ 20 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -3 & -8 & -10 \\ 0 & 13 & 20 & 30 \\ 0 & 1 & 2 & 3 \end{bmatrix} \quad R_3/11$$

$$\sim \begin{bmatrix} 1 & -3 & -8 & -10 \\ 0 & 13 & 20 & 30 \\ 0 & 0 & -6 & -9 \end{bmatrix} \quad R_3 \to R_2 - 13R_3 \quad 0 \ -13 \ -26 \ -39$$

$$\sim \begin{bmatrix} 1 & -3 & -8 & -10 \\ 0 & 13 & 20 & 30 \\ 0 & 0 & -6 & -9 \end{bmatrix}$$

$\rho(A,B) = \rho(A) = 3 =$ no. of Unknowns

$\therefore$ System is consistent & has Unique solution.

$-6z = -9$

$z = 9/6$

$\boxed{z = 3/2}$

$13y + 20z = 30$

$13y + 20\left(\frac{3}{2}\right) = 30$

$13y = 0$

$\boxed{y = 0}$

$x - 3y - 8z = -10$

$x - 0 - 8\left(3/2\right) = -10$

$x - 12 = -10$

$\boxed{x = 2}$

2. S.T the Eqn. $x+y+z=6$, $x+2y+3z=14$, $x+4y+7z=30$ are consistent and solve them.

Solution:

The matrix Eqn. is

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}}_{B}$$

$$[A, B] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 3 & 6 & 24 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 1 & 2 & 8 \end{bmatrix} \quad R_3/3$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$\rho(A) = \rho(A, B) = 2 <$ no of unknowns

$\therefore$ System is consistent and has infinitely many solutions.

Let $\boxed{z = k}$

$y + 2z = 8$

$y + 2k = 8$

$\boxed{y = 8 - 2k}$

$x + y + z = 6$

$x + 8 - 2k + k = 6$

$x + 8 - k = 6$

$\boxed{x = -2 + k}$

$\therefore x = -2 + k$, $y = 8 - 2k$, $z = k$ $\because k$ is any arbitrary real no.

3. Investigate for what values of λ, μ. The system of Equations. $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$

i) no solutions, ii) a unique solution,

iii) Infinitely many solutions.

Solution:

The Matrix eqn is

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}}_{B}$$

$$[A, B] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

(i) no solutions	$\rho(A,B) \neq \rho(A)$	$\lambda = 3,\ \mu \neq 10$
(ii) a unique solution	$\rho(A,B) = \rho(A) =$ no. of unknowns	$\lambda \neq 3,\ \mu \neq 10$
(iii) Infinitely many solutions	$\rho(A,B) = \rho(A) <$ no. of unknowns	$\lambda = 3,\ \mu = 10$

4. For what values of λ & μ, $x + 5y + 3z = 9$, $x + \lambda y + z = 3$, $x + 2y + 2z = \mu$.

Solution:

The Matrix Eqn is

$$\underbrace{\begin{bmatrix} 1 & 5 & 3 \\ 1 & \lambda & 1 \\ 1 & 2 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 9 \\ 3 \\ \mu \end{bmatrix}}_{B}$$

$$[A, B] = \begin{bmatrix} 1 & 5 & 3 & 9 \\ 1 & \lambda & 1 & 3 \\ 1 & 2 & 2 & \mu \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 5 & 3 & 9 \\ 0 & \lambda-5 & -2 & -6 \\ 1 & 2 & 2 & \mu \end{bmatrix} \quad R_2 \to R_2 - R_1$$

$$\sim \begin{bmatrix} 1 & 5 & 3 & 9 \\ 0 & \lambda-5 & -2 & -6 \\ 0 & -3 & -1 & \mu-9 \end{bmatrix} \quad R_3 \to R_3 - R_1$$

$$\sim \begin{bmatrix} 1 & 5 & 3 & 9 \\ 0 & \lambda-5 & -2 & -6 \\ 0 & -3-\frac{(\lambda-5)}{2} & 0 & \mu-6 \end{bmatrix} \quad R_3 \to R_3 - R_2/2$$

$$\sim \begin{bmatrix} 1 & 5 & 3 & 9 \\ 0 & \lambda-5 & -2 & -6 \\ 0 & -\frac{\lambda-1}{2} & 0 & \mu-6 \end{bmatrix}$$

$\rho(A) = 3 = \rho(A, B) =$ no. of unknowns

$-\frac{\lambda - 1}{2} \neq 0$ $\mu - 6 \neq 0$

$\boxed{\mu = 6}$

$\frac{-\lambda}{2} \neq \frac{1}{2}$

$\boxed{\lambda \neq -1}$

CRAMER'S RULE

- Consider the linear eqns. as $a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$, $a_3x + b_3y + c_3z = d_3$

$$x = \frac{\Delta x}{\Delta}, \quad y = \frac{\Delta y}{\Delta}, \quad z = \frac{\Delta z}{\Delta}$$

1. Solve: $x - 3y + 2z = 4$, $2x + y - 3z = -2$, $4x - 5y + z = 5$.

2. $x + y + z = -1$, $x + 2y + 3z = -4$, $x + 3y + 4z = -6$

Solution:

let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{vmatrix}$

$= 1(8 - 9) - 1(4 - 3) + 1(3 - 2)$

$= 1(-1) - 1(1) + 1(1)$

$= -1 - \cancel{1} + \cancel{1}$

$= -1 \neq 0$

let $\Delta x = \begin{vmatrix} -1 & 1 & 1 \\ -4 & 2 & 3 \\ -6 & 3 & 4 \end{vmatrix}$

$= -1(8 - 9) - 1(-16 + 18) + 1(-12 + 12)$

$= -1(-1) - 1(2) + 1(0)$

$= 1 - 2$

$\Delta x = -1$

let $\Delta y = \begin{vmatrix} 1 & -1 & 1 \\ 1 & -4 & 3 \\ 1 & -6 & 4 \end{vmatrix}$

$= 1(-16 + 18) + 1(4 - 3) + 1(-6 + 4)$

$= 1(2) + 1(1) + 1(-2)$

$= \cancel{2} + 1 - \cancel{2}$

$= 1$

let $\Delta z = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & -4 \\ 1 & 3 & -6 \end{vmatrix}$

$= 1(-12 + 12) - 1(-6 + 4) - 1(3 - 2)$

$= 1(0) - 1(-2) - 1(1)$

$= 2 - 1$

$= 1$

By cramer's rule,

$x = \frac{\Delta x}{\Delta} = \frac{-1}{-1}$, $\boxed{x = 1}$

$y = \frac{\Delta y}{\Delta} = \frac{1}{-1}$, $\boxed{y = -1}$

$z = \frac{\Delta z}{\Delta} = \frac{1}{-1}$, $\boxed{z = -1}$

13.07.2018

1. Solve $3x+4y+5z=18$, $2x-y+8z=13$, $5x-2y+7z$ using Cramer's rule:

Solution:

$$\text{let } \Delta = \begin{vmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 7 \end{vmatrix}$$

$= 3(-7+16) - 4(14-40) + 5(-4+5)$

$= 3(9) - 4(-26) + 5(1)$

$= 27 + 102 + 5$

$= 136$

$\neq 0$

$$\text{let } \Delta_x = \begin{vmatrix} 18 & 4 & 5 \\ 13 & -1 & 8 \\ 20 & -2 & 7 \end{vmatrix}$$

$= 18(-7+16) - 4(91-160) + 5(-26+20)$

$= 18(9) - 4(69) + 5(-6)$

$= 162 + 276 - 30$

$\Delta x = 408$

$$\text{let } \Delta y = \begin{vmatrix} 3 & 18 & 5 \\ 2 & 13 & 8 \\ 5 & 20 & 7 \end{vmatrix}$$

$= 3(91-160) - 18(14-40) + 5(40-65)$

$= 3(-69) - 18(-26) + 5(-25)$

$= -207 + 468 - 125$

$= 136$

$$\text{let } \Delta z = \begin{vmatrix} 3 & 4 & 18 \\ 2 & -1 & 13 \\ 5 & -2 & 20 \end{vmatrix}$$

$= 3(-20+26) - 4(40-65) + 18(-4+5)$

$= 3(6) - 4(-25) + 18(1)$

$= 18 + 100 + 18$

$\Delta z = 136$

By CRAMER'S RULE,

$x = \frac{\Delta x}{\Delta}$, $y = \frac{\Delta y}{\Delta}$, $z = \frac{\Delta z}{\Delta}$

$= \frac{408}{136}$, $= \frac{136}{136}$, $= \frac{136}{136}$

$\boxed{x = 3}$, $\boxed{y = 1}$, $\boxed{z = 1}$

$\therefore (x, y, z) = (3, 1, 1)$.

Test for consistency using Cramer's rule

16.07.18

1. If $\Delta \neq 0$, the system is consistent & it has a unique solution

2. If $\Delta = 0$, the system may (or) may not be considered.

$\Delta = 0$, & if any one of $\Delta x, \Delta y, \Delta z$ is non-zero, then the system is consistent.

$\Delta = 0$, & $\Delta x = \Delta y = \Delta z = 0$ then the system is consistent & it has infinitely many solns.

Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $\Delta^t = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$a_1 l + a_2 m + a_3 n = 0$

$b_1 l + b_2 m + b_3 n = 0$

$c_1 l + c_2 m + c_3 n = 0$

then find l, m, n

$a_1 x + b_1 y + c_1 z = d_1$

$a_2 x + b_2 y + c_2 z = d_2$

$a_3 x + b_3 y + c_3 z = d_3$

Condition for consistency

$d_1 l + d_2 m + d_3 n = 0$

1. Determine whether the system of equations $x - 3y + 2z = 4$, $2x + y - 3z = -2$, $4x - 5y + z = 5$

Solution:

The Matrix Eqn. is

$$\begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ 4 & -5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 5 \end{bmatrix}$$

Let $\Delta = \begin{vmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ 4 & -5 & 1 \end{vmatrix}$

$= 1(1-15) + 3(2+12) + 2(-10-4)$

$= 1(-14) + 3(14) + 2(-14)$

$= -14 + 42 - 28$

$\Delta = 0$

$\therefore$ The system may or may not be consistent.

$\Delta x = \begin{vmatrix} 4 & -3 & 2 \\ -2 & 1 & -3 \\ 5 & -5 & 1 \end{vmatrix}$

$= 4(1-15) + 3(-2+15) + 2(10-5)$

$= 56 + 39 + 10$

$= -56 + 49$

$= -7$

$\neq 0$

$\therefore$ The system is inconsistent & it has no solution.

2. Find the value of 'a', so that the system is consistent.

$$2x-3y+4z=a$$
$$x-y+5z=1$$
$$x-2y-z=2.$$

The matrix Eqn. is

$$\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 5 \\ 1 & -2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ 1 \\ 2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 2 & -3 & 4 \\ 1 & -1 & 5 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= 2(1+10)+3(-1-5)+4(-2+1)$$
$$= 22-18-4$$
$$\Delta = 0$$

$\therefore$ The system is may or may not be consistent

$$\Delta^t = \begin{vmatrix} 2 & 1 & 1 \\ -3 & -1 & -2 \\ 4 & 5 & -1 \end{vmatrix}$$

(x) by l, m, n

$$2l+m+n=0$$
$$-3l-m-2n=0$$
$$4l+5m-n=0$$

$$\Rightarrow \begin{vmatrix} l & m & n \\ 2 & 1 & 1 \\ -3 & -1 & -2 \end{vmatrix}$$

$$\frac{l}{\begin{vmatrix} 1 & 1 \\ -1 & -2 \end{vmatrix}} = \frac{-m}{\begin{vmatrix} 2 & 1 \\ -3 & -2 \end{vmatrix}} = \frac{n}{\begin{vmatrix} 2 & 1 \\ -3 & -1 \end{vmatrix}}$$

$$\frac{l}{-2+1} = \frac{-m}{-4+3} = \frac{n}{-2+3}$$

$$\frac{l}{-1} = \frac{m}{1} = \frac{n}{1} \qquad \therefore l=-1, m=1, n=1$$

By condn:

$$d_1 l + d_2 m + d_3 n = 0$$
$$a(-1)+1(1)+2(1)=0$$
$$-a+1+2=0$$
$$-a=-3$$
$$\therefore \boxed{a=3}$$

3. Find the value of 'k' if

$$3x-2y+4z=1$$
$$5x+4y-6z=2$$
$$x-8y+14z=k$$ are consistent and obtain general soln.?

Solution:

$$\begin{bmatrix} 3 & -2 & 4 \\ 5 & 4 & -6 \\ 1 & -8 & 14 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ k \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -2 & 4 \\ 5 & 4 & -6 \\ 1 & -8 & 14 \end{vmatrix}$$

$= 3(56-48) + 2(70+6) + 4(-40-4)$

$= 24 + 152 - 176$

$\Delta = 0$

$$\Delta^t = \begin{vmatrix} 3 & 5 & 1 \\ -2 & 4 & -8 \\ 4 & -6 & 14 \end{vmatrix}$$

(x) by l, m, n

$3l + 5m + n = 0$

$-2l + 4m - 8n = 0 \Rightarrow \begin{vmatrix} l & m & n \\ 3 & 5 & 1 \\ -2 & 4 & -8 \end{vmatrix}$

$4l - 6m + 14n = 0$

$$\frac{l}{\begin{vmatrix} 5 & 1 \\ 4 & -8 \end{vmatrix}} = \frac{-m}{\begin{vmatrix} 3 & 1 \\ -2 & -8 \end{vmatrix}} = \frac{n}{\begin{vmatrix} 3 & 5 \\ -2 & 4 \end{vmatrix}}$$

$$\frac{l}{-40-4} = \frac{-m}{-24+2} = \frac{n}{12+10}$$

$l = -44, \quad m = 22, \quad n = 22$

By condition;

$d_1 l + d_2 m + d_3 n = 0$

$1(-44) + 2(22) + K(22) = 0$

$-44 + 44 + 22K = 0$

$\Delta = 0$

The system is consistent and it has infinitely many solutions

Put $\boxed{z = t}$

$3x - 2y + 4t = 1 \longrightarrow (1)$

$5x + 4y - 6t = 2 \longrightarrow (2)$

$x - 8y + 14t = 0 \longrightarrow (3)$

$(1) \Rightarrow 3x - 2y + 4t = 1$

$(3) \times -3 \Rightarrow -3x + 24y - 42t = 0$

$22y - 38t = 1$

$22y = 38t + 1$

$$\boxed{y = \frac{38t+1}{22}} \longrightarrow (4)$$

$(1) \times 2 \Rightarrow 6x - 4y + 8t = 2$

$(2) \Rightarrow 5x + 4y - 6t = 2$

$11x + 2t = 4$

$11x = 4 - 2t$

$$\boxed{x = \frac{4-2t}{11}}$$

$\therefore (x, y, z) = \left(\frac{4-2t}{11}, \frac{38t+1}{22}, t\right) \quad \because t \in R$

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17/07/18

+ Find the value of k, if the linear eqns $3x-2y+4z=1$,

1. S.T the eqns $x+y+z=a$, $3x+4y+5z=b$, $2x+3y+4z=c$. have (i) no solutions, if $a=b=c=1$

(ii) many solutions, if $a=\frac{1}{2}, b=c=\frac{1}{2}$.

Solution:

$$\text{let } \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{vmatrix}$$

$= 1(16-15) - 1(12-10) + 1(9-8)$

$= 1(1) - 1(2) + 1(1)$

$= 1+1-2$

$\Delta = 0$

$\therefore$ system may or may not be consistent.

$$\Delta x = \begin{vmatrix} a & 1 & 1 \\ b & 4 & 5 \\ c & 3 & 4 \end{vmatrix}$$

$= a(16-15) - 1(4b-5c) + 1(3b-4c)$

$= a(1) - 1(4b-5c) + 1(3b-4c)$

$= a-4b+5c+3b-4c$ if $a=b=c=1$

$\cancel{=1-4(1)+5(1)+3(1)}$

$\cancel{=1-4+5+3}$

$= \underline{a-b+c}$

$= \cancel{1}-\cancel{1}+1$

$\boxed{\Delta x = 1}$

(i) When $a=b=c=1$

$\Delta x = 1$, $\Delta = 0$

hence the system is inconsistent & have no solutions.

(ii) $\Delta = 0$

Condition: $d_1 l + d_2 m + d_3 n = 0$

$$\Delta^t = \begin{vmatrix} 1 & 3 & 2 \\ 1 & 4 & 3 \\ 1 & 5 & 4 \end{vmatrix}$$

(x) by l, m, n

$l+3m+2n=0$

$l+4m+3n=0$

$l+5m+4n=0$

$$\Rightarrow \begin{vmatrix} l & m & n \\ 1 & 3 & 2 \\ 1 & 4 & 3 \end{vmatrix}$$

$$\frac{l}{4-3} = -\frac{m}{3-2} = \frac{n}{4-3}$$

$$\frac{l}{1} = -\frac{m}{1} = \frac{n}{1}$$

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$l = 1,\ m = -1,\ n = 1$

By condn:

$$a(1) + b(-1) + c(1) = 0$$

$$a - b + c = 0$$

$\therefore$ ~~[illegible]~~ $a = c = \frac{1}{2}\ ;\ b = 1 \Rightarrow$

~~[illegible]~~

~~[illegible]~~

$$\frac{1}{2} - 1 + \frac{1}{2} = 0$$

$$1 - 1 = 0$$

$$0 = 0$$

$\therefore$ System is Consistent & has many solutions.

2. Find the value for which eqns. $x - 3y + 2z = 4,\ 2x + y - z = 1,\ 5x - 2y + z = a$ possess a solution

Soln:

let $\Delta = \begin{vmatrix} 1 & -3 & 2 \\ 2 & 1 & -1 \\ 5 & -2 & 1 \end{vmatrix}$

$$= 1(1-2) + 3(2+5) + 2(-4-5)$$

$$= 1(-1) + 3(7) + 2(-9)$$

$$= -1 + 21 = 18$$

$$\Delta = 2$$

$$\Delta \neq 0$$

$\therefore$ The System is Consistent & has Unique solution. 'a' has all finite values.

$\Delta^t = \begin{vmatrix} 1 & 2 & 5 \\ -3 & 1 & -2 \\ 2 & -1 & 1 \end{vmatrix}$

(x) by l, m, n

$$l + 2m + 5n = 0$$
$$-3l + m - 2n = 0 \quad \Rightarrow \begin{vmatrix} l & m & n \\ 1 & 2 & 5 \\ -3 & 1 & -2 \end{vmatrix}$$
$$2l - m + n = 0$$

$$\frac{l}{-4-5} = -\frac{m}{-2+15} = \frac{n}{1+6}$$

$$\frac{l}{-9} = -\frac{m}{13} = \frac{n}{7}$$

$$l = -9,\ m = -13,\ n = 7$$

By condn:

$$-9(4) - 13(1) + 7(a) = 0$$

$$-36 - 13 + 7a = 0$$

$-49 = -5a \neq 0$

$-5a = -49$

$\therefore \boxed{a = \frac{49}{5}}$

$\therefore -49 + 7a = 0$

$7a = 49$

$\boxed{a = 7}$

19/07/2018

UNIT-2

2. MATHEMATICAL LOGIC

1. Statement → Declarative Sentence

* A Declarative sentence is called 'Statement'. A statement can have one and only one of two possible values called 'Truth values'. The truth values are True (T) and False (F)

Eg:

(i) India is a country
- Statement (T)

(ii) Open the door!
- Not a statement

(iii) $1 + 99 = 145$
- Statement (F)

* Connections are 'Connectives'. Simple statements are called Primary Statements. It is possible to construct complicated Statements from Simpler statements by using certain Connecting words or Expression known as Connectives

(i) Conjunction: ($\wedge$)

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

~~(ii) Disjunction~~:

- The conjunction of two statements P & Q is $P \wedge Q$. It has the truth value T when both P & Q have truth value T, otherwise it has truth value F

(ii) Disjunction:

The Disjunction of two statements P & Q is $P \vee Q$. It has the truth value F if both P & Q have truth value F, otherwise it has truth value T

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

(iii) Negation:

The Negation of a statement is generally denotes ("It is not the case") It is denoted by the symbol $\sim$

P	$\sim P$
T	F
F	T

Examples:

(i) Let 'P' be the statement Rama speakes english Q → Rama speaks tamil. Give verbal statements for each of the following

(1) $P \vee q$ - Rama speaks either english or tamil.

(2) $P \wedge q$ - Rama speaks english and tamil.

(3) $P \wedge \sim q$ - Rama speaks english and not tamil

(4) $\sim(\sim p)$ = It is not true that Rama does not speak English

Write each of the following in Symbolic forms:

P → Sathish is poor

Q → Sathish is happy

1. Sathish is poor but not happy

$p \wedge (\sim q)$

2. Sathish is neither rich nor happy

~~$\sim p \vee$~~ $p \wedge \sim q$

3. Sathish is either rich or unhappy

$(\sim p) \vee (\sim q)$.

4. Sathish is either happy or poor

$q \vee p$

19-07-18

1. Let 'P' be the proposition he is tall and 'Q' be the proposition he is handsome. Give a simple verbal sentence which describes each of the following

(i) $p \wedge q$:

He is tall and handsome

(ii) $p \wedge \sim q$:

He is tall and he is not handsome.

(iii) $\sim(\sim p \vee q)$

It is not true that he is not tall or handsome

(iv) $\sim p \wedge \sim q$

He is not tall and not handsome.

(v) $p \vee (\sim p \wedge q)$

He is tall or he is not tall and handsome.

(vi) $\sim(\sim p \vee \sim q)$

It is not true that he is neither tall nor handsome.

2. P- Magesh reads HINDU &

Q- Magesh reads economic times

R- Magesh reads Thinathanthi.

(i) Magesh reads HINDU and economic times but not T.T

$(P \wedge Q) \wedge \sim R$

(ii) It is not true that Magesh reads Economic times but not T.T and HINDU

$\cancel{\sim[Q \wedge (\sim R \wedge \sim P)]}$ $\sim[Q \wedge \sim(R \wedge P)]$

2. Find the truth table of : (i) $\sim p \wedge q$

(ii) $\sim(p \vee \sim q)$

(i)

p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

(ii) $\sim(p \vee \sim q)$

p	q	$\sim q$	$p \vee \sim q$	$\sim(p \vee \sim q)$
T	T	F	T	F
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

23.07.18 Maths

1. Find the truth table for:

(i) $\sim(\sim p \vee \sim q)$

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$\sim(\sim p \vee \sim q)$
T	T	F	F	F	T
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	F

(ii) $(p \wedge \sim q) \vee r$

p	q	r	$\sim q$	$p \wedge \sim q$	$(p \wedge \sim q) \vee r$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	F	F	F
F	F	T	T	F	T
F	F	F	T	F	F

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(iii) $(p\vee\sim r)\wedge(q\vee\sim r)$

p	q	r	$\sim r$	$(p\vee\sim r)$	$(q\vee\sim r)$	$(p\vee\sim r)\wedge(q\vee\sim r)$
T	T	T	F	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	F	F
T	F	F	T	T	T	T
F	T	T	F	F	T	F
F	T	F	T	T	T	T
F	F	T	F	F	F	F
F	F	F	T	T	T	T

(iv) $\sim(p\vee\sim q)\wedge(\sim p\vee r)$

~~p q ~r ~q ~p p∨~q ~(p∨~q)~~

p	q	r	$\sim p$	$\sim q$	$(p\vee\sim q)$	$(\sim p\vee r)$	$\sim(p\vee\sim q)$	$\sim(p\vee\sim q)\wedge(\sim p\vee r)$
T	T	T	F	F	T	T	F	F
T	T	F	F	F	T	T	F	F
T	F	T	F	T	T	F	F	F
T	F	F	F	T	T	F	F	F
F	T	T	T	F	F	T	T	T
F	T	F	T	F	F	T	T	T
F	F	T	T	T	T	T	F	F
F	F	F	T	T	T	T	F	F

Implication ($\rightarrow$)

Let P and Q be two proposition, the implication $P\rightarrow Q$ is the proposition (i.e) F if P is T & Q is F otherwise it is T. In this case P is called hypothesis, Q is called conclusion. The truth table for $P\rightarrow Q$

P	Q	$P\rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

The following are sum of the common usages of the implication.

(i) If P then Q

(ii) P implies Q

(iii) P only if Q

(iv) P is sufficient for Q

(v) Q if P

(vi) Q is necessary for P

Examples:

* If A, B, C is an isosceles triangle with a=b then the angles A and B are equal

* I will be going (Q) to movies provided only my car starts (P)

Biconditional: ($\leftrightarrow$) / ($\Leftrightarrow$)

Let P and Q be two propositions the biconditional $P \leftrightarrow Q$ is the proposition (i.e) T when both P & Q have the same truth values and is F, Otherwise

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

The other ways of expressing Biconditional $P \leftrightarrow Q$

(i) P is necessary and sufficient for Q

(ii) If P then Q and Conversely

Examples:

* Two triangles are congruent if and only if corresponding sides are proportional

* Two lines are parallel if and only if they have the same slope.

Converse, Inverse and Contrapositive:

Converse: $Q \to P$

Inverse: $\sim P \to \sim Q$

Contrapositive: $\sim Q \to \sim P$

P and Q be propositions, $P \to Q$ be conditional proposition then $Q \to P$ is called the converse of proposition and $\sim p \to \sim Q$ is called the inverse of proposition and the proposition $\sim Q \to \sim p$ is called the contra-positive of the proposition

Examples:

* P: A is a square

Q: A is a rectangle

$P \to Q$: If A is a square then it is a rectangle (T)

$Q \to P$: If A is a rectangle, then it is a square (F).

* P: x^2 is odd

Q: x is odd

If x is not odd then x^2 is also not odd

∴ The contrapositive statement

$\sim Q \to \sim P$ is True.

P	$\sim P$	Q	$\sim Q$	$P \to Q$	$Q \to P$	$\sim P \to \sim Q$	$\sim Q \to \sim P$
T	F	T	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	T	F	T	T	T	T	T

Tautology and contradiction:

A compound preposition (i.e) always True no matter what the truth values of prepositions that occur is called 'Tautology'.

A compound preposition (i.e) always False, is called 'contradiction'.

A Preposition (i.e) neither tautology nor contradiction is "Contingency".

A Prepositions P & Q are called Logically Equivalent if $P \leftrightarrow Q$ is a tautology.

02.08.18

Eg:

Truth Table:

$P \vee \sim p$

P	$\sim P$	$P \vee \sim p$	$P \wedge \sim P$
T	F	T	F
F	T	T	F

∴ $P \vee \sim P \equiv T$ (Tautology)

$P \wedge \sim P \equiv F$ (Contradiction)

Algebra of Propositions:

1. Idempotent law:

$P \vee P \equiv P \wedge P\ (P)$

$P \wedge P \equiv P \vee P\ (P)$

2. Associative law:

$P \vee (q \vee r) \equiv (P \vee q) \vee r$

$P \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

3 Commutative law:

$p \vee q \equiv q \vee p$

$p \wedge q \equiv q \wedge p$

4 Distribution laws:

$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

5. Demorgan's law:

$\sim (p \vee q) \equiv \sim p \wedge \sim q$

$\sim (p \wedge q) \equiv \sim p \vee \sim q$

6 Identity laws

I. $p \vee F \equiv p$

$p \wedge F \equiv F$

II. $p \vee T \equiv T$

$p \wedge T \equiv p$

7 Complement laws:

$p \vee \sim p \equiv T$

$p \wedge \sim p \equiv F$

07.08.18

1. P.T $p \vee (p \wedge q) \equiv p$ by using algebra of proposition:

$p \vee (p \wedge q) \equiv (p \wedge T) \vee (p \wedge q)$ [$\because p = p \wedge T$] (Identity law)

$\equiv p \wedge (T \vee q)$ ($\because$ Distributive)

$\equiv p \wedge T$ ($\because$ Identity law)

$\equiv p$

2 P.T $\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$.

$\equiv \sim (p \vee q) \vee (\sim p \wedge q)$

$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$ ($\because$ Demorgan's law)

$\equiv \cancel{\sim p \vee (q \wedge \sim q)}$ $\sim p \wedge (q \vee \sim q)$ ($\because$ Distributive law)

$\equiv \cancel{\sim p \vee F}$ $\sim p \wedge (T)$ ($\because$ Complement law)

$\equiv \cancel{\sim p}$ $\sim p$

3. P.T $p \wedge (p \vee q) \equiv p$

$p \wedge (p \vee q) \equiv (p \vee F) \wedge (p \vee q)$ ($\because$ Identity law)

$\equiv p \vee (F \wedge q)$ (Distributive law)

$\equiv p \vee F$ (Identity law)

$\equiv p$

03.08.18

1. $(p \wedge q) \vee \sim p \equiv \sim p \vee q$

$L.H.S \equiv (p \wedge q) \vee \sim p$

$\equiv \sim p \vee (p \wedge q)$ (∵ Commutative law)

$\equiv (\sim p \vee p) \wedge (\sim p \vee q)$ (∵ Distributive law) (∵ Complement law)

$\equiv T \wedge (\sim p \vee q)$ (∵ Identity law)

~~$\equiv (T \wedge \sim p) \vee (T \wedge q)$~~

$\equiv ($

$\equiv \sim p \vee q$ ∵

$\equiv R.H.S$

2. P.T $p \wedge (\sim p \vee q) \equiv p \wedge q$

$L.H.S \equiv p \wedge (\sim p \vee q)$

$\equiv (p \wedge \sim p) \vee (p \wedge q)$ (∵ Complement law)

$\equiv F \vee (p \wedge q)$

$\equiv p \wedge q$

Hint: $p \to q \equiv \sim p \vee q$

p	q	$\sim p$	$\sim p \vee q$	$p \to q$
T	T	F		
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T
			T	T

3. Write down the negation of each of the following propositions:

i. If he studies he will pass the examination

Solution:

p - He studies

q - He will pass the Examination

Given proposition: $p \to q \equiv \sim p \vee q$

Negation: $\sim (p \to q) \equiv \sim (\sim p \vee q)$

$\equiv p \wedge \sim q$

Answer: If he studies and he will not pass the examination.

ii. If the stock price falls then the Unemployment raises

p - The stock price falls

q - The Unemployment raises.

Given proposition: $p \to q \equiv \sim p \vee q$

Negation: $\sim (p \to q) \equiv \sim (\sim p \vee q)$

$\equiv p \wedge \sim q$

Ans: If the stock price falls and the Unemployment does not raises.

iii) If it snows then he does not drive his car.

p: It snows

q: he drives his car.

Given proposition: $p \rightarrow \sim q \equiv \sim p \vee \sim q$

Negation: $\sim(p \rightarrow \sim q) \equiv \sim(\sim p \vee \sim q)$

$\equiv p \wedge q$

Ans: If it snows and he drives the car.

iv) He swims if and only if the water is warm

p: He swims.

q: the water is warm

Given proposition: $p \leftrightarrow q$

Negation: $\sim(p \leftrightarrow q) \equiv \sim p \leftrightarrow q$

(or)

$p \leftrightarrow \sim q$

Ans:

He does not swim if and only if the water is warm

v). If Rama is rich then Ravi & Roy are happy.

p: Rama is rich

q: Ravi is happy

r: Roy is happy

Given proposition: $p \rightarrow (q \wedge r)$

Negation: $\sim[p \rightarrow (q \wedge r)] \equiv \sim[\sim p \wedge (q \wedge r)]$

$\equiv p \wedge \sim(q \wedge r)$

Ans:

Rama is rich and Ravi and Roy are not happy.

07.06.18

Arguments:

An argument is an assertion that given set of propositions $p_1, p_2, \ldots p_n$ yields another proposition q. The propositions $p_1, p_2, \ldots p_n$ are called 'Premises'. And q is called the 'Conclusion'. The argument is denoted by $p_1, p_2, \ldots p_n \vdash q$.

$\therefore \vdash$ - turnstile

An argument is a valid argument if $p_1 \wedge p_2 \wedge \ldots \wedge p_n \rightarrow q$ is a tautology otherwise it is an invalid argument or fallacy

Eg:

I. Test the validity of the argument.

1. I will get an 'A' grade in this course or I will not graduate. If I don't graduate I will go to the army.
(I got an 'A') thus, I will not go to the army

p – I will get an 'A'

Q – I will graduate

r – I will go to the army

Premises: $p \vee \sim q$, $\sim q \rightarrow r$, p

Conclusion: $\sim r$

Argument: $(p \vee \sim q), (\sim q \rightarrow r), p \vdash \sim r$

T.P.T $(p \vee \sim q) \wedge (\sim q \rightarrow r) \wedge p \rightarrow \sim r$

p	q	r	$\sim q$	$\sim r$	$p \vee \sim q$	$\sim q \rightarrow r$	$(p \vee \sim q) \wedge (\sim q \rightarrow r) \wedge p$
T	T	T	F	F	T	T	T
T	T	F	F	T	T	T	T
T	F	T	T	F	T	T	T
T	F	F	T	T	T	F	F
F	T	T	F	F	F	T	F
F	T	F	F	T	F	T	F
F	F	T	T	F	T	T	F
F	F	F	T	T	T	F	F

$\therefore$ It is not a valid arguments.

2. If 6 is even then 2 does not divide 7. Either 5 is not prime or 2 divides 7. But 5 is a prime. Therefore 6 is odd.

p : 6 is even

q : 2 does not divide 7

r : 5 is a prime

Premises: $p \rightarrow q$, $\sim r \vee \sim q$, $r \rightarrow \sim p$

Conclusion: $\sim p$

Argument: $p \rightarrow q, \sim r \vee \sim q, r \vdash \sim p$

T.P.T $(p \rightarrow q) \wedge (\sim r \vee \sim q) \wedge r \longrightarrow \sim p$ is a tautology

p	q	r	$\sim r$	$\sim q$	$p \rightarrow q$	$\sim r \vee \sim q$	$\sim p$	$(p \rightarrow q) \wedge (\sim r \vee \sim q) \wedge r$	
T	T	T	F	F	T	F	F	F	T
T	T	F	T	F	T	T	F	F	T
T	F	T	F	T	F	T	F	F	T
T	F	F	T	T	F	T	F	F	T
F	T	T	F	F	T	F	T	F	T
F	T	F	T	F	T	T	T	F	T
F	F	T	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T	F	T

∴ It is a valid Argument.

3. On my sister's birthday, I bring her flowers. Either it is mine sister's birthday or I work late. I did not bring my sister's flowers today. Therefore I worked late.

Ans:

P: my sister's birthday.

q: I bring flower

r: I work late.

Premises: $P \to q$, $P \vee r$, $\sim q$

Conclusion: r

Argument: $P \to q,\ P \vee r,\ \sim q \vdash r$

T.P.T: $(P \to q) \wedge (P \vee r) \wedge (\sim q) \to r$

P	q	r	$\sim q$	$P \to q$	$P \vee r$	$(P\to q)\wedge(P\vee r)\wedge(\sim q)$	$(P\to q)\wedge(P\vee r)\wedge(\sim q) \to r$
T	T	T	F	T	T	F	T
T	T	F	F	T	T	F	T
T	F	T	T	F	T	F	T
T	F	F	T	F	T	F	T
F	T	T	F	T	T	F	T
F	T	F	F	T	T	F	T
F	F	T	T	T	F	F	T
F	F	F	T	T	F	F	T

It is a valid argument.

09.08.18

UNIT - 3

SETS, RELATIONS, FUNCTIONS

A Set is a collection of well defined objects. The objects of a set are called 'Elements'.

If a is an element of a set A, we write as $a \in A$.

If b is not an element of a set A, we write as $b \notin A$.

Eg:

1. A set N of all natural numbers.
   $5 \in N$ but $-5 \notin N$

Finite and Infinite Sets:

A set which contains specific no. of distinct elements is called a 'Finite set'. If a set is not finite, it is an 'Infinite set'.

Eg: 1. A set of all integers ÷ by 5
   Infinite set.

2. Set of all natural numbers such that (7) $x \in N$ & $x^2 < 50$
   $\{1, 2, \ldots, 7\}$
   Finite set

Description of Sets:

There are 2 methods of representing sets. They are

i.) Tabulation form

ii.) Set-builder form

Eg: A = {Set of all days in a week}

-Set builder form

A = {Mon, Tue, Wed, Thu, Fri, Sat, Sun}

- Tabulation form

Singleton Set: - A set which contains only one element.

Eg: $A = \{7\}$

Null set :- (Null set / Empty/void set)

A set which contains no element.

Eg: $A = \{\}$ or ϕ

Subset: - A set 'B' is a subset of 'A' if all the elements of B is in A

B is said to be a proper subset of A.

$A = \{1, 2, 3, 4\}$, $B = \{2, 3, 4\}$, $C = \{1, 2, 3, 4\}$

$D = \phi$. $E = \{1, 5\}$

i.) $B \subset A$ - Proper subset

ii.) $C \subseteq A$, $A \subseteq C$ - Improper Subset

iii.) $D \subset A$ - Improper Subset

iv.) $E \not\subset A$ - Not a subset.

P(A) = Set of all subsets of A

$= \{\phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{1, 2, 3, 4\}, \}$

If a set 'A' has 'n' elements then there are 2^n subsets of A, & $2^n - 2 \Rightarrow$ Proper subsets of A.

Equality of Sets:

A and B are said to be equal if $A \subset B$ & $B \subset A$

Equivalent sets:

If A&B have the same no. of elements.

Eg: $A = \{1, 2, 3\}$
$B = \{a, b, c\}$

Disjoint sets

A & B are disjoint if A&B do not have any common elements

Eg $\left.\begin{array}{l} A = \{1, 3, 5, 7\} \\ B = \{2, 4, 6, 8\} \end{array}\right\}$ Disjoint

16.08.18 Operation on Sets:

Union:

$A \cup B = \{x / x \in A \text{ or } x \in B\}$

Intersection:

$A \cap B = \{x / x \in A \text{ \& } x \in B\}$

Difference of Sets:

$A - B = \{x / x \in A \text{ but } x \notin B\}$.

Complement of Set:

$A' = \{x / x \in U \text{ but } x \notin A\}$

Laws of Sets

* Commutative law:

$A \cup B = B \cup A$
$A \cap B = B \cap A$
$A - B \neq B - A$

* Associative law:

$A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$

* Distributive law:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

* Demorgan's law:

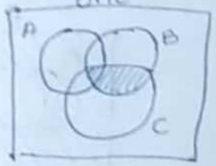
$(A \cup B)' = A' \cap B'$
$(A \cap B)' = A' \cup B'$

1. State and Prove Distributive law using Venn Diagram:

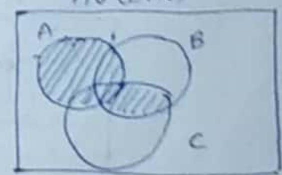
(1) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$L.H.S = A \cup (B \cap C)$

$B \cap C$



$A \cup (B \cap C)$



Equality of Sets:

A and B are said to be equal if $A \subset B$ & $B \subset A$

Equivalent sets:

If A&B have the same no. of elements

Eg: $A = \{1, 2, 3\}$
$B = \{a, b, c\}$

Disjoint sets:

A & B are disjoint if A&B do not have any common elements

Eg $\left.\begin{array}{l} A = \{1, 3, 5, 7\} \\ B = \{2, 4, 6, 8\} \end{array}\right\}$ Disjoint

16.08.18 **Operation on Sets**

Union:

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Intersection:

$A \cap B = \{x \mid x \in A \text{ \& } x \in B\}$

Difference of Sets:

$A - B = \{x \mid x \in A \text{ but } x \notin B\}$

Complement of Set

$A' = \{x \mid x \in U \text{ but } x \notin A\}$

Laws of Sets:

* Commutative law:

$A \cup B = B \cup A$
$A \cap B = B \cap A$
$A - B \neq B - A$

* Associative law:

$A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$

* Distributive law:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

* Demorgan's law:

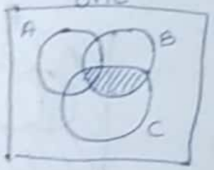
$(A \cup B)' = A' \cap B'$
$(A \cap B)' = A' \cup B'$

1. State and Prove Distributive law using Venn Diagram:

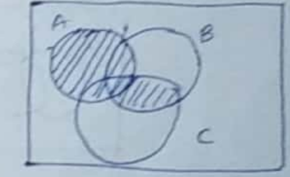
① $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S $= A \cup (B \cap C)$

$B \cap C$

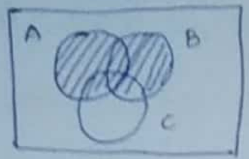


$A \cup (B \cap C)$

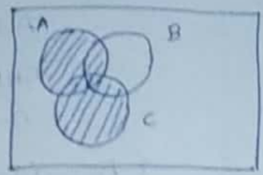


R.H.S = $(A\cup B)\cap(A\cup C)$

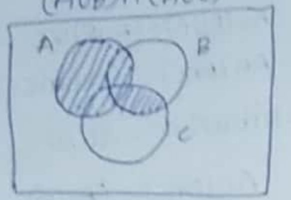
$A\cup B$



$A\cup C$



$(A\cup B)\cap(A\cup C)$



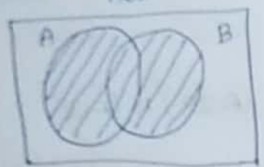
L.H.S = R.H.S

∴ Hence Proved

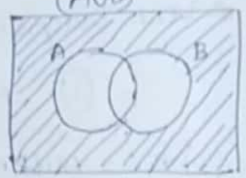
2 State and Prove De-morgan's theorem:

i. $(A\cup B)' = A'\cap B'$

L.H.S ⇒ $A\cup B$

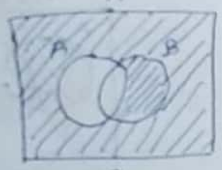


$(A\cup B)'$

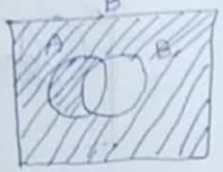


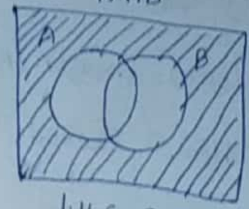
R.H.S ⇒

A'



B'





L.H.S = R.H.S

∴ Hence Proved

3 Prove by Laws:

i. $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

Sol:

Let $x \in A\cup(B\cap C)$

$\Rightarrow x\in A$ or $x\in(B\cap C)$

$= x\in A$ or $(x\in B$ & $x\in C)$

$= x\in A$ or $(x\in B$ & $x\in C)$

$= (x\in A$ or $x\in B)$ & $(x\in A$ or $x\in C)$

$= (x\in A\cup B)\cap(x\in A\cup C)$

$= x\in(A\cup B)\cap x\in(A\cup C)$

$\therefore A\cup(B\cap C)\subseteq(A\cup B)\cap(A\cup C) \rightarrow (1)$

Let $y\in(A\cup B)\cap(A\cup C)$

$\Rightarrow y\in A\cup B$ & $y\in A\cup C$

$\Rightarrow (y\in A)$ or $(y\in B)$ & $(y\in A)$ or $(y\in C)$

$= (y\in A)$ or $[(y\in B)$ & $(y\in C)]$

$= y\in A\cup[y\in B\cap y\in C]$

$= y\in A\cup(B\cap C)$

$\therefore (A\cup B)\cap(A\cup C)\subseteq A\cup(B\cap C) \rightarrow (2)$

from ① & ②,

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$\therefore$ Hence Proved

(ii) $(A \cup B)' = A' \cap B'$

Let $x \in (A \cup B)'$

$\Rightarrow x \notin A \cup B$

$= x \notin A$ or $x \notin B$

$= x \in A'$ & $x \in B'$

$= x \in (A' \cap B')$

$\therefore (A \cup B)' \subset (A' \cap B') \rightarrow$ ①

Let $y \in A' \cap B'$

$\Rightarrow y \in A'$ & $y \in B'$

$= y \notin A$ or $y \notin B$

$= y \notin A \cup B$

$= y \in (A \cup B)' \rightarrow$ ②

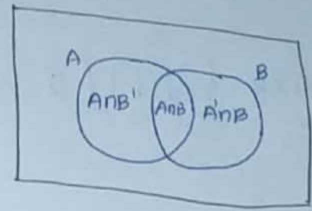
① & ②,

$\therefore (A \cup B)' = A' \cap B'$

Hence Proved

THEOREMS

① If $n(A)$ denotes the no. of element in A then S.T $n(A \cup B) = n(A) + n(B) - n(A \cap B)$



If A & B are disjoint then,

$n(A \cup B) = n(A) + n(B)$

Now, $A \cup B = A \cup (A' \cap B)$

$n(A \cup B) = \cancel{n(A)}\ n\,(A \cup (A' \cap B))$

$= n(A) + \underbrace{n(A' \cap B)}_{①}$ (A & A'∩B are disjoint)

|||ly, $B = (A \cap B) \cup (A' \cap B)$

$n(B) = n\left[(A \cap B) \cup (A' \cap B)\right]$

$= n(A \cap B) + n(A' \cap B)$ ($\because$ A∩B & A'∩B are disjoint)

$n(A' \cap B) = \underbrace{n(B) - n(A \cap B)}_{②}$

Sub ② in ①,

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Hence Proved.

Cartesian Product: $(A \times B)$

If A and B are any two sets.

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

$[\because n(A \times B) = n(A) \cdot n(B)]$

Problems:

1. $A = \{1, 2, 3\}$, Find $(A \times B) \cup (A \times C)$
   $B = \{3, 4, 5\}$
   $C = \{3, 4\}$

$A \times B = \{(1,3), (1,4), (1,5),$
$(2,3), (2,4), (2,5)$
$(3,3), (3,4), (3,5)\}$

$A \times C = \{(1,3), (1,4),$
$(2,3), (2,4)$
$(3,3), (3,4)\}$

$\therefore (A \times B) \cup (A \times C) = \{(1,3), (1,4), (1,5),$
$(2,3), (2,4), (2,5),$
$(3,3), (3,4), (2,5)\}$

2. S.T $(A \times B) \cap (S \times T) = (A \cap S) \times (B \cap T)$

$A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $S = \{1, 3, 4\}$,
$T = \{2, 4, 5\}$

L.H.S $\Rightarrow$

$(A \times B) = \{(1,2), (1,3), (1,4),$
$(2,2), (2,3), (2,4),$
$(3,2), (3,3), (3,4)\}$

$(S \times T) = \{(1,2), (1,4), (1,5),$
$(3,2), (3,4), (3,5),$
$(4,2), (4,4), (4,5)\}$

$(A \times B) \cap (S \times T) = \{(1,2), (1,4), (3,2), (3,4)\}$ → ①

R.H.S $\Rightarrow$

$A \cap S = \{1, 3\}$. $B \cap T = \{2, 4\}$

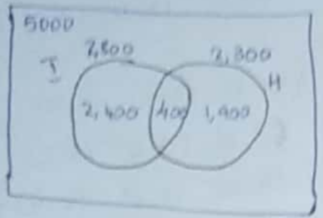
$(A \cap S) \times (B \cap T) = \{(1,2), (1,4), (3,2), (3,4)\}$ → ②

From eqns ① & ②,

$(A \times B) \cap (S \times T) = (A \cap S) \times (B \cap T)$

3. In the survey of 5,000 persons, it was found that 2,800 read Indian express And 2,300 reads HINDU. While 400 reads both papers. How many reads neither Indian express nor HINDU?

Solution:



No. of persons read I.E $\Rightarrow n(I) = 2,800$

No. of persons read Hindu $\Rightarrow n(H) = 2,300$

$n(I \cap H) = 400$

No. of persons reading either I.E or Hindu

$\Rightarrow n(I \cup H) = n(I) + n(H) - n(I \cap H)$

$= 2,800 + 2,300 - 400$

$= 4,700$

No. of persons reading neither Hindu or I.E.

$\Rightarrow 5000 - 4700$

$= 300$

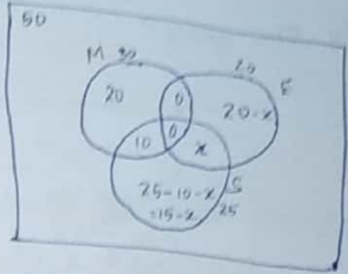
PROBLEMS

1. Out of group of 50 teachers in a high school 30 teach maths, 20 teach English, 25 teach Science, 10 teach both Maths & Science. None teach Maths & English.

i.) How many teach Science & English

ii) How many teach only English

Solution:



By given,

(i) $50 \Rightarrow 20 + 20 - \cancel{x} + 15 - x + 10 + \cancel{x}$

$50 = 40 + 15 + 10 - x$

$50 = 65 - x$

$x = 65 - 50$

$\boxed{x = 15}$

$\therefore$ 15 teachers teach both English and Science.

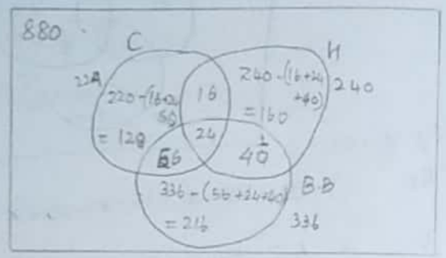
(ii)

Teachers teach English only $= 20 - x$

$= 20 - 15$

$= 5$

2. Out of 880 boys in a School, 220 played Cricket, 240 played hockey, 336 played Basket ball. Of the total 64 played Basket ball & hockey, 80 played cricket & basket ball, 40 played cricket & hockey. 24 played all the three games. How many did not play any of the games and how many played only one game?



Plays only cricket $= 220-(16+24+56)$

$= 124$

Plays only hockey $= 240-(16+24+40)$

$= 160$

Plays only B.B $= 336-(56+24+40)$

$= 216$

No. of played only one game $= 128+160+216$

$= 504$

No. played at least one game:

$=$ One game + two game + three game.

$= 504+(16+56+40)+24$

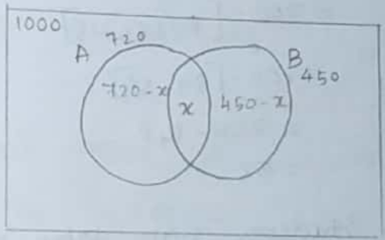
$= 640$

Those who do not play any game

$\Rightarrow 880-640$

$= 240$

3. In a Survey of 1000 consumers, 720 like product A. And 450 like B. What is the least no. that must have liked both the products.



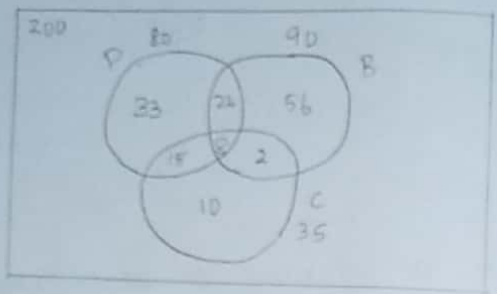
$1000 \Rightarrow 720-\cancel{x}+\cancel{x}+450-x$

$1000 = 1170-x$

$x = 1170-1000$

$\boxed{x = 170}$

4. In a graduate course of 200 students of a college records 80 students have taken Phy, 90 taken Bio, 35 taken che, 32 taken both B & P, 23 taken both C & P, 10 taken both B & C, 8 taken all the three. How many have not taken any of the three?



$$\Rightarrow 200 - [33+56+10+24+15+2+8]$$

$$= 200 - [89+20+24+15]$$

$$= 200 - [89+44+15]$$

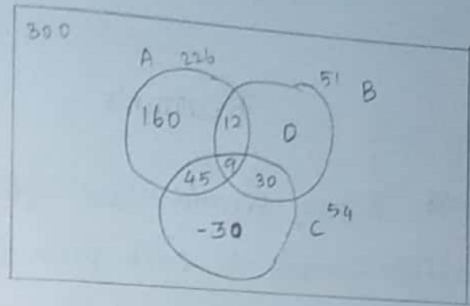
$$= 200 - [89+59]$$

$$= 200 - 148$$

$$= 52$$

∴ Students have taken none of three subjects is 52

5. The company studies the product preferences of 300 consumers. It was found that 226 like product A, 51 like Pro. B, 54 like C, 21 like A & B, 54 like A & C, 39 like B & C. And 9 like all the three. Verify that the study results are correct or not? Assume that each consumer likes atleast one of the 3 products?



Only A $= 226 - (45+21)$
$= 160$

Only B $= 51 - (39+12)$
$= 0$

Only C $= 54 - (45+9+30)$
$= -30$

∴ Since, those who like the product C cannot be in negative.

Hence, the Survey is Wrong

List the elements in the Set:

1.) $\{x \in N,\ x = 2y.\ y \in N,\ 2 < y < 5\}$

$\Rightarrow \{6, 8\}$

2.) $\{x : y \in N,\ x = \frac{y+1}{y}\}$

$\Rightarrow \{2\}$

RELATIONS

A Binary relation are ~~from~~ from a set A to B assigns to each pair (a, b) in A×B exactly one of the following.

i) a is related to b (i.e) aRb

ii) a is not related to b (i.e) $a \not R b$

Eg: (i) $A = \{1, 2, 3\}$, $B = \{2, 5\}$ the $R = (1,2), (1,5) (2,3) (2,5)$

Is not a relation since $3 \notin B$

(ii) $A = \{1, 2, 3, 4\}$ $B = \{2, 3, 5\}$ $R \to$ less than

then $R = \{(1,2), (1,3), (1,5)$
$(2,3), (2,5), (3,5), (4,5)\}$

* Number of distinct relation from A to B

If 'A' has 'm' elements

'B' has 'n' elements

$|A \times B|$ has $m \times n$ elements

then no. of different relation from A to B is 2^{mn}.

* Inverse relation:

If R is relation from A to B.

The inverse relation R^{-1} is the relation from B to A which consist those ordered pairs which when reversed $\in$ to R.

$$R^{-1} = \{(b,a) / (a,b) \in R\}$$

Eg:

$A = \{1, 2\}$, $B = \{a, b\}$, $R = \{(1,a) (2,a) (2,b)\}$

$R^{-1} = \{(a,1), (a,2) (b,2)\}$

* Identity relation:

If A is the set, the relation I A

$= \{(a,a) / a \in A\}$

$A = \{1, 2\}$

$IA = \{(1,1) (2,2)\}$

* Reflexive relation : aRa

Symmetric relation if aRb then bRa

Transitive relation if aRb, bRc then aRc

A relation R in a set A is called reflexive if aRa (i.e) $(a,a) \in R$ $\forall a \in A$

* Symmetric relation:

A relation R in a set A is said to be symmetric if aRb then bRa (i.e) $(a,b) \in R$ then $(b,a) \in R$

* Transitive relation:

A relation R in a set A is transitive if aRb, bRc then aRc (i.e) $(a,b) \in R$ & $(b,c) \in R$ then $(a,c) \in R$

* Anti-Symmetric relation:

A relation R in a set A is said to be anti-symmetric if aRb then $b \not R a$ (or) if aRb then bRa only for $a=b$.

* Equivalence relation:

A relation R is said to be equivalence,

i) if R is reflexive

ii) if R is symmetric

iii) if R is transitive

Eg: Let A be a set of all lines in a plane, R be the relation parallel to

$a \parallel$ to a $\forall a \in A$ (i.e) aRa

$a \parallel$ to b then $b \parallel$ to a (i.e) bRa

$a \parallel$ to b & $b \parallel$ to c then $a \parallel$ to c (i.e) aRc

Hence 'R' is an equivalence relation.

Identity the type of relation:

A = Set of all real numbers

$R = \{(x,y) / |x| = |y|\}$

$R_1 = \{(x,y) / x \geq y\}$

R will be the form of.

$R = \{(a,a), (a,-a) (-a,a) (-a,-a) / \forall a \in A\}$

$R \Rightarrow$

1.) Reflexive.

$aRa \quad |a| = |a|$

2) Symmetric

$aRb \Rightarrow |a| = |b|$

$|b| = |a|$ (i.e) bRa

3.) Transitive

$aRb \Rightarrow |a| = |b|$

$bRc \Rightarrow |b| = |c|$

then,

$|a| = |c|$

hence aRc

$R_1 \Rightarrow$

1) Reflexive

$xRx, \quad x \geq x$

2) Symmetric

$xRy \Rightarrow x \geq y$ then $y \ngeq x$ if $x \neq y$

$\therefore$ R is not Symmetric

3) Transitive

xRy & yRz

$\Rightarrow x \geq y$ & $y \geq z$ then $x \geq z$

$\therefore$ It is not equivalence relation

* Equivalene class:

Let A be a non empty set and R be a equivalence relation in A. Let $a \in A$ be an arbitary element. The element x belongs to A Satisfying xRa consitute a subset A_a Called an equivalence class of a with respective to R.

$A_a = [a] = \{x \in A / xRa\}$

Eg. Let A be a set of all triangles in a plane and R be a equivalence relation define by "x congruent to y", $x, y \in A$ when $a \in A$, the equivalence class of a

$[a]$ = is the set of Δ^{le} ~~argum~~ Congruent to "a"

Let us now find the equivalence classes in the set I of all integers with respect to the equivalence relation "Congruent modulo 5"

$a \equiv b \pmod 5$

$6 \equiv 1 \pmod 5$

$7 \equiv 2 \pmod 5$

$8 \equiv 3 \pmod 5$

$9 \equiv 4 \pmod 5$

$10 \equiv 0 \pmod 5$

06.09.2018

Equivalence Class:

$R \Rightarrow$ Related

Eg:

* I - Set of all integers

R - "Congruent modulo 5"

$I_0 = \{\ldots -10, -5, 0, 5, 10, 15, \ldots\}$

$I_1 = \{\ldots -4, 1, 6, 11, \ldots\}$

$I_2 = \{\ldots -8, -3, 2, 7, 12, \ldots\}$

$I_3 = \{\ldots -7, -2, 3, 8, 13, \ldots\}$

$I_4 = \{\ldots -6, -1, 4, 9, 14, \ldots\}$

where I_0, I_1, I_2, I_3, I_4 forms equivalence classes.

$\therefore I = I_0 \cup I_1 \cup I_2 \cup I_3 \cup I_4$

Properties of Equivalence class:

Let "A" be a non-empty set and "R" be an Equivalence relation in A. Let a, b be an arbitrary element in A

(i) $a \in [a]$

(ii) if $b \in [a]$ then $[b] = [a]$

(iii) $[a] = [b] \iff aRb$

(iv) Either $[a] = [b]$ or $[a] \cap [b] = \phi$

Partition:

The Partition of a set X is a sub-division of X into subsets which are disjoints and whose union is X.

The collection $\{A_1, A_2, \ldots A_n\}$ is a partition of a set X if (i) $X = A_1 \cup A_2 \ldots \cup A_n$

(ii) $\forall i \neq j \quad A_i \cap A_j = \phi$

For Eg:

$N = \{1, 2, \ldots 9\}$

$A_1 = \{1, 2, 5\}, \quad A_2 = \{3, 6, 9\}, \quad A_3 = \{4, 8, 7, 3\}$

$\therefore A_1, A_2, A_3$ is not a Partition of a set ($\because A_2 \cap A_3$)

S.T. the relation of congruence modulo N i.e $a \equiv b \pmod m$ in the set Z (Z-Integers) is an Equivalent relation.

Solution:

$a \equiv b \pmod m$

$a - b = km$

$\boxed{\because 17 \equiv 2 \pmod 5 \quad 17 - 2 = 15 = \underset{m}{5} \times 3}$

1) Reflexive:

T.S.T aRa

$a \equiv a \pmod m$

$a - a = 0 \cdot m = 0$

$\therefore aRa$

(ii) Symmetric:

T.S.T if aRb then bRa

aRb:

$a \equiv b \pmod m$

$a-b = km$

$\Rightarrow -(b-a) = km$

$\Rightarrow b-a = (-k)m$ $\qquad [\because k' = -k]$

$\Rightarrow b-a = k'm$

$b \equiv a \pmod m$

$\Rightarrow bRa$

(iii) Transitive:

T.S.T if aRb, bRc, then aRc.

$a \equiv b \pmod m$, $\qquad b \equiv c \pmod m$

$a-b = km \to (1)$ $\qquad b-c = km \to (2)$

$b = k_1 m + c \to (2)$

Sub (2) in (1),

$a-(k_1 m+c) = km$

$a-c-k_1 m = km$

$a-c = km + k_1 m$

$a-c = (k+k_1)m$

$a-c = k_2 m$

$\Rightarrow a \equiv c \pmod m$

$\therefore aRc$

$\therefore R$ is an Equivalence relation

Find all the Partitions of $X = \{1,2,3,4\}$

(i) $\{1,2,3,4\}$

(ii) $\{\{1\},\{2,3,4\}\}$, $\{\{2\},\{1,3,4\}\}$,
$\{\{3\},\{1,2,4\}\}$, $\{\{4\},\{1,2,3\}\}$

(iii) $\{\{1\},\{2\},\{3,4\}\}$
$\{\{1\},\{3\},\{2,4\}\}$
$\{\{1\},\{4\},\{2,3\}\}$
$\{\{2\},\{3\},\{1,4\}\}$
$\{\{2\},\{4\},\{1,3\}\}$
$\{\{3\},\{4\},\{1,2\}\}$

(iv) $\{\{1\},\{2\},\{3\},\{4\}\}$

07/01/18

1. If R and S are equivalence relation in X P.T $R \cap S$ is an equivalence relation.

Solution

Gn: R & S are equivalence relation in X

T.P $R \cap S$ is an equivalence relation in X

(i) Reflexive:

T.P.T $\forall x \in X$, $(x,x) \in R\cap S$

Since R & S are reflexive

$\forall x \in X$, $(x,x) \in R$ & $(x,x) \in S$

$\Rightarrow (x,x) \in R\cap S$.

(ii) Symmetric:

$(x,y) \in R\cap S$ then $(y,x) \in R\cap S$

$(x,y) \in R\cap S$

$\Rightarrow (x,y) \in R$ & $(x,y) \in S$

$\Rightarrow (y,x) \in R$ & $(y,x) \in S$ ($\because$ R & S are symmetric)

$\Rightarrow (y,x) \in R\cap S$

(iii) Transitive:

T.P.T $(x,y) \in R\cap S$, $(y,z) \in R\cap S$ then $(x,z) \in R\cap S$

$\Rightarrow (x,y) \in R$ & $(x,y) \in S$ $\Rightarrow x \in R\cap S$

$\Rightarrow (y,z) \in R$ & $(y,z) \in S$ $\Rightarrow z \in R\cap S$

$\therefore (x,z) \in R\cap S$

R is an equivalence relation in a set X

② Define a relation $\sim$ on $N\times N$ as if $(a,b), (c,d) \in N\times N$ then $(a,b) \sim (c,d)$ if $a+d = b+c$.

P.T $\sim$ is a equivalence relation.

Solution:

(i) Reflexive:

P.T $(a,b) \sim (a,b)$

$\Rightarrow a+b = b+a$

$\therefore (a,b) \sim (a,b)$

(ii) Symmetric:

P.T $(a,b) \sim (c,d)$ then $(c,d) \sim (a,b)$

$\Rightarrow (a+d) = (b+c)$

$\Rightarrow b+c = a+d$

$\Rightarrow c+b = d+a$

$\therefore (c,d) \sim (a,b)$

(iii) Transitive:

$(a,b) \sim (c,d)$, $(c,d) \sim (e,f)$, then P.T

$\therefore (a,b) \sim (e,f)$

$(a,b) \sim (c,d) \Rightarrow a+d = b+c$

$a-b = c-d \rightarrow ①$

$(c,d) \sim (e,f) \Rightarrow c+f = d+e$

$c-d = e-f \rightarrow ②$

From eqns ① & ②,

$a-b = e-f$

$a+f = e+b$

$\therefore (a,b) \sim (e,f)$

Hence '$\sim$' is an equivalence relation.

PARTIALLY ORDERED SET:

A Set X is called Partially ordered set by a relation 'R' if.

(i) R is reflexive

(ii) R is Antisymmetric.

(iii) R is Transitive.

The relation R is called 'Partial order'

Eg:

① Let 'R' be a relation in the set of natural no.s defined by "x is a multiple of y" then R is a partial order in N.

Solution:

"x is a multiple of y"

(i) Reflexive:

P.T. xRx

$\Rightarrow x = k \cdot x$

$\therefore xRx$ is Proved.

(ii) Anti Symmetric:

P.T. xRy then $y \not R x$

$xRy \Rightarrow \quad y = k \cdot x$

$\Rightarrow x = \frac{1}{k} y$

$\therefore$ x is not a multiple of y.

(iii) Transitive:

$x R y$, $y R z$ then $x R z$

$xRy \Rightarrow \quad y = kx \rightarrow (1)$

$yRz \Rightarrow \quad z = k_1 y \rightarrow (2)$

Sub (1) in (2),

$z = k_1(kx)$

$= k k_1 x$

$z = k_2 x$

x is a multiple of z

Hence Proved

Note:

i. Two elements in a & b, Partially ordered set are not comparable $\Rightarrow$ $(a,b) \in R$ or $(b,a) \in R$ or $a = b$

ii. Every two elements in Partially ordered set are comparable then, the partially order is called 'Total order'.

10.09.2018

Functions:

Range:

Mapped elements' no. only.

Types of functions:

1. Many one (1 img. $\Rightarrow$ so many pre img)
2. Into (1-1) (Unique)
3. Onto ($\because$ range = co-domain)
4. Bijective (Both 1-1 & Onto)
5. Identity (Same element map)
6. Constant
7. Inverse

Definition:

Suppose to each element of a set A there is assigned a unique element of set B. The collection is 'called function or mapping from A to B', denoted by $f: A \rightarrow B$.

Domain = A

Co-domain = B

Range $\Rightarrow f(A) \subseteq B$

Eg:

~~f is from~~

i. $F: R \rightarrow R$ assigns to each rational nos 1 and for irrational number -1.

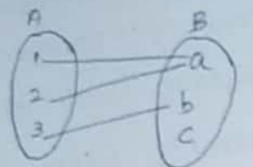
i.e. $f(x) = \begin{cases} 1 ; & x \text{ is rational} \\ -1 ; & x \text{ is irrational} \end{cases}$

1. ~~Into function~~ / Injective / Into function

$F : A \to B$ is a function such that there is atleast one element in B which is not in the range of A (i.e)

$F(A)$ is a proper subset of B

Eg:

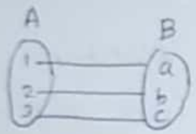


$f(A) = \{a, b\} \subset B$

2 Onto (Surjective) function:

The Range of F = Co-domain

Eg:

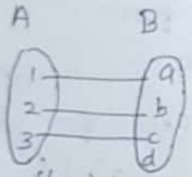


Range $(A) = \{a, b, c\} = B$

3 1-1 function:

$F : A \to B$ is 1-1 if different element in A has different element in B

Eg:



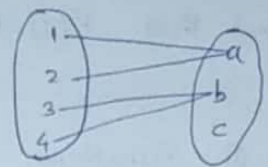
$\therefore f(x_1) = f(x_2)$
$\Rightarrow x_1 = x_2$

4. Many to One:

If Distinct elements of A has the same image in B.

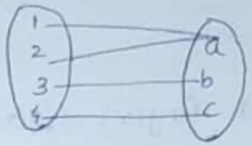
Eg:

①



Many to One & Into

②



Many to One & Onto

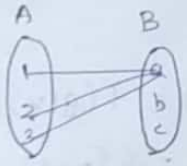
⑤ Identity function

$F : A \to A$ is identity if $f(x) = x \ \forall x \in A$

6. Constant function:

$F : A \to B$ is called a Constant func. if each element of A has the image in B. The range of $A = 1$

Eg:



7. Inverse function:

$F: A \to B$ is 1-1 and Onto

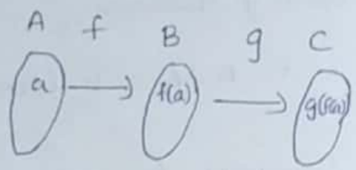
The function F^{-1} which associates to each element $b \in B$, the element $a \in B$ such that $F(a) = b$

Composition of Functions

Consider the function $F: A \to B$ &

$g: B \to C$.

* $f \circ g(x) = f(g(x))$
* $g \circ f(x) = g(f(x))$



Eg.

1. $f: R \to R$ $g: R \to R$ defined that $f(x) = x^2$, $g(x) = x+5$ Find $f \circ g$ & $g \circ f$.

Solution:

i. $F \circ g(x) = f(g(x))$
$= f(x+5)$
$= (x+5)^2$

ii. $g \circ F(x) = g(f(x))$
$= g(x^2)$
$= x^2+5$

Problems:

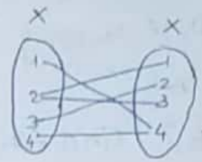
1. $X = \{1, 2, 3, 4\}$

Determine whether the relation is a function or not

i.) $f = \{(2,3), (1,4), (2,1), (3,2), (4,4)\}$
$g = \{(3,1), (4,2), (1,1)\}$
$h = \{(2,1), (3,4), (1,4), \cancel{(2,1)}, (4,4)\}$

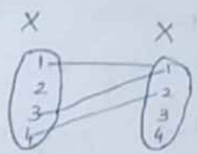
Solution:

$F: X \to X$



It is not a ~~relation~~ function.

$g: X \to X$



It is not a function.

$h: X \to X$

It is a function

Method of testing (1-1) function

$f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$

(or)

$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

Method of testing onto function.

$F: A \to B$

f is onto if $\forall y \in B$ there exists $x \in A$ such that $f(x) = y$.

Problems

1. $A = \{-1, 0, 2, 5, 6, 11\}$

$B = \{-2, -1, 0, 18, 28, 108\}$

& $f(x) = x^2 - x - 2$

Is $f(A) = B$? find $f(A)$

Solution:

$f(-1) = (-1)^2 + 1 - 2$
$= 1 + 1 - 2$
$= 0$

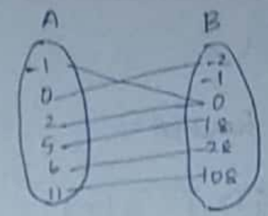
$f(0) = 0 - 0 - 2$
$= -2$

$f(2) = 4 - 2 - 2$
$= 0$

$f(5) = 25 - 5 - 2$
$= 18$

$f(6) = 36 - 6 - 2$
$= 28$

$f(11) = 121 - 11 - 2$
$= 108$



$f(A) = \{-2, 0, 18, 28, 108\}$
$\neq B$

It is a function

$\therefore$ It is not a onto - function

2. Which of the followings funcs are injections, Surjects or Bijections on R the set of real no.s.

(i) $f(x) = -2x$.

* (1-1) function

$f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$

$-\cancel{2}x_1 = -\cancel{2}x_2$

$x_1 = x_2$

$\therefore$ f is 1-1 function

* Onto function

$\forall y \in R \ \exists x \in R \ni f(x) = y$

$\forall y \in R, \ \exists (-y/2) \in R \ni f(-y/2) = y$

$\therefore$ f is 1-1 and onto function

(ii) $g(x) = x^2 - 1$

(i) 1-1 function:

$g(x_1) = g(x_2)$

$\Rightarrow x_1 = x_2$

$x_1^2 - 1 = x_2^2 - 1$

$x_1^2 = x_2^2$

$\therefore x_1 = \pm x_2 \Rightarrow \boxed{x_1 = x_2}$ or $\boxed{x_1 = -x_2}$

g is not 1-1 function

(ii) Onto function:

$\forall y \in R, \exists x \in R \ni g(x) = y$

~~$\forall y \in R$~~, ~~$\exists$~~

$g(x) = y$

$x^2 - 1 = y$

$x^2 = y + 1$

$x = \pm\sqrt{y+1}$

$\therefore$ g is not onto

(iii) $h(x) = \begin{cases} x, & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$

$h(x_1) = h(x_2)$

if x_1 & x_2 are negative

$x_1 = x_2$

if x_1 & x_2 are positive

$x_1^2 = x_2^2$

$x_1 = x_2$

$\therefore h(x)$ is 1-1 but not onto

3. Let 'R' be a set of Real nos,

$F: R \to R$ $F(x) = 2x + 5$ if possible find the inverse func F^{-1}.

<u>Solution</u>

Inverse should be 1-1 & onto

(i) 1-1 function:

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$2x_1 + 5 = 2x_2 + 5$

$\Rightarrow x_1 = x_2$

It is 1-1 function

(ii) Onto function:

$\forall y \in R \ \exists x \in R \ni f(x) = y$

~~$f(x) = 2x+5$~~

$f(x) = y$

$2x + 5 = y$

$2x = y - 5$

$x = \frac{y-5}{2} \in R$

$\therefore F^{-1}$ exists

$F: R \to R$

$F^{-1}: R \to R$

$f^{-1}(y) = \frac{y-5}{2}$

R f^{-1} R R R

4. Is $f(x) = \dfrac{x^2-8x+18}{x^2+4x+30}$ an 1-1 function

H.W

$$f(x_1) = f(x_2)$$

$$\frac{x_1^2-8x_1+18}{x_1^2+4x_1+30} = \frac{x_2^2-8x_2+18}{x_2^2+4x_2+30}.$$

5. $A = \{1, 2, 3\}$

$f : A \to A$

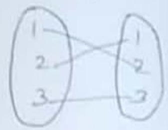
$f(1) = 2$ Find f^2, f^3, f^4, f^{-1}

$f(2) = 1$

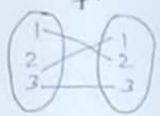
$f(3) = 3$

Sol:

$$f^2(1) = f(f(1)) = f(2) = 1$$



$$f^2(2) = f(f(2)) = f(1) = 2$$



$$f^2(3) = f(f(3)) = f(3) = 3$$

$$f^3(1) = f(f^2(1)) = f(1) = 2$$

$$f^4(1) = f(f^3(1)) = f(2) = 1$$

6. $f(x) = \dfrac{x}{x+1}$, $g(x) = \dfrac{x}{1-x}$.

$$f\circ g(x) = f(g(x))$$
$$= f\left(\frac{x}{1-x}\right)$$
$$= \frac{\frac{x}{1+x}}{1-\frac{x}{1+x}}$$
$$= \frac{\frac{x}{1+x}}{\frac{1+\cancel{x}-\cancel{x}}{1+x}}$$
$$= \frac{x}{\cancel{1+x}} \times \frac{\cancel{1+x}}{1}$$
$$= x$$

* $f(x) = \dfrac{1}{1-x}$ find $f\circ f\circ f(x)$

$$f\circ f\circ f(x) = f\circ f\left(\frac{1}{1-x}\right)$$
$$= f\left[\frac{1}{1-\frac{1}{1-x}}\right]$$
$$= f\left[\frac{1}{\frac{\cancel{1}-x-\cancel{1}}{1-x}}\right]$$
$$= f\left[\frac{1}{\frac{-x}{1-x}}\right]$$
$$= f\left[\frac{1-x}{-x}\right]$$
$$= \frac{1}{1-\left(\frac{1-x}{-x}\right)}$$
$$= \frac{\cancel{1}}{\cancel{x+1+x}} \qquad \frac{1}{1+\frac{1-x}{x}}$$
$$= \frac{1}{\frac{x+1-x}{x}} \Rightarrow \frac{1}{1/x} = x$$

UNIT-IV

Differentiation

Standard formula

Function	Differentiation $\frac{dy}{dx}$
x^n	nx^{n-1}
e^x	e^x
e^{ax}	ae^{ax}
$\log x$	$1/x$
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \cdot \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cdot \cot x$

* Differentiation rules:

$$d(uv) = u\,dv + v\,du$$

$$d(u+v) = du + dv$$

$$d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$

* Differentiate the following.

i) $y = x + \frac{1}{x}$

D. w.r to x

$$\frac{dy}{dx} = \frac{d}{dx}\left(x + \frac{1}{x}\right)$$

$$= \frac{d}{dx}(x) + \frac{d}{dx}\left(\frac{1}{x}\right)$$

$$= 1 + \frac{d}{dx}(x^{-1})$$

$$= 1 + (-1)x^{-2}$$

$$= 1 - x^{-2}$$

$$= 1 - \frac{1}{x^2}$$

ii) $y = 7x^3 + 4x^2 - 3x + 2$

D. w.r to x

$$\frac{dy}{dx} = 21x^2 + 8x - 3$$

iii.) $y = \frac{x^2 + 2x + 3}{\sqrt{x}}$

$$y = \frac{x^2}{(x)^{1/2}} + \frac{2x}{x^{1/2}} + \frac{3}{x^{1/2}}$$

$$\cancel{\frac{dy}{dx}} = \cancel{\frac{2x^1}{\frac{1}{2}(x)^{-1/2}}} + \cancel{\frac{x}{\frac{1}{2}(x)^{-1/2}}} + 0$$

$$\Rightarrow \frac{x^2}{x^{1/2}} + \frac{2x^1}{x^{1/2}} + \frac{3}{x^{1/2}}$$

$$= x^{2-1/2} + 2x^{1-1/2} + 3x^{-1/2}$$

$$y = x^{3/2} + x^{1/2} + 3x^{-1/2}$$

$$\frac{dy}{dx} = \frac{3x^{\frac{3}{2}-1}}{2} + \frac{x^{\frac{1}{2}-1}}{2} + \left(-\frac{x^{-\frac{1}{2}-1}}{2}\right)$$

$$= \frac{3x^{1/2}}{2} + \frac{x^{-1/2}}{2} - \frac{x^{-3/2}}{2}$$

$$= \frac{1}{2}\left(3x^{1/2} + x^{-1/2} - x^{-3/2}\right).$$

iv) $y = x \log x$

$$\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \log x\,(1)$$

$$= 1 + \log x$$

v) $y = 5e^x \cdot x^2 \cos x$

$$\frac{dy}{dx} = 5e^x x^2(-\sin x) + 5e^x \cdot (2x)\cos x + (5e^x x^2 \cos x)$$

$$\Rightarrow 5e^x\left[-x^2 \sin x + 2x\cos x + x^2 \cos x\right]$$

vi) $y = \cancel{5e^x x^2}\ x^3(2x-1)^2$

$$\frac{dy}{dx} = x^3\left[2(2x-1)(2)\right] + (2x-1)^2\left[3x^2\right]$$

$$= (4x^3)(2x-1) + (3x^2)(2x-1)^2$$

$$= x^2(2x-1)\left[4x + 3(2x-1)\right]$$

$$= x^2(2x-1)\left[4x + 6x - 3\right]$$

$$= x^2(2x-1)(10x-3)$$

$$= x^2(10x-3)(2x-1)$$

vii) $y = x(2x-1)(3x+2)$

$$\frac{dy}{dx} = x(2x-1)(3) + x(2)(3x+2) + (1)(2x-1)(3x+2)$$

$$= 3x(2x-1) + 2x(3x+2) + (2x-1)(3x+2)$$

viii) $y = \dfrac{ax+b}{cx+d}$

$$\frac{dy}{dx} = \frac{(cx+d)(a) - (ax+b)(c)}{(cx+d)^2}$$

$$= \frac{acx + ad - (acx + bc)}{(cx+d)^2}$$

$$= \frac{\cancel{acx} + ad - \cancel{acx} - bc}{(cx+d)^2}$$

$$= \frac{ad-bc}{(cx+d)^2}$$

ix) $y = \dfrac{\log x}{\sin x}$

$$\frac{dy}{dx} = \frac{\sin x\left(\frac{1}{x}\right) - \log x(\cos x)}{(\sin x)^2}$$

$$= \frac{\sin x/x - \log x(\cos x)}{\sin^2 x}$$

x) $y = \dfrac{x \cdot e^x}{\sin x}$

$u = x \cdot e^x$, $v = \sin x$

$du = x e^x + e^x(1)$ $\quad dv = \cos x$

$= e^x(x+1)$

$$\frac{dy}{dx} = \frac{\sin x\, e^x(x+1) - x \cdot e^x(\cos x)}{\sin^2 x}$$

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xi.) $y = \frac{1}{\sqrt[3]{4x^2-5x+3}}$

$= \frac{1}{(4x^2-5x+3)^{1/3}}$

$y = (4x^2-5x+3)^{-1/3}$

$\frac{dy}{dx} = -\frac{1}{3}(4x^2-5x+3)^{-\frac{1}{3}-1}(8x-5)$

$= -\frac{1}{3}(4x^2-5x+3)^{-4/3}(8x-5).$

$= -\frac{(8x-5)}{3(4x^2-5x+3)^{4/3}}$

Formulae:

* $\log\left(\frac{a}{b}\right) = \log a - \log b$
* $\log(ab) = \log a + \log b$
* $\log(a^x) = x\log a$

1. $y = \log\left(\frac{1+\cos x}{1-\cos x}\right)$

$y = \log(1+\cos x) - \log(1-\cos x)$

$\frac{dy}{dx} = \frac{1}{1+\cos x}(-\sin x) - \frac{1}{1-\cos x}(\sin x)$

$= \frac{-\sin x}{1+\cos x} - \frac{\sin x}{1-\cos x}$

$= -\sin x\left[\frac{1}{1+\cos x} + \frac{1}{1-\cos x}\right]$

$= -\sin x\left[\frac{1-\cancel{\cos x}+1+\cancel{\cos x}}{1^2-\cos^2 x}\right]$

$= -\cancel{\sin x}\left(\frac{2}{\sin^{\cancel{2}} x}\right)$

$= \frac{-2}{\sin x}$

$= -2\left(\frac{1}{\sin x}\right)$

$= -2\operatorname{cosec} x$

2. $y = \frac{x^2}{x^2+1}$, $x = \sqrt{2z+1}$ $\frac{dy}{dz} = ?$

$y = \frac{2z+1}{2z+1+1}$

$y = \frac{2z+1}{2z+2}$

$\frac{dy}{dz} = \frac{(2z+2)(2)-(2z+1)(2)}{(2z+2)^2}$

$= \frac{\cancel{4z}+4-\cancel{4z}-2}{(2z+2)^2}$

$= \frac{2}{(2z+2)^2}$

Differentiation of logarithmic function:

17-09-2018

$* \log a^x = x \log a \; / \; \log m^n = n \log m$

1. $y = x^y$

Sol:

Taking log on both sides,

$\log y = \log x^y$

$\log y = y \log x$

D. w.r to x,

$\frac{1}{y} \cdot \frac{dy}{dx} = y\left(\frac{1}{x}\right) + \log x \left(\frac{dy}{dx}\right)$

$\frac{1}{y} \cdot \frac{dy}{dx} - \log x \left(\frac{dy}{dx}\right) = \frac{y}{x}$

$\frac{dy}{dx}\left[\frac{1}{y} - \log x\right] = \frac{y}{x}$

$\frac{dy}{dx}\left[\frac{1 - y\log x}{y}\right] = \frac{y}{x}$

$$\frac{dy}{dx} = \frac{y^2}{x(1 - y\log x)}.$$

2. $y = x^{\log(\log x)}$

Sol:

Taking log on both sides,

$\log y = \log x^{\log(\log x)}$

$\log y = \log(\log x) . \log x$

D. w.r to x,

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{\cancel{\log x}} \log(\log x) \frac{1}{x} + \cancel{\log x}\left(\frac{1}{\cancel{\log x}}\right)\frac{1}{x}$

$= \frac{\log(\log x)}{x} + \frac{1}{x}$

$\frac{dy}{dx} = \frac{y}{x}\left[\log(\log x) + 1\right]$

$= \frac{x^{\log(\log x)}}{x}\left[\log(\log x + 1)\right]$

X
Compulsory Question

3. $y = x^{\sin x} + x^{\tan x}$

Sol:

$y = u + v \quad ; \quad u = x^{\sin x}, \; v = x^{\tan x}$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$u = x^{\sin x}$

Taking 'log' on both sides,

$\log u = \log x^{\sin x}$

$\log u = \sin x . \log x$

$$\frac{1}{u}\cdot\frac{du}{dx} = \sin x\,(1/x) + \log x\,(\cos x)$$

$$\frac{du}{dx} = u\left[\sin x/x + \log x(\cos x)\right]$$

$$\frac{du}{dx} = x^{\sin x}\left[\sin x/x + \cos x\cdot\log x\right] \quad \to (1)$$

$$V = x^{\tan x}$$

$$\frac{dv}{dx} = ?$$

Taking 'Log' on both sides,

$$\log V = \log x^{\tan x}$$
$$= \tan x\cdot\log x$$

D w.r to x,

$$\frac{1}{V}\frac{dv}{dx} = \tan x\,(1/x) + \log x\,(\sec^2 x)$$

$$\frac{dv}{dx} = V\left[\tan x/x + \log x\,(\sec^2 x)\right]$$

$$= x^{\tan x}\left[\tan x/x + \sec^2 x\cdot\log x\right] \quad \to (2)$$

(1) + (2) $\Rightarrow$

$$\frac{dy}{dx} = x^{\sin x}\left[\sin x/x + \cos x\cdot\log x\right]$$
$$+ x^{\tan x}\left[\tan x/x + \sec^2 x\cdot\log x\right]$$

4. $$y = \sqrt[4]{\frac{x^5(2x^2-3)}{(1-2x)^2}}$$

$$y = \frac{x^{5/4}(2x^2-3)^{1/4}}{(1-2x)^{2/4}}$$

$$= \frac{x^{5/4}(2x^2-3)^{1/4}}{(1-2x)^{1/2}}$$

$$\log y = \log\left[\frac{x^{5/4}(2x^2-3)^{1/4}}{(1-2x)^{1/2}}\right]$$

$$= \log\left[(x)^{5/4}(2x^2-3)^{1/4}\right] - \log(1-2x)^{1/2}$$

$$= \log x^{5/4} + \log(2x^2-3)^{1/4} - \log(1-2x)^{1/2}$$

$$= \frac{5}{4}\log x + \frac{1}{4}\log(2x^2-3) - \frac{1}{2}\log(1-2x)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{5}{4}\left(\frac{1}{x}\right) + \frac{1}{4}\left(\frac{1}{2x^2-3}\right)(4x) - \frac{1}{2}\left(\frac{1}{1-2x}\right)(-2)$$

$$= y\left[\frac{5}{4x} + \frac{x}{2x^2-3} + \frac{1}{1-2x}\right]$$

$$\Rightarrow \sqrt[4]{\frac{x^5(2x^2-3)}{(1-2x)^2}}\left[\frac{5}{4x} + \frac{x}{2x^2-3} + \frac{1}{1-2x}\right].$$

5. $f(x,y) = 0$

$x^3 + y^3 = 3axy$ find $\frac{dy}{dx}$.

$$3x^2(1) + 3y^2\left(\frac{dy}{dx}\right) = 3a\left[x\frac{dy}{dx} + y(1)\right]$$

$$3x^2 + 3y^2\frac{dy}{dx} = 3a\left[x\frac{dy}{dx} + y\right]$$

$$3x^2 + 3y^2\frac{dy}{dx} = 3ax\frac{dy}{dx} + 3ay$$

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$$3y^2\frac{dy}{dx} - 3ax\frac{dy}{dx} = 3ay - 3x^2$$

$$\frac{dy}{dx}\left[3y^2 - 3ax\right] = 3ay - 3x^2$$

$$\therefore \frac{dy}{dx} = \frac{3ay - 3x^2}{3y^2 - 3ax}.$$

18.09.18

1) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ find $\frac{dy}{dx}$

D.W.r to x,

$$2ax + 2h\left[x\frac{dy}{dx} + y(1)\right] + 2by\frac{dy}{dx} + 2g(1) + 2f\frac{dy}{dx} + 0 = 0$$

$$\frac{dy}{dx}\left[2hx + 2by + 2f\right] = -\left[2ax + 2hy + 2g\right]$$

$$\frac{dy}{dx} = -\frac{\cancel{2}\left[ax + hy + g\right]}{\cancel{2}\left[hx + by + f\right]}$$

$$\therefore \frac{dy}{dx} = -\frac{\left[ax + hy + g\right]}{\left[hx + by + f\right]}$$

2) $x^y = e^{x-y}$ P.T $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

Sol:

Taking 'log' on both sides,

$$\log x^y = \log e^{x-y}$$

$$y\log x = (x-y)\log e \qquad [\because \log e = 1]$$

$$y\log x = x - y$$

$$y + y\log x = x$$

$$y(1+\log x) = x$$

$$y = \frac{x}{1+\log x}$$

$$\frac{dy}{dx} = \frac{(1+\log x)(1) - x\left[0 + \frac{1}{x}\right]}{(1+\log x)^2}$$

$$= \frac{1+\log x - x\left(\frac{1}{x}\right)}{(1+\log x)^2}$$

$$= \frac{\cancel{1} + \log x - \cancel{1}}{(1+\log x)^2}$$

$$\therefore \frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}.$$

Hence Proved.

3. $y = x^{n-1}\log x$, S.T $xy_1 = (n-1)y + x^{n-1}$.

Sol:

$$\frac{dy}{dx} = x^{n-1}\left(\frac{1}{x}\right) + \log x\left[(n-1)x^{n-1-1}(1)\right]$$

$$y_1 = x^{n-1}\left(\frac{1}{x}\right) + \log x\left[(n-1)x^{n-2}\right]$$

Multiply by 'x' on both sides,

$$xy_1 = \cancel{x}\left[x^{n-1}\left(\frac{1}{\cancel{x}}\right)\right] + x\log x\left[(n-1)x^{n-2}\right]$$

$$xy_1 = x^{n-1} + \log x\left[(n-1)x^{n-1}\right]$$

$$xy_1 = x^{n-1} + (n-1)y$$

$$\therefore \boxed{xy_1 = (n-1)y + x^{n-1}}$$

Hence Proved

---

4. $y = \left(\frac{1+x}{1-x}\right)^r$ PT $(1-x^2)y_1 = 2ry$.

$$y = \frac{(1+x)^r}{(1-x)^r}$$

Taking log on both sides,

$$\log y = \log(1+x)^r - \log(1-x)^r$$

$$\log y = r\log(1+x) - r\log(1-x)$$

D.w.r to x,

$$\frac{1}{y}\cdot\frac{dy}{dx} = r\cdot\left(\frac{1}{1+x}\right) - r\left(\frac{1}{1-x}\right)(-1)$$

$$\frac{1}{y}\cdot y_1 = \frac{r}{1+x} + \frac{r}{1-x}$$

$$= \frac{r(1-x) + r(1+x)}{1^2 - x^2}$$

$$= \frac{r - \cancel{rx} + r + \cancel{rx}}{1-x^2}$$

$$\frac{1}{y}\cdot y_1 = \frac{2r}{1-x^2}$$

$$\therefore \boxed{(1-x^2)y_1 = 2ry}.$$

$\therefore$ Hence Proved

5. $\sin y = x\sin(a+y)$ S.T $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

Sol

$$x = \frac{\sin y}{\sin(a+y)}$$

$$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y\cos(a+y)(0+1)}{[\sin(a+y)]^2}$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a+y)\cos y - \sin y\cos(a+y)}$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a} \quad \left[\because \sin A\cos B - \cos A\sin B \Rightarrow \sin(A-B)\right]$$

$\therefore$ Hence Proved

Parametric Differentiation

$$x = f(\theta)\ ;\ y = g(\theta)$$

$$\frac{dy}{dx} = \frac{dy}{d\theta}\cdot\frac{d\theta}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

1. $x = a\cos^3\theta$; $y = a\sin^3\theta$ find $\frac{dy}{dx}$.

D.w.r to x;

$$\frac{dx}{d\theta} = 3a\cos^2\theta(-\sin\theta) = -3a\cos^2\theta\sin\theta \rightarrow (1)$$

$$\frac{dy}{d\theta} = 3a\sin^2\theta(\cos\theta) = 3a\sin^2\theta\cos\theta \rightarrow (2)$$

$$\frac{(2)}{(1)} \Rightarrow \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3a\sin^{\cancel{2}}\theta\cancel{\cos\theta}}{-3a\cos^{\cancel{2}}\theta\cancel{\sin\theta}} \Rightarrow \frac{\sin\theta}{\cos\theta}$$

19.09.2018

1. $x = a(t\cos t - \sin t)$; $y = a(t\sin t + \cos t)$

find $\frac{dy}{dx}$.

Sol

$$\frac{dx}{dt} = a\left[t(-\sin t) + \cos t(1) - \cos t\right]$$

$$= a\left[-t\sin t + 0\right]$$

$$= -at\sin t$$

$$\frac{dy}{dt} = a\left[t\cos t + \sin t(1) - \sin t\right]$$

$$= at\cos t$$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{at\cos t}{-at\sin t}$$

$$= -\cot t$$

2. $x = \frac{2t}{1+t^2}$; $y = \frac{1-t^2}{1+t^2}$ find $\frac{dy}{dx}$.

Sol

$$\frac{dx}{dt} = \frac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2}$$

$$= \frac{2+2t^2-4t^2}{(1+t^2)^2}$$

$$= \frac{2-2t^2}{(1+t^2)^2}$$

$$\frac{dy}{dt} = \frac{(1+t^2)(-2t) - (1-t^2)(2t)}{(1+t^2)^2}$$

$$= \frac{-2t - \cancel{2t^3} - 2t + \cancel{2t^3}}{(1+t^2)^2}$$

$$= \frac{-4t}{(1+t^2)^2}$$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} \Rightarrow \frac{\frac{-4t}{(1+t^2)^2}}{\frac{2-2t^2}{(1+t^2)^2}}$$

$$= \frac{-4t}{2(1-t^2)}$$

$$= \frac{-2t}{1-t^2}$$

Successive Differentiation:

1. $y = a\cos 5x + b\sin 5x$; S.T $\frac{d^2y}{dx^2} + 25y = 0$.

$$\frac{dy}{dx} = a(-\sin 5x)(5) + b\cos 5x\,(5)$$

$$= -5a\sin 5x + 5b\cos 5x$$

$$\frac{d^2y}{dx^2} = -5a(\cos 5x)(5) + 5b(-\sin 5x)(5)$$

$$= -25a\cos 5x - 25b\sin 5x$$

$$= -\left[25a\cos 5x + 25b\sin 5x\right]$$

$$\frac{d^2y}{dx^2} = -25\left[a\cos 5x + b\sin 5x\right]$$

$$= -25y$$

$\therefore \frac{d^2y}{dx^2} + 25y = 0$

Hence Proved.

2. $y = a\cos(\log x) + b\sin(\log x)$ P.T $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Sol:

$\frac{dy}{dx} = -a\sin(\log x)(1/x) + b\cos(\log x)(1/x)$

$= -\frac{a\sin(\log x)}{x} + \frac{b\cos(\log x)}{x}$

$x\frac{dy}{dx} = -a\sin(\log x) + b\cos(\log x)$

$x\left(\frac{d^2y}{dx^2}\right) + \frac{dy}{dx}(1) = -a\cos(\log x)\frac{1}{x} + b[-\sin(\log x)]\frac{1}{x}$

$= -\frac{a\cos(\log x)}{x} - \frac{b\sin(\log x)}{x}$

$= -\frac{1}{x}[a\cos(\log x) + b\sin(\log x)]$

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -[a\cos(\log x) + b\sin(\log x)]$

$= -y$

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Hence Proved.

$\frac{1}{2}(1+x^2)^{-1/2}$

$\frac{(1+x^2)^{-1/2}}{2}$

3. $y = (x + \sqrt{1+x^2})^m$ P.T $(1+x^2)y_2 + xy_1 - m^2y = 0$.

Solution:

$(1+x^2)^{1/2}$

Taking log on both sides,

$\log y = \log(x + \sqrt{1+x^2})^m$

$\log y = m\log(x + \sqrt{1+x^2})$

$\frac{1}{y}\cdot\frac{dy}{dx} = m\left[\frac{1}{x+\sqrt{1+x^2}}\cdot\frac{(1)+(1+x^2)^{-1/2}(2x)}{2}\right]$

$= m\left[\frac{1}{x+\sqrt{1+x^2}} \times \frac{1 + (1+x^2)^{-1/2}(2x)}{2}\right]$

$= m\left[\frac{1}{x+\sqrt{1+x^2}}\right]\left[1 + \left(\frac{1}{2(\sqrt{1+x^2})}(2x)\right)\right]$

$= \frac{m}{x+\sqrt{1+x^2}}\left[1 + \frac{x}{\sqrt{1+x^2}}\right]$

$= \frac{m}{\cancel{x+\sqrt{1+x^2}}}\left[\frac{\cancel{\sqrt{1+x^2}+x}}{\sqrt{1+x^2}}\right]$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{m}{\sqrt{1+x^2}}$

$\frac{dy}{dx} = \frac{my}{\sqrt{1+x^2}}$

$\frac{d^2y}{dx^2} = m\left[\frac{\sqrt{1+x^2}\frac{dy}{dx} - y\cdot\frac{1}{2}(1+x^2)^{-1/2}(2x)}{(\sqrt{1+x^2})^2}\right]$

$\Rightarrow m\frac{\left[\sqrt{1+x^2}\frac{dy}{dx} - \frac{xy}{\sqrt{1+x^2}}\right]}{1+x^2}$

$$\frac{d^2y}{dx^2}(1+x^2) - m\left[\sqrt{1+x^2}\frac{dy}{dx} \cdot \frac{xy}{\sqrt{1+x^2}}\right]$$

$$\sqrt{1+x^2}\,\frac{dy}{dx} = my$$

Squaring on both sides,

$$(1+x^2)\left(\frac{dy}{dx}\right)^2 = m^2y^2$$

D.W.r to x,

$$(1+x^2)2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2(2x) = m^2(2y)\frac{dy}{dx}$$

$$\div 2\frac{dy}{dx} \Rightarrow \quad (1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = m^2y$$

Hence Proved.

Differentiation of Inverse & Signometric forms:

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

Problems:

1. If $y = \frac{1}{2}(\sin^{-1}x)^2$ P.T $(1-x^2)y_2 - xy_1 = 1$

Solution:

D.w.r to x,

$$\frac{dy}{dx} = \frac{1}{2} \times 2\,(\sin^{-1}x)\left(\frac{1}{\sqrt{1-x^2}}\right)$$

$$y_1 = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$$

$$(\sqrt{1-x^2})\,y_1 = \sin^{-1}x$$

Squaring on both sides,

$$(1-x^2)\,y_1^2 = (\sin^{-1}x)^2$$

$$(1-x^2)\,y_1^2 = 2y$$

$$\cancel{(1-x^2)\,2y_1(y_2) = 2}$$

$$(1-x^2)\,2y_1(y_2) + y_1^2(-2x) = 2\left(\frac{dy}{dx}\right)$$

$$(1-x^2)\,2y_1y_2 - 2xy_1^2 = 2y_1$$

$$2y_1\left[(1-x^2)y_2 - xy_1\right] = 2y_1 \cdot 1$$

$$\therefore (1-x^2)y_2 - xy_1 = 1$$

Hence Proved.

$$= x^2 + 6x^2\log x + [-2x^2(-1+3\log x)] + 3x^2$$

$$= x^2 + 6x^2\log x + [2x^2 - 6x^2\log x] + 3x^2$$

$$\sin^{-1} y = m\sin^{-1}x.$$

D.w.r to x,

$$\frac{1}{\sqrt{1-y^2}}\left(\frac{dy}{dx}\right) = m\left(\frac{1}{\sqrt{1-x^2}}\right)..$$

Sq $\Rightarrow$ $$\frac{1}{(1-y^2)}\left(\frac{dy}{dx}\right)^2 = \frac{m^2}{1-x^2}$$

$$(1-x^2)\,y_1^2 = (1-y^2)\,m^2$$

$$(1-x^2)(2y_1y_2) + y_1^2(-2x) = \left[-2y\left(\frac{dy}{dx}\right)m^2\right]$$

$$\cancel{2}y_1\left[(1-x^2)y_2 - xy_1\right] = -\cancel{2}y_1\left[ym^2\right]$$

$$(1-x^2)y_2 - xy_1 = -ym^2.$$

24.09.18

UNIT - $\overline{V}$

INTEGRATION

Standard formulas:

1.) $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

2.) $\int \frac{1}{x} dx = \log x + c$

3.) $\int e^x dx = e^x + c$

4.) $\int \sin x \, dx = -\cos x + c$

5.) $\int \cos x \, dx = \sin x + c$

6.) $\int \sec^2 x \, dx = \tan x + c$

7.) $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

8.) $\int \sec x \cdot \tan x = \sec x + c$

9.) $\int \operatorname{cosec} x \cdot \cot x \, dx = -\operatorname{cosec} x + c$

10.) $\int \frac{dx}{1+x^2} = \tan^{-1} x + c$

11.) $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c$

12.) $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$

13.) $\int dx = x + c$

Standard formulas / rules:

1. $\int K f(x) \, dx = K \int f(x) \, dx$

2. $\int (f(x) \pm g(x)) \, dx = \int f(x) \, dx \pm \int g(x) \, dx$

3. $\int u dv = uv - \int v du$.

[∵ I LATE]

PROBLEMS:

1. $\int \frac{(x^2+2x-1)}{\sqrt{x}} dx$

$\Rightarrow \int (x^2+2x-1)(x^{-1/2}) \, dx$

$\Rightarrow \int [x^2 \cdot x^{-1/2} + 2x \cdot x^{-1/2} - x^{-1/2}] \, dx$

$= \int (x^{3/2} + 2x^{1/2} - x^{-1/2}) \, dx$

$= \frac{x^{5/2}}{5/2} + \frac{2x^{3/2}}{3/2} - \frac{x^{1/2}}{1/2} + C$

$= \frac{2x^{5/2}}{5} + \frac{4x^{3/2}}{3} - 2x^{1/2} + C$.

2. $\int (x-1)(x+2) \, dx$.

$\Rightarrow \int (x^2+2x-x-2) \, dx$

$= \frac{x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^2}{2} - 2x + C$

$= \frac{x^3}{3} + \frac{x^2}{2} + x^2 - 2x + C \Rightarrow \frac{x^3}{3} + \frac{x^2}{2} - 2x + C$

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3. $\int (x+\frac{1}{x})^3 dx$

$\because (a+b)^3 = \cancel{(a+b)(a^2-ab+b^2)}\ a^3+3a^2b+3ab^2+b^3$

$\int (x+\frac{1}{x})^3 dx = \int \left[x^3+3x^2(\frac{1}{x})+3x(\frac{1}{x^2})+(\frac{1}{x^3})\right] dx$

$= \int (x^3+3x+\frac{3}{x}+\frac{1}{x^3}) dx$

$= \int [x^3+3x+\cancel{3x}\ \frac{3}{x}+x^{-3}] dx$

$= \frac{x^4}{4}+\frac{3x^2}{2}+3\log x+\frac{x^{-2}}{(-2)}+c$

$= \frac{x^4}{4}+\frac{3x^2}{2}+3\log x-\frac{x^{-2}}{2}+c$

$= \frac{x^4}{4}+\frac{3x^2}{2}+3\log x-\frac{1}{2x^2}+c$

4. $\int \frac{1}{1-\cos x} dx$

$\Rightarrow \int (\frac{1}{1-\cos x}) dx$

$= \int \left[\frac{1}{1-\cos x} \times \frac{1+\cos x}{1+\cos x}\right] dx$

$= \int \frac{(1+\cos x)}{(1-\cos^2 x)} dx$

$= \int \left(\frac{1+\cos x}{\sin^2 x}\right) dx$

$= \int \left(\frac{1}{\sin^2 x}+\frac{\cos x}{\sin^2 x}\right) dx$

$= \int \left(\operatorname{cosec}^2 x+\frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}\right) dx$

$= \int (\operatorname{cosec}^2 x+\cot x\cdot \operatorname{cosec} x) dx$

$= -\cot x - \operatorname{cosec} x + c$.

5. $\int \frac{1}{\sin^2 x\cos^2 x} dx$.

$\Rightarrow \int \frac{1}{\sin^2 x}\cdot\frac{1}{\cos^2 x} dx$

~~$\int \operatorname{cosec}^2 x.\ \sec^2 x\ dx$.~~

$\Rightarrow \int \frac{(\sin^2 x+\cos^2 x)}{\sin^2 x\cdot\cos^2 x} dx$

$= \int \left[\frac{\cancel{\sin^2 x}}{\cancel{\sin^2 x}\cdot\cos^2 x}+\frac{\cancel{\cos^2 x}}{\sin^2 x\cancel{\cos^2 x}}\right] dx$

$= \int \left[\frac{1}{\cos^2 x}+\frac{1}{\sin^2 x}\right] dx$

$= \int \sec^2 x+\operatorname{cosec}^2 x\ dx$

$= \tan x - \cot x + c$

6. $\int \sqrt{1+\sin 2x}\, dx$.

$\Rightarrow \int \sqrt{\sin^2 x+\cos^2 x + 2\sin x \cos x}\ dx$

$= \int \sqrt{(\sin x+\cos x)^2}\ dx$

$= \int (\sin x+\cos x)\, dx$

$= -\cos x + \sin x + C$

$= \sin x - \cos x + C$

25.09.18 Maths . C.W

Formulas:

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A \sin B = \sin(A+B) - \sin(A-B)$

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$\sin 2x = 2\sin x \cos x$ $\quad$ $\sin 3x = 3\sin x - 4\sin^3 x$

$\sin^2 x = \frac{1-\cos 2x}{2}$ $\quad$ $\sin^3 x = \frac{3\sin x - \sin 3x}{4}$

$\cos^2 x = \frac{1+\cos 2x}{2}$ $\quad$ $\cos^3 x = \frac{\cos 3x + 3\cos x}{4}$

$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$

$\int \frac{1}{(ax+b)} dx = \frac{\log(ax+b)}{a} + C$

$\int \sin(ax+b)\, dx = \frac{-\cos(ax+b)}{a} + C$

PROBLEMS:

1. $\int (\sin 3x \cos 2x)\, dx$ $\quad$ $[\because 2\sin A\cos B]$

$\Rightarrow \frac{\sin(3x+2x) + \sin(3x-2x)}{2}$

$\Rightarrow \frac{\sin 5x}{2} + \frac{\sin x}{2}$

$= \int \frac{\sin 5x}{2} dx + \int \frac{\sin x}{2} dx$

$= \frac{1}{2}\int \sin 5x\, dx + \frac{1}{2}\int \sin x\, dx$

$= \frac{1}{2}\frac{(-\cos 5x)}{5} + \frac{1}{2}(-\cos x) + C$

$= -\frac{1}{2}\left[\frac{\cos 5x}{5} + \cos x\right] + C$

2. $\int (\sin 2x \sin 5x)\, dx$ $\quad$ $[\because 2\sin A \sin B]$

$\Rightarrow \frac{\cos(2x-5x) - \cos(2x+5x)}{2}$

$= \frac{\cos(-3x)}{2} - \frac{\cos(7x)}{2}$

$= \frac{1}{2}(\cos 3x - \cos 7x)$

$= \frac{1}{2}\left[\int (\cos 3x)\, dx - \int (\cos 7x) dx\right]$

$= \frac{1}{2}\left[\frac{\sin 3x}{3} - \frac{\sin 7x}{7}\right] + C$

$= \frac{\sin 3x}{6} - \frac{\sin 7x}{14} + C$

3. $\int \sin^3 x \, dx$

$\sin^3 x = \frac{3\sin x - \sin 3x}{4}$

$\int (\sin^3 x) dx = \int \frac{3}{4}(\sin x) dx - \int \frac{1}{4}(\sin 3x) dx$

$= \frac{3}{4} \int \sin x \, dx - \frac{1}{4} \int \sin 3x \, dx$

$= \frac{3}{4}(-\cos x) - \frac{1}{4}\left[-\frac{\cos 3x}{3}\right] + C$

$= -\frac{3}{4}(\cos x) + \frac{1}{12}(\cos 3x) + C$

$= \frac{1}{4}\left[-3\cos x + \frac{\cos 3x}{3}\right] + C$

4. $\int \cos^4 x \, dx$

$= (\cos^2 x)^2$

$= \left(\frac{1+\cos 2x}{2}\right)^2$

$= \frac{1}{4}(1+\cos 2x)^2$

$= \frac{1}{4}\left[1 + 2\cos 2x + \cos^2 2x\right]$

$= \frac{1}{4}\left[1 + 2\cos 2x + \frac{1+\cos 2(2x)}{2}\right]$

$= \frac{1}{4}\left[1 + 2\cos 2x + \frac{1}{2} + \frac{\cos 4x}{2}\right]$

$= \frac{1}{4}\left[\frac{3}{2} + 2\cos 2x + \frac{1}{2}\cos 4x\right]$

$\int \cos^4 x \, dx = \frac{1}{4} \int \left[\frac{3}{2} + 2\cos 2x + \frac{1}{2}\cos 4x\right] dx$

$= \frac{1}{4}\left[\frac{3x}{2} + (\cancel{2})\frac{\sin 2x}{\cancel{2}} + \frac{1}{2}\left(\frac{\sin 4x}{4}\right)\right] + C$

$= \frac{1}{4}\left[\frac{3x}{2} + \sin 2x + \frac{\sin 4x}{4}\right] + C.$

5. $\int \frac{x}{(ax+b)^{1/3}} dx$

let $(ax+b) = t \Rightarrow \boxed{x = \frac{t-b}{a}}$

$a \cdot dx + 0 = dt$

$\boxed{dx = \frac{dt}{a}}$

$\int \frac{x}{(ax+b)^{1/3}} dx \Rightarrow \int \frac{\left(\frac{t-b}{a}\right)}{t^{1/3}} \frac{dt}{a}$

$= \frac{1}{a} \int \frac{(t-b)}{a t^{1/3}} dt$

$= \frac{1}{a^2} \int (t-b) \cdot t^{-1/3} dt$

$= \frac{1}{a^2} \int (t \cdot t^{-1/3} - b \cdot t^{-1/3}) dt$

$= \frac{1}{a^2} \int (t^{2/3} - b \cdot t^{-1/3}) dt$

$= \frac{1}{a^2}\left[\frac{t^{5/3}}{5/3} - b\left(\frac{t^{2/3}}{2/3}\right)\right] + C$

$= \frac{1}{a^2}\left[\frac{3t^{5/3}}{5} - \frac{3bt^{2/3}}{2}\right] + C.$

$$= \frac{1}{a^2}\left[\frac{3(ax+b)^{5/3}}{5} - \frac{3b(ax+b)^{2/3}}{2}\right] + C.$$

6. $\int \frac{3x+2}{(3x^2+4x+7)^3} dx$

$\Rightarrow$ let $t = (3x^2+4x+7)$ $\Rightarrow$ $t - 7 = 3x^2 + 4x$

$\Rightarrow x = \frac{t-3x^2-7}{4}$

$dt = 6x + 4(1)$

$\frac{dt}{dx} = 6x+4$ $\Rightarrow (6x+4)dx$

$\Rightarrow dt = 2(3x+2)dx$

$\frac{dt}{2} = (3x+2)dx$

$\int \frac{3x+2}{(3x^2+4x+7)^3} dx = \int$

$\Rightarrow \int \frac{1}{t^3} \frac{dt}{2}$

$= \frac{1}{2} \int t^{-3} dt$

$= \frac{1}{2}\left[\frac{t^{-2}}{-2}\right] + C$

$= -\frac{1}{4}(3x^2+4x+7)^{-2} + C$

$= -\frac{1}{4(3x^2+4x+7)^2} + C$

04.10.18

Maths CW

Integration By parts:

$$\int u\,dv = uv - \int v\,du$$

ILATE

Bernoullie's formula (Use when algebraic & trigonometric forms are given)

$$\int u\,dv = uv - u'v_1 + u''v_2 - u'''v_3 \cdots$$

$u \to$ Diff $\quad v \to$ Integration

PROBLEMS

1. $\int e^{2x} \cos x\,dx$
   U V

2. $\int x^3 \sin x\,dx$
   U V

$$\int u\,dv = uv_1 - u'v_2 + u''v_3 - u'''v_4$$

$\cancel{\int x^3 \sin x = x^3 \sin x - 3x^2(-\cos x) + 6x(-\sin x) - 6(\cos x) + C}$

$\cancel{= x^3 \sin x + 3x^2\cos x - 6x\sin x - 6\cos x + C}$

$$\int x^3 \sin x\,dx = x^3(-\cos x) - 3x^2(-\sin x) + 6x(\cos x) - 6(\sin x) + C.$$

$$= -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + C.$$

③ $\int \underset{u}{e^x} \, (\underset{v}{\sin x + \cos x}) \, dx$

Sol:

6. By Bernouli's theorem,

$$\int uv \, dx = uv_1 - u'v_2 + u''v_3 - \dots$$

$u = e^x$, $v = \sin x + \cos x$

$u' = e^x$ $\quad v_1 = -\cos x + \sin x$

$u'' = e^x$ $\quad v_2 = -\sin x - \cos x$

$v_3 = \cos x - \sin x$

$\Rightarrow e^x(\sin x - \cos x) - e^x(-\sin x - \cos x) + e^x(\cos x - \sin x) + c$

$= e^x(\sin x - \cos x) + e^x(\sin x + \cos x) + e^x(\cos x - \sin x) + c$