

Part 1

You are expected to show/derive each weight update rule in detail and in a step-by-step fashion by utilizing the backpropagation algorithm for both regression and classification problems represented here.

Regression problem:

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$$SE(y, y') = (y - y')^2$$

$$O_0^{(0)} = 1, O_1^{(0)} = x_1, O_2^{(0)} = x_2$$

$$O_0^{(1)} = 1$$

$$O_k^{(1)} = \sigma\left(\sum_{i=0} O_i^{(0)} \cdot a_{ik}^{(0)}\right)$$

$$O_1^{(1)} = \sigma\left(1 \cdot a_{01}^{(0)} + x_1 \cdot a_{11}^{(0)} + x_2 \cdot a_{21}^{(0)}\right)$$

$$= \frac{1}{1 + e^{-(a_{01}^{(0)} + x_1 a_{11}^{(0)} + x_2 a_{21}^{(0)})}}$$

$$O_2^{(1)} = \sigma\left(1 \cdot a_{02}^{(0)} + x_1 \cdot a_{12}^{(0)} + x_2 \cdot a_{22}^{(0)}\right)$$

$$= \frac{1}{1 + e^{-(a_{02}^{(0)} + x_1 a_{12}^{(0)} + x_2 a_{22}^{(0)})}}$$

$$O_3^{(1)} = \sigma\left(1 \cdot a_{03}^{(0)} + x_1 \cdot a_{13}^{(0)} + x_2 \cdot a_{23}^{(0)}\right)$$

$$= \frac{1}{1 + e^{-(a_{03}^{(0)} + x_1 a_{13}^{(0)} + x_2 a_{23}^{(0)})}}$$

$$O_0^{(2)} = \sum_{k=0} O_k^{(1)} \cdot a_{k0}^{(1)}$$

$$= 1 \cdot a_{00}^{(1)} + O_1^{(1)} \cdot a_{10}^{(1)} + O_2^{(1)} \cdot a_{20}^{(1)} + O_3^{(1)} \cdot a_{30}^{(1)}$$

Weights = $a_{ik}^{(0)} = a_{ik}^{(0)} - \alpha \frac{\delta SE(y, O_0^{(2)})}{\delta a_{ik}^{(0)}}$

$$\frac{\delta (y - O_0^{(2)})^2}{\delta a_{ik}} = \frac{2(y - O_0^{(2)}) \cdot \delta(O_0^{(2)})}{\delta a_{ik}}$$

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$$1 \quad a_{01}^{(0)} = a_{01}^{(0)} - \alpha (2(y - O_1^{(1)})) \cdot \frac{\delta O_1^{(1)}}{\delta a_{01}}$$

$$a_{01}^{(0)} = a_{01}^{(0)} - \alpha (2(y - O_1^{(1)})) \cdot \frac{\delta \sigma(a_{01}^{(0)} + x_1 a_{11}^{(0)} + x_2 a_{21}^{(0)})}{\delta a_{01}}$$

$$a_{01}^{(0)} = a_{01}^{(0)} - \alpha (2(y - O_1^{(1)})) \cdot \frac{e^{-a_{01}^{(0)} - x_1 a_{11}^{(0)} - x_2 a_{21}^{(0)}}}{(1 + e^{-a_{01}^{(0)} - x_1 a_{11}^{(0)} - x_2 a_{21}^{(0)}})^2}$$

$$a_{01}^{(0)} = a_{01}^{(0)} + 2\alpha (y - O_1^{(1)}) \frac{e^{-a_{01}^{(0)} - x_1 a_{11}^{(0)} - x_2 a_{21}^{(0)}}}{(1 + e^{-a_{01}^{(0)} - x_1 a_{11}^{(0)} - x_2 a_{21}^{(0)}})^2}$$

$$2 \quad a_{11}^{(0)} = a_{11}^{(0)} + 2\alpha (y - O_1^{(1)}) \cdot \frac{\delta O_1^{(1)}}{\delta a_{11}^{(0)}}$$

$$a_{11}^{(0)} = a_{11}^{(0)} + 2\alpha (y - O_1^{(1)}) \cdot \frac{e^{-a_{01}^{(0)} - x_1 a_{11}^{(0)} - x_2 a_{21}^{(0)}}}{(1 + e^{-a_{01}^{(0)} - x_1 a_{11}^{(0)} - x_2 a_{21}^{(0)}})^2}$$

$$3 \quad a_{21}^{(0)} = a_{21}^{(0)} + 2\alpha (y - O_1^{(1)}) \cdot \frac{\delta O_1^{(1)}}{\delta a_{21}^{(0)}}$$

$$a_{21}^{(0)} = a_{21}^{(0)} + 2\alpha (y - O_1^{(1)}) \cdot \frac{e^{-a_{01}^{(0)} - x_1 a_{11}^{(0)} - x_2 a_{21}^{(0)}}}{(1 + e^{-a_{01}^{(0)} - x_1 a_{11}^{(0)} - x_2 a_{21}^{(0)}})^2}$$

$$4 \quad a_{02}^{(0)} = a_{02}^{(0)} + 2\alpha (y - O_2^{(1)}) \cdot \frac{\delta O_2^{(1)}}{\delta a_{02}}$$

$$a_{02}^{(0)} = a_{02}^{(0)} + 2\alpha (y - O_2^{(1)}) \cdot \frac{e^{-a_{02}^{(0)} - x_1 a_{12}^{(0)} - x_2 a_{22}^{(0)}}}{(1 + e^{-a_{02}^{(0)} - x_1 a_{12}^{(0)} - x_2 a_{22}^{(0)}})^2}$$

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$$5 \quad a_{12}^{(0)} = a_{12}^{(0)} + 2\alpha(y - O_2^{(1)}) \cdot \frac{\delta O_2^{(1)}}{\delta a_{12}^{(0)}}$$

$$a_{12}^{(0)} = a_{12}^{(0)} + 2\alpha(y - O_2^{(1)}) \cdot \frac{\kappa_1 e^{-a_{02}^{(0)} - \kappa_1 a_{12}^{(0)} - \kappa_2 a_{22}^{(0)}}}{(1 + e^{-a_{02}^{(0)} - \kappa_1 a_{12}^{(0)} - \kappa_2 a_{22}^{(0)}})^2}$$

$$6 \quad a_{22}^{(0)} = a_{22}^{(0)} + 2\alpha(y - O_2^{(1)}) \cdot \frac{\delta O_2^{(1)}}{\delta a_{22}^{(0)}}$$

$$a_{22}^{(0)} = a_{22}^{(0)} + 2\alpha(y - O_2^{(1)}) \cdot \frac{\kappa_2 e^{-a_{02}^{(0)} - \kappa_1 a_{12}^{(0)} - \kappa_2 a_{22}^{(0)}}}{(1 + e^{-a_{02}^{(0)} - \kappa_1 a_{12}^{(0)} - \kappa_2 a_{22}^{(0)}})^2}$$

$$7 \quad a_{03}^{(0)} = a_{03}^{(0)} + 2\alpha(y - O_3^{(1)}) \cdot \frac{\delta O_3^{(1)}}{\delta a_{03}^{(0)}}$$

$$a_{03}^{(0)} = a_{03}^{(0)} + 2\alpha(y - O_3^{(1)}) \cdot \frac{e^{-a_{03}^{(0)} - \kappa_1 a_{13}^{(0)} - \kappa_2 a_{23}^{(0)}}}{(1 + e^{-a_{03}^{(0)} - \kappa_1 a_{13}^{(0)} - \kappa_2 a_{23}^{(0)}})^2}$$

$$8 \quad a_{13}^{(0)} = a_{13}^{(0)} + 2\alpha(y - O_3^{(1)}) \cdot \frac{\delta O_3^{(1)}}{\delta a_{13}^{(0)}}$$

$$a_{13}^{(0)} = a_{13}^{(0)} + 2\alpha(y - O_3^{(1)}) \cdot \frac{\kappa_1 e^{-a_{03}^{(0)} - \kappa_1 a_{13}^{(0)} - \kappa_2 a_{23}^{(0)}}}{(1 + e^{-a_{03}^{(0)} - \kappa_1 a_{13}^{(0)} - \kappa_2 a_{23}^{(0)}})^2}$$

$$9 \quad a_{23}^{(0)} = a_{23}^{(0)} + 2\alpha(y - O_3^{(1)}) \cdot \frac{\delta O_3^{(1)}}{\delta a_{23}^{(0)}}$$

$$a_{23}^{(0)} = a_{23}^{(0)} + 2\alpha(y - O_3^{(1)}) \cdot \frac{\kappa_2 e^{-a_{03}^{(0)} - \kappa_1 a_{13}^{(0)} - \kappa_2 a_{23}^{(0)}}}{(1 + e^{-a_{03}^{(0)} - \kappa_1 a_{13}^{(0)} - \kappa_2 a_{23}^{(0)}})^2}$$

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$$10 \quad a_{00}^{(1)} = a_{00}^{(0)} - \alpha (2(y - O_0^{(2)})) \cdot \frac{\delta O_0^{(2)}}{\delta a_{00}^{(0)}}$$

$$a_{00}^{(1)} = a_{00}^{(0)} - \alpha (2(y - O_0^{(2)})) \cdot -1$$

$$a_{00}^{(1)} = a_{00}^{(0)} + 2\alpha (y - O_0^{(2)})$$

$$11 \quad a_{10}^{(1)} = a_{10}^{(0)} + 2\alpha (y - O_0^{(2)}) \cdot \frac{\delta O_0^{(2)}}{\delta a_{10}^{(0)}}$$

$$a_{10}^{(1)} = a_{10}^{(0)} + 2\alpha (y - O_0^{(2)}) \cdot O_1^{(1)}$$

$$a_{10}^{(1)} = a_{10}^{(0)} + 2\alpha (y - O_0^{(2)}) \cdot \frac{1}{1 + e^{-a_{01}^{(0)} - x_{11}^{(0)} - x_{21}^{(0)}}}$$

$$12 \quad a_{20}^{(1)} = a_{20}^{(0)} + 2\alpha (y - O_0^{(2)}) \cdot \frac{\delta O_0^{(2)}}{\delta a_{20}^{(0)}}$$

$$a_{20}^{(1)} = a_{20}^{(0)} + 2\alpha (y - O_0^{(2)}) \cdot O_2^{(1)}$$

$$a_{20}^{(1)} = a_{20}^{(0)} + 2\alpha (y - O_0^{(2)}) \cdot \frac{1}{1 + e^{-a_{02}^{(0)} - x_{12}^{(0)} - x_{22}^{(0)}}}$$

$$13 \quad a_{30}^{(1)} = a_{30}^{(0)} + 2\alpha (y - O_0^{(2)}) \cdot \frac{\delta O_0^{(2)}}{\delta a_{30}^{(0)}}$$

$$a_{30}^{(1)} = a_{30}^{(0)} + 2\alpha (y - O_0^{(2)}) \cdot O_3^{(1)}$$

$$a_{30}^{(1)} = a_{30}^{(0)} + 2\alpha (y - O_0^{(2)}) \cdot \frac{1}{1 + e^{-a_{03}^{(0)} - x_{13}^{(0)} - x_{23}^{(0)}}}$$

Classification problem:

$$CE(L, L') = - \sum_i l_i \log(l'_i)$$

$$a_{kn}^{(1)} = a_{kn}^{(0)} - \alpha \frac{\partial CE}{\partial a_{kn}^{(0)}}$$

$$O_0^{(0)} = 1, O_1^{(0)} = x_1, O_2^{(0)} = x_2$$

$$O_k^{(1)} = \sigma\left(\sum_{i=0} O_i^{(0)} \cdot a_{ik}^{(0)}\right)$$

$$O_0^{(1)} = 1$$

$$O_1^{(1)} = \sigma(1 \cdot a_{01}^{(0)} + x_1 \cdot a_{11}^{(0)} + x_2 \cdot a_{21}^{(0)})$$

$$O_1^{(1)} = \frac{1}{1 + e^{-(a_{01}^{(0)} + x_1 a_{11}^{(0)} + x_2 a_{21}^{(0)})}}$$

$$O_2^{(1)} = \frac{1}{1 + e^{-(a_{02}^{(0)} + x_1 a_{12}^{(0)} + x_2 a_{22}^{(0)})}}$$

$$O_3^{(1)} = \frac{1}{1 + e^{-(a_{03}^{(0)} + x_1 a_{13}^{(0)} + x_2 a_{23}^{(0)})}}$$

$$\frac{\partial CE}{\partial a_{kn}} = - \sum_{i=0} l_i \log(l'_i)$$

$$= \sum \frac{l_i}{l'_i} \cdot \frac{\delta(l'_i)}{\delta a_{kn}}$$

$$CE = O^T = - \sum_{i=0}^3 l_i \log(O_i^{(1)})$$

$$CE = (l_0 \log(O_0^{(1)}) + l_1 \log(O_1^{(1)}) + l_2 \log(O_2^{(1)}) + l_3 \log(O_3^{(1)}))$$

$$\begin{aligned} 1 \quad a_{01}^{(1)} &= a_{01}^{(0)} - \alpha \frac{\delta CE}{\delta a_{01}^{(0)}} \\ &= a_{01}^{(0)} - \alpha \left(l_0 \log(1) + l_1 \log\left(\frac{1}{1 + e^{-a_{01}^{(0)} - \eta_1 a_{11}^{(0)} - \eta_2 a_{21}^{(0)}}}\right) \right. \\ &\quad \left. + l_2 \log\left(\frac{1}{1 + e^{-a_{01}^{(0)} - \eta_1 a_{11}^{(0)} - \eta_2 a_{21}^{(0)}}}\right) \right. \\ &\quad \left. + l_3 \log\left(\frac{1}{1 + e^{-a_{01}^{(0)} - \eta_1 a_{11}^{(0)} - \eta_2 a_{21}^{(0)}}}\right) \right) \end{aligned}$$

$$a_{01}^{(1)} = a_{01}^{(0)} + \alpha \cdot \frac{l_1 e^{-a_{01}^{(0)} - \eta_1 a_{11}^{(0)} - \eta_2 a_{21}^{(0)}}}{1 + e^{-a_{01}^{(0)} - \eta_1 a_{11}^{(0)} - \eta_2 a_{21}^{(0)}}}$$

$$2 \quad a_{11}^{(1)} = a_{11}^{(0)} + \alpha \cdot \frac{l_1 \eta_1 e^{-a_{01}^{(0)} - \eta_1 a_{11}^{(0)} - \eta_2 a_{21}^{(0)}}}{1 + e^{-a_{01}^{(0)} - \eta_1 a_{11}^{(0)} - \eta_2 a_{21}^{(0)}}}$$

$$3 \quad a_{21}^{(1)} = a_{21}^{(0)} + \alpha \cdot \frac{l_1 \eta_2 e^{-a_{01}^{(0)} - \eta_1 a_{11}^{(0)} - \eta_2 a_{21}^{(0)}}}{1 + e^{-a_{01}^{(0)} - \eta_1 a_{11}^{(0)} - \eta_2 a_{21}^{(0)}}}$$

$$4 \quad a_{02}^{(1)} = a_{02}^{(0)} + \alpha \cdot \frac{l_2 e^{-a_{01}^{(0)} - \eta_1 a_{11}^{(0)} - \eta_2 a_{21}^{(0)}}}{1 + e^{-a_{01}^{(0)} - \eta_1 a_{11}^{(0)} - \eta_2 a_{21}^{(0)}}}$$

$$5 \quad a_{12}^{(1)} = a_{11}^{(0)} + \alpha \cdot l_2 \eta_1 \neq 4$$

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$$6 \quad a_{22}^{(2)} = a_{22}^{(1)} + \alpha \frac{L_2 \eta_2 e^{-a_{02}^{(1)} - \eta_1 a_{12}^{(1)} - \eta_2 a_{22}^{(1)}}}{1 + e^{-a_{02}^{(1)} - \eta_1 a_{12}^{(1)} - \eta_2 a_{22}^{(1)}}}$$

$$7 \quad a_{03}^{(2)} = a_{03}^{(1)} + \alpha \frac{L_3 e^{-a_{03}^{(1)} - \eta_1 a_{13}^{(1)} - \eta_2 a_{23}^{(1)}}}{1 + e^{-a_{03}^{(1)} - \eta_1 a_{13}^{(1)} - \eta_2 a_{23}^{(1)}}}$$

$$8 \quad a_{13}^{(2)} = a_{13}^{(1)} + \alpha \frac{L_3 \eta_1 e^{-a_{03}^{(1)} - \eta_1 a_{13}^{(1)} - \eta_2 a_{23}^{(1)}}}{1 + e^{-a_{03}^{(1)} - \eta_1 a_{13}^{(1)} - \eta_2 a_{23}^{(1)}}}$$

$$4 \quad a_{23}^{(2)} = a_{23}^{(1)} + \alpha \frac{L_3 \eta_2 e^{-a_{03}^{(1)} - \eta_1 a_{13}^{(1)} - \eta_2 a_{23}^{(1)}}}{1 + e^{-a_{03}^{(1)} - \eta_1 a_{13}^{(1)} - \eta_2 a_{23}^{(1)}}}$$

$$O_n^{(2)} = \text{softmax}(X_0^{(1)}, X^{(2)}) = \frac{e^{x_n^{(2)}}}{e^{x_0^{(2)}} + e^{x_1^{(2)}} + e^{x_2^{(2)}}}$$

$$X_0^{(2)} = \sum_{k=0} O_k^{(1)} \cdot a_{kn}^{(1)} \\ = O_0^{(1)} \cdot a_{00}^{(1)} + O_1^{(1)} \cdot a_{10}^{(1)} + O_2^{(1)} \cdot a_{20}^{(1)} + O_3^{(1)} \cdot a_{30}^{(1)}$$

$$X_1^{(2)} = O_0^{(1)} \cdot a_{01}^{(1)} + O_1^{(1)} \cdot a_{11}^{(1)} + O_2^{(1)} \cdot a_{21}^{(1)} + O_3^{(1)} \cdot a_{31}^{(1)}$$

$$X_2^{(2)} = O_0^{(1)} \cdot a_{02}^{(1)} + O_1^{(1)} \cdot a_{12}^{(1)} + O_2^{(1)} \cdot a_{22}^{(1)} + O_3^{(1)} \cdot a_{32}^{(1)}$$

$$O_0^{(2)} = \frac{e^{x_0^{(2)}}}{e^{x_0^{(2)}} + e^{x_1^{(2)}} + e^{x_2^{(2)}}}$$

$$O_1^{(2)} = \frac{e^{x_1^{(2)}}}{e^{x_0^{(2)}} + e^{x_1^{(2)}} + e^{x_2^{(2)}}}$$

$$O_2^{(2)} = \frac{e^{x_2^{(2)}}}{e^{x_0^{(2)}} + e^{x_1^{(2)}} + e^{x_2^{(2)}}}$$

This is
Child abuse!!!

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$$CF = -\sum_{i=0}^2 (r_i \log(O_i^{(2)}))$$

$$= -\left(L_0 \log \frac{e^{x_0^{(2)}}}{e^{x_0^{(2)}} + e^{x_1^{(2)}} + e^{x_2^{(2)}}} + L_1 \log \frac{e^{x_1^{(2)}}}{e^{x_0^{(2)}} + e^{x_1^{(2)}} + e^{x_2^{(2)}}} + L_2 \log \frac{e^{x_2^{(2)}}}{e^{x_0^{(2)}} + e^{x_1^{(2)}} + e^{x_2^{(2)}}} \right)$$

b $a_{oo}^{(1)} = a_{oo}^{(0)} - \alpha \cdot \frac{CF}{\partial a_{oo}^{(0)}}$

$a_{oo}^{(1)} = a_{oo}^{(0)} + \alpha \cdot \text{on PC}$

```
clc; clear; close all; format long;
```

```
%% Variable Definition
```

```
syms O0 O1 O2 O3
```

```
syms a00 a10 a20 a30 a01 a11 a21 a02 a12 a22 a03 a13 a23 a31 a32
```

```
syms x1 x2
```

```
syms L0 L1 L2
```

```
syms b0 b1 b2 % the softmax function
```

```
% Df = diff(f,var) % Derivates the function f with respect to var
```

```
%% Equations to Derivate
```

```
O0 = 1;
```

```
O1 = 1/(1+exp(-(a01+x1*a11+x2*a21)));
```

```
O2 = 1/(1+exp(-(a02+x1*a12+x2*a22)));
```

```
O3 = 1/(1+exp(-(a03+x1*a13+x2*a23)));
```

```
b0 = O0*a00+O1*a10+O2*a20+O3*a30;
```

```
b1 = O0*a01+O1*a11+O2*a21+O3*a31;
```

```
b2 = O0*a02+O1*a12+O2*a22+O3*a32;
```

```
C = -( ...  
    +L0*log(exp(b0)/(exp(b0)+exp(b1)+exp(b2))) ...  
    +L1*log(exp(b1)/(exp(b0)+exp(b1)+exp(b2))) ...  
    +L2*log(exp(b2)/(exp(b0)+exp(b1)+exp(b2))) ...  
    );
```

```
dC = diff(C,a00);
```

Figure 1 taking the derivative of CE

Tries to derivate and the results are way too long to write down hence probably what am doing is wrong however working makes sense.

if this answer is acceptable then yes we do the same thing for other weights and we get each weight value.