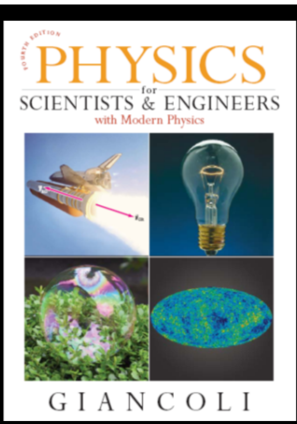


Lecture PowerPoints

Chapter 21

Physics for Scientists and Engineers, with Modern Physics, 4th edition

Giancoli

Chapter 21 Electric Charge and Electric Field

Units of Chapter 21

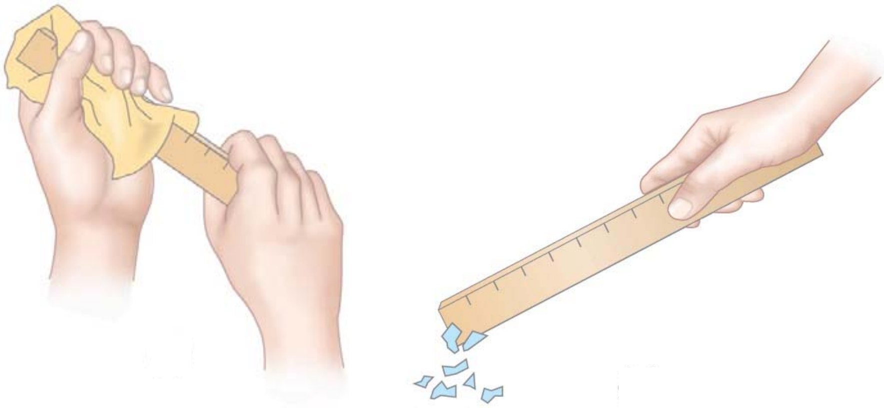
- Static Electricity; Electric Charge and Its Conservation
- Electric Charge in the Atom
- Insulators and Conductors
- Induced Charge; the Electroscope
- Coulomb's Law
- The Electric Field
- Electric Field Calculations for Continuous Charge Distributions

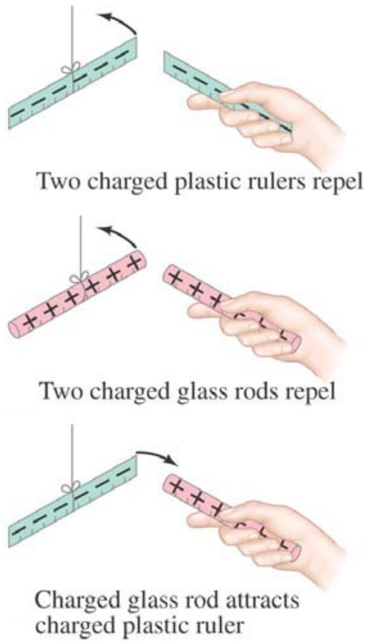
Units of Chapter 21

- Field Lines
- Electric Fields and Conductors
- Motion of a Charged Particle in an Electric Field

21-1 Static Electricity; Electric Charge and Its Conservation

Objects can be charged by rubbing





21-1 Static Electricity; Electric Charge and Its Conservation

Charge comes in two types,

positive and negative;

like charges repel and opposite charges attract.

21-1 Static Electricity; Electric Charge and Its Conservation

Conservation of Electric Charge

Electric charge is conserved – the arithmetic sum of the total charge cannot change in any interaction.

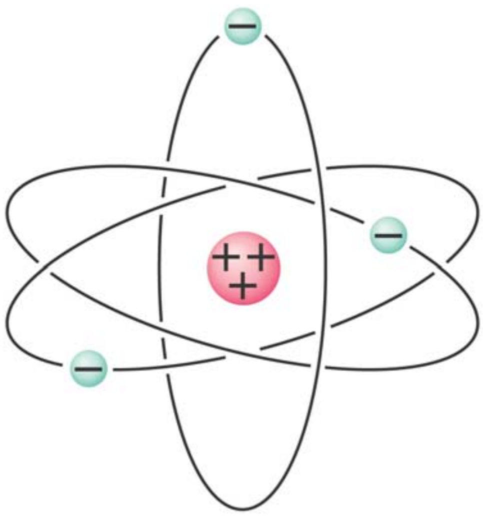
Ie. No net electric charge can be created or destroyed.

21-2 Electric Charge in the Atom

Atom:

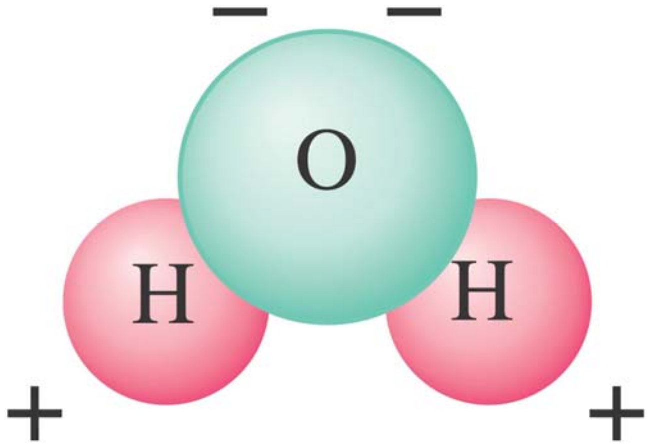
Nucleus (small, massive, positive charge)

Electron cloud (large, very low density, negative charge)



21-2 Electric Charge in the Atom

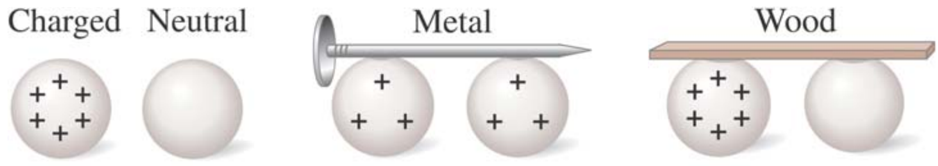
Polar molecule: neutral overall, but charge not evenly distributed



21-3 Insulators and Conductors

Conductor:	Insulator:
Charge flows freely	Almost no charge flows
Metals	Most other materials

Some materials are semiconductors.

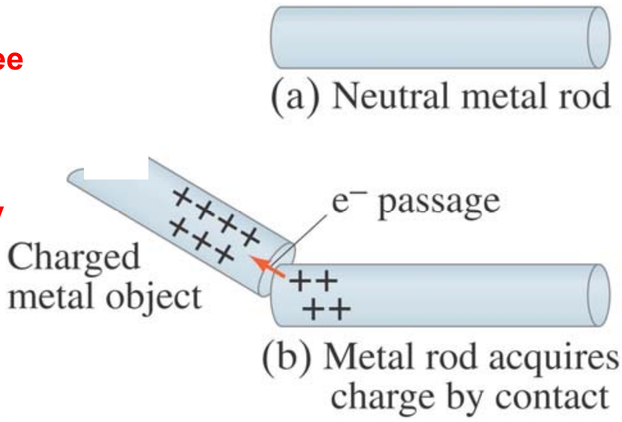


21-4 Induced Charge; the Electroscope

Metal objects can be charged by conduction:

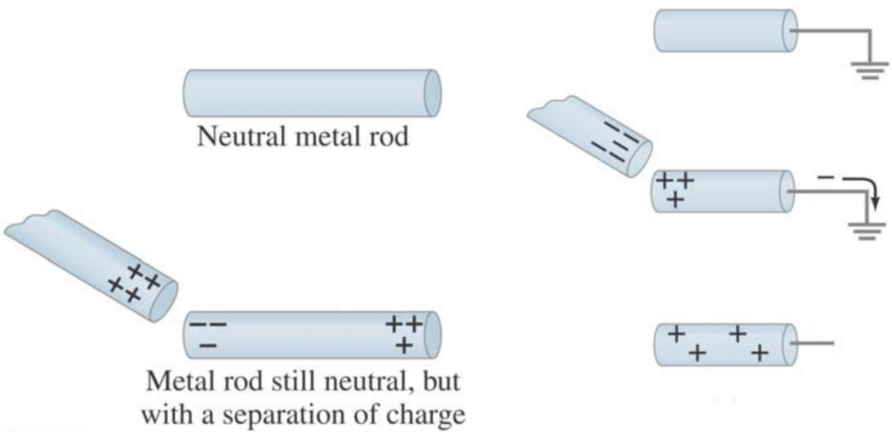
Note that only electrons (e^-) are free to move.

Positive charges (protons) are tightly bound in atomic nucleus.



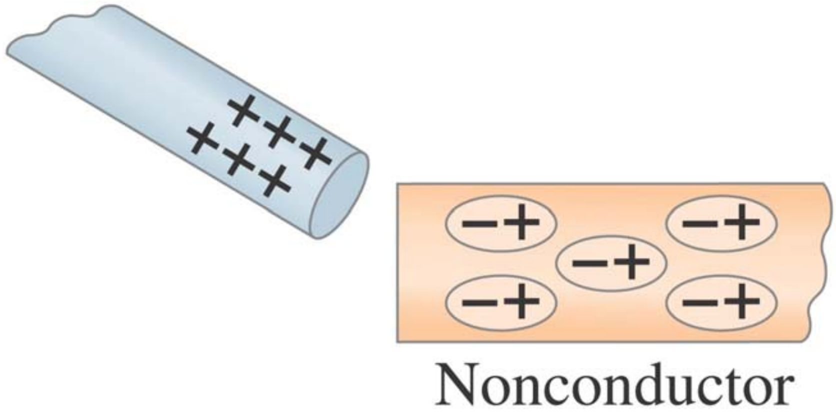
21-4 Induced Charge; the Electroscope

They can also be charged by induction, either while connected to ground or not:



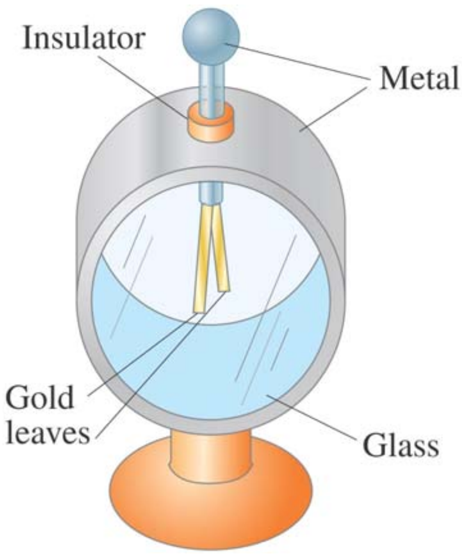
21-4 Induced Charge; the Electroscope

Nonconductors won't become charged by conduction or induction, but will experience charge separation:



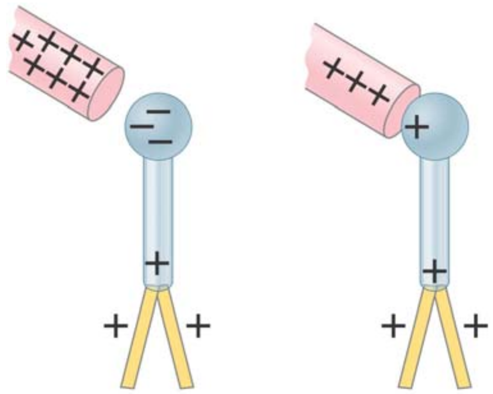
21-4 Induced Charge; the Electroscope

The electroscope can be used for detecting charge.



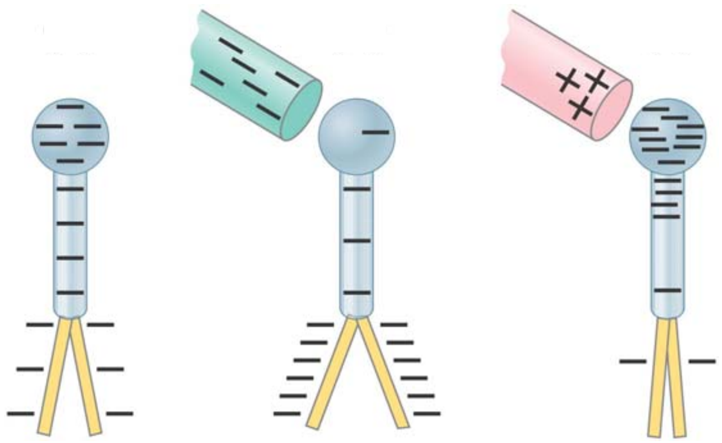
21-4 Induced Charge; the Electroscope

The electroscope can be charged either by conduction or by induction.

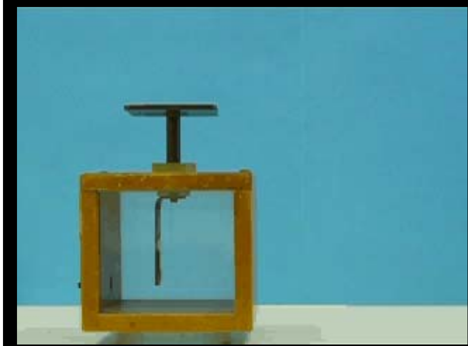


21-4 Induced Charge; the Electroscope

The charged electroscope can then be used to determine the sign of an unknown charge.



Consider electroscope shown below.
Both positive (BLACK) and negative (WHITE) objects cause divergence of leaves.



Charging by induction

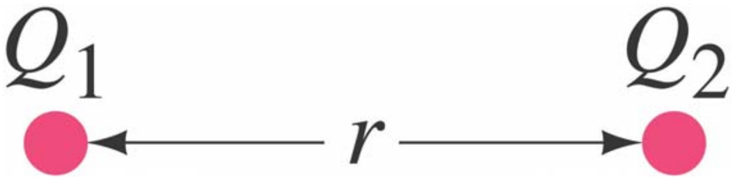
Charge on electroscope is opposite to charge on object brought near electroscope.

Leaves collapse – charge opposite to charge on electroscope

Leaves diverge further – charge the same as charge on electroscope

21-5 Coulomb's Law

Experiment shows that the electric force between two charges is proportional to the product of the charges and inversely proportional to the distance between them.



21-5 Coulomb's Law

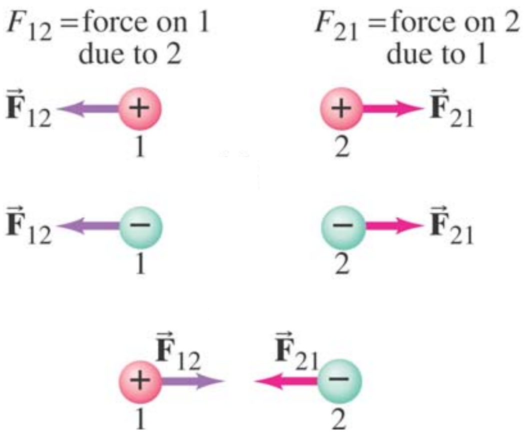
Coulomb's law:

$$F = k\frac{Q_1 Q_2}{r^2}$$

This equation gives the magnitude of the force between two charges.

21-5 Coulomb's Law

The force is along the line connecting the charges, and is attractive if the charges are opposite, and repulsive if they are the same.



21-5 Coulomb's Law

Unit of charge: coulomb, C.

The proportionality constant in Coulomb's law is then:

$$k = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$$

$$\approx 9.0 \times 10^9\ \mathrm{N.m^2/C^2}$$

Charges produced by rubbing are typically around a microcoulomb:

$$1\ \mu\mathrm{C} = 10^{-6}\ \mathrm{C}.$$

21-5 Coulomb's Law

Charge on the electron:

$$e = 1.602 \times 10^{-19}\ \mathrm{C}.$$

Electric charge is quantized in units of the electron charge.

21-5 Coulomb's Law

The proportionality constant *k* can also be written in terms of ε_0, the permittivity of free space:

$$F = \frac{1}{4\pi\epsilon_0}\frac{Q_1Q_2}{r^2},$$

where

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}.$$

21-5 Coulomb’s Law

Conceptual Example 21-1: Which charge exerts the greater force?

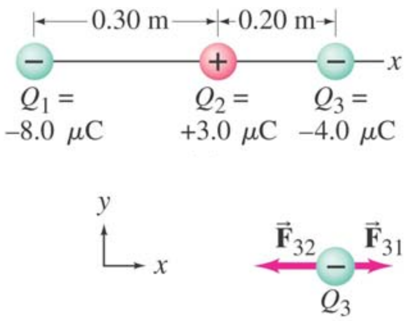
Two positive point charges, $Q_1 = 50\ \mu C$ and $Q_2 = 1\ \mu C$, are separated by a distance ℓ. Which is larger in magnitude, the force that Q_1 exerts on Q_2 or the force that Q_2 exerts on Q_1?

Coulomb’s Law and Newton’s third Law both tell us that the forces are equal in magnitude.

21-5 Coulomb’s Law

Example 21-2: Three charges in a line.

Three charged particles are arranged in a line, as shown. Calculate the net electrostatic force on particle 3 (the -4.0 μC on the right) due to the other two charges.



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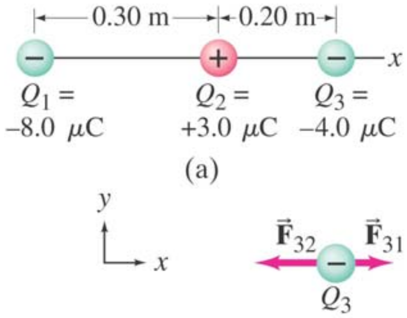


$$F_{31} = \frac{kQ_3Q_1}{r_{31}^2} = \frac{9.0\times10^9 \times 4\times10^{-6} \times 8\times10^{-6}}{0.50^2}$$

$$= 1.2 \text{ N to right } (+\text{ve})$$

$$F_{32} = \frac{kQ_3Q_2}{r_{32}^2} = \frac{9.0\times10^9 \times 4\times10^{-6} \times 3\times10^{-6}}{0.20^2}$$

$$= 2.7 \text{ N to left } (-\text{ve})$$

$$F = F_{31} - F_{32} = 1.2 - 2.7 = -1.5 \text{ N}$$

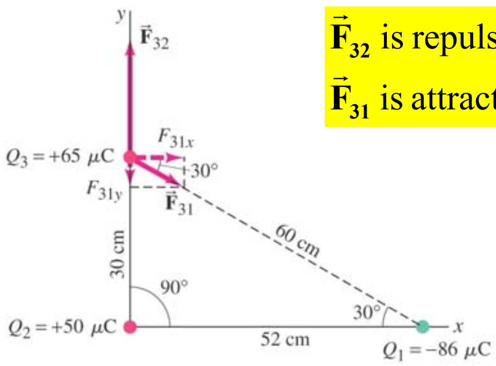
Ans : $\underline{F = 1.5 \text{ N to left}}$

Note: Coulomb's law gives the magnitude of the force – Use the diagram to show the direction.

21-5 Coulomb's Law

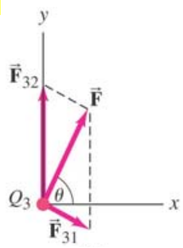
Example 21-3: Electric force using vector components.

Calculate the net electrostatic force on charge Q_3 shown in the figure due to the charges Q_1 and Q_2.



$\vec{F}_{32}$ is repulsive

$\vec{F}_{31}$ is attractive



Example 21-3: Electric force using vector components.

$$F_{32x} = 0$$

$$F_{32y} = \frac{kQ_3Q_2}{r_{32}^2} = 325\ \mathrm{N}$$

$$F_{31} = \frac{kQ_3Q_1}{r_{31}^2} = 140\ \mathrm{N}$$

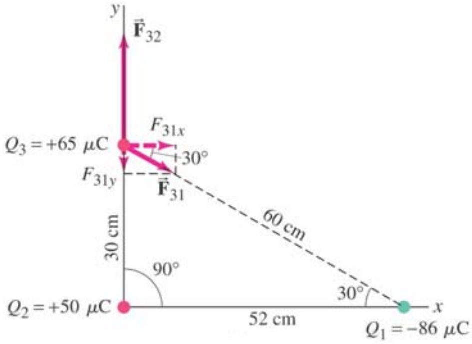
$$F_{31x} = F_{31}\cos 30° = 121\ \mathrm{N}$$

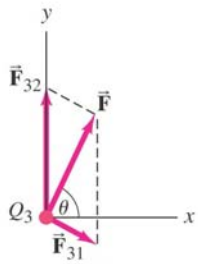
$$F_{31y} = F_{31}\sin 30° = 70\ \mathrm{N}$$

$$F_x = F_{32x} + F_{31x} = 0 + 121 = 121\ \mathrm{N}$$

$$F_y = F_{32y} - F_{31y} = 325 - 70 = 255\ \mathrm{N}$$

$$\boxed{F = \sqrt{F_x^2 + F_y^2} = 280\ \mathrm{N} \qquad \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = 65°}$$

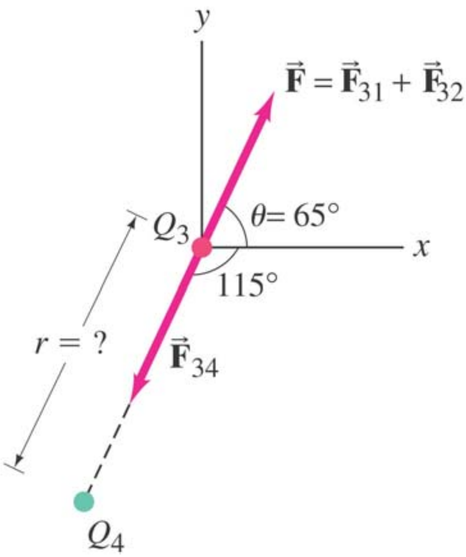




21-5 Coulomb's Law

Conceptual Example 21-4: Make the force on Q_3 zero.

In the figure, where could you place a fourth charge, Q_4 = -50 μC, so that the net force on Q_3 would be zero?



21-5 Coulomb's Law

Using previous result :

$F = 280$ N at 65° to x axis

F_{34} must be the same magnitude as F

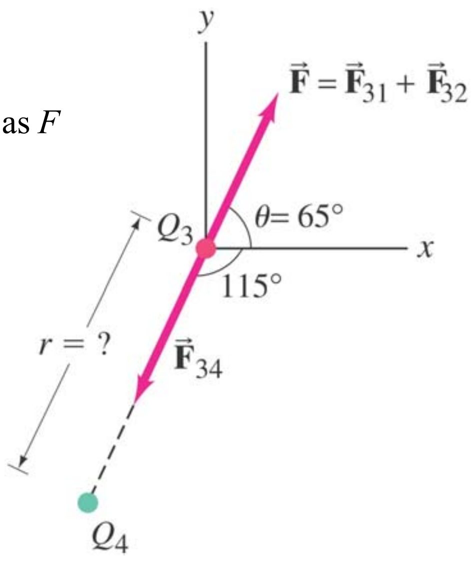
but in the opposite direction

ie. angle is $180° - 65° = 115°$

to x axis as shown

$$F = \frac{kQ_3Q_4}{r_{34}^2} \rightarrow r_{34} = \sqrt{\frac{kQ_3Q_4}{F}}$$

$$= 0.32 \text{ m} = 32 \text{ cm from } Q_3$$



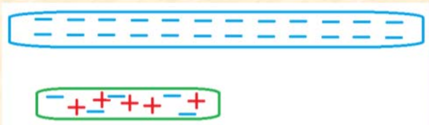
Some Additional Q & A Examples

1. Why is it so?

1. A plastic ruler is rubbed on fur. Explain:
   (a) How the ruler becomes charged in terms of movement of charges.
   (b) Why the ruler can pick up small pieces of paper.
   (c) Why the bits of paper often “jump” off soon after being picked up.

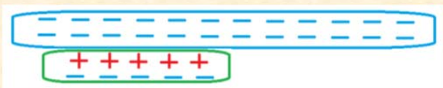
1. (a) Negative charges (electrons) move from fur to plastic giving plastic a negative charge and leaving fur with electron deficit → positive charge.

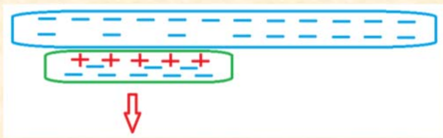
   (b) The paper is neutral. When ruler is brought near paper electrons are repelled from the ruler and move to the bottom surface of the paper. This leaves a net positive charge on the top surface of the paper which is closer to the ruler than the negative charge on the bottom surface hence there is an attractive force between the ruler and the paper.



1. A plastic ruler is rubbed on fur. Explain:
   (a) How the ruler becomes charged in terms of movement of charges.
   (b) Why the ruler can pick up small pieces of paper.
   (c) Why the bits of paper often “jump” off soon after being picked up.

   (c) Once the paper is in contact with the paper, electrons can transfer from the ruler to the paper giving the paper a net negative charge. This causes a repulsive force and the paper “jumps” off.





2. If you walk briskly across a carpet, you often experience a spark upon touching a door knob. (a) What causes this? (b) How might it be prevented?

   The soles of your shoes are usually a good insulator and can become charged by friction when walking across the carpet which in turn induces a charge on your body. Your body can be charged to several thousand volts in this way. On touching the (uncharged conducting) door knob a spark can jump from your hand to the door knob. This could be prevented by spraying the carpet with an anti-static spray.

3. Why do electrostatic experiments not work well on humid days?

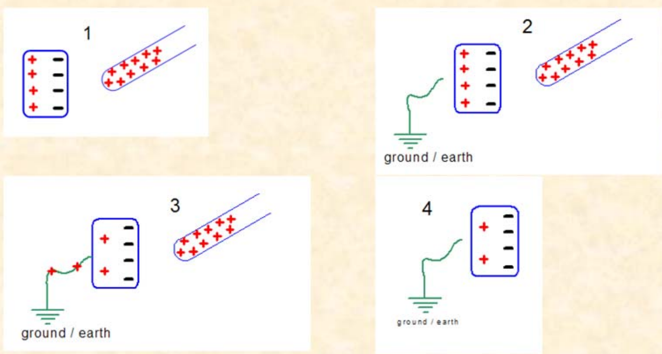
   Moist (humid) air is a better conductor than dry air and allows charge to leak away by conduction thought the air.

4. A positive charge is brought very near an uncharged isolated conductor. The conductor is then grounded while the charge is kept near. Is the conductor charged positively, negatively, or not at all if

   (a) the charge is taken away and then the ground connection is removed and
   (b) the ground connection is removed and then the charge is taken away?

(a) If the charge is taken away with the earth still connected the conductor will not have any charge as it is connected to ground. Once the ground is removed the conductor will still be uncharged.

(b) Diagrams 1 to 4 show that the conductor will finally have a net negative charge.



5. How could you determine the sign of the charge on a charged, isolated rod?

Easiest way is to use known charges, polystyrene rubbed on wool becomes negative, glass rubbed on silk becomes positive. If you bring a negatively charged polystyrene rod near the unknown rod: Attraction → unknown is positive
Repulsion → unknown is negative

6. (a) A positively charged glass rod attracts an object suspended by a nonconducting thread. Can we conclude that the object is negatively charged?
(b) A positively charged glass rod repels a similarly suspended object. Can we conclude that this object is positively charged?

Ans: (a) No. The object could be negatively charged but an uncharged object can still be attracted. See explanation for question 1.
(b) Yes. Repulsion can only occur when there are like charges so the object must be positive.

7. An electron (charge = -e) circulates around a helium nucleus (charge = +2e) in a helium atom. Which particle exerts the larger force on the other?

The forces are equal in magnitude – Newton's Third law.

8. What would be the electrostatic force between two 1.00 C charges separated by a distance of (a) 1.00 m and (b) 1.00 km if such a configuration could be set up?

$$\mathbf{F} = \frac{\mathbf{kQ_1Q_2}}{\mathbf{r^2}}$$

$\mathbf{k} = 9 \times 10^9$ Nm^2/C^2 $\mathbf{Q_1} = \mathbf{Q_2} = 1$ C

(a) r = 1 m

$$\mathbf{F} = \frac{9\times10^9\times1\times1}{1^2} = 9\times10^9 \text{ N}$$

(b) r = 1 km = 1000 m

$$\mathbf{F} = \frac{9\times10^9\times1\times1}{1000^2} = 9\times10^3 = 9000 \text{ N}$$

9. A charge of +3.00 x 10^{-6} C is 120 mm distant from a second charge of -1.50 x 10^{-6} C. Calculate the magnitude of the force on each charge.

$$\mathbf{F} = \frac{\mathbf{kQ_1Q_2}}{\mathbf{r^2}}$$

$$\mathbf{F} = \frac{9\times10^{9}\times3\times10^{-6}\times1.5\times10^{-6}}{(0.12)^2} = 2.8\text{ N}$$

k = 9 x 10^9 Nm^2/C^2
Q_1 = +3 x 10^{-6} C
Q_2 = -1.5 x 10^{-6} C
r = 120 mm = 0.12 m

Note that as the charges are opposite in sign the force is attractive however this in not asked for in the question.

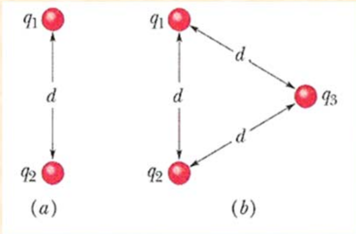
10. What must be the distance between point charge q_1 = 26.0 μC and point charge q_2 = -47.0 μC in order that the electrostatic force between them have a magnitude of 5.70 N?

k = 9 x 10^9 Nm^2/C^2
q_1 = 26 μC = 26 x 10^{-6} C
q_2 = -47 μC = -47 x 10^{-6} C
F = 5.7 N

$$\mathbf{F} = \frac{\mathbf{kQ_1Q_2}}{\mathbf{r^2}}$$

$$\mathbf{r} = \sqrt{\frac{\mathbf{kQ_1Q_2}}{\mathbf{F}}} = \sqrt{\frac{9\times10^{9}\times26\times10^{-6}\times47\times10^{-6}}{5.7}} = 1.39\text{ m}$$

11. The diagram shows two charges, q_1 and q_2, held a fixed distance d apart.
(a) What is the magnitude of the electrostatic force that acts on q_1? Assume that $q_1 = q_2 = 20.0\ \mu C$ and $d = 1.50$ m
(b) A third charge $q_3 = 20.\mu C$ is brought in and placed as shown. What now is the magnitude of the electrostatic force on q_1?

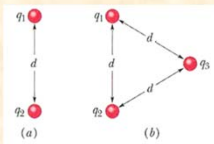


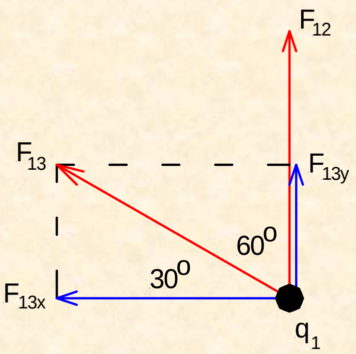
(a) $\mathbf{F} = \frac{\mathbf{kQ_1Q_2}}{\mathbf{r^2}}$

$\mathbf{k} = 9 \times 10^9\ Nm^2/C^2$
$\mathbf{q_1} = \mathbf{q_2} = \mathbf{q_3} = 20\ \mu C = 20 \times 10^{-6}\ C$
$\mathbf{r} = \mathbf{d} = 1.5$ m

$$\mathbf{F} = \frac{9\times10^9 \times 20\times10^{-6} \times 20\times10^{-6}}{1.5^2} = 1.6\ N$$

(b) Force on $\mathbf{q_1}$ due to $\mathbf{q_2}$ = 1.6 N
Force on $\mathbf{q_1}$ due to $\mathbf{q_3}$ = 1.6 N (**Q** and **r** are the same)
The vector diagram shows the components of the force:





$\mathbf{F_{12}} = 1.6$ N
$\mathbf{F_{13}} = 1.6$ N

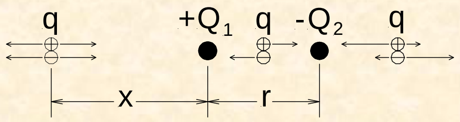
$\mathbf{F_{13x}} = -1.6 \cos 30° = -1.39$ N
$\mathbf{F_{13y}} = 1.6 \sin 30° = 0.8$ N

$\mathbf{q_1} = 20\ \mu C$

Total force on $\mathbf{q_1}$ is:
$\mathbf{F_x} = -1.39$ N
$\mathbf{F_y} = 1.6 + 0.8 = 2.4$ N

Magnitude:
$$\mathbf{F} = \sqrt{\mathbf{F_x^2} + \mathbf{F_y^2}} = \sqrt{1.39^2 + 2.4^2} = 2.77\ N$$

12. Two fixed charges, +1.0 µC, and -3.0 µC, are 100 mm apart. Where can a third charge be located so that no net electrostatic force acts on it?



Q_1 = 1 µC = 1 x 10^{-6} C
Q_2 = -3 µC = -3 x 10^{-6} C
q can have any value (+ or -)
r = 100 mm = 0.1 m

Charge **q**, whether + or -, can only be placed to the left of **Q_1** as this is the only point where the forces can cancel.

$$\mathbf{F} = \frac{\mathbf{kQ_1q}}{x^2} = \frac{\mathbf{kQ_2q}}{(x+0.1)^2} \quad \rightarrow \quad \frac{\mathbf{Q_1}}{x^2} = \frac{\mathbf{Q_2}}{(x+0.1)^2} \quad \rightarrow \quad \frac{1}{x^2} = \frac{3}{(x+0.1)^2}$$

$$\therefore 3x^2 = x^2 + 0.2x + 0.01 \quad \rightarrow \quad 2x^2 - 0.2x - 0.01 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-0.2) \pm \sqrt{0.2^2 - 4 \times 2 \times (-0.01)}}{2 \times 2} = \frac{0.2 \pm 0.346}{4}$$

$$\text{as x cannot be negative: } x = \frac{0.2 + 0.346}{4} = 0.137 \text{ m} = 137 \text{ mm to left of } +1\,\mu\text{C charge}$$

13. Charges of +1 µC and -2 µC are placed 10 mm apart. What is the force acting between them? Is the force attractive or repulsive?

$$\mathbf{F} = \frac{\mathbf{kQ_1Q_2}}{\mathbf{r^2}} = \frac{\mathbf{9 \times 10^9 \times 10^{-6} \times -2 \times 10^{-6}}}{\mathbf{\left(10^{-2}\right)^2}} = \mathbf{-180\ N}$$

$\therefore$ Force is 180 N attraction (unlike charges)

r = 10 mm = 10^{-2} m
k = 9 x 10^9 Nm^2/C^2
Q_1 = +1 µC = +10^{-6} C
Q_2 = - 2 µC = + 2 x 10^{-6} C

14. How many electrons must be removed from a neutral metal sphere to give it a net charge of +1 µC?

Q = ne

$$n = \frac{Q}{e} = \frac{10^{-6}}{1.6 \times 10^{-19}} = 6.25 \times 10^{12} \text{ electrons}$$

Q = 1 µC = 10^{-6} C
n = integer
e = 1.6×10^{-6} C

15. Given that k = 9×10^9 N.m^2/C^2, find the repulsive force between two electrons spaced 1 nm apart.

$Q = e = 1.6\times10^{-19}$ C
$r = 1$ nm $= 10^{-9}$ m

$$F = \frac{kQ_1Q_2}{r^2} = \frac{ke^2}{r^2} = \frac{9\times10^9\times\left(1.6\times10^{-19}\right)^2}{\left(10^{-9}\right)^2} = 2.3\times10^{-10}\text{ N}$$

16. For the electrons in question 15, what initial acceleration will these electrons experience if there are no other forces present?

From Newton's Second Law: **F** = m**a**

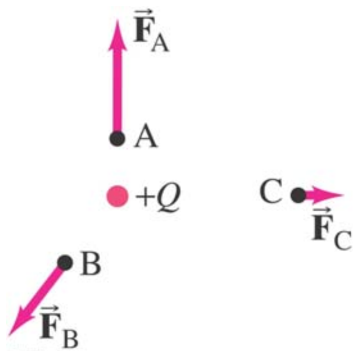
a = **F**/m = $(2.3\times10^{-10})/(9.1\times10^{-31}) = 2.5\times10^{20}$ m/s^2

21-6 The Electric Field

The electric field is defined as the force on a small charge, divided by the magnitude of the charge:

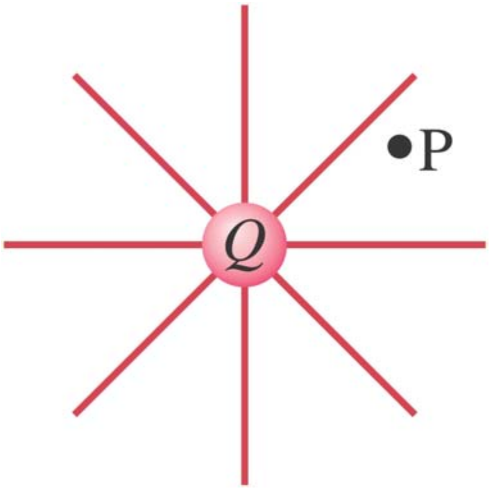
$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q}.$$

Force exerted by charge Q on a small test charge, q, placed at points A, B, and C



21-6 The Electric Field

An electric field surrounds every charge.



21-6 The Electric Field

For a point charge:

$$E = \frac{F}{q} = \frac{kqQ}{r^2 \cdot q}$$

$$\boxed{E = \frac{kQ}{r^2}}$$

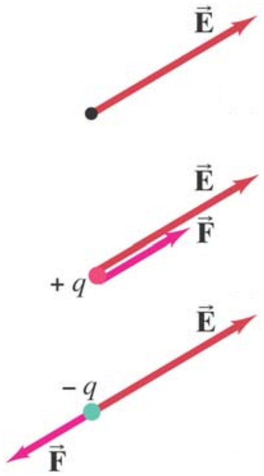
$$\text{or, in terms of } \varepsilon_0 \quad \left(\mathrm{k} = \frac{1}{4\pi\,\varepsilon_0} \right)$$

$$\boxed{E = \frac{Q}{4\pi\,\varepsilon_0\,r^2}}$$

21-6 The Electric Field

Force on a point charge in an electric field:

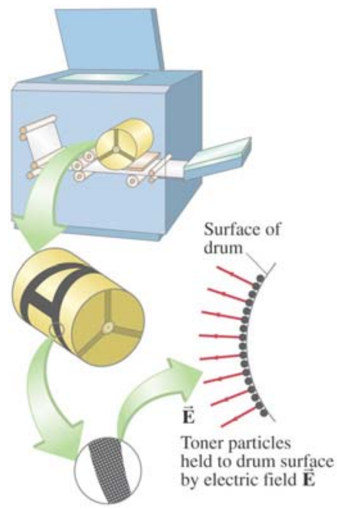
$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}.$$



21-6 The Electric Field

Example 21-5: Photocopy machine.

A photocopy machine works by arranging positive charges (in the pattern to be copied) on the surface of a drum, then gently sprinkling negatively charged dry toner (ink) particles onto the drum. The toner particles temporarily stick to the pattern on the drum and are later transferred to paper and "melted" to produce the copy. Suppose each toner particle has a mass of 9.0×10^{-16} kg and carries an average of 20 extra electrons to provide an electric charge. Assuming that the electric force on a toner particle must exceed twice its weight in order to ensure sufficient attraction, compute the required electric field strength near the surface of the drum.

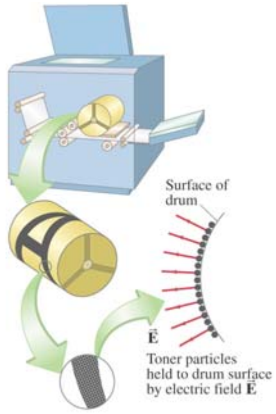


21-6 The Electric Field

Example 21-5: Photocopy machine.

Mass of each toner particle = 9.0 x 10^{-16} kg and has an average of 20 extra electrons to provide an electric charge.

Assuming electric force on a toner particle must exceed twice its weight in order to ensure sufficient attraction, compute the required electric field strength near the surface of the drum.



$$\text{Charge } q = 20e = 20 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-18}\ \text{C}$$

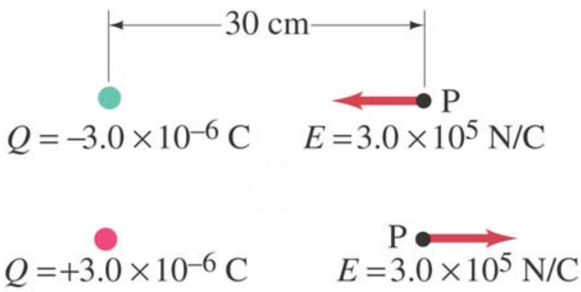
$$\text{and } F = qE = 2mg$$

$$\therefore \quad E = \frac{2mg}{q} = \frac{2 \times 9.0 \times 10^{-16} \times 9.8}{3.2 \times 10^{-18}} = 5.5 \times 10^{3}\ \text{N/C}$$

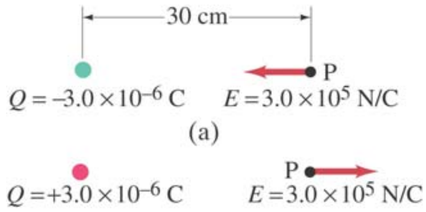
21-6 The Electric Field

Example 21-6: Electric field of a single point charge.

Calculate the magnitude and direction of the electric field at a point P which is 30 cm to the right of a point charge *Q* = -3.0 x 10^{-6} C.



21-6 The Electric Field



$$\text{For negative charge: } E = \frac{kQ}{r^2}$$

$$= \frac{9.0 \times 10^{9} \times 3.0 \times 10^{-6}}{0.3^2} = 3.0 \times 10^{5} \text{ N/C to LEFT}$$

Note that the direction of E is the same as the force on a POSITIVE charge

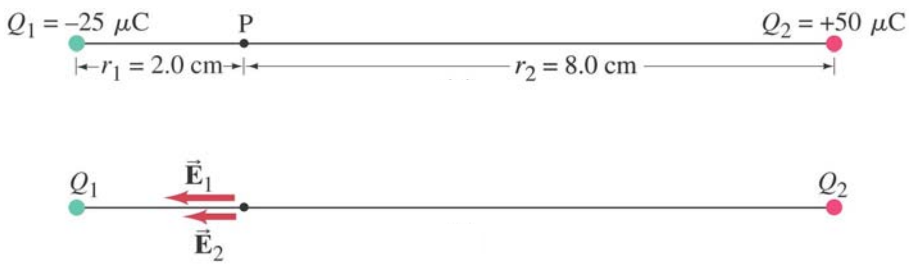
For positive charge: E has the same magnitude $= 3.0 \times 10^{5}$ N/C to RIGHT

Example 21-7: *E* at a point between two charges.

Two point charges are separated by a distance of 10.0 cm. One has a charge of -25 μC and the other +50 μC.

(a) Determine the direction and magnitude of the electric field at a point P between the two charges that is 2.0 cm from the negative charge.

(b) If an electron (mass = 9.11 x 10^{-31} kg) is placed at rest at P and then released, what will be its initial acceleration (direction and magnitude)?



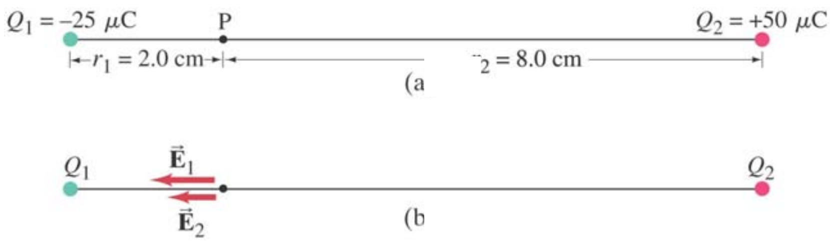
Example 21-7: *E* at a point between two charges.

(a) Determine the direction and magnitude of the electric field at a point P between the two charges that is 2.0 cm from the negative charge.

$$\text{For } Q_1 \quad E_1 = \frac{kQ_1}{r_1^2} = \frac{9.0\times10^9 \times 25\times10^{-6}}{0.02^2} = 5.62\times10^8 \text{ N/C to left}$$

$$\text{For } Q_2 \quad E_2 = \frac{kQ_2}{r_2^2} = \frac{9.0\times10^9 \times 50\times10^{-6}}{0.08^2} = 7.0\times10^7 \text{ N/C to left}$$

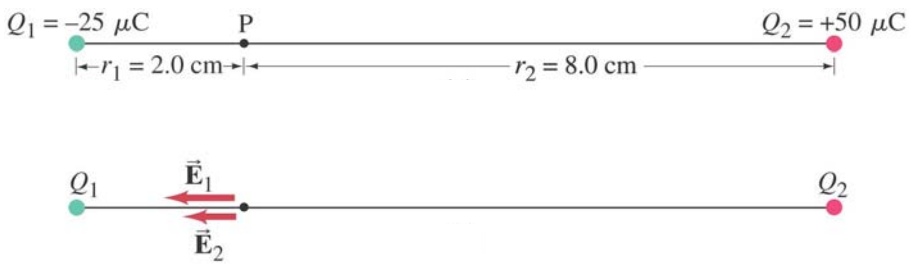
$$E = E1 + E2 = 6.3\times10^8 \text{ N/C to left}$$



(b) If an electron (mass = 9.11 x 10^{-31} kg) is placed at rest at P and then released, what will be its initial acceleration (direction and magnitude)?

$$a = \frac{F}{m} = \frac{Eq}{m} = \frac{6.3\times10^8 \times 1.6\times10^{-19}}{9.1\times10^{-31}} = 1.1\times10^{20} \text{ m/s}^2 \text{ to right}$$

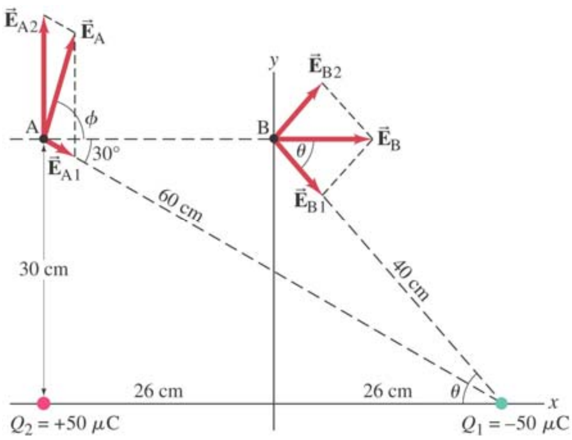
NB : Force on negative charge is in opposite direction to $\vec{\mathbf{E}}$



21-6 The Electric Field

Example 21-8: $\vec{E}$ above two point charges.

Calculate the total electric field (a) at point A and (b) at point B in the figure due to both charges, Q_1 and Q_2.



21-6 The Electric Field

Problem solving in electrostatics: electric forces and electric fields

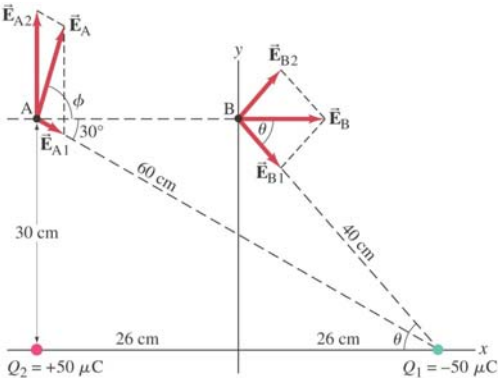
1. Draw a diagram; show all charges, with signs, and electric fields and forces with directions.

2. Calculate forces using Coulomb's law.

3. Add forces vectorially to get result.

4. Check your answer!

Total electric field point A



$$E_{A1} = \frac{kQ_1}{r_1^2} = 1.25\times10^6 \text{ N/C}$$

$$E_{A1x} = E_{A1}\cos 30° = 1.08\times10^6 \text{ N/C}$$

$$E_{A1y} = -E_{A1}\sin 30° = -6.25\times10^6 \text{ N/C}$$

$$E_{A2} = \frac{kQ_2}{r_2^2} = 5.00\times10^6 \text{ N/C}$$

$$E_{A2x} = 0$$

$$E_{A2y} = E_{A2} = 5.00\times10^6 \text{ N/C}$$

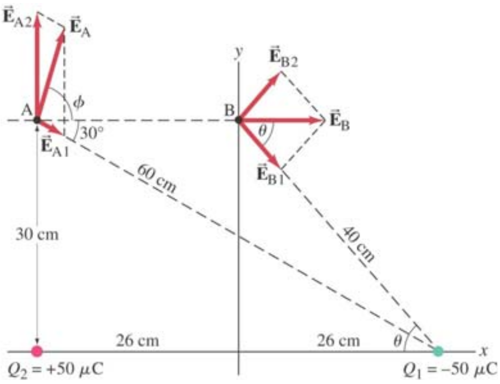
$$E_{Ax} = E_{A1x} + E_{A2x} = 1.08\times10^6 + 0 = 1.08\times10^6 \text{ N/C}$$

$$E_{Ay} = E_{A1y} + E_{A2y} = -6.25\times10^5 + 5.00\times10^6 = 4.34\times10^6 \text{ N/C}$$

$$E_A = \sqrt{E_{Ax}^2 + E_{Ay}^2} = 4.5\times10^6 \text{ N/C}$$

$$\phi = tan^{-1}\left(\frac{E_{Ay}}{E_{Ax}}\right) = 76°$$

Total electric field point B



$$|E_{B1}| = |E_{B2}| \quad (r_1 = r_2)$$

$$E_{B1} = E_{B2} = \frac{kQ_1}{r_1^2} = 2.8\times10^6 \text{ N/C}$$

$$\theta = tan^{-1}\left(\frac{30}{26}\right) = 49°$$

$$E_{B1x} = E_{B1}\cos 49° = 1.8\times10^6 \text{ N/C}$$

$$E_{B1y} = -E_{B1}\sin 49° = -2.1\times10^6 \text{ N/C}$$

$$E_{B2x} = E_{B2}\cos 49° = 1.8\times10^6 \text{ N/C}$$

$$E_{B2y} = E_{B1}\sin 49° = 2.1\times10^6 \text{ N/C}$$

$$E_{Bx} = E_{B1x} + E_{B2x} = 3.6\times10^6 \text{ N/C}$$

$$E_{By} = E_{B1y} + E_{B2y} = 0$$

$$E_B = \sqrt{E_{Bx}^2 + E_{By}^2} = E_{Bx} = 3.6\times10^6 \text{ N/C}$$

along x axis $(E_{By} = 0)$

21-7 Electric Field Calculations for Continuous Charge Distributions

A continuous distribution of charge may be treated as a succession of infinitesimal (point) charges. The total field is then the integral of the infinitesimal fields due to each bit of charge:

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2}.$$

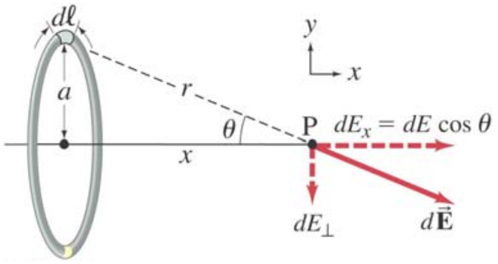
$$\vec{\mathbf{E}} = \int d\vec{\mathbf{E}}.$$

Remember that the electric field is a vector; you will need a separate integral for each component.

21-7 Electric Field Calculations for Continuous Charge Distributions

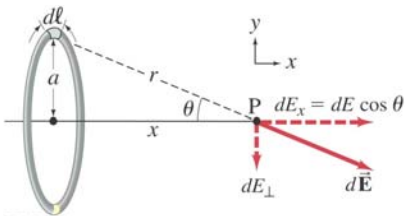
Example 21-9: A ring of charge.

A thin, ring-shaped object of radius a holds a total charge $+Q$ distributed uniformly around it. Determine the electric field at a point P on its axis, a distance x from the center. Let λ be the charge per unit length (C/m).



Example 21-9: A ring of charge.

Because P is on the axis, the transverse components of E ($dE_{\perp}$) must add to zero, by symmetry.



The **direction** of the electric field due to one infinitesimal length $d\ell$ of the charged ring is shown in Fig.

Coulomb's law. The electric field, $d\vec{\mathbf{E}}$, due to this particular segment of the ring of length $d\ell$ has magnitude
$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2}.$$

The whole ring has length (circumference) of $2\pi a$, so the charge on a length $d\ell$ is
$$dQ = Q\left(\frac{d\ell}{2\pi a}\right) = \lambda\, d\ell$$
where $\lambda = Q/2\pi a$ is the charge per unit length.

Now we write dE as
$$dE = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, d\ell}{r^2}.$$

Add **vectorially** and use **symmetry**:
The vector $d\vec{\mathbf{E}}$ has components dE_x along the x axis and $dE_{\perp}$ perpendicular to the x axis.
sum (integrate) around the entire ring.
The total field is then
$$E = E_x = \int dE_x = \int dE\cos\theta = \frac{1}{4\pi\epsilon_0}\lambda\int\frac{d\ell}{r^2}\cos\theta.$$

Since $\cos\theta = x/r$, where $r = (x^2 + a^2)^{\frac{1}{2}}$, we have
$$E = \frac{\lambda}{(4\pi\epsilon_0)}\frac{x}{(x^2 + a^2)^{\frac{3}{2}}}\int_0^{2\pi a} d\ell = \frac{1}{4\pi\epsilon_0}\frac{\lambda x(2\pi a)}{(x^2 + a^2)^{\frac{3}{2}}}$$
$$E = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2 + a^2)^{\frac{3}{2}}}.$$

check reasonableness, at great distances, $x \gg a$, this result reduces to $E = Q/(4\pi\epsilon_0 x^2)$.
We would expect this result because at great distances the ring would appear to be a point charge
Also result gives $E = 0$ at $x = 0$,
because all components will cancel at the center of the circle.

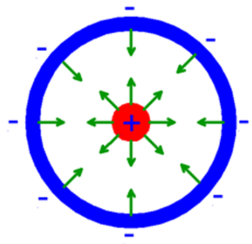
21-7 Electric Field Calculations for Continuous Charge Distributions

Conceptual Example 21-10: Charge at the center of a ring.

Imagine a small positive charge placed at the center of a nonconducting ring carrying a uniformly distributed negative charge. Is the positive charge in equilibrium if it is displaced slightly from the center along the axis of the ring, and if so is it stable? What if the small charge is negative? Neglect gravity, as it is much smaller than the electrostatic forces.

Conceptual Example 21-10: Charge at the center of a ring.

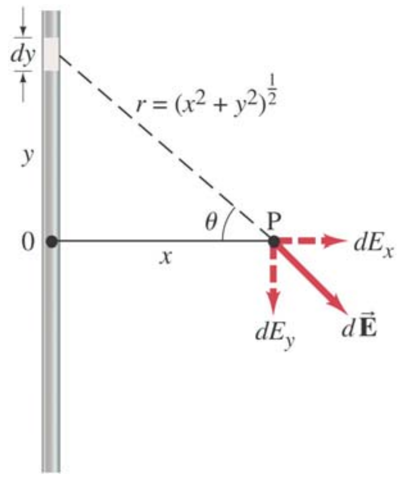
The positive charge is in equilibrium because there is no net force on it, by symmetry. If the positive charge moves away from the center of the ring along the axis in either direction, the net force will be back towards the center of the ring and so the charge is in stable equilibrium. A negative charge at the center of the ring would feel no net force, but is in unstable equilibrium because if it moved along the ring's axis, the net force would be away from the ring and the charge would be pushed farther away.



21-7 Electric Field Calculations for Continuous Charge Distributions

Example 21-11: Long line of charge.

Determine the magnitude of the electric field at any point P a distance x from a very long line (a wire, say) of uniformly distributed charge. Assume x is much smaller than the length of the wire, and let λ be the charge per unit length (C/m).



Example 21-11: Long line of charge.

Coordinate system: wire on y axis

Segment dy has charge $dQ = \lambda dy$

The field $d\vec{\mathbf{E}}$ at point $\mathbf{P}$ due to dy is:

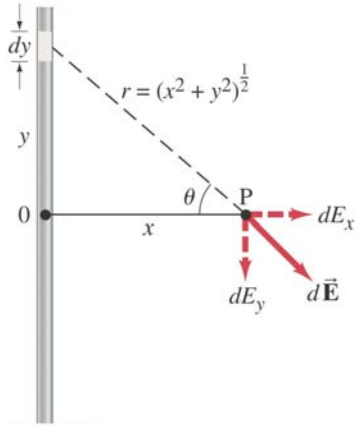
$$dE = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{\lambda dy}{4\pi\varepsilon_0\left(x^2 + y^2\right)}$$

Because $\mathbf{O}$ is the midpoint of the wire,

$$E_y = \int dE_y = 0$$

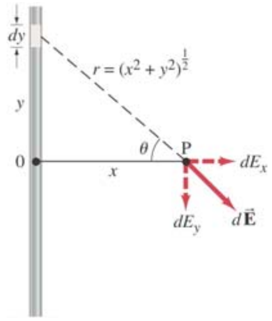
x direction: $dE_x = dE\cos\theta$

$$E = E_x = \int dE\cos\theta = \frac{\lambda}{4\pi\varepsilon_0}\int\frac{\cos\theta\, dy}{x^2 + y^2}$$



Example 21-11:

Long line of charge.



The integration here is over y, along the wire, with x treated as constant. We must now write θ as a function of y, or y as a function of θ. We do the latter: since $y = x\tan\theta$, then $dy = x\,d\theta/\cos^2\theta$. Furthermore, because $\cos\theta = x/\sqrt{x^2+y^2}$, then $1/(x^2+y^2) = \cos^2\theta/x^2$ and our integrand above is $(\cos\theta)(x\,d\theta/\cos^2\theta)(\cos^2\theta/x^2) = \cos\theta\,d\theta/x$. Hence

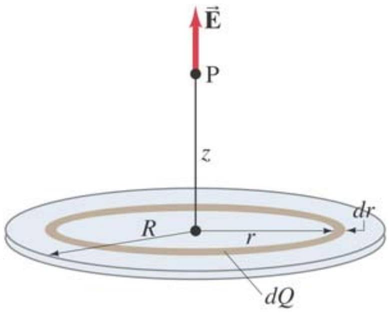
$$E = \frac{\lambda}{4\pi\epsilon_0}\frac{1}{x}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta = \frac{\lambda}{4\pi\epsilon_0 x}(\sin\theta)\bigg|_{-\pi/2}^{\pi/2} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{x},$$

where we have assumed the wire is extremely long in both directions ($y \to \pm\infty$) which corresponds to the limits $\theta = \pm\pi/2$. Thus the field near a long straight wire of uniform charge decreases inversely as the first power of the distance from the wire.

21-7 Electric Field Calculations for Continuous Charge Distributions

Example 21-12: Uniformly charged disk.

Charge is distributed uniformly over a thin circular disk of radius R. The charge per unit area (C/m^2) is σ. Calculate the electric field at a point P on the axis of the disk, a distance z above its center.



Example 21-12: Uniformly charged disk.

APPROACH We can think of the disk as a set of concentric rings. We can then apply the result of Example 21–9 to each of these rings, and then sum over all the rings.

SOLUTION For the ring of radius r shown, the electric field has magnitude (see result of Example 21–9)

$$dE = \frac{1}{4\pi\epsilon_0}\frac{z\,dQ}{(z^2 + r^2)^{\frac{3}{2}}}$$

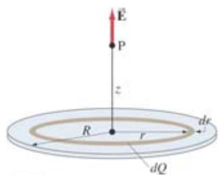
where we have written dE (instead of E) for this thin ring of total charge dQ. The ring has area $(dr)(2\pi r)$ and charge per unit area $\sigma = dQ/(2\pi r\,dr)$. We solve this for dQ $(= \sigma\,2\pi r\,dr)$ and insert it in the equation above for dE:

$$dE = \frac{1}{4\pi\epsilon_0}\frac{z\sigma 2\pi r\,dr}{(z^2 + r^2)^{\frac{3}{2}}} = \frac{z\sigma r\,dr}{2\epsilon_0(z^2 + r^2)^{\frac{3}{2}}}.$$

Now we sum over all the rings, starting at $r = 0$ out to the largest with $r = R$:

$$E = \frac{z\sigma}{2\epsilon_0}\int_0^R \frac{r\,dr}{(z^2 + r^2)^{\frac{3}{2}}} = \frac{z\sigma}{2\epsilon_0}\left[-\frac{1}{(z^2 + r^2)^{\frac{1}{2}}}\right]_0^R$$

$$= \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{(z^2 + R^2)^{\frac{1}{2}}}\right].$$



Example 21-12: Uniformly charged disk.

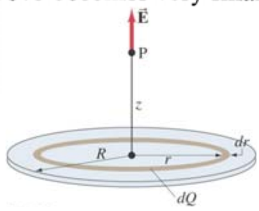
$$E = \frac{z\sigma}{2\epsilon_0}\int_0^R \frac{r\,dr}{\left(z^2 + r^2\right)^{\frac{3}{2}}} = \frac{z\sigma}{2\epsilon_0}\left[-\frac{1}{\left(z^2 + r^2\right)^{\frac{1}{2}}}\right]_0^R$$

$$= \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\left(z^2 + R^2\right)^{\frac{1}{2}}}\right].$$

This gives the magnitude of $\vec{\mathbf{E}}$ at any point z along the axis of the disk. The direction of each $d\vec{\mathbf{E}}$ due to each ring is along the z axis (as in Example 21–9), and therefore the direction of $\vec{\mathbf{E}}$ is along z. If Q (and σ) are positive, $\vec{\mathbf{E}}$ points away from the disk; if Q (and σ) are negative, $\vec{\mathbf{E}}$ points toward the disk.

If the radius of the disk in Example 21–12 is much greater than the distance of our point P from the disk (i.e., $z \ll R$) then we can obtain a very useful result: the second term in the solution above becomes very small, so

$$E = \frac{\sigma}{2\epsilon_0}. \qquad \text{[infinite plane]}$$



21-7 Electric Field Calculations for Continuous Charge Distributions

In the previous example, if we are very close to the disk (that is, if $z << R$), the electric field is:

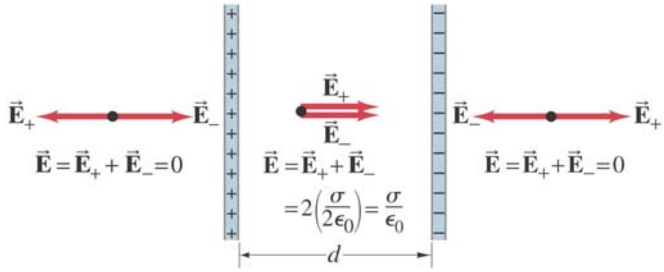
$$E = \frac{\sigma}{2\epsilon_0}. \qquad \text{[infinite plane]}$$

This is the field due to an infinite plane of charge.

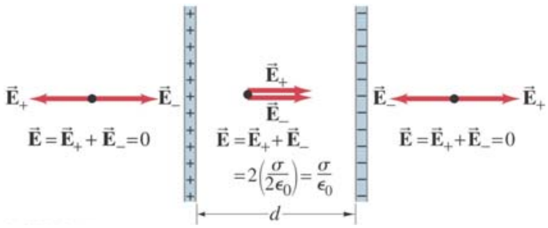
21-7 Electric Field Calculations for Continuous Charge Distributions

Example 21-13: Two parallel plates.

Determine the electric field between two large parallel plates or sheets, which are very thin and are separated by a distance d which is small compared to their height and width. One plate carries a uniform surface charge density σ and the other carries a uniform surface charge density $-\sigma$ as shown (the plates extend upward and downward beyond the part shown).



Example 21-13: Two parallel plates.



Each plate sets up an electric field of magnitude $\dfrac{\sigma}{2\varepsilon_0}$

Between plates: $E = E_+ + E_- = \dfrac{\sigma}{2\varepsilon_0} + \dfrac{\sigma}{2\varepsilon_0} = \dfrac{\sigma}{\varepsilon_0}$

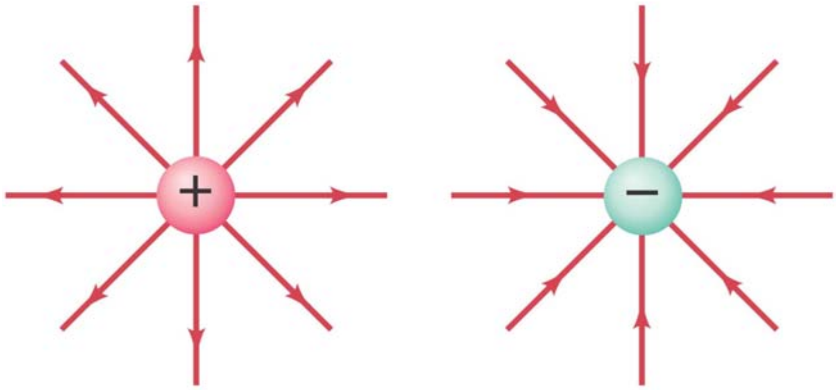
The field is uniform.

Outside the plates the fields cancel:

$$E = E_+ + E_- = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

21-8 Field Lines

The electric field can be represented by field lines. These lines start on a positive charge and end on a negative charge.



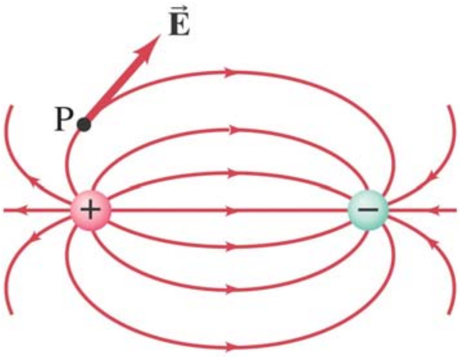
21-8 Field Lines

The number of field lines starting (ending) on a positive (negative) charge is proportional to the magnitude of the charge.

The electric field is stronger where the field lines are closer together.

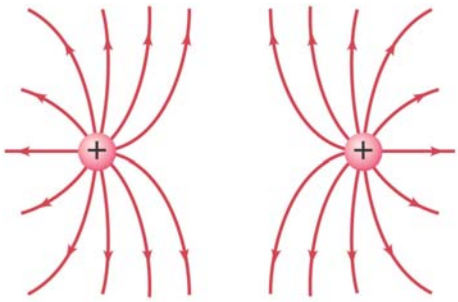
21-8 Field Lines

Electric dipole: two equal charges, opposite in sign:

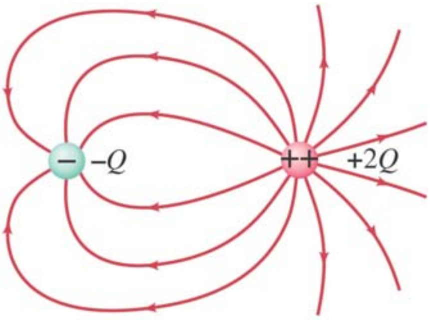


Note that the tangent to a field line at any point $\mathbf{P}$ gives the direction of the electric field $\vec{E}$ at $\mathbf{P}$ and the direction of the force acting on a positive charge placed at $\mathbf{P}$.

21-8 Field Lines – Other Examples

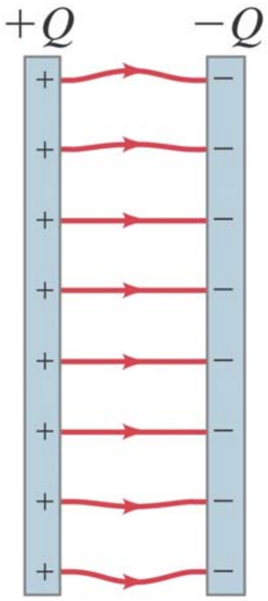


Opposite charges



Unequal charges

21-8 Field Lines



The electric field between two closely spaced, oppositely charged parallel plates is constant.

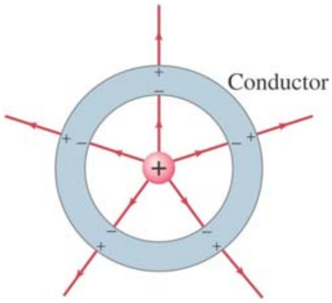
21-8 Field Lines

Summary of field lines:

1. Field lines indicate the direction of the field; the field is tangent to the line.
2. The magnitude of the field is proportional to the density of the lines.
3. Field lines start on positive charges and end on negative charges; the number is proportional to the magnitude of the charge.

21-9 Electric Fields and Conductors

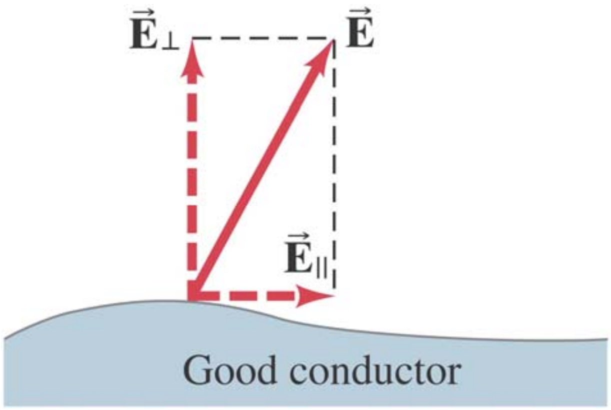
The static electric field inside a conductor is zero – if it were not, the charges would move.
(Proof of this is given in Chapter 22 Gauss's Law. Check this if you like but Gauss's Law is not in course)



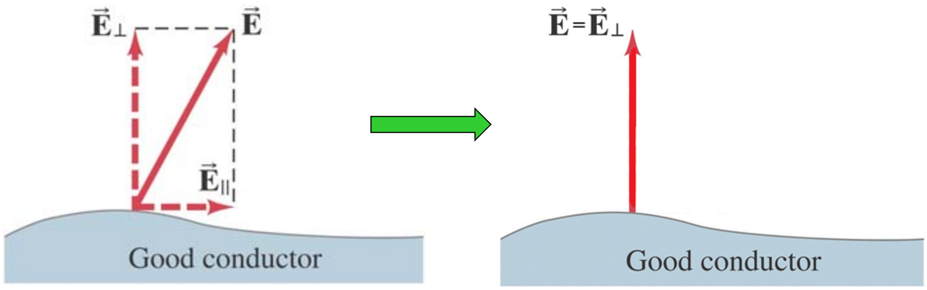
The net charge on a conductor resides on its outer surface.

21-9 Electric Fields and Conductors

The electric field is perpendicular to the surface of a conductor – again, if it were not, charges would move.



21-9 Electric Fields and Conductors

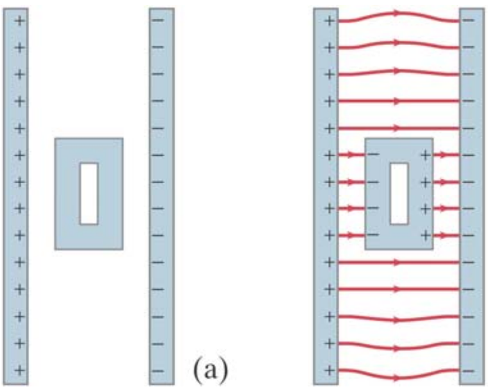


If the electric field at the surface of a conductor had a component parallel to the surface $E_{||}$, the latter would accelerate electrons into motion. In the static case, $E_{||}$ must be zero, and the electric field must be perpendicular to the conductor's surface: $E = E_\perp$

21-9 Electric Fields and Conductors

Conceptual Example 21-14: Shielding, and safety in a storm.

A neutral hollow metal box is placed between two parallel charged plates as shown. What is the field like inside the box?



The field inside the box is zero. This is why it can be relatively safe to be inside an automobile during an electrical storm.

21-10 Motion of a Charged Particle in an Electric Field

The force on an object of charge q in an electric field $\vec{\mathbf{E}}$ is given by:

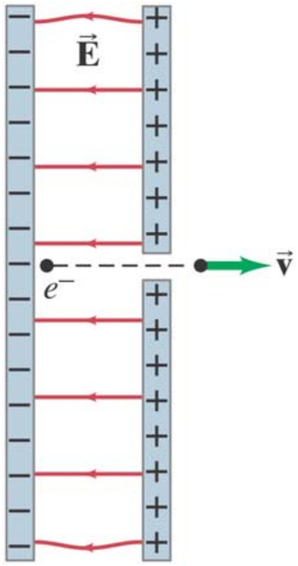
$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

Therefore, if we know the mass and charge of a particle, we can describe its subsequent motion in an electric field.

21-10 Motion of a Charged Particle in an Electric Field

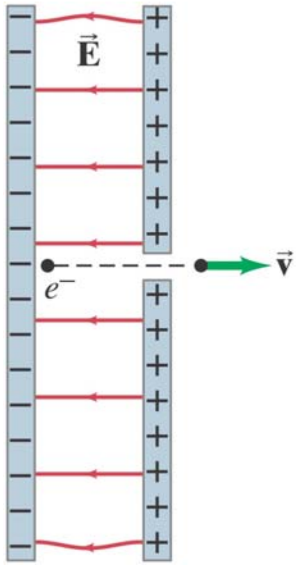
Example 21-15: Electron accelerated by electric field.

An electron (mass m = 9.11 x 10^{-31} kg) is accelerated in the uniform field $\vec{\mathbf{E}}$ (E = 2.0 x 10^4 N/C) between two parallel charged plates. The separation of the plates is 1.5 cm. The electron is accelerated from rest near the negative plate and passes through a tiny hole in the positive plate. (a) With what speed does it leave the hole? (b) Show that the gravitational force can be ignored. Assume the hole is so small that it does not affect the uniform field between the plates.



Example 21-15: Electron accelerated by electric field.

(a) With what speed does it leave the hole?



$$F = Eq = ma \quad \rightarrow \quad a = \frac{Eq}{m}$$

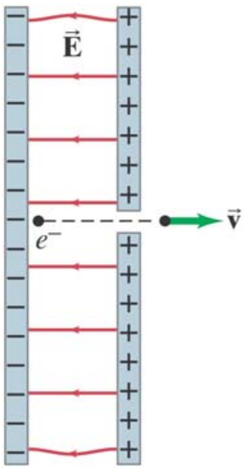
$$= \frac{2.0\times10^{4}\times1.6\times10^{-19}}{9.1\times10^{-31}} = 3.5\times10^{15}\ \text{m/s}^2$$

$$v^2 = v_0^2 + 2ax \quad (v_0 = 0)$$

$$v = \sqrt{2ax} = \sqrt{2\times3.5\times10^{15}\times0.015} = \underline{1.0\times10^{7}\ \text{m/s}}$$

Example 21-15: Electron accelerated by electric field.

(b) Show that the gravitational force can be ignored.



[Alternative (and better) way of showing] cf. text

Time for e$^-$ to travel 1.5 cm

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \quad (x_0 = 0,\ v_0 = 0)$$

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2\times0.015}{3.5\times10^{15}}} = 2.9\times10^{-9}\ \text{s}$$

Distance electron falls in y direction in t s:

$$y = y_0 + v_{0y} t + \tfrac{1}{2}gt^2 \quad (y_0 = 0,\ v_0 = 0)$$

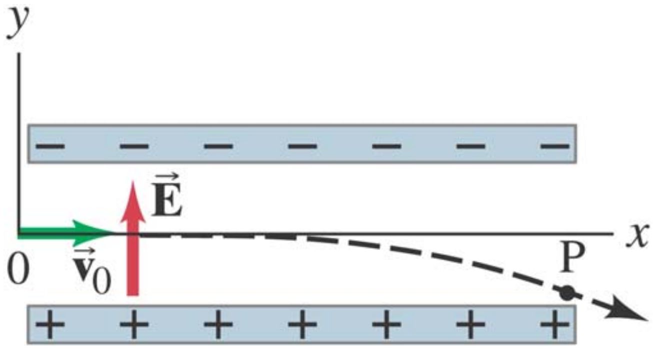
$$y = \frac{9.8\times\left(2.9\times10^{-9}\right)^2}{2} = 4.1\times10^{-17}\ \text{m}$$

$\therefore$ Gravitational effect is negligible.

21-10 Motion of a Charged Particle in an Electric Field

Example 21-16: Electron moving perpendicular to $\vec{E}$.

Suppose an electron traveling with speed v_0 = 1.0 x 10^7 m/s enters a uniform electric field $\vec{E}$, which is at right angles to v_0 as shown. Describe its motion by giving the equation of its path while in the electric field. Ignore gravity.



21-10 Motion of a Charged Particle in an Electric Field

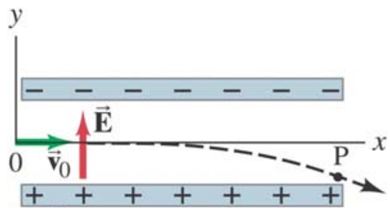
Example 21-16: Electron moving perpendicular to $\vec{E}$.

x direction : No force $\rightarrow\ v_{0x} = \text{constant} = v_0$

y direction : $F = Eq = Ee = ma_y \quad (q = e = -1.6\times10^{-19}\ \text{C})$

$$\therefore \quad a_y = -\frac{Ee}{m}$$

$$\therefore \quad x = v_0 t \quad \rightarrow \quad t = \frac{x}{v_0}$$



$$\& \quad y = y_0 + v_{0y}t + \tfrac{1}{2}at^2 \quad (y_0 = 0, v_{0y} = 0)$$

$$\therefore \quad y = \frac{at^2}{2} = \frac{-Ee}{2m}\cdot\left(\frac{x}{v_0}\right)^2 = \frac{-Eex^2}{2mv_0^2}$$

ie. $y \propto x^2 \quad \rightarrow$ Parabolic motion

Summary of Chapter 21

- Two kinds of electric charge – positive and negative.
- Charge is conserved.
- Charge on electron:

$$e = 1.602 \text{ x } 10^{-19}\ \text{C}.$$

- Conductors: electrons free to move.
- Insulators: nonconductors.

Summary of Chapter 21

- Charge is quantized in units of *e*.
- Objects can be charged by conduction or induction.
- Coulomb's law: $F = k\frac{Q_1 Q_2}{r^2}$. [magnitudes]

$$F = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2},$$

where

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2.$$

- Electric field is force per unit charge: $\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q}$.

Summary of Chapter 21

• Electric field of a point charge:

$$E = \frac{F}{q} = \frac{kqQ/r^2}{q}$$

$$E = k\frac{Q}{r^2}. \qquad \text{[single point charge]}$$

• Electric field can be represented by electric field lines.

• Static electric field inside conductor is zero; surface field is perpendicular to surface.