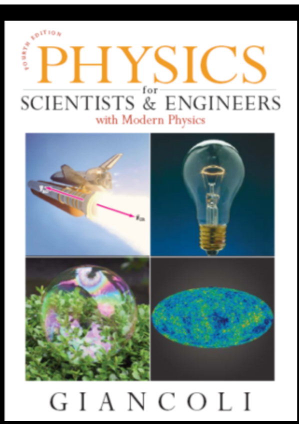


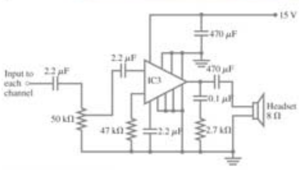
Lecture PowerPoints

Chapter 26

Physics for Scientists and Engineers, with Modern Physics, 4th edition

Giancoli

Chapter 26
DC Circuits



Units of Chapter 26

- EMF and Terminal Voltage
- Resistors in Series and in Parallel
- Kirchhoff's Rules
- Series and Parallel EMFs; Battery Charging
- Circuits Containing Resistor and Capacitor (*RC* Circuits)
- Electric Hazards
- Ammeters and Voltmeters

26-1 EMF and Terminal Voltage

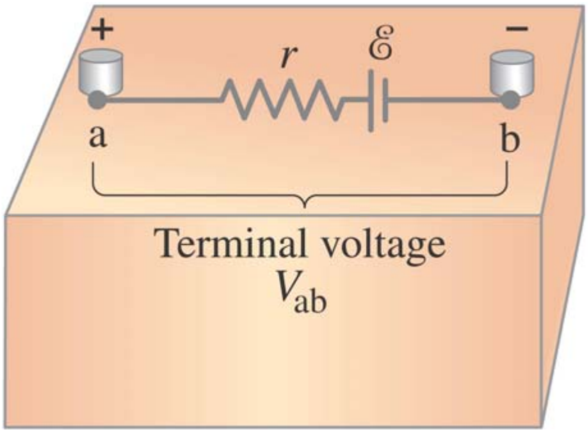
Electric circuit needs battery or generator to produce current – these are called sources of emf.

Battery is a nearly constant voltage source, but does have a small internal resistance, which reduces the actual voltage from the ideal emf:

$$V_{\mathrm{ab}} = \mathscr{E} - Ir.$$

26-1 EMF and Terminal Voltage

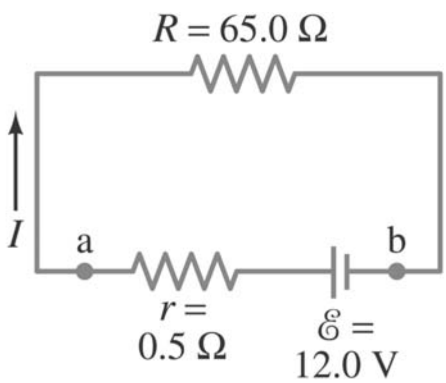
This resistance behaves as though it were in series with the emf.



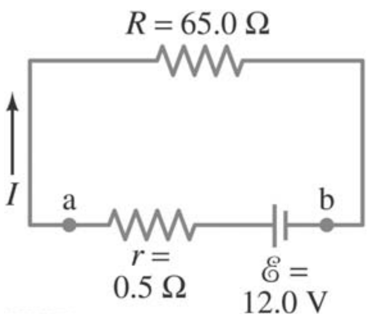
26-1 EMF and Terminal Voltage

Example 26-1: Battery with internal resistance.

A 65.0 Ω resistor is connected to the terminals of a battery whose emf is 12.0 V and whose internal resistance is 0.5 Ω. Calculate (a) the current in the circuit, (b) the terminal voltage of the battery, V_{ab}, and (c) the power dissipated in the resistor R and in the battery's internal resistance r.



Example 26-1: Battery with internal resistance.



(a) the current in the circuit

The Total Resistance:

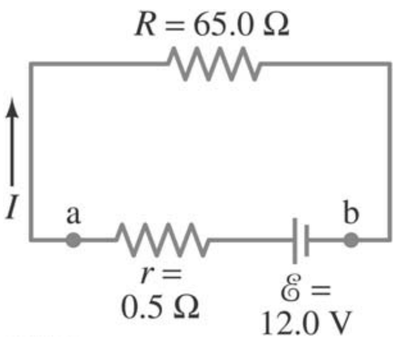
$R_{TOT} = R + r$ (See 26-2 for proof)

$$\mathscr{E} = IR_{TOT} \rightarrow I = \frac{\mathscr{E}}{R_{TOT}} = \frac{12.0}{65.5} = 0.183\text{ A} = 183\text{ mA}$$

(b) the terminal voltage of the battery, V_{ab}

$$V_{ab} = \mathscr{E} - Ir = 12.0 - 0.183 \times 0.5 = 11.9\text{ V}$$

Example 26-1: Battery with internal resistance.



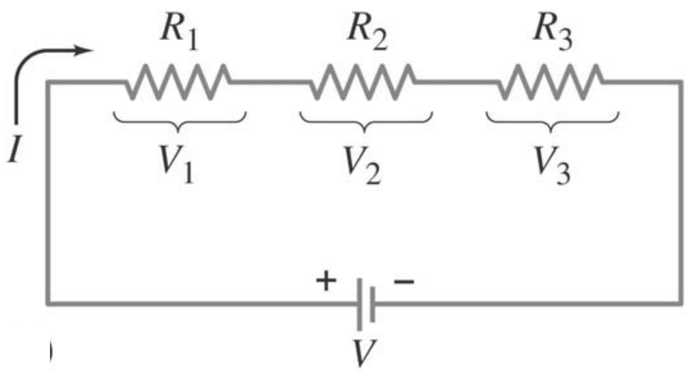
(c) the power dissipated in the resistor R and in the battery's internal resistance r.

Power in Resistor: $P = I^2R = 0.183^2 \times 65.0 = 2.18\text{ W}$

Power in Internal Resistance: $P = I^2R = 0.183^2 \times 0.5 = 0.02\text{ W}$

26-2 Resistors in Series and in Parallel

A series connection has a single path from the battery, through each circuit element in turn, then back to the battery.



26-2 Resistors in Series and in Parallel

The current through each resistor is the same; the voltage depends on the resistance. The sum of the voltage drops across the resistors equals the battery voltage:

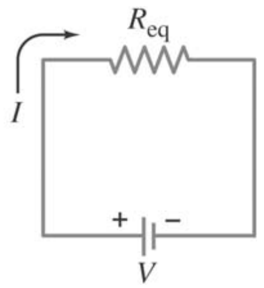
$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$$

$$R_{\text{eq}} = \frac{V}{I} = R_1 + R_2 + R_3$$

26-2 Resistors in Series and in Parallel

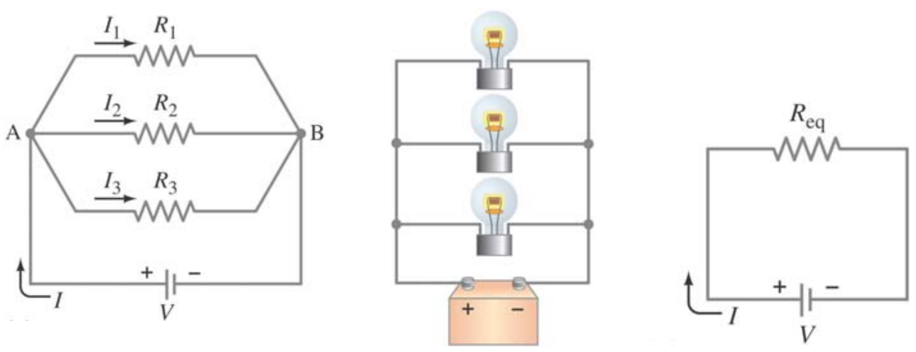
From this we get the equivalent resistance (that single resistance that gives the same current in the circuit): For Series Resistors

$$R_{eq} = R_1 + R_2 + R_3$$



26-2 Resistors in Series and in Parallel

A parallel connection splits the current; the voltage across each resistor is the same:



26-2 Resistors in Series and in Parallel

The total current is the sum of the currents across each resistor:

$$I = I_1 + I_2 + I_3,$$

$$\frac{V}{R_{\text{eq}}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}.$$

26-2 Resistors in Series and in Parallel

This gives the reciprocal of the equivalent resistance:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Note that for 2 resistors in parallel

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Multiply each term by $R_{\text{eq}} R_1 R_2$

$$\frac{R_{\text{eq}} R_1 R_2}{R_{\text{eq}}} = \frac{R_{\text{eq}} R_1 R_2}{R_1} + \frac{R_{\text{eq}} R_1 R_2}{R_2}$$

Solve for R_{eq}

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2}$$

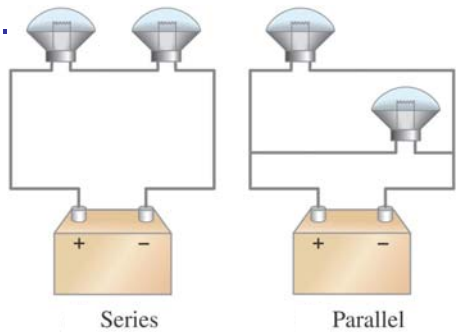
26-2 Resistors in Series and in Parallel

An analogy using water may be helpful in visualizing parallel circuits. The water (current) splits into two streams; each falls the same height, and the total current is the sum of the two currents. With two pipes open, the resistance to water flow is half what it is with one pipe open.

26-2 Resistors in Series and in Parallel

Conceptual Example 26-2: Series or parallel?

(a) The lightbulbs in the figure are identical. Which configuration produces more light? (b) Which way do you think the headlights of a car are wired? Ignore change of filament resistance R with current.



Conceptual Example 26-2: Series or parallel?

(a) The lightbulbs in the figure are identical. Which configuration produces more light?

$$\text{Series}: \; R_{eq} = R + R = 2R$$

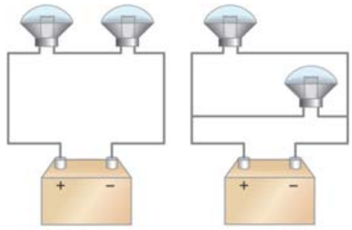
$$P_{series} = \frac{V^2}{R_{eq}} = \frac{V^2}{2R}$$

$$\text{Parallel}: \; \frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \rightarrow R_{eq} = \frac{R}{2}$$

$$P_{parallel} = \frac{V^2}{R_{eq}} = \frac{2V^2}{R}$$

$$\frac{P_{parallel}}{P_{series}} = \frac{2V^2}{R} \times \frac{2R}{V^2} = 4$$

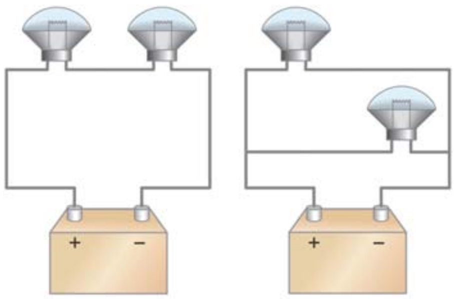
$\rightarrow$ Parallel connection is brighter



(1) Series (2) Parallel

Conceptual Example 26-2: Series or parallel?

(b) Which way do you think the headlights of a car are wired? Ignore change of filament resistance *R* with current.



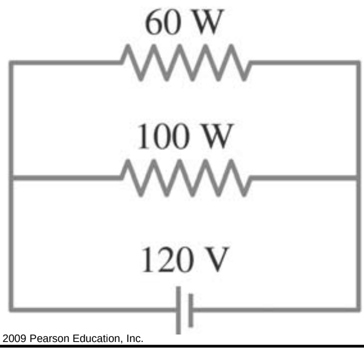
(1) Series (2) Parallel

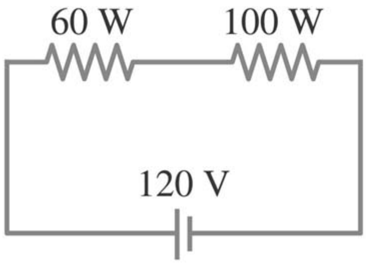
They are wired in parallel, so that if one light burns out the other one still stays on

26-2 Resistors in Series and in Parallel

Conceptual Example 26-3: An illuminating surprise.

A 100 W, 120 V lightbulb and a 60 W, 120 V lightbulb are connected in two different ways as shown. In each case, which bulb glows more brightly? Ignore change of filament resistance with current (and temperature).

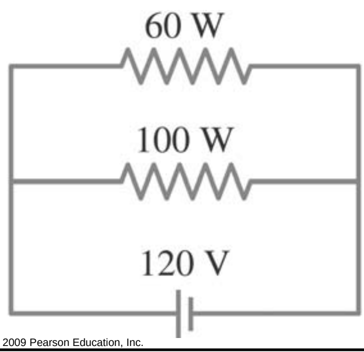




26-2 Resistors in Series and in Parallel

Conceptual Example 26-3: An illuminating surprise.

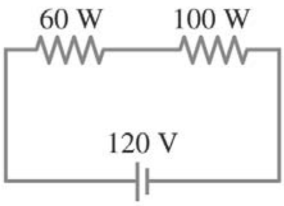
A 100 W, 120 V lightbulb and a 60 W, 120 V lightbulb are connected in two different ways as shown. In each case, which bulb glows more brightly? Ignore change of filament resistance with current (and temperature).



Each bulb sees the full 120 V drop, as they are designed to do, so the 100 W bulb is brighter

Conceptual Example 26-3: An illuminating surprise.

A 100 W, 120 V lightbulb and a 60 W, 120 V lightbulb are connected in two different ways as shown. In each case, which bulb glows more brightly? Ignore change of filament resistance with current (and temperature).

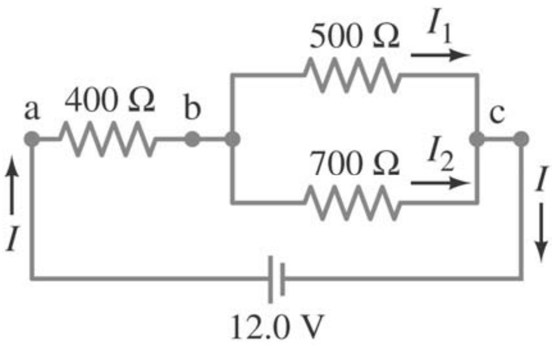


$P = V^2/R$, so at constant voltage the bulb dissipating more power will have lower resistance. In series, then, the 60 W bulb – whose resistance is higher – will be brighter. (More of the voltage will drop across it than across the 100 W bulb)

26-2 Resistors in Series and in Parallel

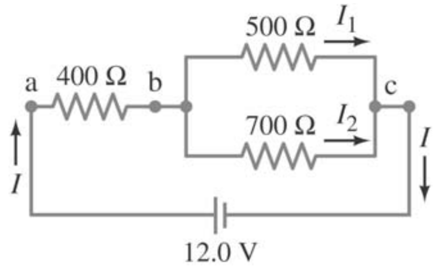
Example 26-4: Circuit with series and parallel resistors.

How much current is drawn from the battery shown?



Example 26-4: Circuit with series and parallel resistors.

How much current is drawn from the battery shown?



$$R_{bc} = \frac{R_1 R_2}{R_1 + R_2} = \frac{500 \times 700}{500 + 700} = 292\,\Omega$$

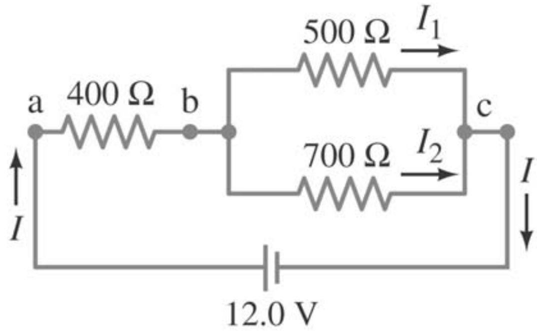
$$R_{ac} = R_{ab} + R_{bc} = 400 + 292 = 692\,\Omega$$

$$I = \frac{V}{R_{ac}} = \frac{12.0}{692} = 0.0173\text{ A} = 17.3\text{ mA}$$

26-2 Resistors in Series and in Parallel

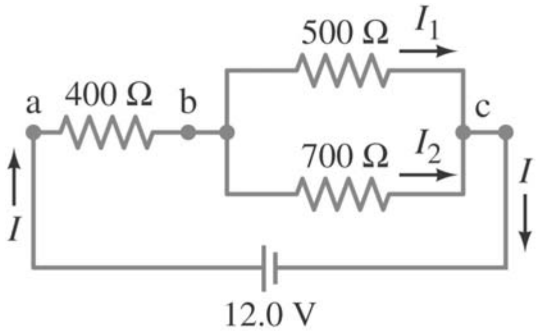
Example 26-5: Current in one branch.

What is the current through the 500 Ω resistor shown? (Note: This is the same circuit as in the previous problem.) The total current in the circuit was found to be 17.3 mA.



Example 26-5: Current in one branch.

What is the current through the 500 Ω resistor shown? From previous example $I = 17.3$ mA and $R_{bc} = 292\ \Omega$.



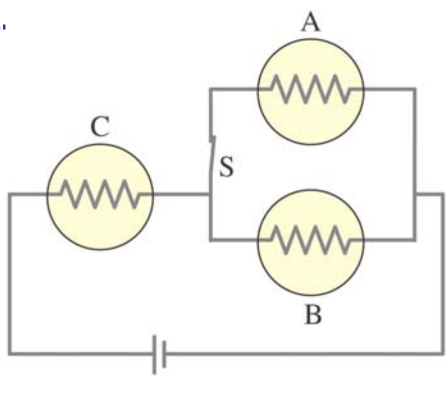
$$V_{bc} = IR_{bc} = 0.0173 \times 292 = 5.0\text{ V}$$

$$I_1 = \frac{V_{bc}}{R} = \frac{5.0}{500} = 0.010\text{ A} = 10\text{ mA}$$

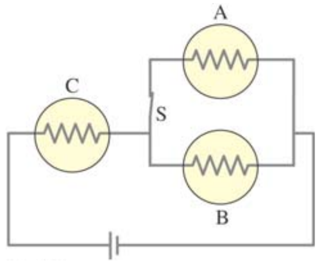
26-2 Resistors in Series and in Parallel

Conceptual Example 26-6: Bulb brightness in a circuit.

The circuit shown has three identical lightbulbs, each of resistance R. (a) When switch S is closed, how will the brightness of bulbs A and B compare with that of bulb C? (b) What happens when switch S is opened? Use a minimum of mathematics in your answers.



Conceptual Example 26-6: Bulb brightness in a circuit.



(a) When S is closed, the bulbs in parallel have an equivalent resistance equal to half that of the series bulb. Therefore, the voltage drop across them is smaller. In addition, the current splits between the two of them. Bulbs A and B will be equally bright, but much dimmer than C.

(b) With switch S open, no current flows through A, so it is dark. B and C are now equally bright, and each has half the voltage across it, so C is somewhat dimmer than it was with the switch closed, and B is brighter.

26-2 Resistors in Series and in Parallel

Example 26-7: A two-speed fan.

One way a multiple-speed ventilation fan for a car can be designed is to put resistors in series with the fan motor. The resistors reduce the current through the motor and make it run more slowly. Suppose the current in the motor is 5.0 A when it is connected directly across a 12-V battery. (a) What series resistor should be used to reduce the current to 2.0 A for low-speed operation? (b) What power rating should the resistor have?

Example 26-7: A two-speed fan.

Current in the motor is 5.0 A when it is connected directly across a 12 V battery.

(a) What series resistor should be used to reduce the current to 2.0 A for low-speed operation?

$$\text{Resistance of motor: } R_M = \frac{V}{I} = \frac{12}{5.0} = 2.4\,\Omega$$

$$\text{For current to be 2.0 A: } R_{eq} = R_M + R = \frac{V}{I} = \frac{12}{2.0} = 6.0\,\Omega$$

$$R = 6.0 - R_M = 6.0 - 2.4 = 3.6\,\Omega$$

(b) What power rating should the resistor have?

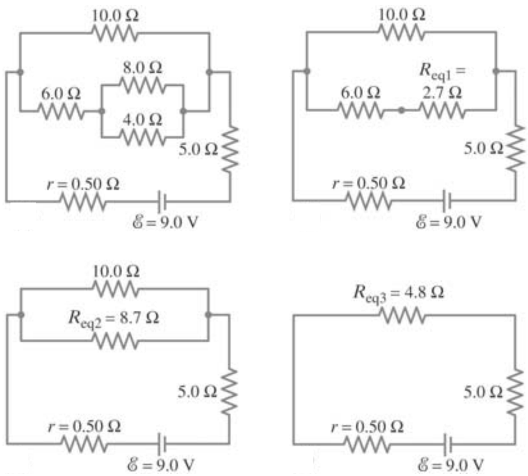
$$\text{Power in } R\text{: } P = I^2 R = 2.0^2 \times 3.6 = 14.4\text{ W}$$

Use (eg.) 20 W resistor

26-2 Resistors in Series and in Parallel

Example 26-8: Analyzing a circuit.

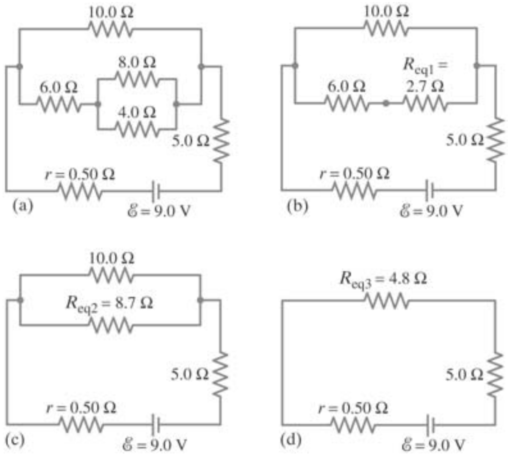
A 9.0 V battery whose internal resistance r is 0.50 Ω is connected in the circuit shown. (a) How much current is drawn from the battery? (b) What is the terminal voltage of the battery? (c) What is the current in the 6.0 Ω resistor?



Example 26-8: Analyzing a circuit.

(a) How much current is drawn from the battery?

By replacing series & parallel combinations as shown reduce the circuit to a single series combination across the battery.

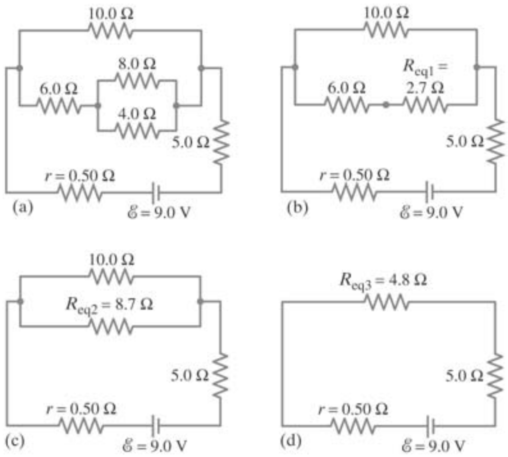


$$I = \frac{\mathscr{E}}{R_{eq}} = \frac{\mathscr{E}}{r + R_{eq3} + 5.0} = \frac{9.0}{0.50 + 4.8 + 5.0} = 0.87\ \mathrm{A}$$

Example 26-8: Analyzing a circuit.

(b) What is the terminal voltage of the battery?

From (a) $I = 0.87$ A



$$V_T = \mathscr{E} - Ir = 9.0 - 0.87 \times 0.50 = 8.6\ \mathrm{V}$$

Example 26-8: Analyzing a circuit.

(c) What is the current in the 6.0 Ω resistor?

Consider (d)

Total V shared by all resistors

$$\text{Voltage across } R_{eq3} = \frac{R_{eq3} \times \mathscr{E}}{R_{TOT}}$$

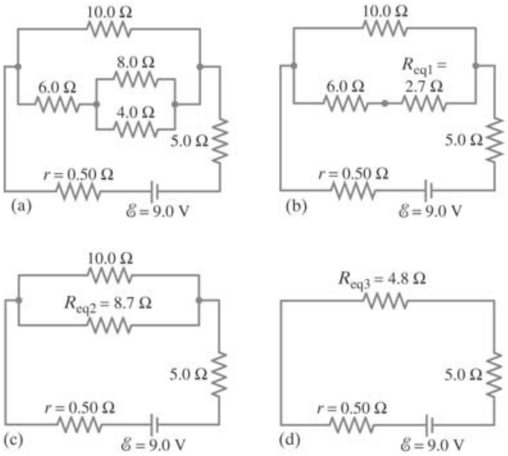
$$= \frac{4.8 \times 9.0}{4.8 + 5.0 + 0.5} = 4.19 \text{ V}$$

$$\text{Voltage across } R_{eq2} = 4.19 \text{ V}$$

$$\text{Voltage across } 6\,\Omega = \frac{6.0 \times 4.19}{6.0 + 2.9}$$

$$= 2.89 \text{ V}$$

$$I_{6\Omega} = \frac{V}{R} = \frac{2.89}{6.0} = 0.48 \text{ A}$$

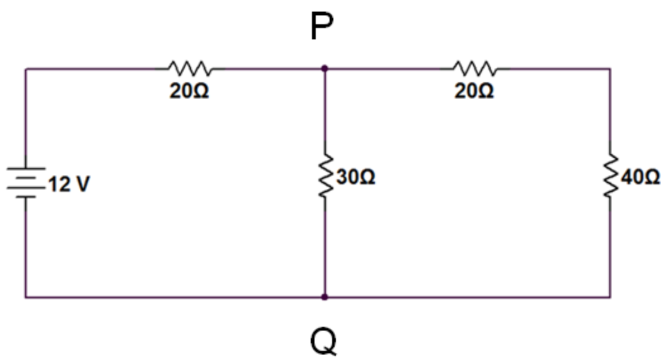


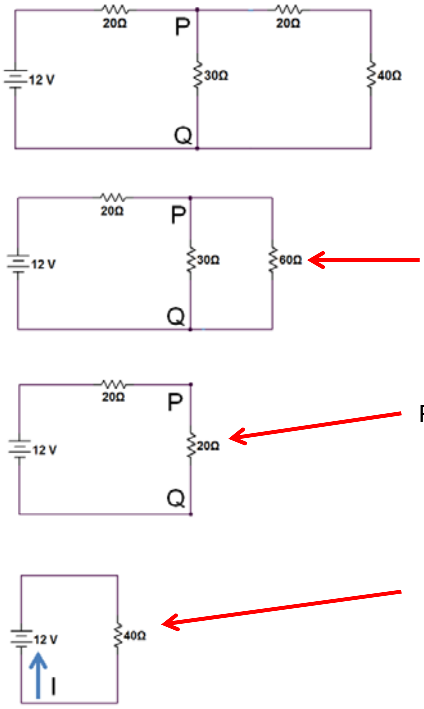
Example 2: Circuit Analysis

Many electric circuits can be solved by simplifying the circuit to replace series and parallel combinations and using Ohm's Law. eg.

Find the current in each branch of the circuit and the potential difference of point **P** with respect to **Q** ($\mathbf{V_{PQ}}$).

(Ans: 0.3 A through battery, 0.2 A through 30 Ω, 0.1 A through 40 Ω, V_{PQ} = +6V)





Simplify circuit by replacing series and parallel combinations and then work from simplest circuit back to the original circuit

Series: $R = R_1 + R_2 + \ldots$

Parallel: $1/R = 1/R_1 + 1/R_2 + \ldots$

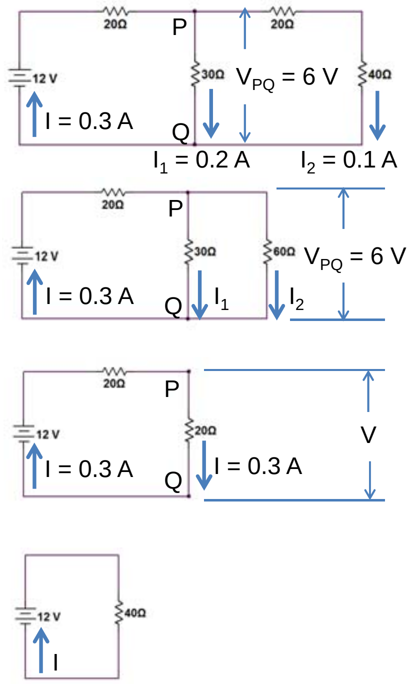
Ohm's Law: $V = IR$

Replace 20 Ω, 40 Ω series combination (60 Ω)

Replace 30 Ω, 60 Ω parallel combination (60 Ω)

Replace 20 Ω, 20 Ω series combination (40 Ω)
And calculate I

$I = V/R = 12/40 = 0.3$ A



$I_1 = V/R = 6/30 = 0.2$ A

$I_2 = V/R = 6/60 = 0.1$ A

$V = V_{PQ} = IR = 0.3 \times 20 = 6$ V

$I = V/R = 12/40 = 0.3$ A

Many circuits cannot be solved using the above approach, particularly circuits having multiple loops and voltage sources. For these circuits Kirchhoff's Rules (or Kirchhoff's Laws) can be used.

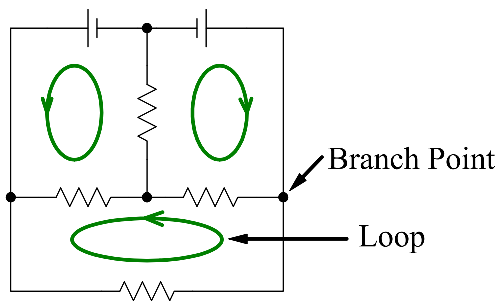
KIRCHHOFF'S RULES

For circuits that cannot be solved by simplifying using series/parallel combinations of resistors and using Ohm's Law Kirchhoff's Rules provide a solution.

Kirchhoff's Rules are based on two fundamental laws:

A: Conservation of Charge, and
B: Conservation of Energy.

Kirchhoff's rules analyse the circuit in terms of **branch points** and **loops**.



A Branch Point (•) is any point where three or more conductors join.

A Loop is any closed path in the circuit that does not pass through any part of the circuit more than once. In analysing a circuit there must be sufficient loops to pass through every circuit component at least once.

Conservation of electric charge gives rise to the **Point Rule**. Since charge cannot be created or destroyed, the total current (rate of flow of charge) flowing into a junction must equal the total current flowing out of the junction.

Conservation of energy requires that the sum of the voltage drops (IR products) in a closed loop equals the sum of the voltage sources in the same loop (the **Loop Rule**).

Point Rule: The sum of the currents flowing into a junction is zero.

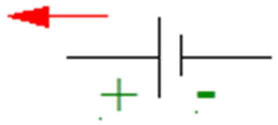
Sign Convention: Inwards currents are positive, outwards currents are negative.

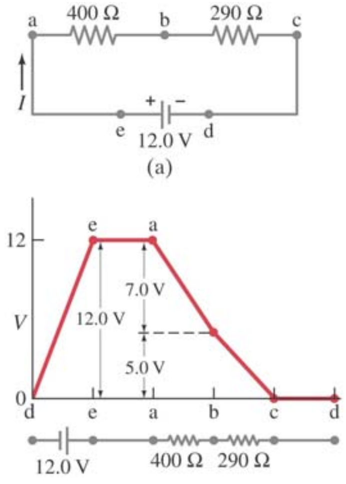
$$\sum \mathrm{I} = 0$$

Loop Rule: The sum of the **e.m.f.'s** (battery voltages) in any loop equals the sum of the **IR** products in the same loop.

Sign Convention: e.m.f.'s and currents in direction of travel around a loop are positive. The direction of a voltage source is always away from its positive terminal.

$$\sum \mathrm{V} = \sum \mathrm{IR}$$



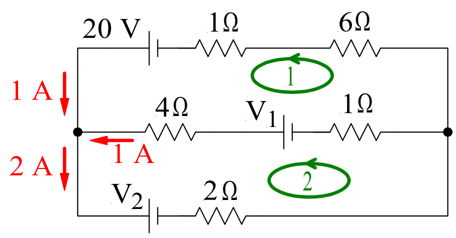


26-3 Kirchhoff's Rules

Problem Solving: Kirchhoff's Rules

1. Label each current, including its direction.
2. Identify unknowns.
3. Apply junction and loop rules; you will need as many independent equations as there are unknowns.
4. Solve the equations, being careful with signs. If the solution for a current is negative, that current is in the opposite direction from the one you have chosen.

Example 1:



Find: V_1 and V_2

Note that the direction of travel around the loop is arbitrary. If you changed the direction it would change all signs in the equations but the answer would be the same.

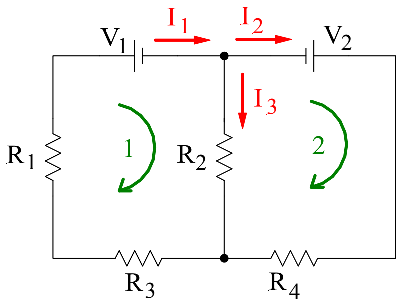
Loop 1: $20 - V_1 = 6\times1 + 1\times1 - 4\times1 - 1\times1 = 6 + 1 - 4 - 1 = 2$

$\therefore \quad V_1 = 20 - 2 = \mathbf{18\ V}$

Loop 2: $V_1 - V_2 = 18 - V_2 = 1\times1 + 4\times1 + 2\times2$

$\therefore \quad V_2 = 18 - 9 = \mathbf{9\ V}$

Example 2:



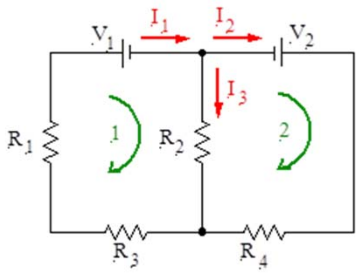
$V_1 = 6\ V$
$V_2 = 12\ V$
$R_1 = 100\ \Omega$
$R_2 = 120\ \Omega$
$R_3 = 270\ \Omega$
$R_4 = 330\ \Omega$

Point Rule: $\sum I = 0 \quad \Rightarrow \quad I_1 - I_2 - I_3 = 0 \quad \Rightarrow \quad I_3 = I_1 - I_2$

Loop Rule: $\sum V = \sum IR$ Note that there are only two independent variables (I_1 & I_2) so that only two equations are required.

Loop 1: $R_2I_3 + R_3I_1 + R_1I_1 = -V_1$
$\therefore$ $R_2(I_1 - I_2) + R_3I_1 + R_1I_1 = -V_1$
$\therefore$ $(R_1 + R_2 + R_3)I_1 - R_2I_2 = -V_1$
$\therefore$ $490\ I_1 - 120\ I_2 = -6$ - (1)

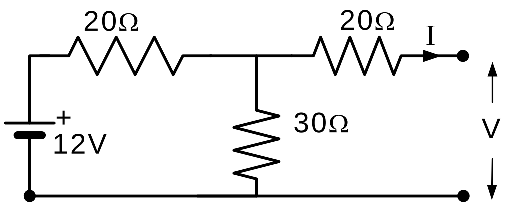
Loop 2: $R_4I_2 - R_2I_3 = V_2$
$\therefore$ $R_4I_2 - R_2(I_1 - I_2) = V_2$
$\therefore$ $-R_2I_1 + (R_2 + R_4)I_2 = V_2$
$\therefore$ $-120\ I_1 + 450\ I_2 = 12$ - (2)



Solving equations (1) & (2) and calculating I_3 using Point Rule gives:
$I_1 = -0.0061\ A$ $I_2 = 0.0250\ A$ $I_3 = -0.0311\ A$
Note that the negative signs for I_1 and I_3 tell us that the currents are in the **OPPOSITE DIRECTION** to that shown on the diagram. Since we generally don't know the directions of the currents before solving the problem, we just arbitrarily assign directions. If you get a negative answer for a current you just reverse the direction of the current.

Answer: $I_1 = 0.0061\ A \leftarrow$ $I_2 = 0.0250\ A \rightarrow$ $I_3 = 0.0311\ A \uparrow$

A trick question: Find V and I.



I = 0 as no current in open loop

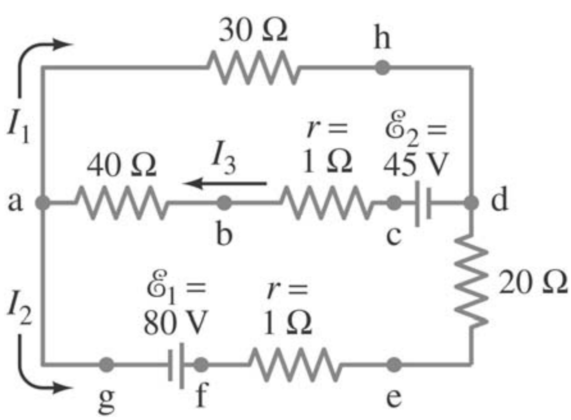
V across 30 Ω is the same as **V** as no voltage drop (**I** = 0) across 20 Ω on right.

$\mathbf{I_{30\Omega}} = \mathbf{V/R_{TOT}} = 12/(20 + 30) = 0.24$ A
$\therefore\ \mathbf{V} = \mathbf{I_{30\Omega}R_{30\Omega}} = 0.24 \times 30 = 7.2$ V

26-3 Kirchhoff's Rules

Example 26-9: Using Kirchhoff's rules.

Calculate the currents I_1, I_2, and I_3 in the three branches of the circuit in the figure.



Example 26-9: Using Kirchhoff's rules.

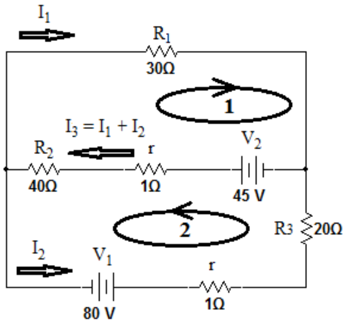
Calculate the currents I_1, I_2, and I_3 in the three branches of the circuit in the figure.

Point Rule gives: $I_3 = I_1 + I_2$

Loop 1: $V_2 = r(I_1 + I_2) + R_2(I_1 + I_2) + R_1 I_1$
Loop 2: $V_1 + V_2 = r(I_1 + I_2) + R_2(I_1 + I_2) + rI_2 + R_3 I_2$

$45 = 1(I_1 + I_2) + 40(I_1 + I_2) + 30I_1$
$80 + 45 = 1(I_1 + I_2) + 40(I_1 + I_2) + 1.I_2 + 20I_2$

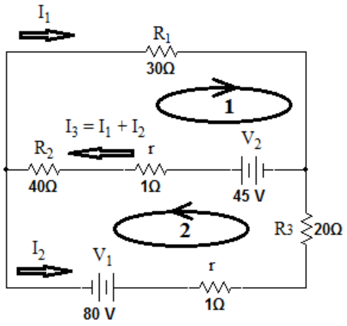
$71I_1 + 41I_2 = 45$
$41I_1 + 62I_2 = 125$



Solve gives:

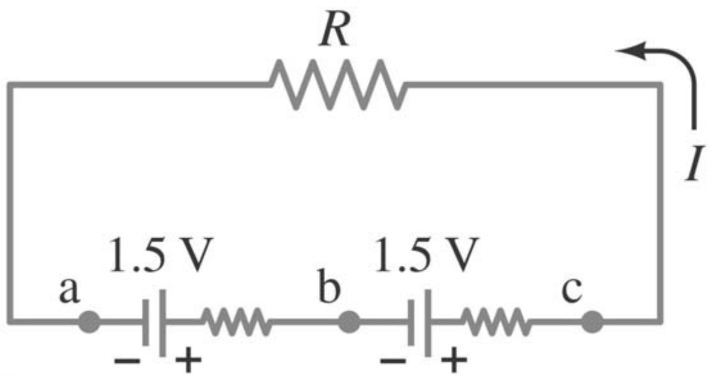
$I_1 = -0.86$ A
$I_2 = 2.59$ A
$I_3 = I_1 + I_2 = 1.73$ A

Note that I_1 is NEGATIVE
$\rightarrow$ I_1 is to LEFT



26-4 Series and Parallel EMFs; Battery Charging

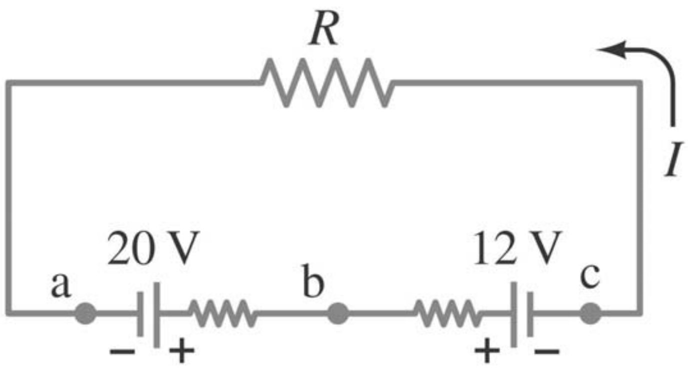
EMFs in series in the same direction: total voltage is the sum of the separate voltages.



$\Sigma\mathscr{E} = 3.0\ \mathrm{V}$

26-4 Series and Parallel EMFs; Battery Charging

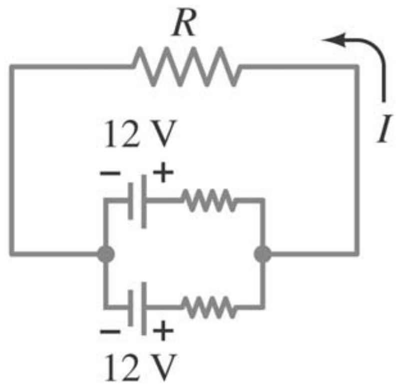
EMFs in series, opposite direction: total voltage is the difference, but the lower-voltage battery is charged (assuming rechargeable battery).



$\Sigma\mathscr{E} = 20 - 12 = 8\mathrm{V}$

26-4 Series and Parallel EMFs; Battery Charging

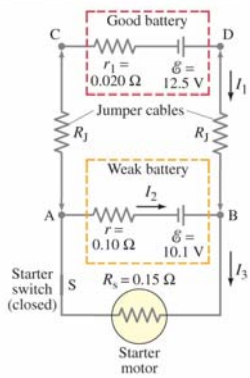
EMFs in parallel only make sense if the voltages are the same; this arrangement can produce more current than a single emf.



26-4 Series and Parallel EMFs; Battery Charging

Example 26-10: Jump starting a car.

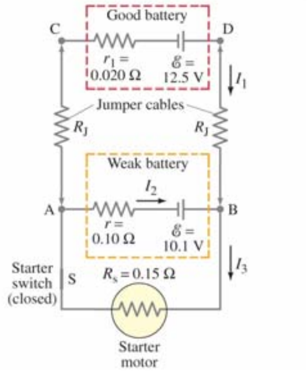
A good car battery is being used to jump start a car with a weak battery. The good battery has an emf of 12.5 V and internal resistance 0.020 Ω. Suppose the weak battery has an emf of 10.1 V and internal resistance 0.10 Ω. Each copper jumper cable is 3.0 m long and 0.50 cm in diameter, and can be attached as shown. Assume the starter motor can be represented as a resistor R_s = 0.15 Ω. Determine the current through the starter motor (a) if only the weak battery is connected to it, and (b) if the good battery is also connected.



26-4 Series and Parallel EMFs; Battery Charging

Example 26-10: Jump starting a car.

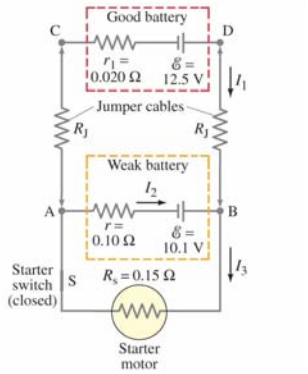
A good car battery is being used to jump start a car with a weak battery. The good battery has an emf of 12.5 V and internal resistance 0.020 Ω. Suppose the weak battery has an emf of 10.1 V and internal resistance 0.10 Ω. Each copper jumper cable is 3.0 m long and 0.50 cm in diameter, and can be attached as shown. Assume the starter motor can be represented as a resistor R_s = 0.15 Ω. Determine the current through the starter motor (a) if only the weak battery is connected to it.



$$I_3 = \frac{\mathscr{E}}{r + R_S} = \frac{10.1}{0.10 + 0.15} = 40.4 \text{ A}$$

Example 26-10: Jump starting a car.

Determine the current through the starter motor (b) if the good battery is also connected.



In this case we need to use Kirchhoff's laws.

But first need to calculate R_J.

Resistivity: $\rho_{CU} = 1.68\times10^{-8}$ Ω.m
Diameter: $D = 5\times10^{-3}$ m
Area: $A = \pi D^2/4 = 1.96\times10^{-5}$ m^2

$$R_J = \rho \ell / A = 0.0026\ \Omega$$

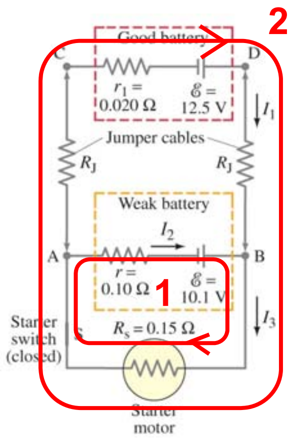
Example 26-10: Jump starting a car.

$$\text{Point Rule: } \Sigma I = 0 \rightarrow I_1 + I_2 - I_3 = 0 \quad \text{-}(1)$$

$$\text{Loop Rule: } \Sigma RI = \Sigma \mathscr{E}$$

$$\text{Loop 1: } rI_2 + r_S I_3 = \mathscr{E}$$
$$\therefore 0.10 I_2 + 0.15 I_3 = 10.1 \quad \text{-}(2)$$

$$\text{Loop 2: } R_J I_1 + r_1 I_1 + R_J I_1 + R_S I_3 = \mathscr{E}$$
$$\therefore 0.0026 I_1 + 0.020 I_1 + 0.0026 I_1 + 0.15 I_3 = 12.5$$
$$\therefore 0.025 I_1 + 0.15 I_3 = 12.5 \quad \text{-}(3)$$



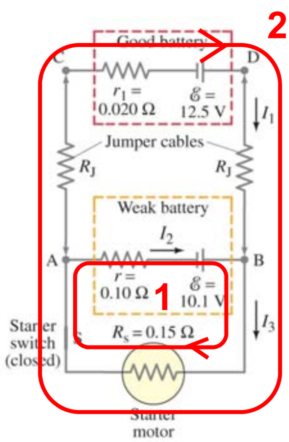
Example 26-10: Jump starting a car.

Using matrix equations:

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 0.1 & 0.15 \\ 0.025 & 0 & 0.15 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 10.1 \\ 12.5 \end{bmatrix}$$

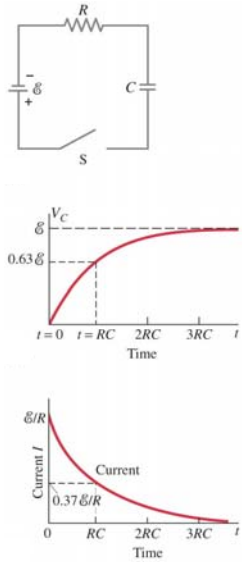
$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 0 & 0.1 & 0.15 \\ 0.025 & 0 & 0.15 \end{vmatrix} = 0.0213$$

$$I_3 = \frac{\begin{vmatrix} 1 & 1 & 0 \\ 0 & 0.1 & 10.1 \\ 0.025 & 0 & 12.5 \end{vmatrix}}{\Delta} = \frac{1.503}{0.0213} = 71\text{ A}$$



26-5 Circuits Containing Resistor and Capacitor (*RC* Circuits)

When the switch is closed, the capacitor will begin to charge. As it does, the voltage across it increases, and the current through the resistor decreases.



26-5 Circuits Containing Resistor and Capacitor (*RC* Circuits)

To find the voltage as a function of time, we write the equation for the voltage changes around the loop:

$$\mathscr{E} = IR + \frac{Q}{C}.$$

Since $Q = dI/dt$, we can integrate to find the charge as a function of time:

$$Q = C\mathscr{E}\left(1 - e^{-t/RC}\right).$$

26-5 Circuits Containing Resistor and Capacitor (*RC* Circuits)

The voltage across the capacitor is $V_C = Q/C$:

$$V_C = \mathscr{E}\left(1 - e^{-t/RC}\right).$$

The quantity RC that appears in the exponent is called the time constant of the circuit:

$$\tau = RC.$$

26-5 Circuits Containing Resistor and Capacitor (*RC* Circuits)

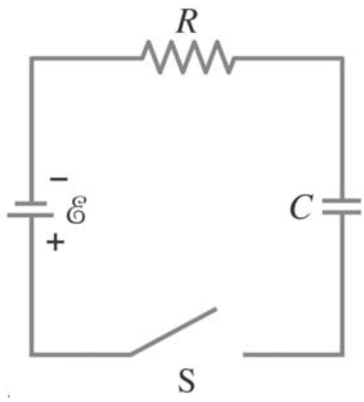
The current at any time t can be found by differentiating the charge:

$$I = \frac{dQ}{dt} = \frac{\mathscr{E}}{R} e^{-t/RC}.$$

26-5 Circuits Containing Resistor and Capacitor (*RC* Circuits)

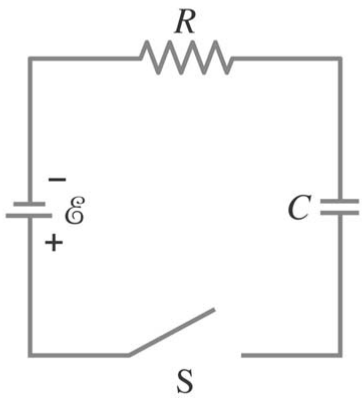
Example 26-11: *RC* circuit, with emf.

The capacitance in the circuit shown is $C = 0.30\ \mu F$, the total resistance is 20 kΩ, and the battery emf is 12 V. Determine (a) the time constant, (b) the maximum charge the capacitor could acquire, (c) the time it takes for the charge to reach 99% of this value, (d) the current I when the charge Q is half its maximum value, (e) the maximum current, and (f) the charge Q when the current I is 0.20 its maximum value.



Example 26-11: *RC* circuit, with emf.

$C = 0.30\ \mu F$
$R = 20\ k\Omega$
$\mathcal{E} = 12\ V$



Determine (a) the time constant

$$\tau = RC = 20\times10^{-3}\times0.30\times10^{-6}$$
$$= 6.0\times10^{-3}\ \mathrm{s} = 6.0\ \mathrm{ms}$$

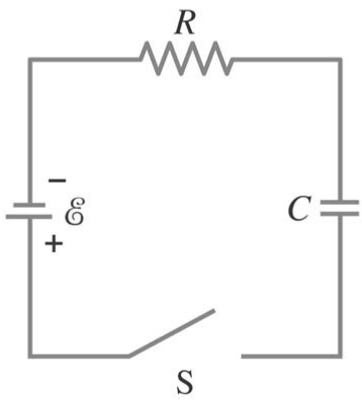
(b) the maximum charge the capacitor could acquire

$$Q = C\mathcal{E} = 0.3\times10^{-6}\times12$$
$$= 3.6\times10^{-6}\ \mathrm{C} = 3.6\ \mu\mathrm{C}$$

Example 26-11: *RC* circuit, with emf.

Determine (c) the time it takes for the charge to reach 99% of Q_{max}

$C = 0.30\ \mu\text{F}$
$R = 20\ \text{k}\Omega$
$\mathscr{E} = 12\ \text{V}$



$$Q = C\mathscr{E}\left(1 - e^{-t/RC}\right) = Q_{max}\left(1 - e^{-t/RC}\right)$$

$$Q/Q_{max} = 0.99 = 1 - e^{-t/RC}$$

$$e^{-t/RC} = 1 - 0.99 = 0.01$$

$$-t/RC = \ln 0.01$$

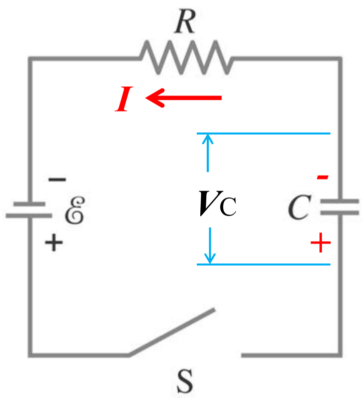
$$t = -RC \ln 0.01$$

$$= -6.0\times10^{-3}(-4.6) = 28\times10^{-3}\ \text{s} = 28\ \text{ms}$$

Example 26-11: *RC* circuit, with emf.

Determine (d) the current *I* when the charge *Q* is half its maximum value

$C = 0.30\ \mu\text{F}$
$R = 20\ \text{k}\Omega$
$\mathscr{E} = 12\ \text{V}$



$$Q = Q_{max}/2 = \frac{3.6\times10^{-6}}{2} = 1.8\times10^{-6}\ \text{C}$$

$$Q = CV_C \rightarrow V_C = \frac{Q}{C} = \frac{1.8\times10^{-6}}{0.3\times10^{-6}} = 6.0\ \text{V}$$

Apply Kirchhoff's Law to Loop (S is closed)

Note: V_C opposes charging current

$$\mathscr{E} - V_C = IR \rightarrow I = \frac{\mathscr{E} - V_C}{R}$$

$$= \frac{12-6}{20\times10^3} = 300\times10^{-6}\ \text{A} = 300\ \mu\text{A}$$

Example 26-11: *RC* circuit, with emf.

Determine (e) the maximum current

$C = 0.30\ \mu\text{F}$
$R = 20\ \text{k}\Omega$
$\mathscr{E} = 12\ \text{V}$

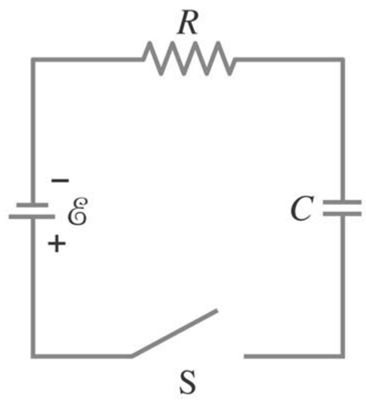
$$I_{\max} \text{ when } V_C = 0$$

$$I_{\max} = \frac{\mathscr{E}}{R} = \frac{12}{20\times10^{3}} = 600\times10^{-6}\ \text{A} = 600\ \mu\text{A}$$

(f) the charge Q when the current I is 0.20 its maximum value.

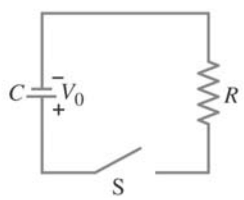
$0.2I_{\max} = 120\times10^{-6}$ A

$$\mathscr{E} = IR + Q/C \rightarrow Q = C(\mathscr{E} - IR)$$
$$= 0.3\times10^{-6}\left(12 - 120\times10^{-6}\times20\times10^{3}\right)$$
$$= 2.9\times10^{-6}\ \text{C} = 2.9\ \mu\text{C}$$

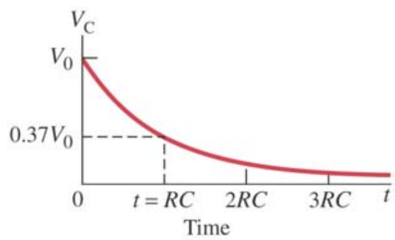


26-5 Circuits Containing Resistor and Capacitor (*RC* Circuits)

If an isolated charged capacitor is connected across a resistor, it discharges:



$$Q = Q_0 e^{-t/RC}.$$



26-5 Circuits Containing Resistor and Capacitor (*RC* Circuits)

Once again, the voltage and current as a function of time can be found from the charge:

$$V_C = V_0 e^{-t/RC}$$

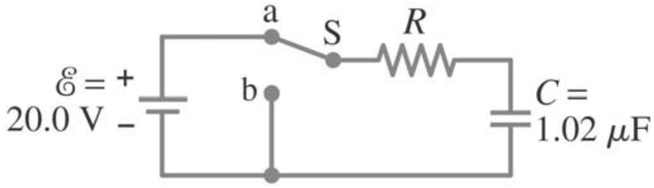
and

$$I = -\frac{dQ}{dt} = \frac{Q_0}{RC} e^{-t/RC} = I_0 e^{-t/RC}.$$

26-5 Circuits Containing Resistor and Capacitor (*RC* Circuits)

Example 26-12: Discharging RC circuit.

In the RC circuit shown, the battery has fully charged the capacitor, so $Q_0 = C\mathcal{E}$. Then at $t = 0$ the switch is thrown from position a to b. The battery emf is 20.0 V, and the capacitance $C = 1.02\ \mu$F. The current I is observed to decrease to 0.50 of its initial value in 40 μs. (a) What is the value of Q, the charge on the capacitor, at $t = 0$? (b) What is the value of R? (c) What is Q at $t = 60\ \mu$s?

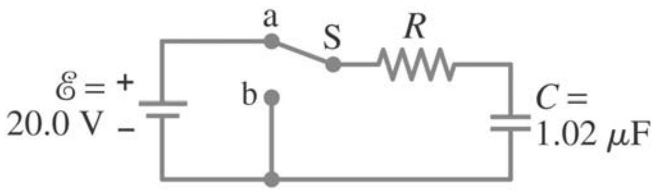


Example 26-12: Discharging *RC* circuit.

In the *RC* circuit shown, the battery has fully charged the capacitor, so *C*. Then at $t = 0$ the switch is thrown from position a to b. The battery emf is 20.0 V, and the capacitance $C = 1.02\ \mu F$. The current I is observed to decrease to 0.50 of its initial value in 40 μs.

(a) What is the value of Q, the charge on the capacitor, at $t = 0$?

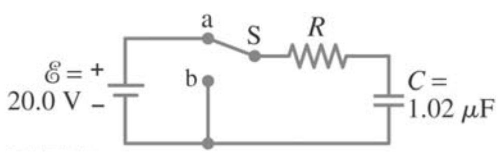
$$Q_0 = C\mathcal{E} = 1.02\times10^{-6}\times20.0 = 20.4\times10^{-6}\ \mathrm{C} = 20.4\ \mu\mathrm{C}$$



Example 26-12: Discharging *RC* circuit.

In the *RC* circuit shown, the battery has fully charged the capacitor, so $Q_0 = C\mathcal{E}$. Then at $t = 0$ the switch is thrown from position a to b. The battery emf is 20.0 V, and the capacitance $C = 1.02\ \mu F$. The current I is observed to decrease to 0.50 of its initial value in 40 μs.

(b) What is the value of R?



$$I = I_0 e^{-t/RC} \rightarrow I/I_0 = 0.5 = e^{-t/RC}$$

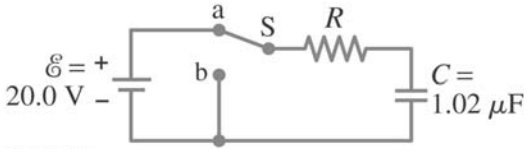
$$\frac{-t}{RC} = \ln 0.5 = -0.693$$

$$\therefore R = \frac{t}{0.693C} = \frac{40\times10^{-6}}{0.693\times1.02\times10^{-6}} = 57\ \Omega$$

Example 26-12: Discharging *RC* circuit.

In the *RC* circuit shown, the battery has fully charged the capacitor, so $Q_0 = C\mathcal{E}$. Then at $t = 0$ the switch is thrown from position a to b. The battery emf is 20.0 V, and the capacitance C = 1.02 μF. The current I is observed to decrease to 0.50 of its initial value in 40 μs.

(c) What is Q at t = 60 μs?



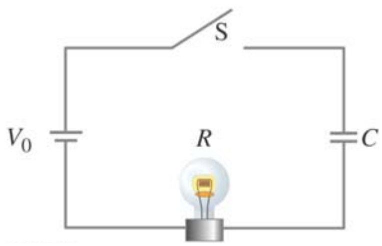
$$Q = Q_0 e^{-t/RC} = 20.4\times10^{-6}\, e^{\frac{-60\times10^{-6}}{57\times1.02\times10^{-6}}}$$

$$= 7.3\times10^{-6}\ \mathrm{C} = 7.3\ \mu\mathrm{C}$$

26-5 Circuits Containing Resistor and Capacitor (*RC* Circuits)

Conceptual Example 26-13: Bulb in *RC* circuit.

In the circuit shown, the capacitor is originally uncharged. Describe the behavior of the lightbulb from the instant switch S is closed until a long time later.



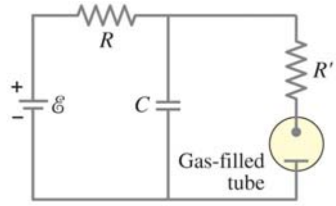
When the switch is closed, the current is large and the bulb is bright. As the capacitor charges, the bulb dims; once the capacitor is fully charged the bulb is dark

26-5 Circuits Containing Resistor and Capacitor (*RC* Circuits)

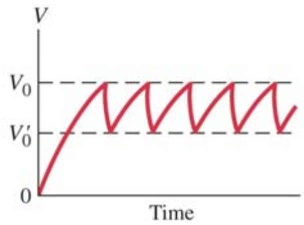
Example 26-14: Resistor in a turn signal.

Estimate the order of magnitude of the resistor in a turn-signal circuit.

An *RC* circuit, coupled with a gas-filled tube as a switch, can produce a repeating "sawtooth" voltage, as shown.



(a)

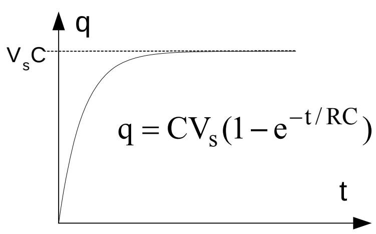


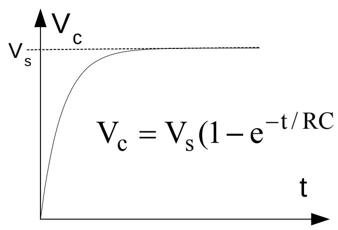
Solution: A turn signal flashes about twice per second; if we use a 1 μF capacitor, we need a 500 kΩ resistor to get an 0.5 s time constant

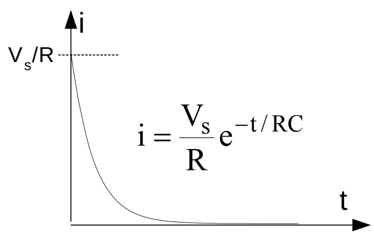
$$\tau = RC \rightarrow R = \frac{\tau}{C} = \frac{0.5}{10^{-6}} = 5\times10^{5}\ \Omega = 500\ \mathrm{k}\Omega$$

Summary of Charging-Discharging of Capacitors

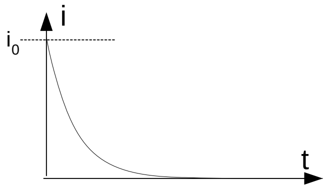
Charging characteristics



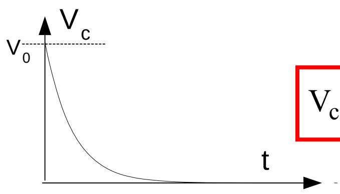




Discharge characteristics



$$i = -i_0 e^{-t/RC}$$

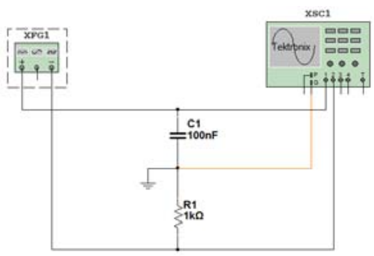


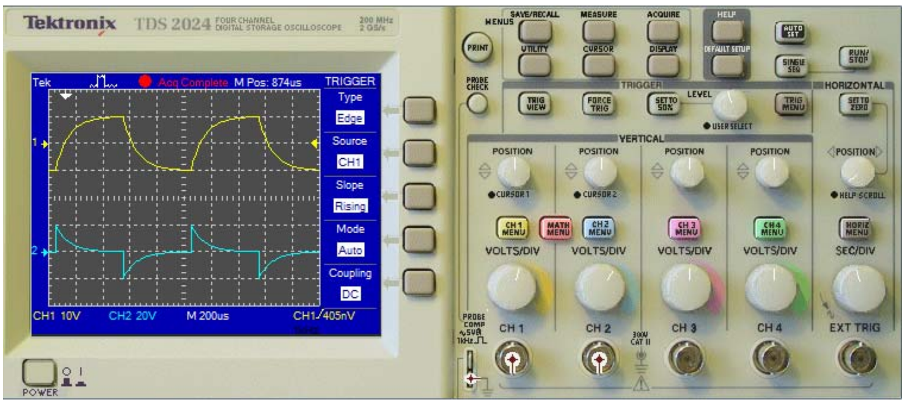
$$V_c = V_0 e^{-t/RC}$$

Circuit to show charging-discharging of capacitor. A square wave is applied across the CR circuit.

Top trace is V_C

Bottom trace is V_R

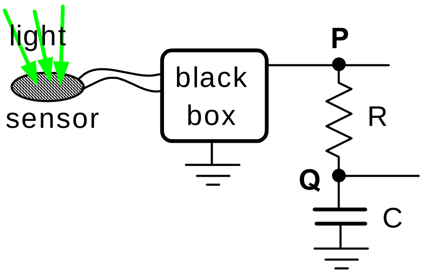




In the circuit below, a signal from a light sensor causes the voltage (with respect to ground) at point P to go from zero to 5 V instantaneously. Design criteria require that point Q reaches 3V, 1 ms later.

select values of R and C which will achieve this.

(ans. RC = 0.00109 - many possible choices for R & C.)



Solution on next slide

For capacitor charging:

$$\mathbf{V_C} = \mathbf{V_S}\left(1 - e^{-\mathbf{t}/\mathbf{RC}}\right)$$

$$\therefore \frac{\mathbf{V_C}}{\mathbf{V_S}} = 0.6 = 1 - e^{-\mathbf{t}/\mathbf{RC}}$$

$$\therefore e^{-\mathbf{t}/\mathbf{RC}} = 1 - 0.6 = 0.4$$

$$\therefore -\mathbf{t}/\mathbf{RC} = ln(0.4) = -0.9163$$

$$\therefore \mathbf{RC} = \frac{t}{0.9163} = \frac{10^{-3}}{0.9163} = 0.00109 = 1.09\times10^{-3}$$

$\mathbf{V_S} = 5$ V
$\mathbf{V_C} = 3$ V

$\mathbf{V_C}/\mathbf{V_S} = 3/5 = 0.6$

$\mathbf{t} = 1$ ms $= 10^{-3}$ s

There are many possible values for **R** and **C**
e.g. if **C** = 1 μF

then $$R = \frac{1.09\times10^{-3}}{10^{-6}} = 1.09\times10^{3}\ \Omega = 1.09\ \mathrm{k}\Omega$$

26-6 Electric Hazards

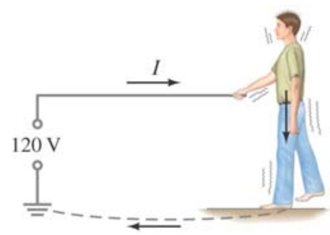
Most people can "feel" a current of 1 mA; a few mA of current begins to be painful. Currents above 10 mA may cause uncontrollable muscle contractions, making rescue difficult. Currents around 100 mA passing through the torso can cause death by ventricular fibrillation.

Higher currents may not cause fibrillation, but can cause severe burns.

Household voltage can be lethal if you are wet and in good contact with the ground. Be careful!

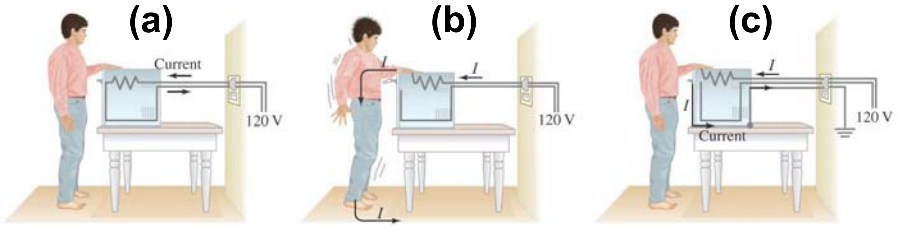
26-6 Electric Hazards

A person receiving a shock has become part of a complete circuit.



26-6 Electric Hazards

Faulty wiring and improper grounding can be hazardous. Make sure electrical work is done by a professional.



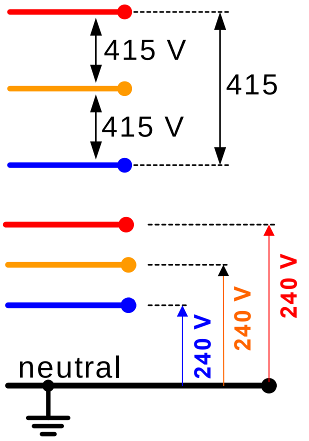
(a) An electric oven operating normally with a 2-prong plug. (b) Short to the case with ungrounded case: shock. (c) Short to the case with the case grounded by a 3-prong plug

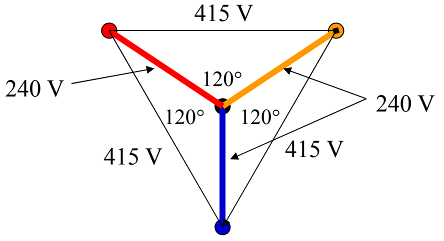
Domestic Power

- In Australia domestic power is
- 240 V AC (RMS) at 50Hz
- The maximum current from normal power points is 10 A (RMS).
  - The maximum power is therefore 2400 W ($P = IV$).
  - Plug in appliances must draw $\leq$ 2400 W
- 15 A power points are available (dedicated fuse).
- High power appliances (stoves, air conditioners etc) must be “wired in” with their own fuse.
- Other countries have different standards eg:
  - USA 110 V, 60Hz Europe 220 V, 50Hz

Three Phase Power

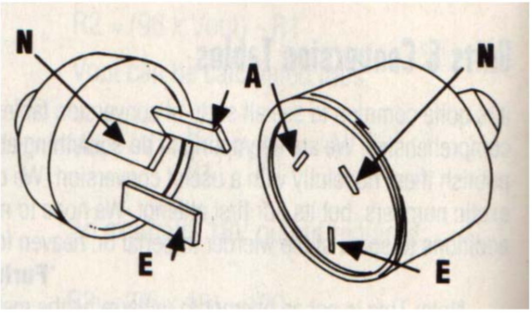
- Power is not carried along the transmission lines in streets at 240 V – this is inefficient.
- Power comes from the sub station in 3 separate lines which are out of phase by 120° - This is called **3 phase power**
  - Needs 4 wires, 3 “active” (thick) to carry the current and 1 “neutral” (thin)
- 415 V, 3 phase power is used extensively in industry.
- Advantages:
  - Cost – needs less wire
  - 3 phase electric motors start and run better and are more compact than single phase.





- The 3 active lines are separated from each other by a potential difference of 415 V.
- Each active line is 240 V above the neutral line.
- The neutral line is “grounded”.
- Your house is connected to one of the active lines and the neutral.

Power Plugs



Active (line)	Brown
Neutral	Blue
Earth (ground)	Green/Yellow

- **Old Code**:

Active – Red
Neutral – Black
Earth – Green

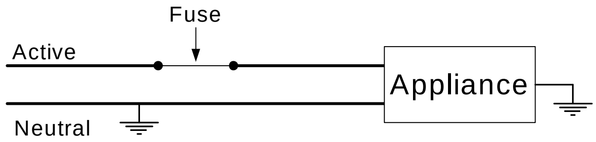
15 A plugs have large earth pins.

Earthing

- The **earth** pin of each power outlet is connected to wiring in the house which is **grounded** ie. it is connected to a metal stake which is buried in the ground.
- The neutral line coming into the house is **"earthed"** at the switch board (it has also been earthed at substations on the way)
- Inside each appliance the earth wire is connected to the casing of the appliance.
- This provides a first level of safety through the use of **fuses**.

Fuses

- A fuse is simply a thin wire, mounted in a holder.
- The size and material of the wire is chosen so that it melts when the current in it exceeds a given value.
- The fuse is placed in "**series**" in the ACTIVE line.



- If the appliance draws too much current, the fuse "blows".
- The fuse protects the house wiring from overloading.
- Many appliances have their own fuse .
- Fuses are primarily for the protection of electrical equipment.

Note: Mostly use circuit breakers now which are resetable electro-mechanical devices.

Examples

- All the power points in a kitchen are on the same circuit which has a 15 A fuse.
- How many 200 W appliances can be plugged in and operated at one time? (ans. 18)

$V = 240$ V & $I_{max} = 15$ A

$P_{max} = VI_{max} = 240 \times 15 = 3600$ W

$P_{max}/200 = 3600/200 = 18$

- If two 1500 W jugs plugged in and turned on at the same time, how much current can be drawn from a third power point before the fuse blows? (ans. 2.5 A)

Remaining Power = 3600 - 2×1500 = 600 W

$I = P/V = 600/240 = 2.5$ A

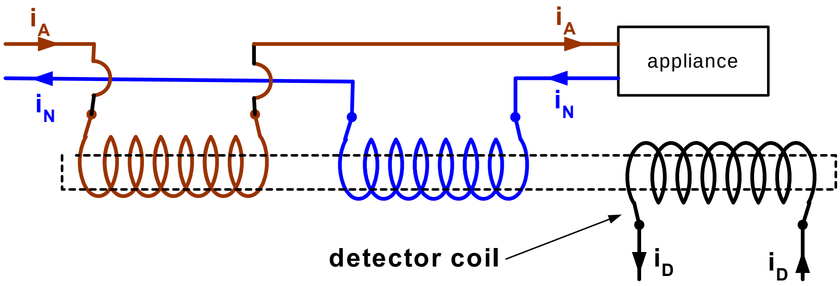
Safety for humans

- Fuses also provide a level of safety for people using electrical equipment and appliances.
- If the casing of an appliance becomes connected to the active line due to a fault, a very large current will flow through the active line to earth.
- The fuse will blow very quickly and isolate the appliance from the active line.
- Without the earth, current can continue to flow to earth through a person who is touching the case
- A fuse will not protect you if you touch an active (live) wire directly.

Residual Current Devices

(Also known as Earth Leakage Protection)

- These devices monitor simultaneously the current in the active line and in the neutral line.
- If $i_{(active)} \neq i_{(neutral)}$ then current must be leaking down to earth, from somewhere it shouldn't; eg. through a person accidentally connected to active.
- The device is designed to "trip out" if the difference between the two currents is greater than 30 mA.
- "Trip time" < 30 ms. (milli second)
  - No damage to the heart can occur in this time



- Normally $i_N = i_A$ and $i_D = 0$
- However if $i_N \neq i_A$, then: $i_D > 0$
  - Electronic circuitry monitoring detector coil turns mains off at switchboard if $i_D > 0$

Double insulation

- Another form of protection is **double insulation**. This method reduces the likelihood of a connection from the live line to the person.
- In addition to the normal insulation required in its electric circuitry, exposed parts of an appliance are often made of insulating materials.
- Eg if the outer casing of an electric drill is made from a good electrical insulator then an internal fault cannot result in an electric shock for a person holding the drill.

Effects of Electric Shock.

- 1 - 5mA — Threshold of sensation.
- 10 - 20 mA — Involuntary muscle contractions. (can't let go)
- 20 - 100 mA — Pain, breathing difficulties.
- 100 - 300 mA — Ventricular fibrillation. (possible death)
- >300 mA — Respiratory paralysis, burns, unconsciousness.

Skin Resistance.

- About **500 kΩ** for **dry skin**.
- Down to about **1 kΩ** for **wet skin**.
- Also depends on "contact pressure."
    - Highly variable from person to person.
    - Drastically reduced if the skin is broken.

examples

1. What is the lowest voltage that may cause a heart attack in adverse conditions?

   If R_{skin} = 1 kΩ & I = 100 mA

   $V = IR = 0.1\times10^3 = 100$ V

2. What range of currents may be expected from a 240 V electric shock?

   Dry skin: $R_{skin} \approx 500$ kΩ $\rightarrow$ $I = V/R \approx 240/5\times10^5 = 480\times10^{-6}$ A $= 480$ μA

   Wet skin: $R_{skin} \approx 1$ kΩ $\rightarrow$ $I = V/R \approx 240/10^3 = 240\times10^{-3}$ A $= 240$ mA

   Range from about 480 μA (negligible) to about 240 mA (lethal)

Electrocution is not the only electrical hazard.

- Large amounts of energy are available from batteries and capacitors.
- When there is a "short" this energy may be dumped in a very short time.
  - Result: burns, ignition of inflammable materials
- Also, be aware V_{peak} ~1.4 times V_{rms}

Summary of Chapter 26

- A source of emf transforms energy from some other form to electrical energy.
- A battery is a source of emf in parallel with an internal resistance.
- Resistors in series:

$$R_{eq} = R_1 + R_2 + R_3. \qquad \text{[series]}$$

Summary of Chapter 26

- Resistors in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

- Kirchhoff's rules:
  1. Sum of currents entering a junction equals sum of currents leaving it.
  2. Total potential difference around closed loop is zero.

Summary of Chapter 26

- *RC* circuit has a characteristic time constant:

$$\tau = RC.$$

- To avoid shocks, don't allow your body to become part of a complete circuit.

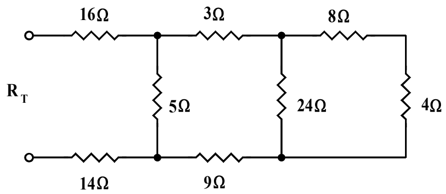
Appendix - Sample Questions & Answers

1. A 1 kΩ resistor is rated at ¼ W. What is the current in the resistor when it is dissipating its maximum allowed power?

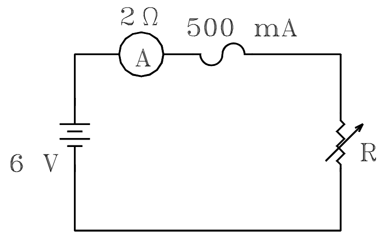
2. A 100 Ω resistor has a power rating of 5 W. What is the maximum safe voltage that can be applied across the resistor? What is the current through the resistor when it is dissipating its maximum allowed power?

3. What power rating is required for a 100 Ω resistor if 10 V is to be applied across it?

4. Find the total resistance of the resistor ladder network shown at right.



5. An ammeter with a resistance of 2 Ω has a full scale deflection of 250 mA. A 500 mA fuse is placed in series to protect the meter. What is the minimum value of resistance that should be used in the circuit if the current is not to exceed 250 mA? If R is reduced below this minimum at what value of R would the fuse be expected to blow?



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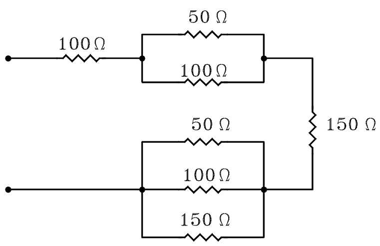
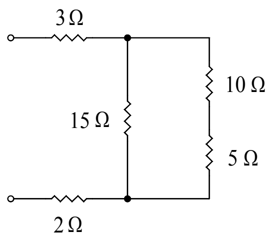


What is the effective resistance of the circuit at left?

7. What is the effective resistance of the circuit at right?



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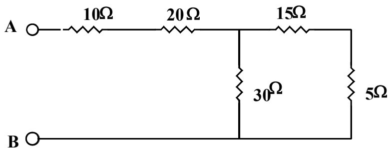
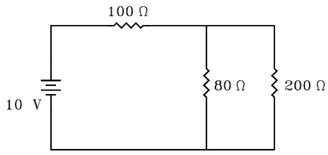


Calculate the effective resistance between A-B for the circuit at left.

9. Calculate the current in the 200 Ω resistor for the circuit at right.



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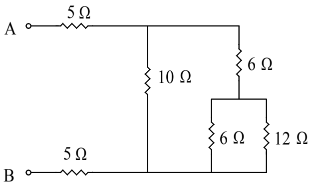
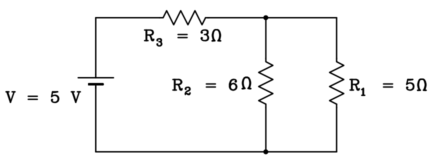


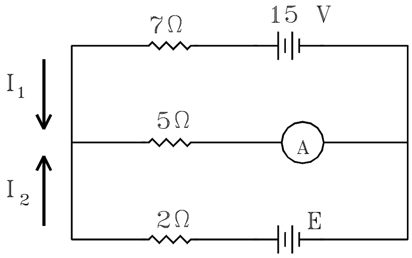
(a) What is the resistance between A and B?

(b) If a 7.5 V battery is connected across AB what is the current through the battery?

11. Calculate the current through all resistors for the circuit at right.

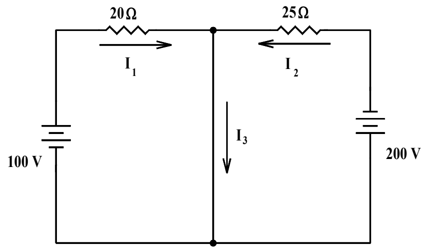


12.



The ammeter in the circuit shown reads 2 A. Find the currents I_1 and I_2 and the value of E.

13. Find the short circuit current I_3 for the circuit at right.



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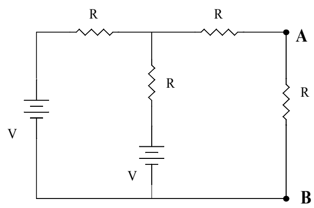
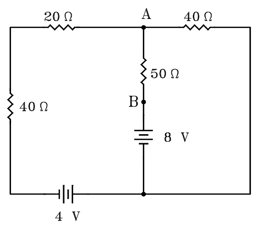


If V = 10 V, and
R = 100 Ω,

Use Kirchhoff's laws to determine the voltage across AB.

Hint: Calculate current between AB first.

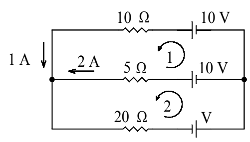
15.



Find all currents for the circuit at left. What would be the reading of a voltmeter placed across points A and B in the circuit?

16. (a) What is the current in the 20 Ω resistor?

(b) Write an equation for loop 2 using Kirchhoff's Loop Rule and hence find the voltage required for the battery in series with the 20 Ω resistor if the currents are as shown in the diagram.



Solutions to Sample Questions

1. A 1 kΩ resistor is rated at ¼ W. What is the current in the resistor when it is dissipating its maximum allowed power?

$P = I^2R$ | **R** = 1 kΩ = 1000 Ω
∴ **$I = \sqrt{(P/R)}$** $= \sqrt{(0.25/1000)} = 0.016$ A = 16 mA | **P** = 0.25 W

2. A 100 Ω resistor has a power rating of 5 W. What is the maximum safe voltage that can be applied across the resistor? What is the current through the resistor when it is dissipating its maximum allowed power?

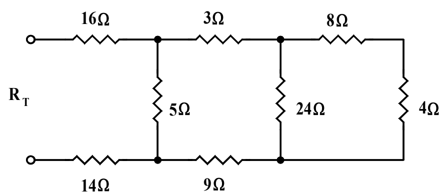
$P = V^2/R$ | **R** = 100 Ω
∴ **$V = \sqrt{(PR)}$** $= \sqrt{(5 \times 100)} = 22.4$ V | **P** = 5 W

& **$P = I^2R$**
∴ **$I = \sqrt{(P/R)}$** $= \sqrt{(5/100)} = 0.224$ A = 224 mA

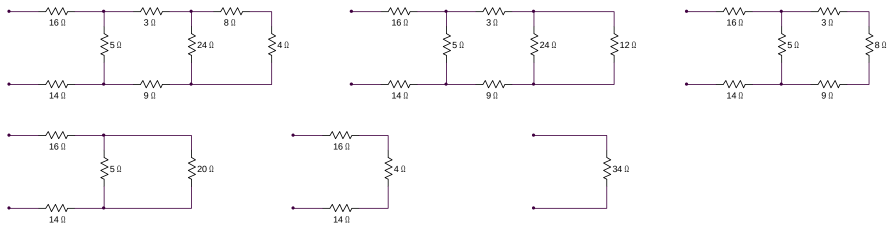
3. What power rating is required for a 100 Ω resistor if 10 V is to be applied across it?

$P = V^2/R$ $= 10^2/100 = 1$ W | **R** = 100 Ω **V** = 10 V

4. Find the total resistance of the resistor ladder network shown at right.

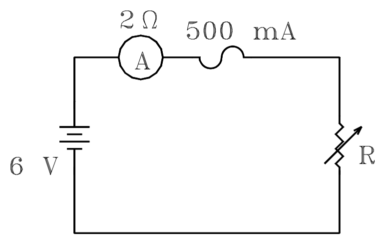


Replace series and parallel combinations (Series: **$R = R_1 + R_2 + \ldots$** & Parallel: **$1/R = 1/R_1 + 1/R_2 + \ldots$**) and redraw circuit one step at a time.



Ans: 34 Ω

5. An ammeter with a resistance of 2 Ω has a full scale deflection of 250 mA. A 500 mA fuse is placed in series to protect the meter. What is the minimum value of resistance that should be used in the circuit if the current is not to exceed 250 mA? If R is reduced below this minimum at what value of R would the fuse be expected to blow?



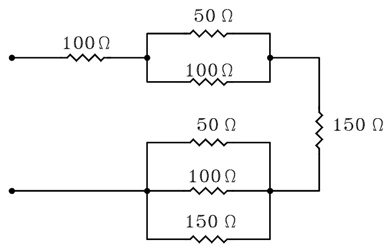
$\mathbf{V = I_{FSD}R_{TOT}}$
$\therefore\ 6 = 0.25\ \mathbf{R_{TOT}} \rightarrow \mathbf{R_{TOT}} = 6/0.25 = 24\ \Omega$

$\therefore\ \mathbf{R_{TOT} = R_A + R}$
$\therefore\ \mathbf{R = R_{TOT} - R_A} = 24 - 2 = 22\ \Omega$

For fuse to blow: $\mathbf{I} = 500$ mA $= 0.5$ A
& $\mathbf{V = IR_{TOT}} \rightarrow \mathbf{R_{TOT} = V/I} = 6/0.5 = 12\ \Omega$
$\therefore\ \mathbf{R = R_{TOT} - R_A} = 12 - 2 = 10\ \Omega$

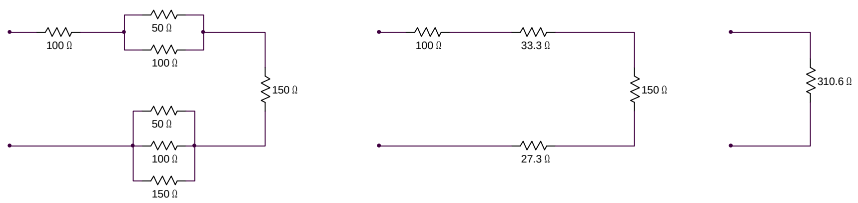
$\mathbf{V = 6\ V}$
$\mathbf{R_A} = 2\ \Omega$
$\mathbf{I_{FSD}} = 250$ mA $= 0.25$ A

6.



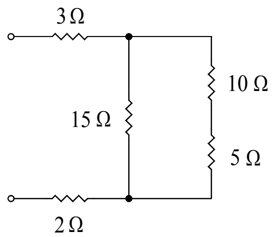
What is the effective resistance of the circuit at left?

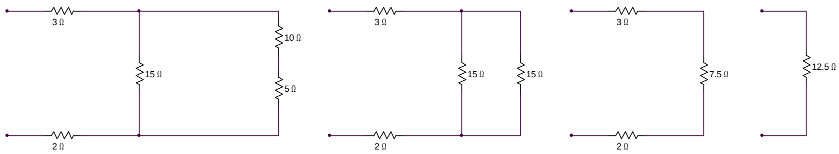
Replace series and parallel combinations (Series: $\mathbf{R = R_1 + R_2 + \ldots}$ & Parallel: $\mathbf{1/R = 1/R_1 + 1/R_2 + \ldots}$) and redraw circuit one step at a time.



Ans: 310.6 Ω

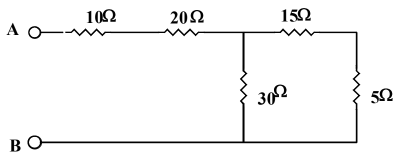
7. What is the effective resistance of the circuit at right?

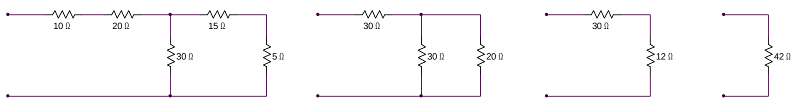




Ans: 12.5 Ω

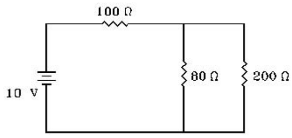
8. Calculate the effective resistance between A-B for the circuit at left.

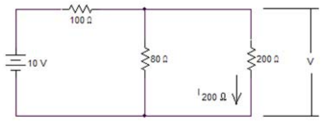


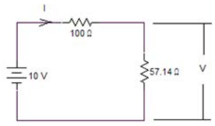


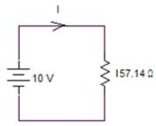
Ans: 42 Ω

9. **Calculate the current in the 200 Ω resistor for the circuit at right.**







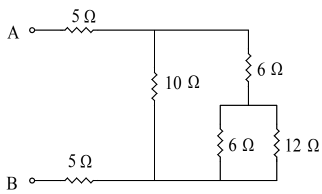


I = V/R = 10/157.14 = 0.0636 A

∴ **V = IR** = 0.0636 x 57.14 = 3.64 V

∴ $I_{200\Omega}$ **= V/R** = 3.64/200 = 0.018 A = 18 mA

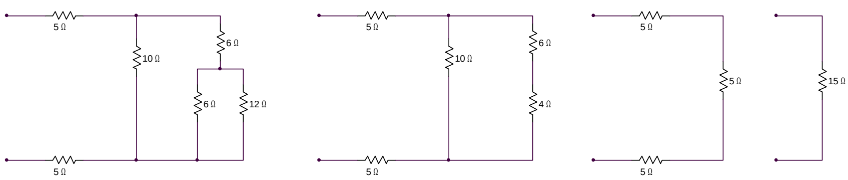
10.



(a) What is the resistance between A and B?

(b) If a 7.5 V battery is connected across AB what is the current through the battery?

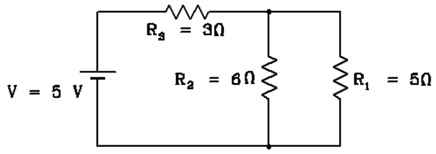
(a)



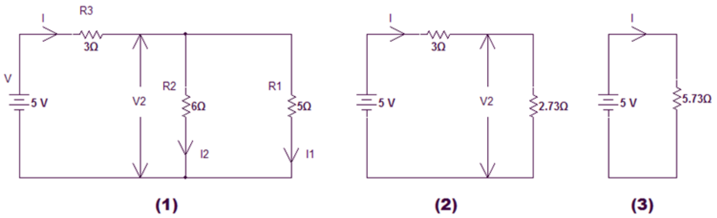
Ans: 15 Ω

(b) $\mathbf{V = I_{Batt}R_{eq}} \rightarrow \mathbf{I_{Batt} = V/R_{eq}} = 7.5/15 = 0.5\text{ A} = 500\text{ mA}$

11. **Calculate the current through all resistors for the circuit at right.**



Simplify circuit by replacing series/parallel combinations.



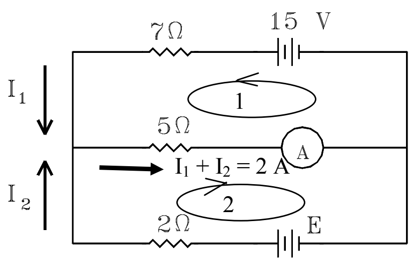
Circuit (3) → Total current I = V/R = 5/5.73 = 0.873 A = **873 mA = Current through R_3**

Circuit (2) → V_2 = IR = 0.873 x 2.73 = 2.38 V

Circuit (1) → $I_1 = V_2/R_1$ = 2.38/5 = 0.476 A = **476 mA = Current through R_1**

$I_2 = V_2/R_2$ = 2.38/6 = 0.397 A = **397 mA = Current through R_2**

12.



The ammeter in the circuit shown reads 2 A. Find the currents I_1 and I_2 and the value of E.

Using conventions from additional notes on Kirchhoff's Rules which are easier to apply sign convention in loops: $\sum \mathbf{V} = \sum \mathbf{IR}$

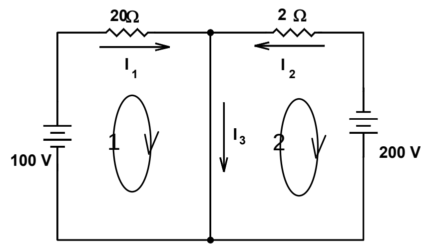
Loop 1: $15 = 7\mathbf{I_1} + 5(\mathbf{I_1} + \mathbf{I_2}) = 7\mathbf{I_1} + 5 \times 2 \rightarrow 7\mathbf{I_1} = 15 - 10 = 5 \rightarrow$ **I_1 = 0.714 A = 714 mA**

Since $\mathbf{I_1} + \mathbf{I_2} = 2 \rightarrow \mathbf{I_2} = 2 - \mathbf{I_1} = 2 - 0.714 =$ **1.286 A**

Loop 2: $\mathbf{E} = 2\mathbf{I_2} + 5(\mathbf{I_1} + \mathbf{I_2}) = 2\mathbf{I_2} + 5 \times 2 = 2\mathbf{I_2} + 10 = 2 \times 1.286 + 10 =$ **12.57 V**

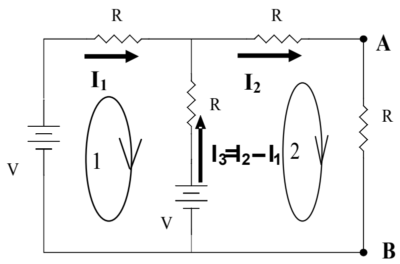
13. Find the short circuit current I_3 for the circuit at right.

$\sum IR = \sum V$



Loop 1: $20\mathbf{I_1} = 100 \rightarrow \mathbf{I_1} = 100/20 = 5$ A
Loop 2: $-25\mathbf{I_2} = 200 \rightarrow \mathbf{I_2} = -200/25 = -8$ A
Short Circuit Current: $\mathbf{I_3} = \mathbf{I_1} + \mathbf{I_2} = 5 - 8 = -3$ A
Short Circuit Current ($\mathbf{I_3}$) = **3 A Upwards**

14.



If V = 10 V, and
R = 100 Ω,

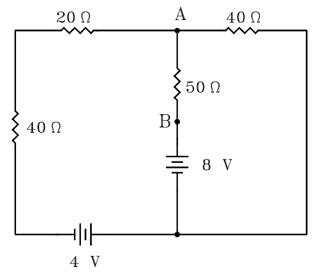
Use Kirchhoff's laws to determine the voltage across AB.

Hint: Calculate current between AB first.

$\sum IR = \sum V$

Loop 1: $\mathbf{RI_1} - \mathbf{R(I_2 - I_1)} = \mathbf{V} - \mathbf{V} \rightarrow 2\mathbf{I_1} - \mathbf{I_2} = 0$ - (1)
Loop 2: $\mathbf{R(I_2 - I_1)} + \mathbf{RI_2} + \mathbf{RI_2} = \mathbf{V} \rightarrow -10\mathbf{I_1} + 30\mathbf{I_2} = 1$ - (2)
(1) x 5 + (2) $\rightarrow$ $\mathbf{I_2} = 1/25 = 0.04$ A = 40 mA

$\mathbf{V_{AB}} = \mathbf{I_2R} = 0.04 \times 100 = \mathbf{4\ V}$

15. 

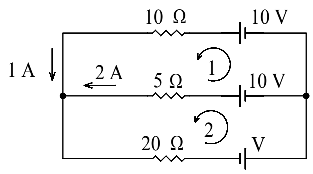
Find all currents for the circuit at left. What would be the reading of a voltmeter placed across points A and B in the circuit?

Using similar method to above: **Current in 20 Ω = 91.9 mA to LEFT**
Current in 40 Ω (Top Right) = 37.8 mA to LEFT
Current in 50 Ω = 129.78 mA UPWARDS

$\mathbf{V_{AB} = -6.49\ V}$ (Note: As current is upwards **B** is at a higher potential than **A** → $\mathbf{V_{BA}}$ = +6.49 V)

16. (a) What is the current in the 20 Ω resistor?

(b) Write an equation for loop 2 using Kirchhoff's Loop Rule and hence find the voltage required for the battery in series with the 20 Ω resistor if the currents are as shown in the diagram.



(a) $\mathbf{I_{20\,\Omega}}$ = 1 + 2 = **3 A to RIGHT**

(b) Loop 2: 5 x 2 + 3 x 20 = 10 + **V** → **V = 60 V**