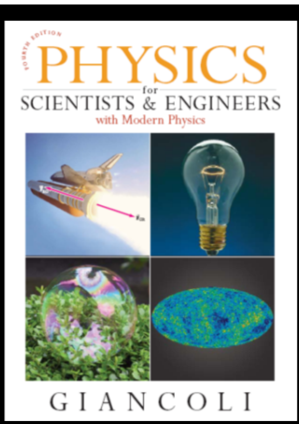


Lecture PowerPoints

Chapter 23

Physics for Scientists and Engineers, with Modern Physics, 4th edition

Giancoli

Chapter 23
Electric Potential

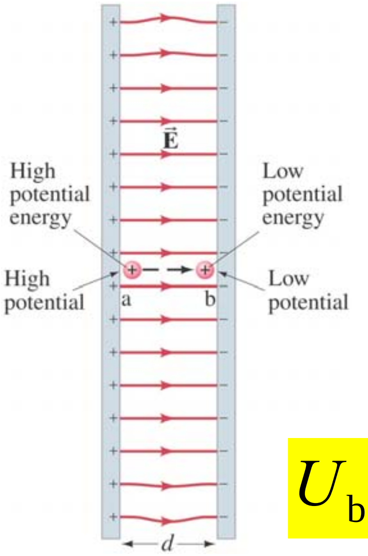
Units of Chapter 23

- Electric Potential Energy and Potential Difference
- Relation between Electric Potential and Electric Field
- Electric Potential Due to Point Charges
- Potential Due to Any Charge Distribution
- Equipotential Surfaces

Units of Chapter 23

- $\vec{E}$ Determined from V
- Electrostatic Potential Energy; the Electron Volt
- Cathode Ray Tube: TV and Computer Monitors, Oscilloscope

23-1 Electrostatic Potential Energy and Potential Difference



The electrostatic force is conservative – potential energy can be defined.

The work done by the electric field is given by

$$W = Fd = qEd$$

Change in electric potential energy is negative of work done by electric force:

$$U_b - U_a = -W = -qEd$$

23-1 Electrostatic Potential Energy and Potential Difference

Electric potential is defined as potential energy per unit charge:

$$V_a = \frac{U_a}{q}$$

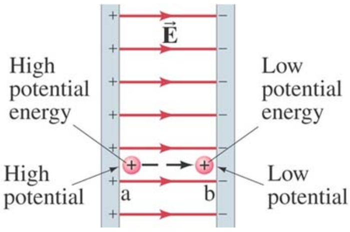
Unit of electric potential: the volt (V):

1 V = 1 J/C.

23-1 Electrostatic Potential Energy and Potential Difference

Only changes in potential can be measured, allowing free assignment of $V = 0$:

$$V_{\mathrm{ba}} = \Delta V = V_{\mathrm{b}} - V_{\mathrm{a}} = \frac{U_{\mathrm{b}} - U_{\mathrm{a}}}{q} = -\frac{W_{\mathrm{ba}}}{q}$$



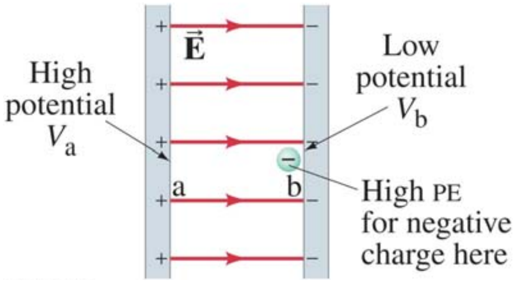
+ve charge moves from high to low potential

-ve charge moves from low to high potential

23-1 Electrostatic Potential Energy and Potential Difference

Conceptual Example 23-1: A negative charge.

Suppose a negative charge, such as an electron, is placed near the negative plate at point b, as shown here. If the electron is free to move, will its electric potential energy increase or decrease? How will the electric potential change?

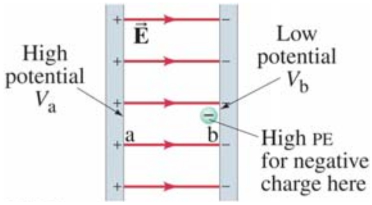


Conceptual Example 23-1: A negative charge.

Suppose a negative charge, such as an electron, is placed near the negative plate at point b, as shown here. If the electron is free to move, will its electric potential energy increase or decrease? How will the electric potential change?

$$U_a < U_b$$

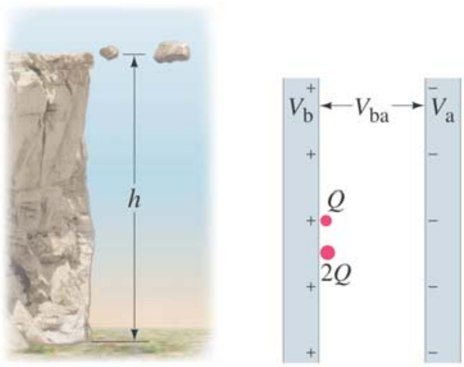
$$V_{ab} = V_a - V_b > 0$$



The electron will move towards the positive plate if released, thereby increasing its kinetic energy. Its potential energy must therefore decrease. However, it is moving to a region of higher potential V; the potential is determined only by the existing charge distribution and not by the point charge. U and V have different signs due to the negative charge.

23-1 Electrostatic Potential Energy and Potential Difference

Analogy between gravitational and electrical potential energy:



(a) (b)

(a) Two rocks are at the same height. The larger rock has more potential energy.

(b) Two charges have the same electric potential. The 2Q charge has more potential energy.

23-1 Electrostatic Potential Energy and Potential Difference

Electrical sources such as batteries and generators supply a constant potential difference. Here are some typical potential differences, both natural and manufactured:

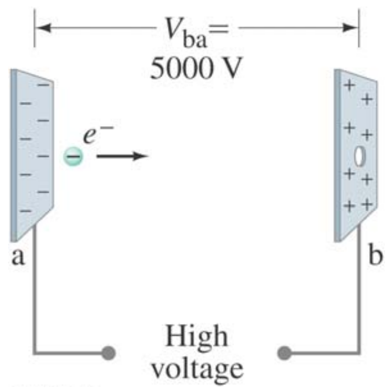
TABLE 23–1 Some Typical Potential Differences (Voltages)

Source	Voltage (approx.)
Thundercloud to ground	10^8 V
High-voltage power line	10^5–10^6 V
Power supply for TV tube	10^4 V
Automobile ignition	10^4 V
Household outlet	10^2 V
Automobile battery	12 V
Flashlight battery	1.5 V
Resting potential across nerve membrane	10^{-1} V
Potential changes on skin (EKG and EEG)	10^{-4} V

23-1 Electrostatic Potential Energy and Potential Difference

Example 23-2: Electron in CRT.

Suppose an electron in a cathode ray tube is accelerated from rest through a potential difference $V_b - V_a = V_{ba} = +5000$ V. (a) What is the change in electric potential energy of the electron? (b) What is the speed of the electron ($m = 9.1 \times 10^{-31}$ kg) as a result of this acceleration?

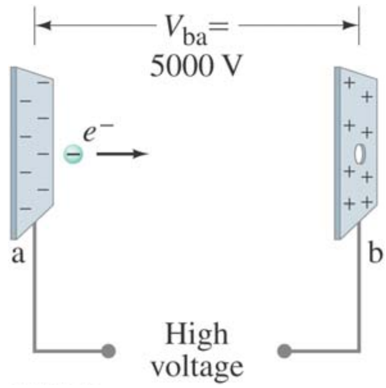


Example 23-2: Electron in CRT.

Suppose an electron in a cathode ray tube is accelerated from rest through a potential difference

$V_b - V_a = V_{ba} = +5000$ V.

(a) What is the change in electric potential energy of the electron?



e^- accelerates towards the +ve plate with a decrease in potential energy:

$$\Delta U = qV_{ba}$$

Loss in potential energy = gain in kinetic energy.

(a) $q = -e = -1.6\times10^{-19}$ C

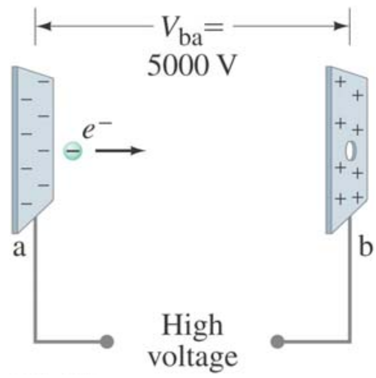
$\Delta U = qV_{ba} = -1.6\times10^{-19}\times5000 = -8.0\times10^{-16}$ J

Example 23-2: Electron in CRT.

Suppose an electron in a cathode ray tube is accelerated from rest through a potential difference

$V_b - V_a = V_{ba} = +5000$ V.

(b) What is the speed of the electron ($m = 9.1 \times 10^{-31}$ kg) as a result of this acceleration?



$$\text{Conservation of Energy}: \ \Delta K + \Delta U = 0$$

$$\Delta K = -\Delta U$$

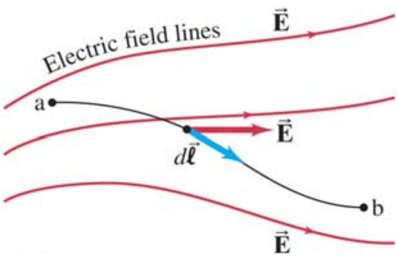
$$\tfrac{1}{2}mv^2 - 0 = -q(V_b - V_a) = -qV_{ba} \quad (K_{initial} = 0)$$

$$v = \sqrt{-\frac{2qV_{ba}}{m}} = \sqrt{-\frac{2\times(-1.6\times10^{-19})\times5000}{9.1\times10^{-31}}} = 4.2\times10^7 \text{ m/s}$$

23-2 Relation between Electric Potential and Electric Field

The general relationship between a conservative force and potential energy:

$$U_b - U_a = -\int_a^b \vec{\mathbf{F}} \cdot d\vec{\ell}.$$



Substituting the potential difference and the electric field:

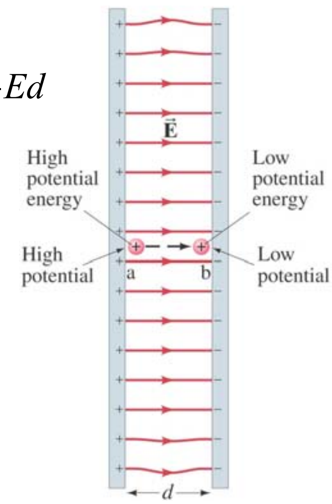
$$V_{ba} = V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\ell}.$$

23-2 Relation between Electric Potential and Electric Field

The simplest case is a uniform field:

$$V_{ba} = V_b - V_a = -\int_a^b \vec{\mathbf{E}} \bullet d\vec{\ell} = -E\int_a^b d\ell = -Ed$$

$$V_{ba} = -Ed$$



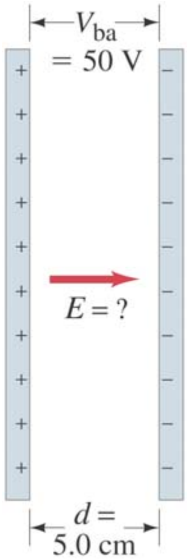
23-2 Relation between Electric Potential and Electric Field

Example 23-3: Electric field obtained from voltage.

Two parallel plates are charged to produce a potential difference of 50 V. If the separation between the plates is 0.050 m, calculate the magnitude of the electric field in the space between the plates.

Since $V_{ba} = -Ed$

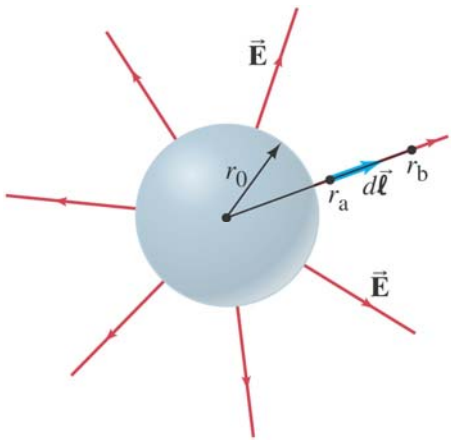
$$\left|\vec{\mathbf{E}}\right| = E = \frac{V}{d} = \frac{50}{0.05} = 1000\ \mathrm{V/m}$$



23-2 Relation between Electric Potential and Electric Field

Example 23-4: Charged conducting sphere.

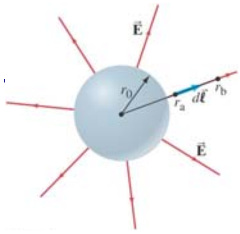
Determine the potential at a distance r from the center of a uniformly charged conducting sphere of radius r_0 for
(a) $r > r_0$,
(b) $r = r_0$,
(c) $r < r_0$.
The total charge on the sphere is Q.



Example 23-4: Charged conducting sphere.

Determine the potential at a distance r from the center of a uniformly charged conducting sphere of radius r_0 for

(a) $r > r_0$



The total charge on the sphere is Q

We found using Gauss's Law that $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

$$V_b - V_a = -\int_{r_a}^{r_b} \vec{E} \bullet d\vec{\ell} = -\frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

Set $d\ell = dr$ and let $V = 0$ at $r = \infty$

(this is an arbitary convention $\rightarrow V = 0$

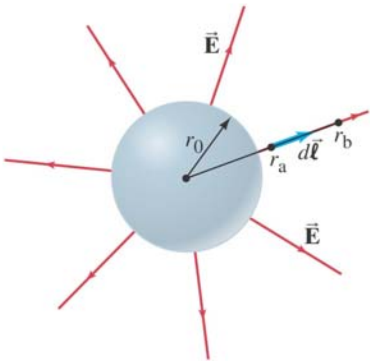
at a large distance from any charge)

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Example 23-4: Charged conducting sphere.

Determine the potential at a distance r from the center of a uniformly charged conducting sphere of radius r_0 for

(b) $r = r_0$



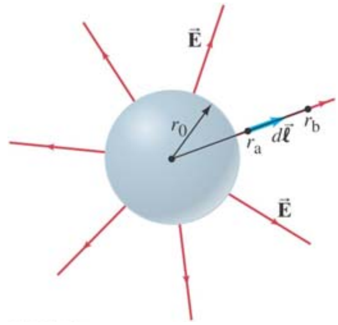
The total charge on the sphere is Q

$$\text{As } r \text{ approaches } r_0: \; V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0}$$

Example 23-4: Charged conducting sphere.

Determine the potential at a distance r from the center of a uniformly charged conducting sphere of radius r_0 for

(c) $r < r_0$



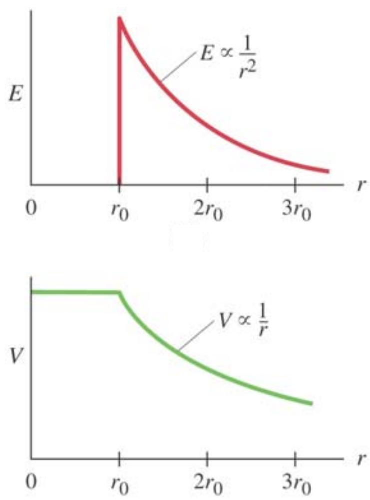
The total charge on the sphere is Q

For points inside the sphere there is **NO** electric field (Gauss's Theorem)

So V is the same as at surface: $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0}$

23-2 Relation between Electric Potential and Electric Field

The previous example gives the electric potential as a function of distance from the surface of a charged conducting sphere, which is plotted here, and compared with the electric field:



23-2 Relation between Electric Potential and Electric Field

Example 23-5: Breakdown voltage.

In many kinds of equipment, very high voltages are used. A problem with high voltage is that the air can become ionized due to the high electric fields: free electrons in the air (produced by cosmic rays, for example) can be accelerated by such high fields to speeds sufficient to ionize O_2 and N_2 molecules by collision, knocking out one or more of their electrons. The air then becomes conducting and the high voltage cannot be maintained as charge flows. The breakdown of air occurs for electric fields of about 3.0×10^6 V/m. (a) Show that the breakdown voltage for a spherical conductor in air is proportional to the radius of the sphere, and (b) estimate the breakdown voltage in air for a sphere of diameter 1.0 cm.

Example 23-5: Breakdown voltage.

The breakdown of air occurs for electric fields of about 3×10^6 V/m. (Note that this is for dry air – in humid conditions this drops to about 2×10^6 V/m) (a) Show that the breakdown voltage for a spherical conductor in air is proportional to the radius of the sphere, and (b) estimate the breakdown voltage in air for a sphere of diameter 1 cm ($r = 0.5$ cm $= 5\text{x}10^{-3}$ m).

$$\text{(a)}\ V = \frac{kQ}{r_0} \quad and \quad E = \frac{kQ}{r_0^2}$$

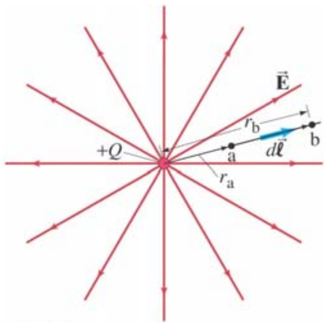
$$\therefore \quad k = \frac{r_0 V}{Q} = \frac{r_0^2 E}{Q} \quad \rightarrow \quad V = r_0 E$$

$$\text{(b)}\ V = 5\times10^{-3} \times 3\times10^{6} = 15000\ \text{V} = 15\ \text{kV}$$

23-3 Electric Potential Due to Point Charges

To find the electric potential due to a point charge, we integrate the field along a field line:

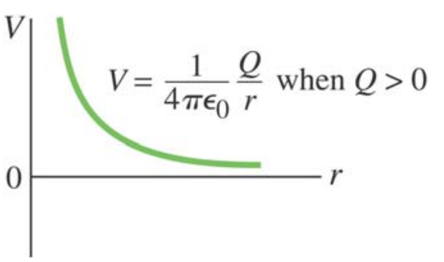
$$V_b - V_a = -\int_{r_a}^{r_b} \vec{\mathbf{E}} \cdot d\vec{\ell} = -\frac{Q}{4\pi\epsilon_0}\int_{r_a}^{r_b} \frac{1}{r^2}\,dr = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r_b} - \frac{Q}{r_a}\right).$$

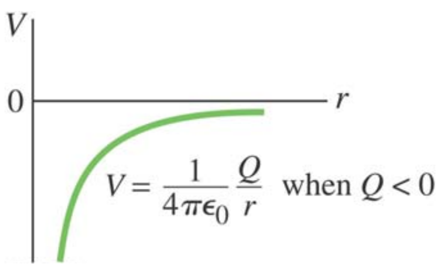


23-3 Electric Potential Due to Point Charges

Setting the potential to zero at $r = \infty$ gives the general form of the potential due to a point charge:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}. \qquad \left[\begin{array}{c}\text{single point charge;} \\ V = 0 \text{ at } r = \infty\end{array}\right]$$





Example 23-6: Work required to bring two positive charges close together.

What minimum work must be done by an external force to bring a charge q = 3.00 μC from a great distance away (take $r = \infty$) to a point 0.500 m from a charge Q = 20.0 μC?

$$W = \Delta U = q\left(V_b - V_a\right)$$

$$\& \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{kQ}{r}$$

$$\therefore \quad W = q\left(\frac{kQ}{r_b} - \frac{kQ}{r_a}\right) \qquad r_b = 0.500\ \text{m}; \quad r_a \to \infty$$

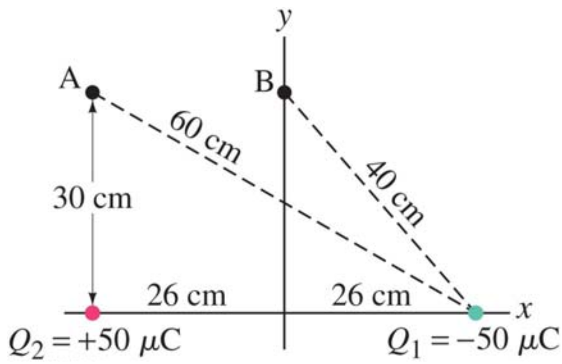
$$\text{Let } q = 3\times10^{-6}\ \text{C}; \quad Q = 20\times10^{-6}\ \text{C}$$

$$\therefore \quad W = 3\times10^{-6}\left(\frac{8.99\times10^{9}\times20\times10^{-6}}{0.500}\right) = 1.08\ \text{J}$$

23-3 Electric Potential Due to Point Charges

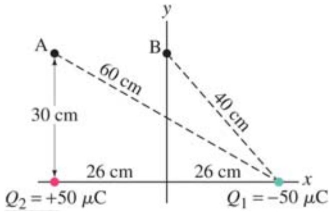
Example 23-7: Potential above two charges.

Calculate the electric potential (a) at point A in the figure due to the two charges shown, and (b) at point B.



Example 23-7: Potential above two charges.

Calculate the electric potential (a) at point A in the figure due to the two charges shown.



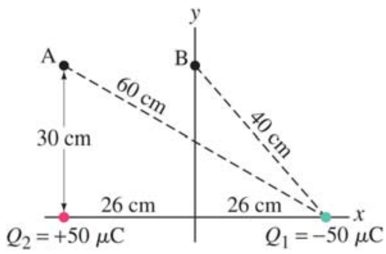
Note that electric potential is a scalar so **direction does not have to be taken into account**.

$$V_A = V_{A1} + V_{A2} = k\left(\frac{Q_1}{r_{1A}} + \frac{Q_2}{r_{2A}}\right)$$

$$= 9.0\times10^9\left(\frac{-50\times10^{-6}}{0.60} + \frac{50\times10^{-6}}{0.30}\right) = 7.5\times10^5\ \text{V}$$

Example 23-7: Potential above two charges.

Calculate the electric potential (a) at point A in the figure due to the two charges shown, and (b) at point B.



Could use same method as part (a) but note that point B is equidistant to 2 equal magnitude charges (one + & one -) so that the potentials due the both charges will cancel $\rightarrow V_B = 0$
This would be true for all points along the perpendicular bisector.

23-4 Potential Due to Any Charge Distribution

The potential due to an arbitrary charge distribution can be expressed as a sum or integral (if the distribution is continuous):

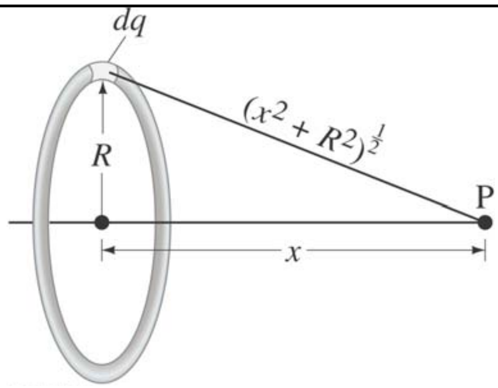
$$V_a = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{Q_i}{r_{ia}}.$$

or

$$V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r}.$$

Example 23-8: Potential due to a ring of charge.

A thin circular ring of radius R has a uniformly distributed charge Q. Determine the electric potential at a point P on the axis of the ring a distance x from its center.



$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{x^2+R^2}}\int dq = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x^2+R^2}}$$

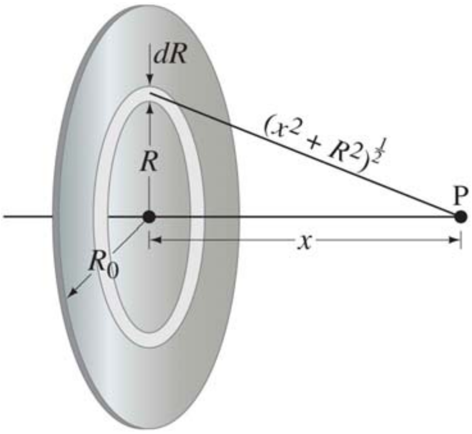
Note that if $x >> R$ this reduces to $V = \frac{Q}{4\pi\varepsilon_0 x}$

which is the result for a single point charge.

23-4 Potential Due to Any Charge Distribution

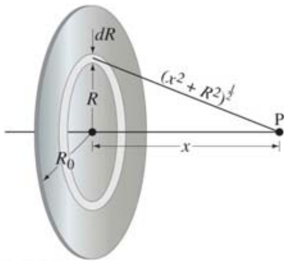
Example 23-9: Potential due to a charged disk.

A thin flat disk, of radius R_0, has a uniformly distributed charge Q. Determine the potential at a point P on the axis of the disk, a distance x from its center.



Example 23-9: Potential due to a charged disk.

Divide the disk into thin rings of radius R and thickness dR and integrate over disk using result for circular ring.



Charge density $\sigma = \dfrac{Q}{\pi R_0^2}$

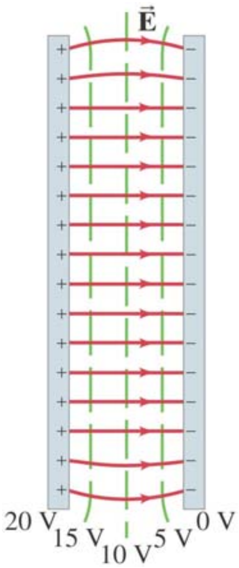
Charge dq on ring of thickness dr: $dq = \sigma dA = \dfrac{Q(2\pi R dR)}{\pi R_0^2} = \dfrac{2QRdR}{R_0^2}$

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{\sqrt{x^2+R^2}} = \frac{2Q}{4\pi\varepsilon_0 R_0^2}\int_0^{R_0}\frac{RdR}{\sqrt{x^2+R^2}}$$

$$= \frac{Q}{2\pi\varepsilon_0 R_0^2}\left[\sqrt{x^2+R^2}\right]_{R=0}^{R=R_0} = \frac{Q}{2\pi\varepsilon_0 R_0^2}\left(\sqrt{x^2+R_0^2}-x\right)$$

Note that if $x \gg R_0$ this must reduce to $\dfrac{Q}{4\pi\varepsilon_0 x}$ as for point charge.

23-5 Equipotential Surfaces

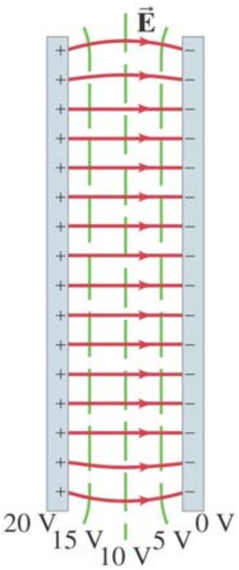


An equipotential is a line or surface over which the potential is constant.

Electric field (RED) lines are perpendicular to equipotentials (GREEN).

The surface of a conductor is an equipotential.

23-5 Equipotential Surfaces



$$\Delta V = \int \vec{\mathbf{E}} \bullet d\vec{\ell} = \int E\cos\theta\, d\ell$$

On an equipotential surface $\Delta V = 0$

In a region where $E \neq 0$ the path $d\ell$ along an equipotential must have

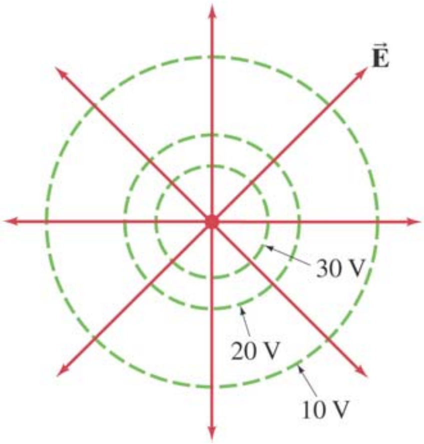
$\cos\theta = 0 \quad \rightarrow \quad \theta = 90°$

Equipotential lines are perpendicular to the electric field lines.

23-5 Equipotential Surfaces

Example 23-10: Point charge equipotential surfaces.

For a single point charge with $Q = 4.0 \times 10^{-9}$ C, sketch the equipotential surfaces (or lines in a plane containing the charge) corresponding to $V_1 = 10$ V, $V_2 = 20$ V, and $V_3 = 30$ V.

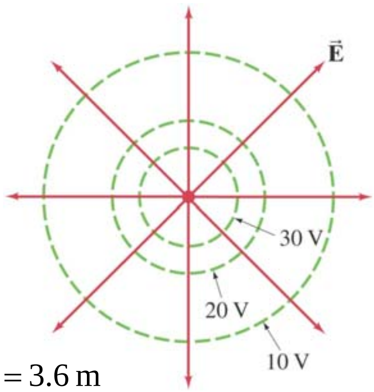


23-5 Equipotential Surfaces

Example 23-10: Point charge equipotential surfaces.

$Q = 4.0 \times 10^{-9}$ C

$V_1 = 10$ V, $V_2 = 20$ V, and $V_3 = 30$ V.



$$V = \frac{kQ}{r} \quad \rightarrow \quad r = \frac{kQ}{V}$$

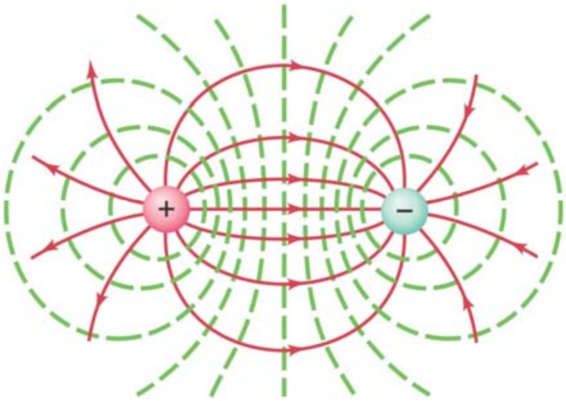
$$V_1 = 10\,\mathrm{V}: \quad \rightarrow \quad r_1 = \frac{9.0\times10^{9} \times 4.0\times10^{-9}}{10} = 3.6\,\mathrm{m}$$

$$V_2 = 20\,\mathrm{V}: \quad \rightarrow \quad r_2 = \frac{9.0\times10^{9} \times 4.0\times10^{-9}}{20} = 1.8\,\mathrm{m}$$

$$V_3 = 30\,\mathrm{V}: \quad \rightarrow \quad r_3 = \frac{9.0\times10^{9} \times 4.0\times10^{-9}}{30} = 1.2\,\mathrm{m}$$

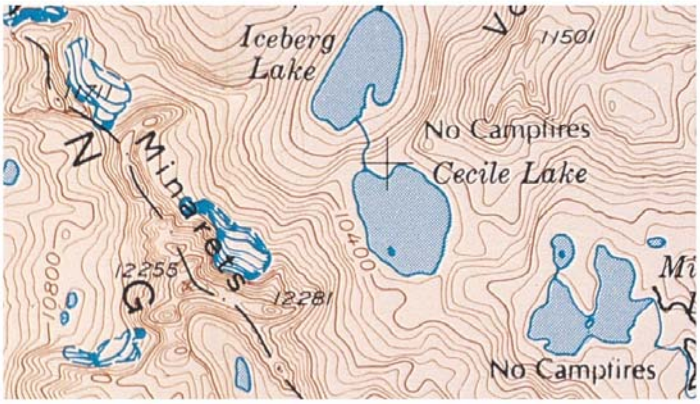
23-5 Equipotential Surfaces

Equipotential surfaces are always perpendicular to field lines; they are always closed surfaces (unlike field lines, which begin and end on charges).



23-5 Equipotential Surfaces

A gravitational analogy to equipotential surfaces is the topographical map – the lines connect points of equal gravitational potential (altitude).



23-7 $\vec{E}$ Determined from V

If we know the field, we can determine the potential by integrating. Inverting this process, if we know the potential, we can find the field by differentiating:

$$E_\ell = -\frac{dV}{d\ell}.$$

This is a vector differential equation; here it is in component form:

$$E_x = -\frac{\partial V}{\partial x}, \qquad E_y = -\frac{\partial V}{\partial y}, \qquad E_z = -\frac{\partial V}{\partial z}.$$

Note that $\frac{\partial V}{\partial x}$ is a **partial derivative**

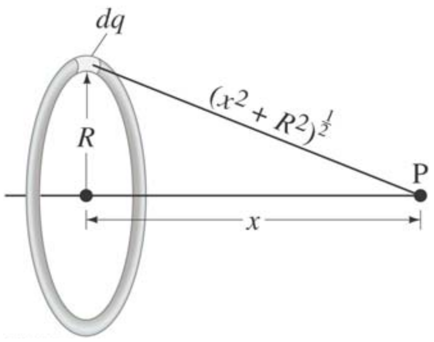
ie. derivative with y and z held constant.

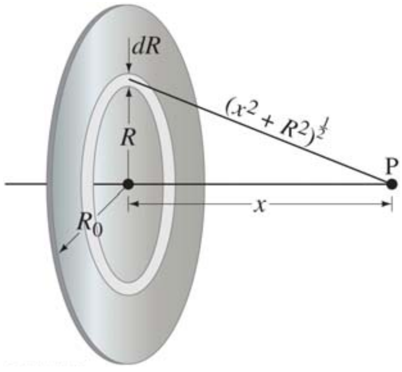
Similarly for y and z directions.

23-7 $\vec{E}$ Determined from V

Example 23-11: $\vec{\mathbf{E}}$ for ring and disk.

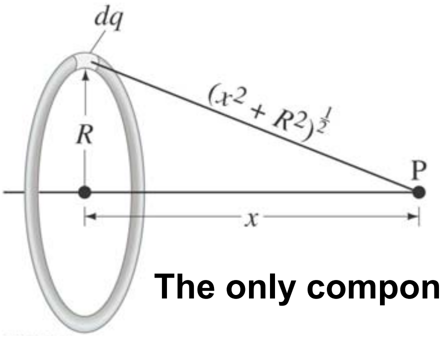
Use electric potential to determine the electric field at point P on the axis of (a) a circular ring of charge and (b) a uniformly charged disk.





23-7 $\vec{E}$ Determined from V

Example 23-11: $\vec{E}$ for ring.



We have shown that for a ring V at point P is:

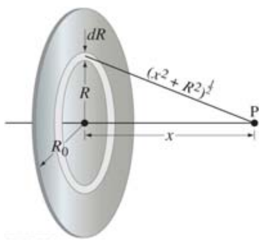
$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x^2 + R^2}}$$

The only component of $\vec{E}$ is in the x direction

$$E_x = -\frac{\partial V}{\partial x} = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{\left(x^2 + R^2\right)^{\frac{3}{2}}}$$

This is the same result that we obtained in chapter 21

Example 23-11: $\vec{E}$ for disk.



From our result for the potential due to a charged disk

$$V = \frac{Q}{2\pi\varepsilon_0 R_0^2}\left(\sqrt{x^2 + R_0^2} - x\right)$$

$$E_x = -\frac{\partial V}{\partial x} = \frac{Q}{2\pi\varepsilon_0 R_0^2}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

For points very close to the disk, $x << R_0$

$$E_x \approx \frac{Q}{2\pi\varepsilon_0 R_0^2} = \frac{\sigma}{2\varepsilon_0}$$

where $\sigma = \dfrac{Q}{\pi R_0^2}$ = surface charge density.

This is the same result as we obtained in chapter 21.

23-8 Electrostatic Potential Energy; the Electron Volt

The potential energy of a charge in an electric potential is $U = qV$. To find the electric potential energy of two charges, imagine bringing each in from infinitely far away. The first one takes no work, as there is no field. To bring in the second one, we must do work due to the field of the first one; this means the potential energy of the pair is:

$$U = Q_2 V = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r_{12}}.$$

23-8 Electrostatic Potential Energy; the Electron Volt

One electron volt (eV) is the energy gained by an electron moving through a potential difference of one volt:

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}.$$

The electron volt is often a much more convenient unit than the joule for measuring the energy of individual particles.

Question: **Calculate the velocity of an electron when accelerated through a potential difference of 1000 V.**

1 eV = 1.6×10^{-19} J
$m_e = 9.1\times10^{-31}$ kg

Electron gains 1000 eV of kinetic energy

$$K = \tfrac{1}{2}m_e v^2 \quad \text{(need to convert to J)}$$

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2\times1000\times1.6\times10^{-19}}{9.1\times10^{-31}}} = 1.9\times10^{7}\ \text{m/s}$$

23-8 Electrostatic Potential Energy; the Electron Volt

Example 23-12: Disassembling a hydrogen atom.

Calculate the work needed to "disassemble" a hydrogen atom. Assume that the proton and electron are initially separated by a distance equal to the "average" radius of the hydrogen atom in its ground state, 0.529×10^{-10} m, and that they end up an infinite distance apart from each other.

The total energy of the electron in atom is potential + kinetic. After separation the total energy is zero.

Example 23-12: Disassembling a hydrogen atom.

$$E_{\text{total}} = U + K$$

$$U = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r} = \frac{-e^2}{4\pi\varepsilon_0 r}$$

Assume e^- is in circular orbit

Centripetal Force = Electric Force

$$\frac{m_e v^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2} \rightarrow v^2 = \frac{e^2}{4\pi\varepsilon_0 m_e r}$$

$$K = \tfrac{1}{2} m_e v^2 = \frac{e^2}{8\pi\varepsilon_0 r}$$

$$E_{\text{total}} = U + K = \frac{-e^2}{4\pi\varepsilon_0 r} + \frac{e^2}{8\pi\varepsilon_0 r} = \frac{-e^2}{8\pi\varepsilon_0 r}$$

$$= \frac{-(1.6\times10^{-19})^2}{8\pi\times8.85\times10^{-12}\times0.529\times10^{-10}} = -2.18\times10^{-18}\ \text{J} = -13.6\,\text{eV}$$

Which is the measured ionization energy for hydrogen.

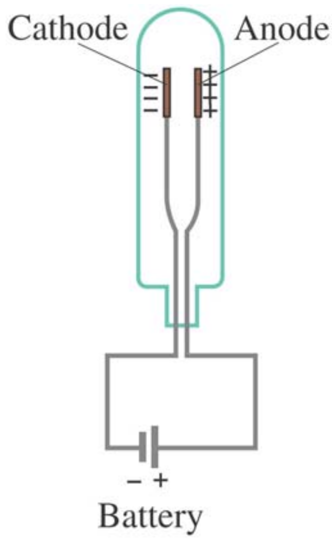
$r = 0.529 \times 10^{-10}$ m

$e = 1.6\times10^{-19}$ C

$\varepsilon_0 = 8.85\times10^{-12}\ C^2/N.m^2$

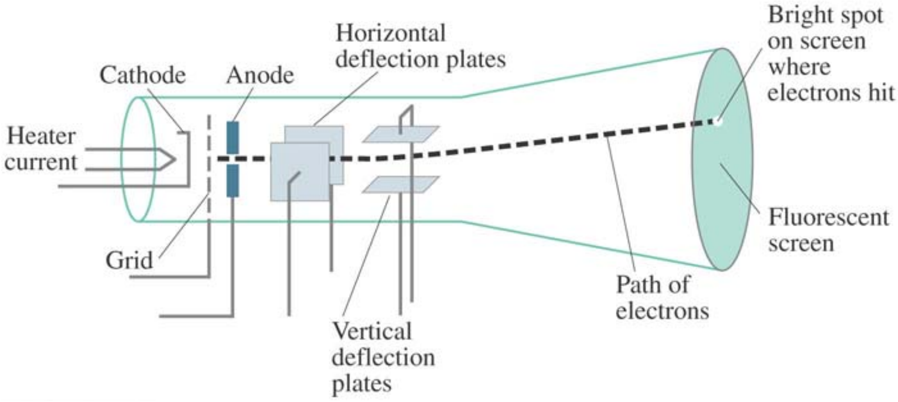
23-9 Cathode Ray Tube: TV and Computer Monitors, Oscilloscope

A cathode ray tube contains a wire cathode that, when heated, emits electrons. A voltage source causes the electrons to travel to the anode.



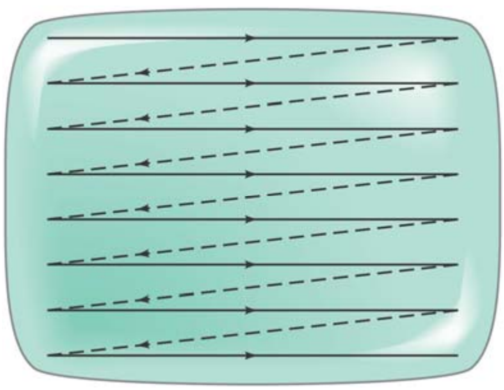
23-9 Cathode Ray Tube: TV and Computer Monitors, Oscilloscope

The electrons can be steered using electric or magnetic fields.



23-9 Cathode Ray Tube: TV and Computer Monitors, Oscilloscope

Televisions and computer monitors (except for LCD and plasma models) have a large cathode ray tube as their display. Variations in the field steer the electrons on their way to the screen.



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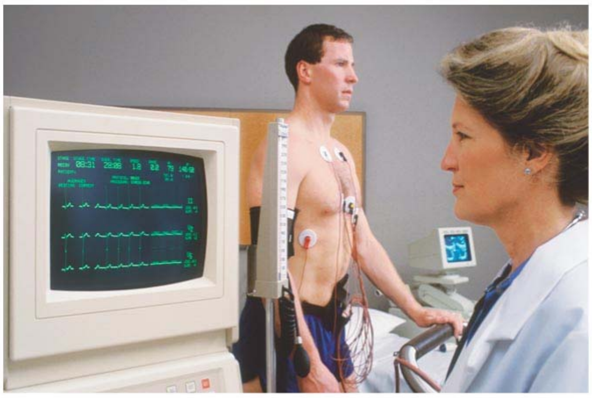
23-9 Cathode Ray Tube: TV and Computer Monitors, Oscilloscope

An oscilloscope displays an electrical signal on a screen, using it to deflect the beam vertically while it sweeps horizontally.



Summary of Chapter 23

• Electric potential is potential energy per unit charge:

$$V_{ba} = \Delta V = V_b - V_a = \frac{U_b - U_a}{q} = -\frac{W_{ba}}{q}.$$

• Potential difference between two points:

$$V_{ba} = V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\ell}.$$

• Potential of a point charge:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}. \qquad \begin{bmatrix}\text{single point charge;} \\ V = 0 \text{ at } r = \infty\end{bmatrix}$$

Summary of Chapter 23

- Equipotential: line or surface along which potential is the same.
- Electric dipole potential is proportional to $1/r^2$.
- To find the field from the potential:

$$E_x = -\frac{\partial V}{\partial x}, \qquad E_y = -\frac{\partial V}{\partial y}, \qquad E_z = -\frac{\partial V}{\partial z}.$$