CHAPTER 22
GAUSS'S LAW

TWO BASIC CONCEPTS

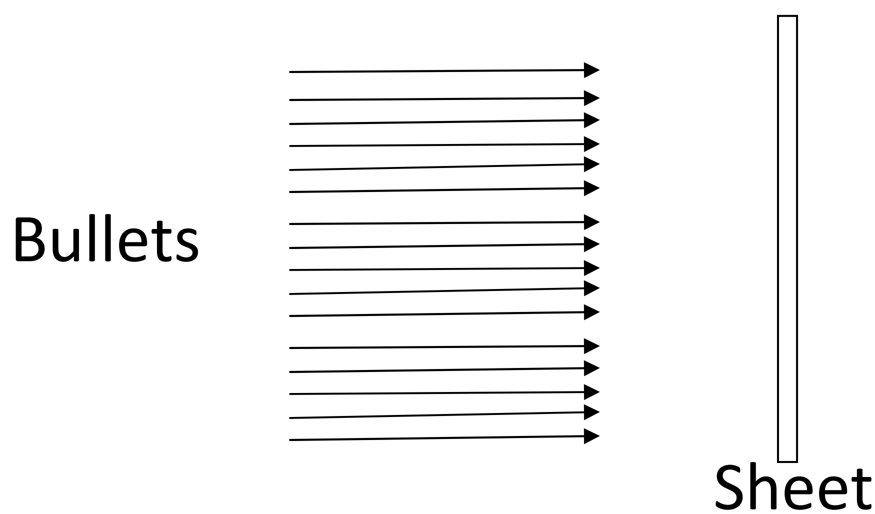
ELECTRIC FLUX

AND

GAUSS'S LAW

ELECTRIC FLUX

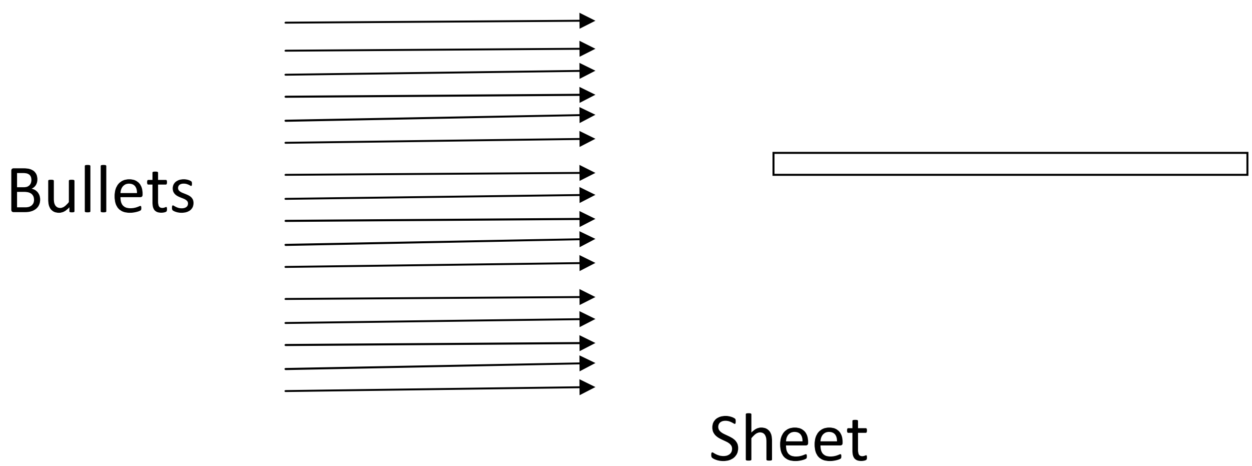
Consider a number of guns at the left shooting to the right.



Flux of bullets is number of bullets times area of sheet perpendicular to bullets.

$$FLUX_{BULLETS} = N_{BULLETS}\ x\ A_{SHEET}$$

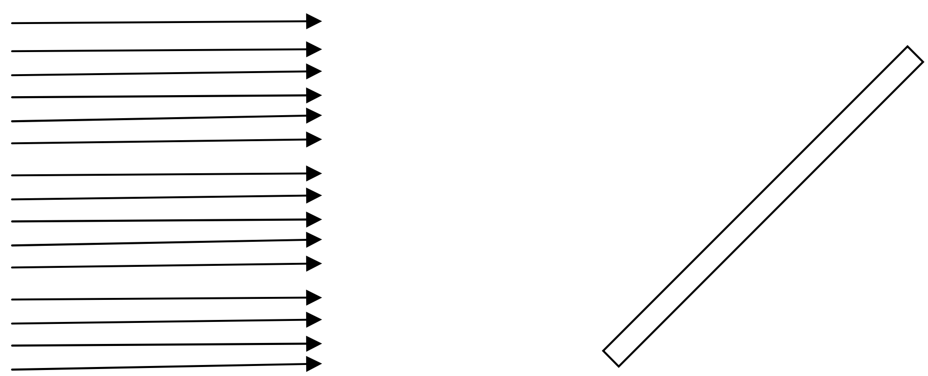
Turn sheet on edge.



Flux now near zero since area of sheet is parallel to bullets.

Turn sheet to angle.

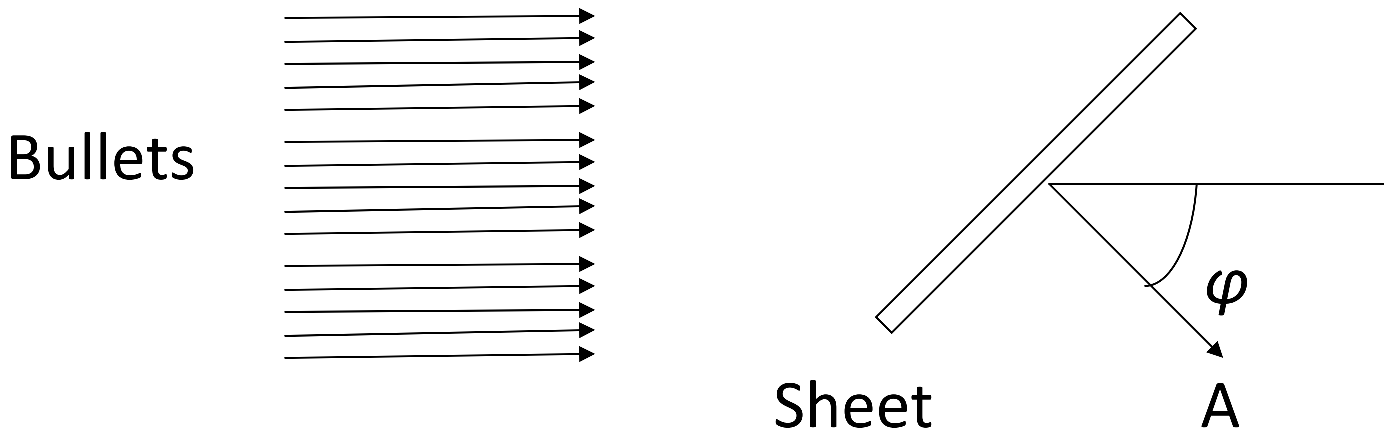
Bullets



Sheet

Flux somewhere between previous two values.

Assign area vector to sheet.

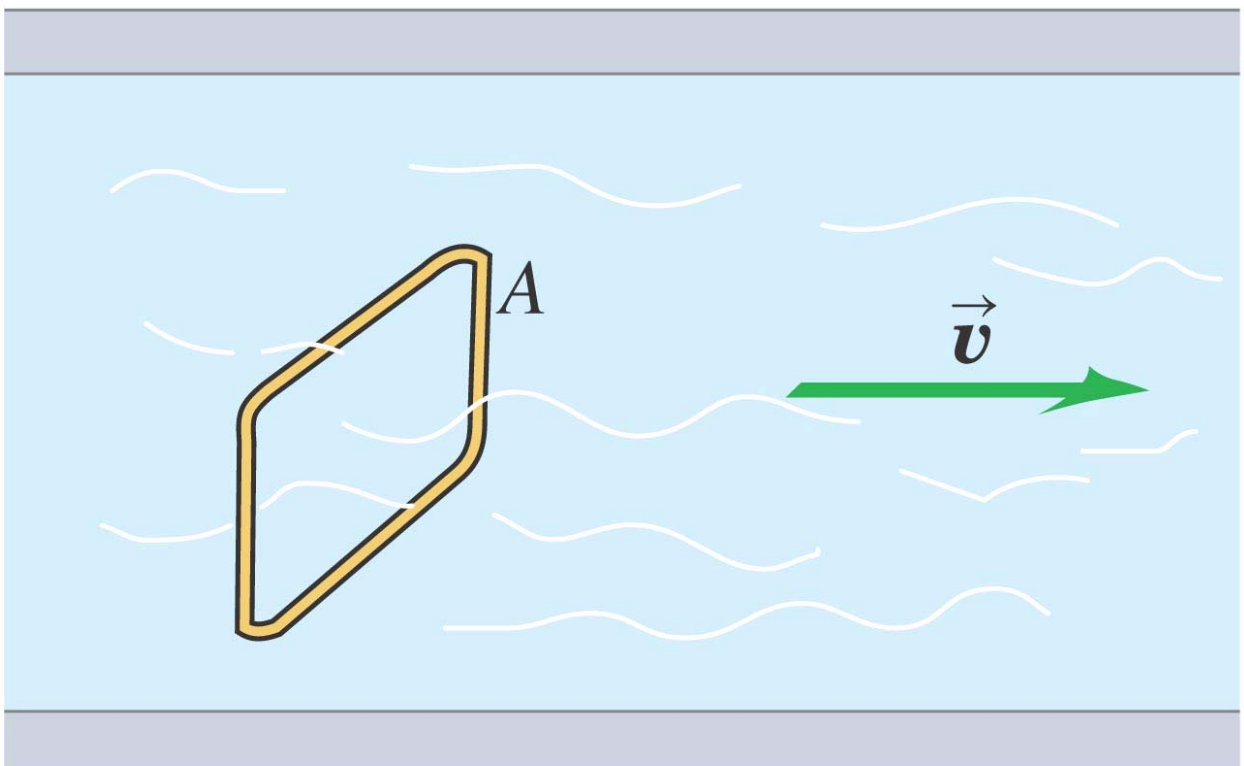


Flux will be

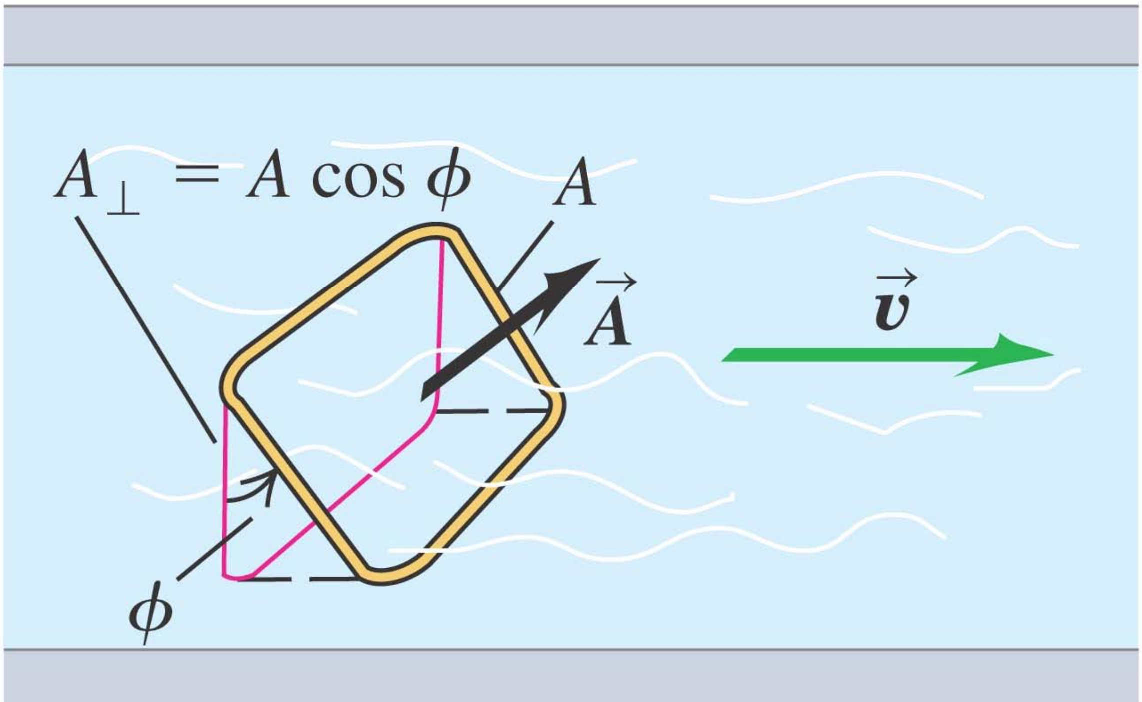
$$\Phi_{Bullets} = N_{Bullets} A cos\varphi$$

OR THINK ABOUT FLUX THE WAY YOUR BOOK DOES.

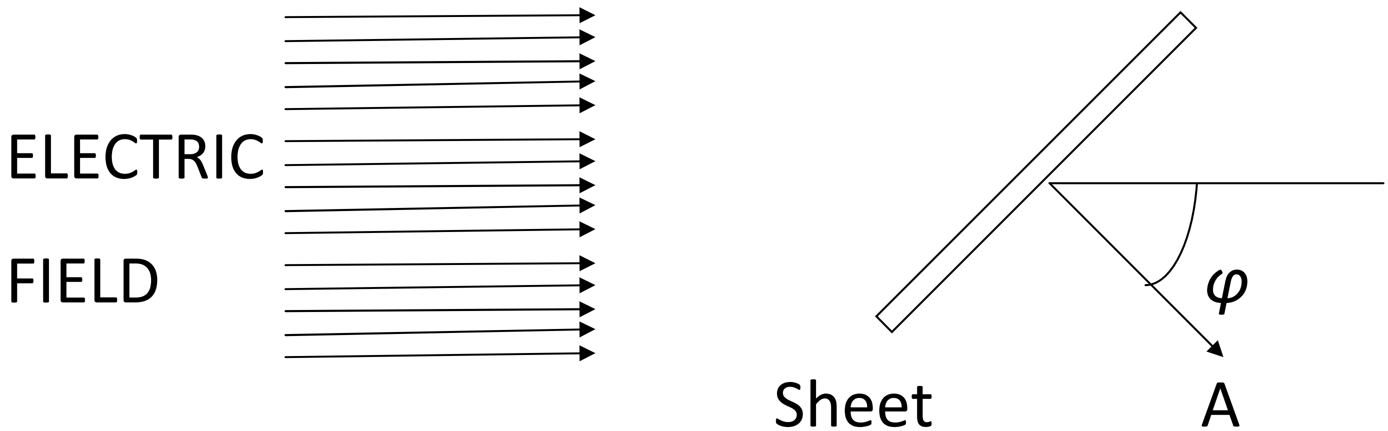
(a) A wire rectangle in a fluid



(b) The wire rectangle tilted by an angle ϕ



FOR ELECTRIC FLUX



$$\varphi = EAcos\varphi = E \cdot A$$

WHAT WE HAVE DISCUSSED IS FOR A UNIFORM ELECTRIC FIELD.

If the field varies we must use differential calculus.

Small increment of flux $d\varphi$

$$d\varphi = Ecos\varphi dA$$

Then integrate

$$\Phi = \int Ecos\varphi dA = \int E \cdot dA$$

GAUSS'S LAW

We will state the law then work some examples. After that we will do an example to justify the law.

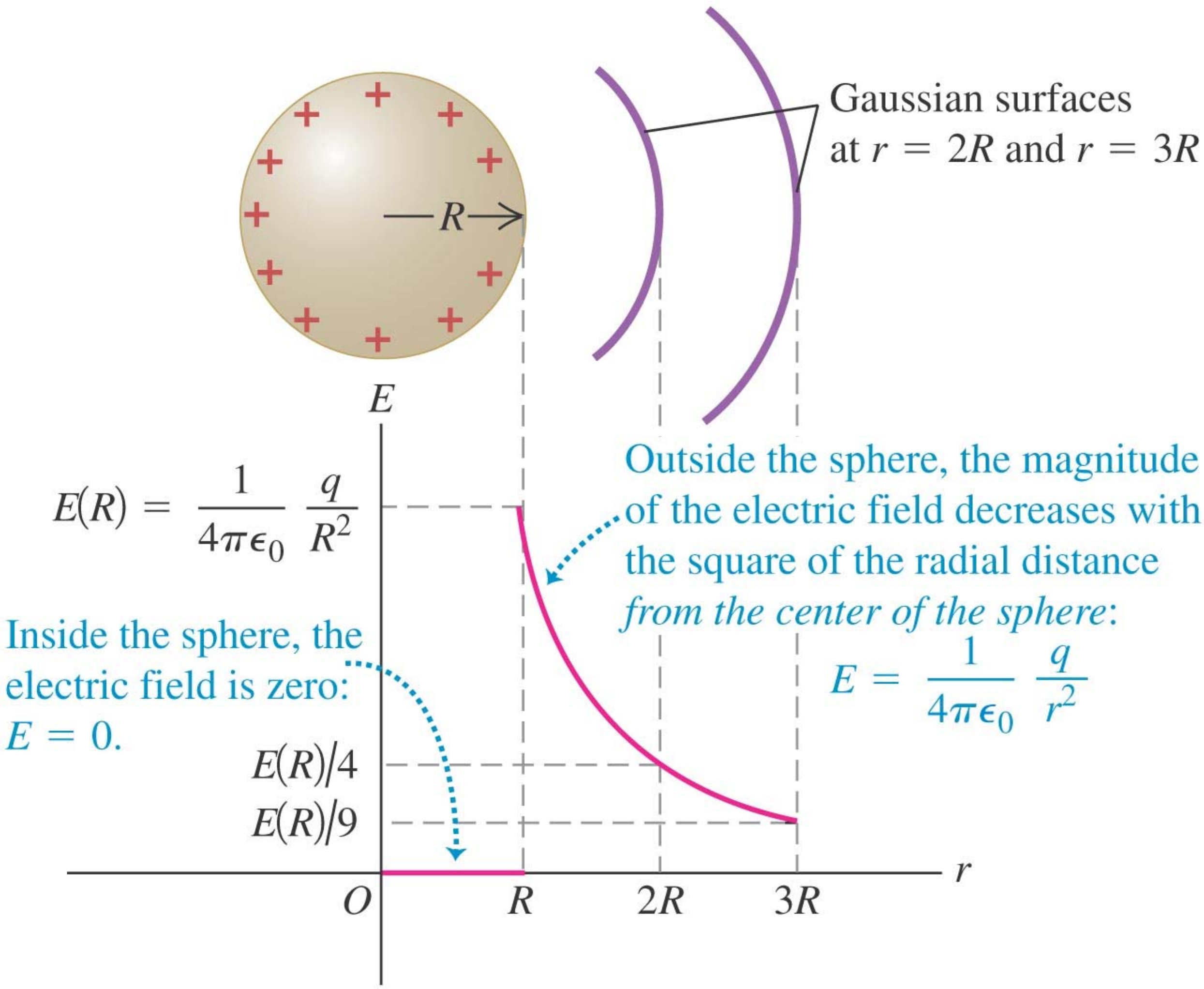
Statement of Gauss's law:

The flux through a closed surface is equal to the net charge enclosed by the surface divided by ε_0.

Equation:

$$\oint E \cdot dA = \frac{Q_{en}}{\varepsilon_0}$$

Consider a conducting sphere of radius R with charge q on the surface.



Draw a Gaussian surface around this sphere with radius r.

Apply Gauss's Law

$$\oint E \cdot dA = \frac{Q_{en}}{\varepsilon_0}$$

$$\oint EdAcos0 = \frac{Q_{en}}{\varepsilon_0}$$

$$E\oint dA = \frac{Q_{en}}{\varepsilon_0}$$

$$Q_{en} = q$$

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

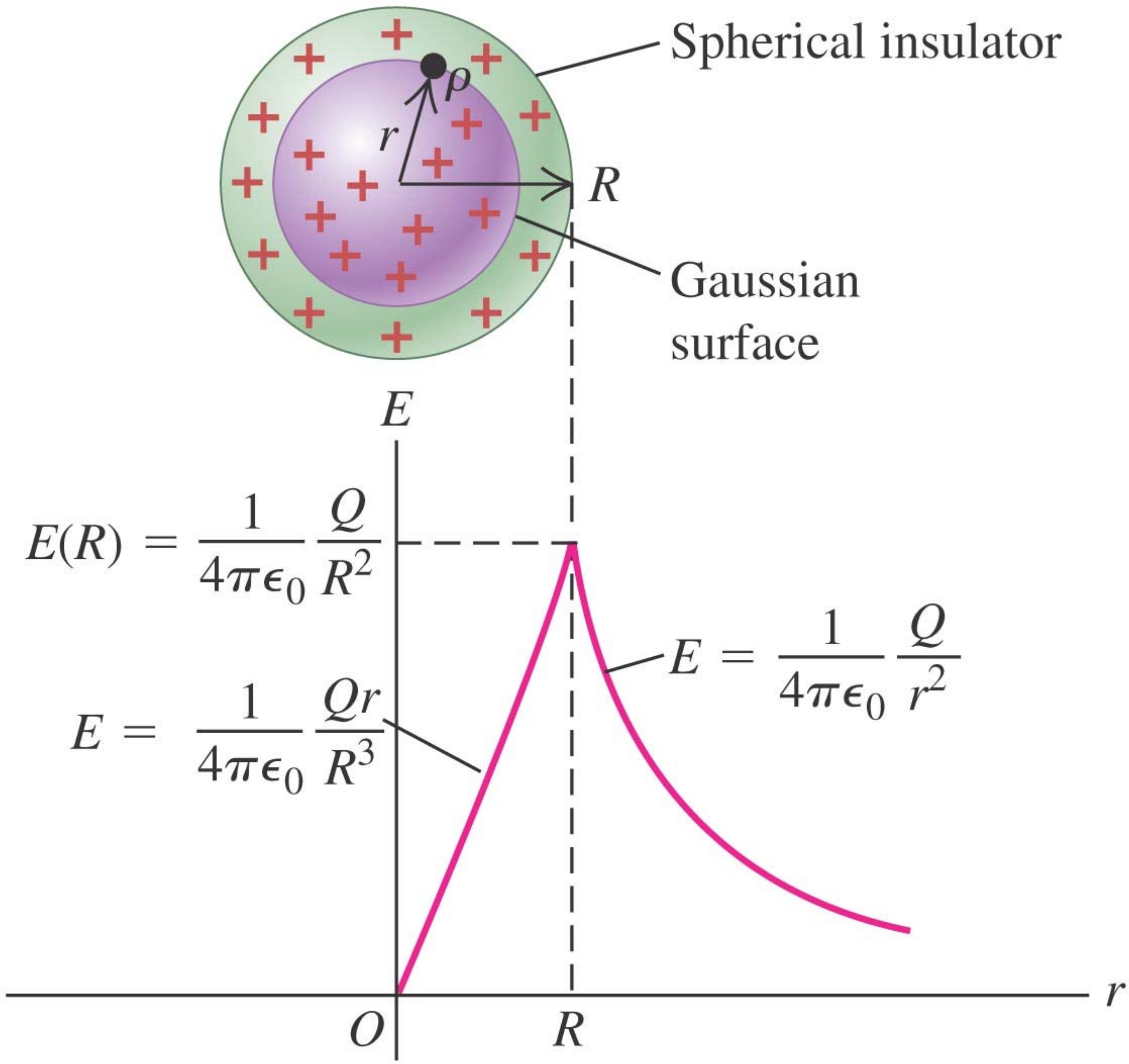
$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

The same equation we had for a point charge.

Example 22.9

Positive electric charge Q is distributed uniformly throughout the volume of an insulating sphere with radius R.

a. Find the electric field at point p where $r < R$.
b. Find the electric field at point p where $r > R$.


Spherical insulator
ρ
r
R
Gaussian surface
E
$E(R) = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}$
$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$
$E = \frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3}$
r
O
R

a. Radius of Gaussian Surface = r

Gauss's Law

$$\oint E \cdot dA = \frac{Q_{en}}{\varepsilon_0}$$

Need charge enclosed by surface.

Charge density $\rho = \frac{Total\ Charge}{Total\ Volume} = \frac{Q}{\frac{4}{3}\pi R^3}$

Charge enclosed by Gaussian Surface

$$Q_{en} = \rho\ x\ volume\ enclosed$$

$$Q_{en} = \rho\left(\frac{4}{3}\pi r^3\right) = \frac{Q}{\frac{4}{3}\pi R^3}\left(\frac{4}{3}\pi r^3\right) = \frac{r^3}{R^3}Q$$

$$\oint E \cdot dA = \frac{\frac{r^3}{R^3}Q}{\varepsilon_0}$$

$$E\int dA = \frac{\frac{r^3}{R^3}Q}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{\frac{r^3}{R^3}Q}{\varepsilon_0}$$

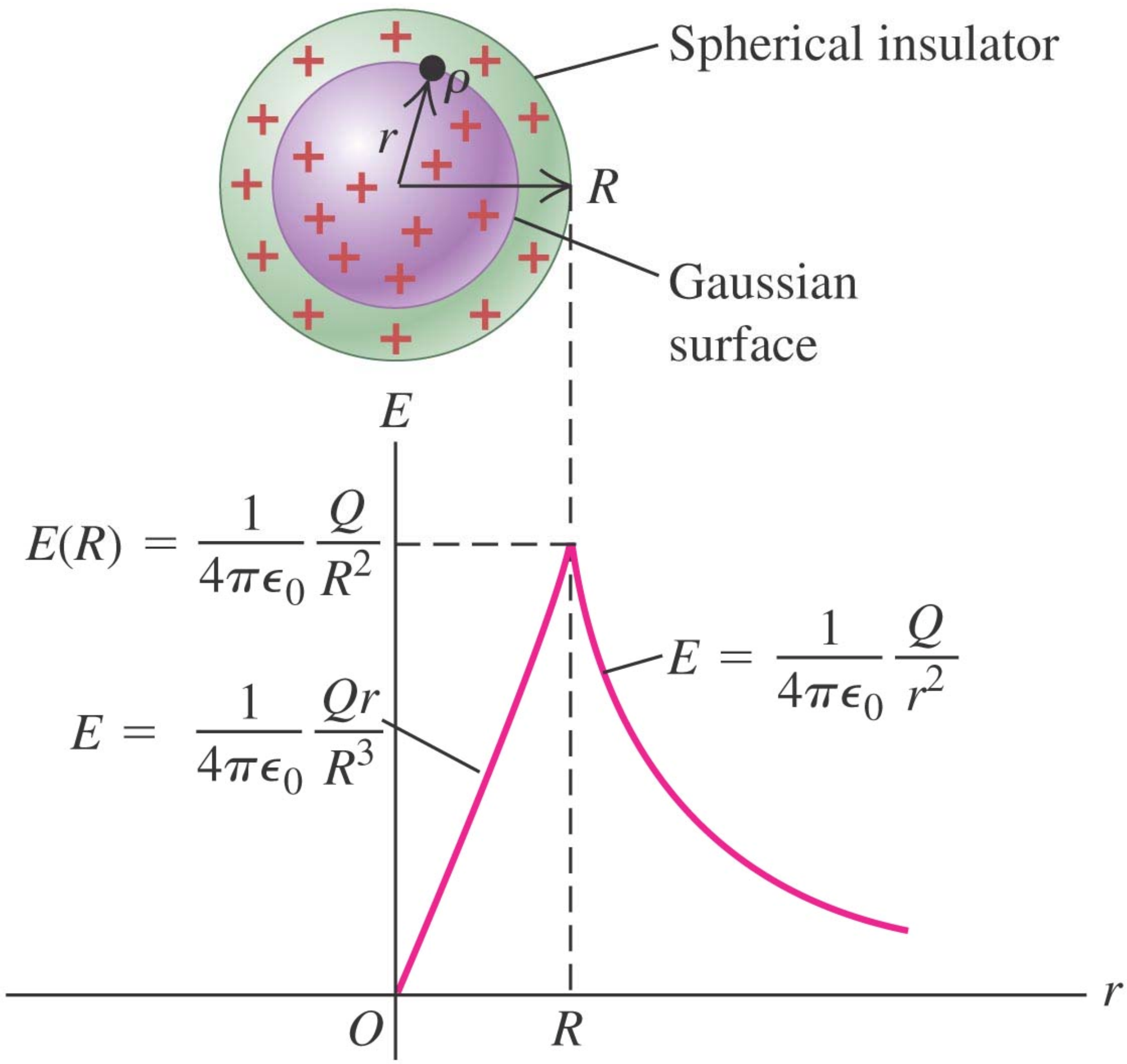
$$E = \frac{\frac{\frac{r^3}{R^3} Q}{\varepsilon_0}}{4\pi r^2}$$

$$E == \frac{1}{4\pi\varepsilon_0} \frac{Qr}{R^3}$$

For r < R

Part b.

Draw Gaussian Surface outside sphere.



$$\oint E \cdot dA = \frac{Q_{en}}{\varepsilon_0}$$

$$Q_{en} = Q$$

Integral same as before

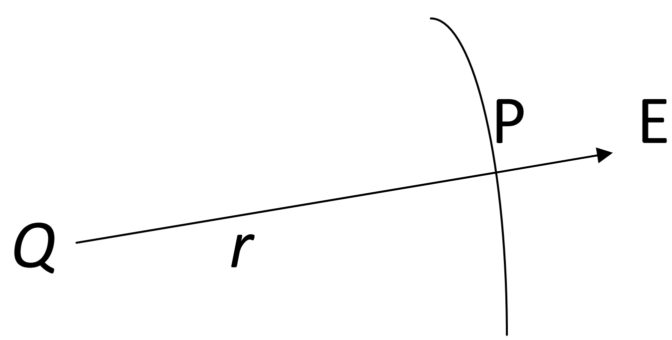
$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

JUSTIFY GAUSS'S LAW

Coulomb's Law $\xrightarrow{yields}$ Gauss's Law

Consider point charge Q



At P $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$

Visualize a sphere centered on Q and with radius r passing through P.

Integrate $E \cdot dA$ over surface.

$$\oint E \cdot dA = \oint \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} dAcos0$$

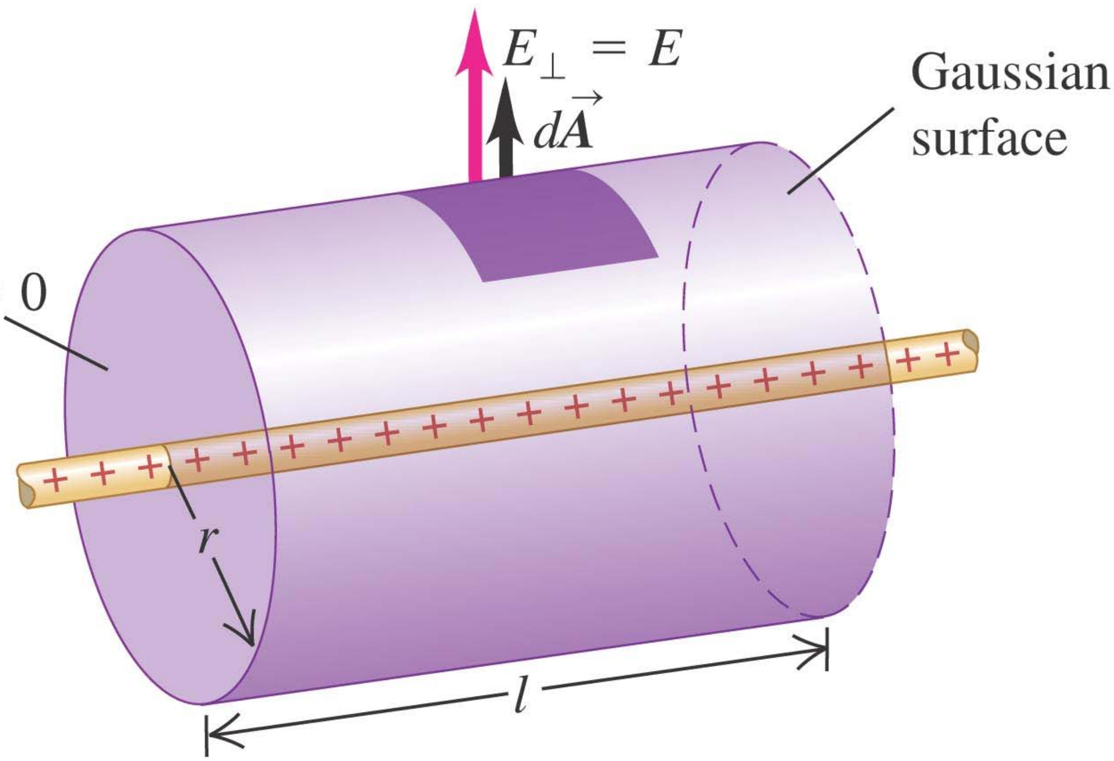
$$\oint E \cdot dA = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \oint dA$$

$$\oint E \cdot dA = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} (4\pi r^2)$$

$$\oint E \cdot dA = \frac{Q_{en}}{\varepsilon_0}$$

Other Geometries

Line of charge



Charge on wire? Charge per unit length λ

Choose Gaussian surface – cylinder

$$\oint E \cdot dA = \frac{Q_{en}}{\varepsilon_0}$$

Cylinder made up of two ends and cylindrical surface.

$$\int_{end} E \cdot dA + \int_{cyl} E \cdot dA + \int_{end} E \cdot dA = \frac{Q_{en}}{\varepsilon_0}$$

$$\int_{end} EdAcos90$$

$$+ \int_{cyl} EdAcos0$$

$$+ \int_{end} EdAcos90 = \frac{Q_{en}}{\varepsilon_0}$$

$$\int_{cyl} EdA = \frac{Q_{en}}{\varepsilon_0}$$

Charge enclosed

$$= (charge/length)(length\ enclosed)$$

$$Q_{en} = \lambda l$$

$$\int_{cyl} EdA = \frac{\lambda l}{\varepsilon_0}$$

$$E \int_{cyl} dA = \frac{\lambda l}{\varepsilon_0}$$

$$E(2\pi rl) = \frac{\lambda l}{\varepsilon_0}$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

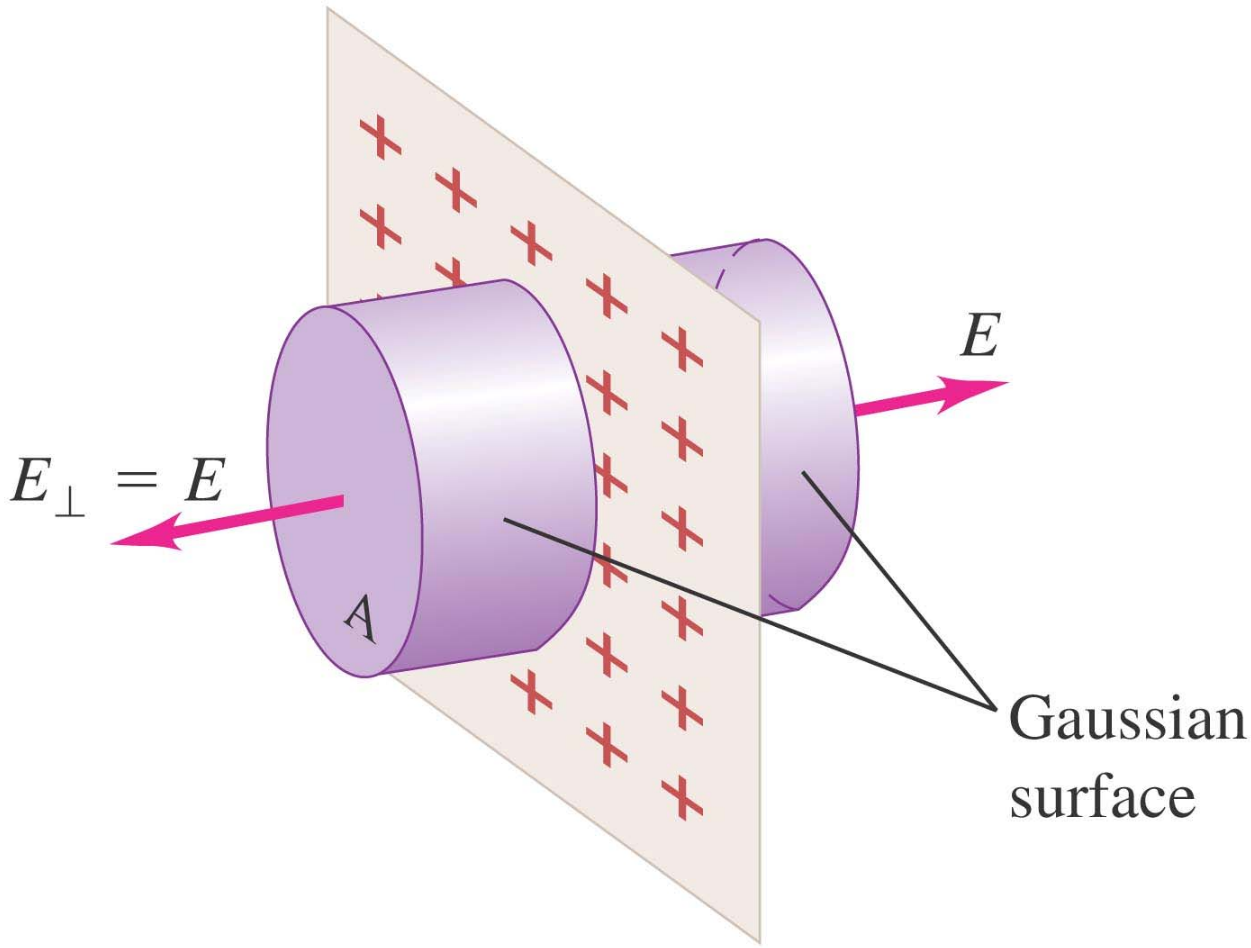
Infinite sheet of charge.

Remember in Chapter 21 we got

$$E = \frac{\sigma}{2\varepsilon_0}$$

where σ was the charge per unit area.

Choose cylinder for Gaussian surface.



$$\oint E \cdot dA = \frac{Q_{en}}{\varepsilon_0}$$

Cylinder made up of two ends and cylindrical surface.

$$\int_{end} E \cdot dA + \int_{cyl} E \cdot dA + \int_{end} E \cdot dA = \frac{Q_{en}}{\varepsilon_0}$$

Cylinder made up of two ends and cylindrical surface.

$$\int_{end} EdAcos0 + \int_{cyl} EdAcos90 + \int_{end} EdAcos0 = \frac{Q_{en}}{\varepsilon_0}$$

$$\int_{end} EdA + \int_{cyl} 0dA + \int_{end} EdA = \frac{Q_{en}}{\varepsilon_0}$$

$$E\int_{end} dA + E\int_{end} dA = \frac{Q_{en}}{\varepsilon_0}$$

$$EA + EA = \frac{Q_{en}}{\varepsilon_0}$$

Charge enclosed

$$Q_{en} = \sigma A$$

$$EA + EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$