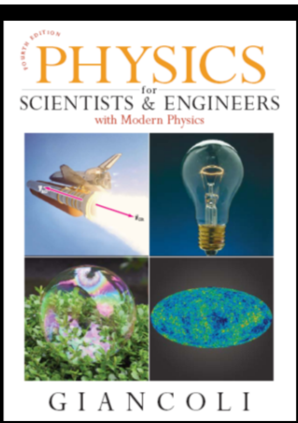


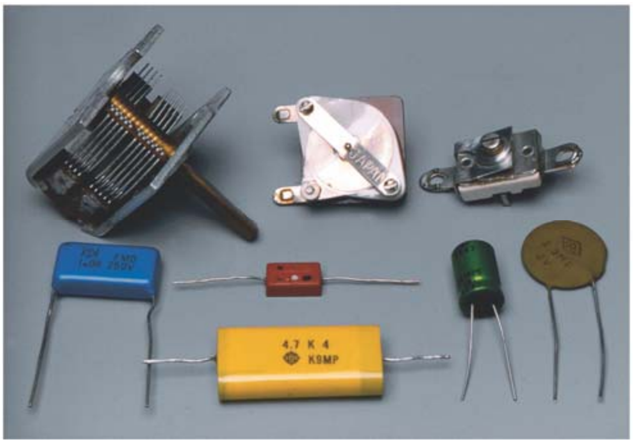
Lecture PowerPoints

Chapter 24

Physics for Scientists and Engineers, with Modern Physics, 4th edition

Giancoli

Chapter 24
Capacitance, Dielectrics, Electric Energy Storage

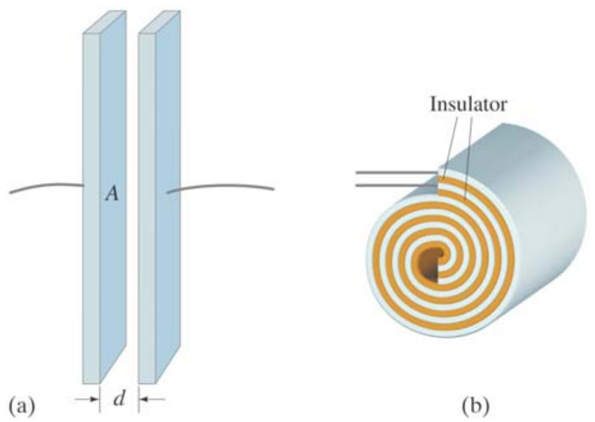


Units of Chapter 24

- Capacitors
- Determination of Capacitance
- Capacitors in Series and Parallel
- Electric Energy Storage
- Dielectrics

24-1 Capacitors

A capacitor consists of two conductors that are close but not touching. The space between the plates is an insulator (dielectric). A capacitor has the ability to store electric charge.

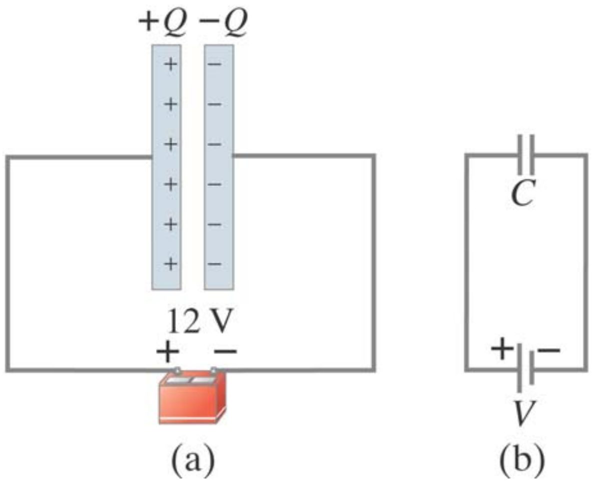


(a) parallel plate capacitor

(b) cylindrical capacitor
(rolled up parallel plate)

24-1 Capacitors

(a) Parallel-plate capacitor connected to battery. (b) is a circuit diagram.



24-1 Capacitors

When a capacitor is connected to a battery, the charge on its plates is proportional to the voltage:

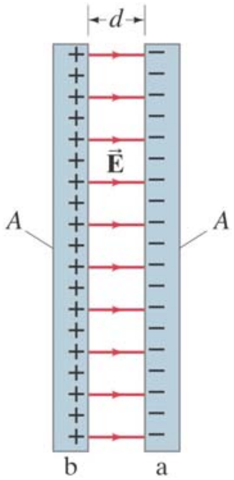
$$Q = CV \quad \rightarrow \quad C = \frac{Q}{V}$$

The quantity C is called the capacitance.

Unit of capacitance: the farad (F):

$$1\ \mathrm{F} = 1\ \mathrm{C/V}$$

24-2 Determination of Capacitance



As shown in chapter 21 the electric field between two parallel plates is:

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{where } \sigma = \text{charge density} = \frac{Q}{A}$$

$$\therefore \quad E = \frac{Q}{\varepsilon_0 A}$$

From chapter 23 we have relation between $\vec{\mathbf{E}}$ and V

$$V = V_{\mathrm{BA}} = V_{\mathrm{B}} - V_{\mathrm{A}} = -\int \vec{\mathbf{E}} \bullet d\vec{\ell}$$

but have shown that $V_{\mathrm{AB}} = \text{-}Ed \quad \rightarrow \quad V_{\mathrm{BA}} = Ed = \dfrac{Qd}{\varepsilon_0 A}$

Capacitance: $C = \dfrac{Q}{V} = \dfrac{\varepsilon_0 A}{d}$ [parallel plate capacitor]

24-2 Determination of Capacitance

Example 24-1: Capacitor calculations.

(a) **Calculate the capacitance of a parallel plate capacitor whose plates are 20 cm × 3.0 cm and are separated by a 1.0 mm air gap.**

(b) **What is the charge on each plate if a 12 V battery is connected across the two plates?**

(c) **What is the electric field between the plates?**

(d) **Estimate the area of the plates needed to achieve a capacitance of 1 F, given the same air gap *d*.**

Example 24-1: Capacitor calculations.

(a) Calculate the capacitance of a parallel plate capacitor whose plates are 20 cm × 3.0 cm and are separated by a 1.0 mm air gap.

$$C = \frac{\varepsilon_0 A}{d} \qquad \left[\varepsilon_0 = 8.85\times10^{-12}\ \mathrm{C^2/N.m^2}\right]$$

$$= \frac{8.85\times10^{-12}\times0.2\times0.03}{0.001} = 5.3\times10^{-11}\ \mathrm{F} = 53\ \mathrm{pF}$$

(b) What is the charge on each plate if a 12 V battery is connected across the two plates?

$$Q = CV = 5.3\times10^{-11}\times12 = 6.4\times10^{-10}\ \mathrm{C} = 640\ \mathrm{pC}$$

Example 24-1: Capacitor calculations.

(c) What is the electric field between the plates?

$$E = \frac{V}{d} = \frac{12}{0.001} = 12000\ \mathrm{V/m}$$

(d) Estimate the area of the plates needed to achieve a capacitance of 1 F, given the same air gap d.

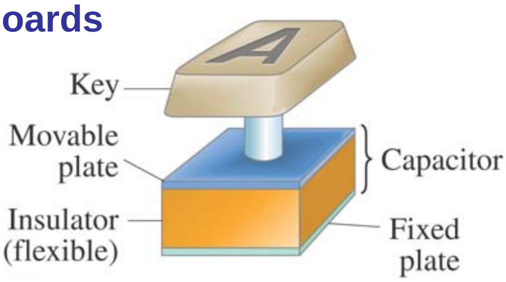
$$\varepsilon_0 = 8.85\times10^{-12}\ \mathrm{C^2/N.m^2}$$

$$C = \frac{\varepsilon_0 A}{d} \rightarrow A = \frac{Cd}{\varepsilon_0} = \frac{1.0\times10^{-3}}{8.85\times10^{-12}} = 1.1\times10^{8}\ \mathrm{m^2} = 110\ \mathrm{km^2}$$

24-2 Determination of Capacitance

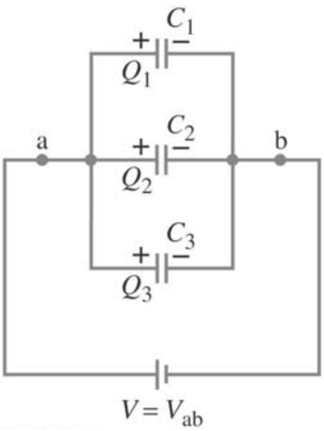
Capacitors are now made with capacitances of 1 farad or more, but they are not parallel-plate capacitors. Instead, they are activated carbon, which acts as a capacitor on a very small scale. The capacitance of 0.1 g of activated carbon is about 1 farad.

Some computer keyboards use capacitors; depressing the key changes the capacitance, which is detected in a circuit.



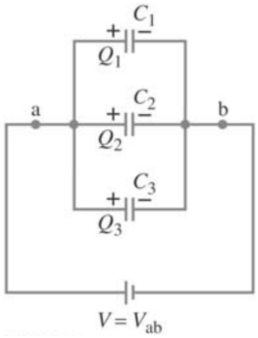
24-3 Capacitors in Series and Parallel

Capacitors in parallel have the same voltage across each one. The equivalent capacitor is one that stores the same charge when connected to the same battery:



$$C_{eq} = C_1 + C_2 + C_3. \qquad \text{[parallel]}$$

24-3 Capacitors in Parallel



All capacitors have the same voltage across their plates.

Total Charge $Q = Q_1 + Q_2 + Q_3 = C_1V + C_2V + C_3V = V(C_1 + C_2 + C_3)$

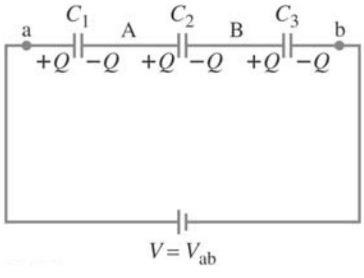
$$C_{eq} = \frac{Q}{V} = C_1 + C_2 + C_3$$

For any number of capacitors: $C_{eq} = \sum_{i=1}^{n} C_i$

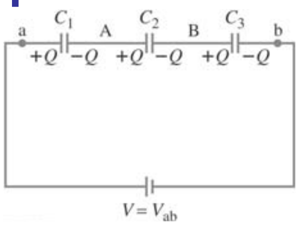
24-3 Capacitors in Series

Capacitors in series have the same charge. In this case, the equivalent capacitor has the same charge across the total voltage drop. Note that the formula is for the inverse of the capacitance and not the capacitance itself!

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}. \qquad \text{[series]}$$



24-3 Capacitors in Series



All capacitors have the same charge on their plates.

Total Voltage $V = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$

$$\frac{1}{C_{eq}} = \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

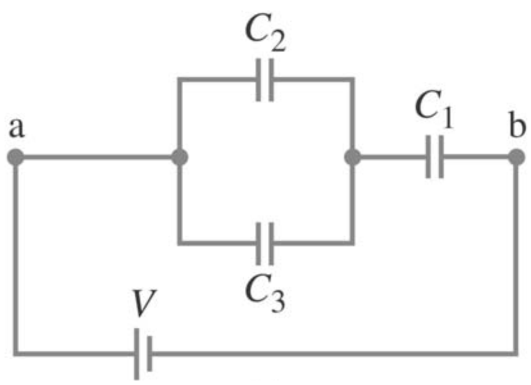
For any number of capacitors: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$

C_{eq} is always smaller than the smallest capacitor.

24-3 Capacitors in Series and Parallel

Example 24-5: Equivalent capacitance.

Determine the capacitance of a single capacitor that will have the same effect as the combination shown.

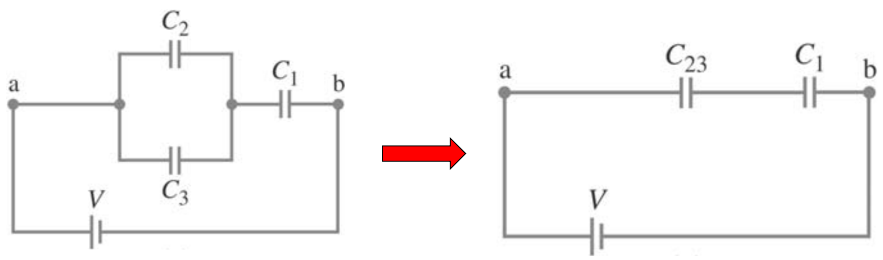


Example 24-5: Equivalent capacitance.

Determine the capacitance of a single capacitor that will have the same effect as the combination shown.

First replace C_2 and C_3 in parallel with equivalent capacitor

$$C_{23} = C_2 + C_3$$

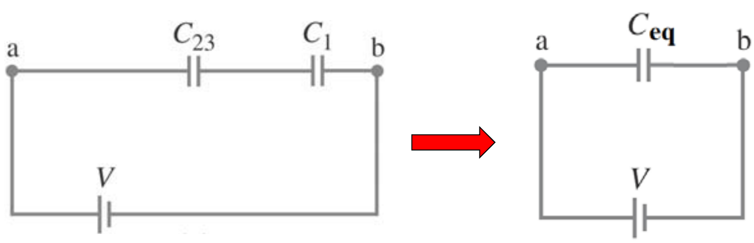


Example 24-5: Equivalent capacitance.

Determine the capacitance of a single capacitor that will have the same effect as the combination shown.

Then calculate the equivalent capacitance of C_{23} and C_1 in series

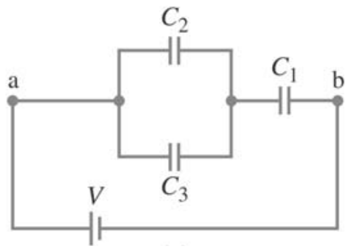
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{23}} \rightarrow C_{eq} = \frac{C_1 C_{23}}{C_1 + C_{23}}$$



Example 24-5: Equivalent capacitance.

Determine the capacitance of a single capacitor that will have the same effect as the combination shown.

If all capacitors are equal: $C = C_1 = C_2 = C_3$



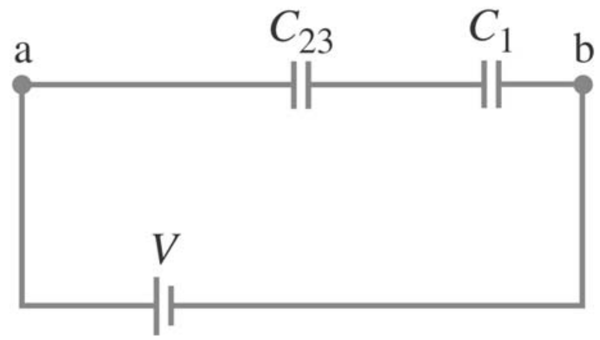
$$C_{23} = C_2 + C_3 = 2C$$

$$C_{\text{eq}} = \frac{C_1 C_{23}}{C_1 + C_{23}} = \frac{C(2C)}{C + 2C} = \frac{2C}{3}$$

24-3 Capacitors in Series and Parallel

Example 24-6: Charge and voltage on capacitors.

Determine the charge on each capacitor and the voltage across each, assuming C = 3.0 μF and the battery voltage is V = 4.0 V.



Example 24-6: Charge and voltage on capacitors.

Determine the charge on each capacitor and the voltage across each, assuming *C* = 3.0 μF and the battery voltage is *V* = 4.0 V.

From previous example

$$C_{\text{eq}} = \frac{2C}{3} = \frac{2\times 3.0}{3} = 2.0\ \mu\text{F} = 2.0\times 10^{-6}\ \text{F}$$

Working backwards from previous example

the charge that leaves the battery with C_{eq} across it is

$$Q = C_{\text{eq}}V = 2.0\times 10^{-6}\times 4.0 = 8.0\times 10^{-6}\ \text{C}$$

Capacitors in series have the same charge on their plates

Example 24-6: Charge and voltage on capacitors.

Determine the charge on each capacitor and the voltage across each, assuming *C* = 3.0 μF and the battery voltage is *V* = 4.0 V.

$$\therefore V_1 = \frac{Q}{C_1} = \frac{8.0\times 10^{-6}}{3.0\times 10^{-6}} = 2.7\ \text{V}$$

C_2 & C_3 are in parallel and have the same V across their plates.

$\therefore C_{23}$ share the charge $Q \rightarrow Q_2 = Q_3 = 4.0\times 10^{-6}\ \text{C}$

$$\therefore V_2 = \frac{Q}{2C_{23}} = \frac{8.0\times 10^{-6}}{2\times 3.0\times 10^{-6}} = 1.3\ \text{V}$$

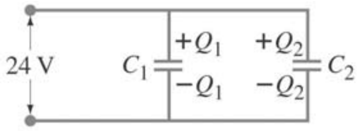
$$\therefore V_3 = \frac{Q}{2C_{23}} = \frac{8.0\times 10^{-6}}{2\times 3.0\times 10^{-6}} = 1.3\ \text{V}$$

Note that $V_{\text{ab}} = 2.7 + 1.3 = 4.0\ \text{V} = \text{Battery voltage}$

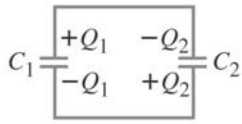
24-3 Capacitors in Series and Parallel

Example 24-7: Capacitors reconnected.

Two capacitors, C_1 = 2.2 μF and C_2 = 1.2 μF, are connected in parallel to a 24 V source as shown. After they are charged they are disconnected from the source and from each other and then reconnected directly to each other, with plates of opposite sign connected together. Find the charge on each capacitor and the potential across each after equilibrium is established.



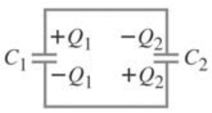
Initial configuration.



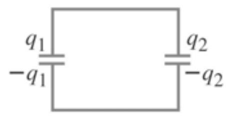
At the instant of reconnection only.

A short time later.

Example 24-7: Capacitors reconnected.
C_1 = 2.2 μF and C_2 = 1.2 μF, V = 24 V



(b) At the instant of reconnection only.



(c) A short time later.

When the capacitors are connected to the voltage source

charge is given by : $Q = CV$

$\rightarrow Q_1 = C_1V = 2.2 \times 24 = 52.8\,\mu\text{C}\ \&\ Q_2 = C_2V = 1.2 \times 24 = 28.8\,\mu\text{C}$

After reconnection

1. Voltage V' is the same across each capacitor

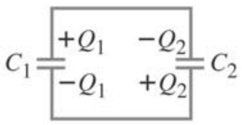
2. The charge on each capacitor is given by $q = CV$

$\rightarrow q_1 = C_1V' \quad \& \quad q_2 = C_2V'$

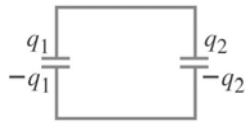
3. The sum of charges on the two capacitors equals the total charge they started with

Example 24-7: Capacitors reconnected.

C_1 = 2.2 μF and C_2 = 1.2 μF, V = 24 V



(b) At the instant of reconnection only.



(c) A short time later.

The total charge on the upper plates in both figures must be the same

$q_1 + q_2 = Q_1 - Q_2 = 52.8 - 28.8 = 24.0\ \mu\text{C}$

Combining equations gives

$$V' = \frac{q_1 + q_2}{C_1 + C_2} = \frac{24.0\times 10^{-6}}{3.4\times 10\times} = 7.1\ \text{V}$$

$q_1 = C_1 V' = 2.2\times 7.1 = 16\ \mu\text{C}$

$q_2 = C_2 V' = 1.2\times 7.1 = 8.5\ \mu\text{C}$

24-4 Electric Energy Storage

A charged capacitor stores electric energy; the energy stored is equal to the work done to charge the capacitor:

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV.$$

24-4 Electric Energy Storage

A charged capacitor stores electric energy; the energy stored is equal to the work done to charge the capacitor:

$$U = W = \int_0^Q V\,dq = \frac{1}{C}\int_0^Q q\,dq = \tfrac{1}{2}\frac{Q^2}{C}$$

$$\text{using } Q = CV \text{ we can write:}$$

$$U = \tfrac{1}{2}\frac{Q^2}{C} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV$$

24-4 Electric Energy Storage

Example 24-8: Energy stored in a capacitor.

A camera flash unit stores energy in a 150 μF capacitor at 200 V.
(a) How much electric energy can be stored?
(b) What is the power output if nearly all this energy is released in 1.0 ms?

24-4 Electric Energy Storage

Example 24-8: Energy stored in a capacitor.

A camera flash unit stores energy in a 150 μF capacitor at 200 V.

(a) How much electric energy can be stored?

$$U = \tfrac{1}{2} C V^2 = \frac{150 \times 10^{-6} \times (200)^2}{2} = 3.0\ \mathrm{J}$$

(b) What is the power output if nearly all this energy is released in 1.0 ms?

$$\text{Power}:\ P = \frac{U}{t} = \frac{3.0}{10^{-3}} = 3000\ \mathrm{W}$$

24-4 Electric Energy Storage

Conceptual Example 24-9: Capacitor plate separation increased.

A parallel plate capacitor carries charge Q and is then disconnected from a battery. The two plates are initially separated by a distance d. Suppose the plates are pulled apart until the separation is $2d$. How has the energy stored in this capacitor changed?

$$C = \frac{\varepsilon_0 A}{d} \quad \text{If } d \to 2d \text{ then } C \to C/2$$

but the charge is the same

$$U = \tfrac{1}{2}\frac{Q^2}{C} \to \tfrac{1}{2}\frac{Q^2}{C/2} = \frac{Q^2}{C}$$

$\to$ Energy is doubled.

24-4 Electric Energy Storage

Example 24-10: Moving parallel capacitor plates.

The plates of a parallel-plate capacitor have area A, separation x, and are connected to a battery with voltage V. While connected to the battery, the plates are pulled apart until they are separated by $3x$. (a) What are the initial and final energies stored in the capacitor? (b) How much work is required to pull the plates apart (assume constant speed)? (c) How much energy is exchanged with the battery?

24-4 Electric Energy Storage

Example 24-10: Moving parallel capacitor plates.

Area = A, Separation $x \to 3x$, Battery voltage = V.

(a) What are the initial and final energies stored in the capacitor?

$$C = \frac{\varepsilon_0 A}{d} \quad (\text{Note that } V \text{ is constant} = V_{\text{batt}})$$

$$U_1 = \tfrac{1}{2} C_1 V^2 = \frac{\varepsilon_0 A V^2}{2x}$$

$$U_2 = \tfrac{1}{2} C_2 V^2 = \frac{\varepsilon_0 A V^2}{2 \times 3x} = \frac{\varepsilon_0 A V^2}{6x}$$

Change in energy

$$\Delta U_{\text{cap}} = U_2 - U_1 = \frac{\varepsilon_0 A V^2}{6x} - \frac{\varepsilon_0 A V^2}{2x} = -\frac{\varepsilon_0 A V^2}{3x}$$

24-4 Electric Energy Storage

Example 24-10: Moving parallel capacitor plates.

Area = A, Separation $x \to 3x$, Battery voltage = V.

(b) How much work is required to pull the plates apart (assume constant speed)?

$$W = \int_x^{3x} F\, d\ell = \int_x^{3x} QE\, d\ell$$

We need the force on one plate due to E only

$\rightarrow$ use $E = V/2\ell$ (by symmetry this is $\frac{1}{2}E = V/\ell$

$$W = \int_x^{3x} F\, d\ell = \int_x^{3x} QE\, d\ell = \frac{\varepsilon_0 A V^2}{x}\int_x^{3x}\frac{d\ell}{\ell^2} = \frac{\varepsilon_0 A V^2}{3x}$$

24-4 Electric Energy Storage

Example 24-10: Moving parallel capacitor plates.

Area = A, Separation $x \to 3x$, Battery voltage = V.

(c) How much energy is exchanged with the battery?

Work on System $W = \Delta U_{\text{cap}} + \Delta U_{\text{batt}}$

$$\Delta U_{\text{batt}} = W - \Delta U_{\text{cap}} = \frac{\varepsilon_0 A V^2}{3x} - \frac{-\varepsilon_0 A V^2}{3x} = \frac{2\varepsilon_0 A V^2}{3x}$$

$\Rightarrow$ Charge flows back into the battery.

24-4 Electric Energy Storage

Heart defibrillators use electric discharge to "jump-start" the heart, and can save lives.

24-5 Dielectrics

A dielectric is an insulator, and is characterized by a dielectric constant K.

Capacitance of a parallel-plate capacitor filled with dielectric:

$$C = \frac{K\varepsilon_0 A}{d}$$

Using the dielectric constant, we define the permittivity:

$$\varepsilon = K\varepsilon_0$$

24-5 Dielectrics

TABLE 24–1
Dielectric Constants (at 20°C)

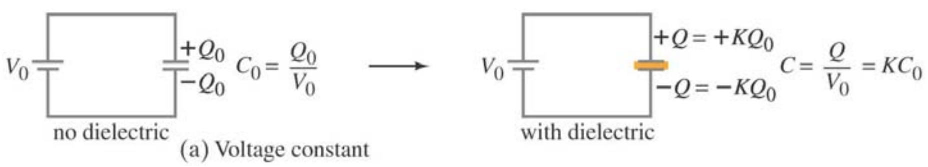
Material	Dielectric constant K	Dielectric strength (V/m)
Vacuum	1.0000	
Air (1 atm)	1.0006	3×10^6
Paraffin	2.2	10×10^6
Polystyrene	2.6	24×10^6
Vinyl (plastic)	2–4	50×10^6
Paper	3.7	15×10^6
Quartz	4.3	8×10^6
Oil	4	12×10^6
Glass, Pyrex	5	14×10^6
Porcelain	6–8	5×10^6
Mica	7	150×10^6
Water (liquid)	80	
Strontium titanate	300	8×10^6

Dielectric strength is the maximum field a dielectric can experience without breaking down.

The higher the dielectric constant, the higher the capacitance. e.g. for the same area and plate separation a strontium titanate dielectric would produce a capacitance 300 times that if the gap was air filled.

24-5 Dielectrics

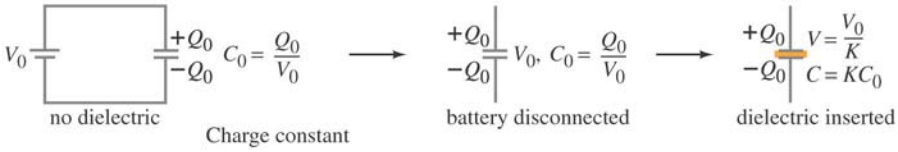
Here are two experiments where we insert and remove a dielectric from a capacitor. In the first, the capacitor is connected to a battery, so the voltage remains constant. The capacitance increases, and therefore the charge on the plates increases as well.



(a) Voltage constant

24-5 Dielectrics

In this second experiment, we charge a capacitor, disconnect it, and then insert the dielectric. In this case, the charge remains constant. Since the dielectric increases the capacitance, the potential across the capacitor drops.



24-5 Dielectrics

Example 24-11: Dielectric removal.

A parallel plate capacitor, filled with a dielectric with $K = 3.4$, is connected to a $100\ \mathrm{V}$ battery. After the capacitor is fully charged, the battery is disconnected. The plates have area $A = 4.0\ \mathrm{m}^2$ and are separated by $d = 4.0\ \mathrm{mm}$.

(a) Find the capacitance, the charge on the capacitor, the electric field strength, and the energy stored in the capacitor.

(b) The dielectric is carefully removed, without changing the plate separation nor does any charge leave the capacitor. Find the new values of capacitance, electric field strength, voltage between the plates, and the energy stored in the capacitor.

24-5 Dielectrics

Example 24-11: Dielectric removal.
$K = 3.4$, $V = 100$, $A = 4.0\ \mathrm{m}^2$ and $d = 4.0$ mm.
(a) Find the capacitance, the charge on the capacitor, the electric field strength, and the energy stored in the capacitor. ($\varepsilon_0 = 8.85\times10^{-12}\ \mathrm{C^2/N.m^2}$)

$$C = K\varepsilon_0 A/d = 3.0\times10^{-8}\ \mathrm{F}$$
$$Q = CV = 3.0\times10^{-6}\ \mathrm{C}$$
$$E = V/d = 25000\ \mathrm{V/m} = 25\ \mathrm{kV/m}$$
$$U = \tfrac{1}{2} CV^2 = 1.5\times10^{-4}\ \mathrm{J}$$

24-5 Dielectrics

Example 24-11: Dielectric removal.
$K = 3.4$, $V = 100$, $A = 4.0\ \mathrm{m}^2$ and $d = 4.0$ mm.
(b) The dielectric is carefully removed, without changing the plate separation nor does any charge leave the capacitor. Find the new values of capacitance, electric field strength, voltage between the plates, and the energy stored in the capacitor. ($\varepsilon_0 = 8.85\times10^{-12}\ \mathrm{C^2/N.m^2}$)

$C = \varepsilon_0 A/d = 8.9\times10^{-9}\ \mathrm{F}$ [$Q = 3.0\times10^{-6}$ C (no change)]
$$V = Q/C = 340\ \mathrm{V}$$
$$E = V/d = 8.5\times10^{4}\ \mathrm{V/m} = 85\ \mathrm{kV/m}$$
$$U = \tfrac{1}{2} CV^2 = \tfrac{1}{2}\times 8.9\times10^{-9}\times(340)^2 = 5.1\times10^{-4}\ \mathrm{J}$$

The increase in energy comes from work to remove the dielectric

Some Sample Q & A

1. A 5 μF capacitor is charged up to 5 V, what is the charge on each plate. (ans. 25 μC)

$C = 5\ \mu F,\ V = 5\ V$
$C = q/V \rightarrow q = CV = 5 \times 10^{-6} \times 5 = 25 \times 10^{-6}\ C = 25\ \mu C$

2. What is the capacitance of a pair of 2 metre square parallel metal plates separated by a 10 mm thick plastic sheet having a dielectric constant of 3.2 (ε_0=8.85x10^{-12} F/m)? (ans. 11.3x10^{-9} F)

$$C = \frac{K\varepsilon_0 A}{d} = \frac{3.2 \times 8.85 \times 10^{-12} \times 4}{0.01} = 1.13 \times 10^{-8}\ \mathrm{F} = 11.3\ \mathrm{nF}$$

$K = 3.2$
$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m
$d = 10$ mm $= 0.01$ m
$A = 2 \times 2 = 4\ m^2$

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3. What happens if the plates of a charged capacitor are pulled apart?

Since $C = \frac{q}{V}$ **and** $C = K\varepsilon_0 \frac{A}{d}$

So the voltage on a charged parallel plate capacitor is:

$$V = \frac{q}{K\varepsilon_0}\frac{d}{A}$$

K and ε_0 are constants and q and A do not change.
∴ If d increases then V will also increase.

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4. In a certain application (eg. Electronic flash) a capacitor is required to store and then deliver 60 J of energy.

   What value of capacitance is required if the supply voltage is 100 V ? (ans. 12000 μF)

$U = 60$ J
$V = 100$ V

$U = \frac{1}{2}CV^2 \rightarrow C = 2U/V^2$
$= (2 \times 60)/100^2 = 12 \times 10^{-3}$ F $= 12$ mF (12000 μF)

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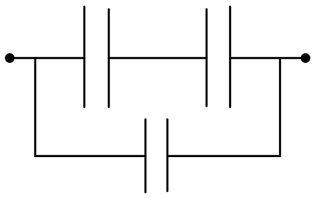
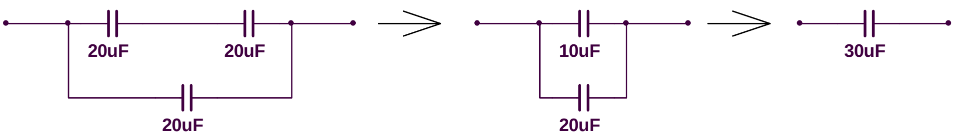
5. What is the equivalent capacitance of the arrangement if the value of each capacitor is 20 μF ?
   (ans. 30 μF)





Replace series capacitors: $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{20} + \frac{1}{20} = \frac{1}{10}$$

$$C_{eq} = 10\ \mu\text{F}$$

For parallel capacitors: $C_{eq} = C_1 + C_2 = 10 + 20 = 30\ \mu$F

Summary of Chapter 24

• Capacitor: nontouching conductors carrying equal and opposite charge.

• Capacitance:

$$Q = CV.$$

• Capacitance of a parallel-plate capacitor:

$$C = \frac{Q}{V} = \epsilon_0 \frac{A}{d}. \qquad \text{[parallel-plate capacitor]}$$

Summary of Chapter 24

• Capacitors in parallel:

$$C_{eq} = C_1 + C_2 + C_3. \qquad \text{[parallel]}$$

• Capacitors in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}. \qquad \text{[series]}$$

Summary of Chapter 24

- A dielectric is an insulator.
- Dielectric constant gives ratio of total field to external field.
- For a parallel-plate capacitor:

$$C = K\epsilon_0 \frac{A}{d}. \qquad \text{[parallel-plate capacitor]}$$