



a)  $\bar{V}_1 = 1.0 \angle 0^\circ \text{ pu}$ ,  $\bar{V}_2 = 0.98 \angle -2^\circ \text{ pu}$   
 $\bar{V}_3 = \frac{1}{1.03} \bar{V}_2 = -0.95 \angle -2^\circ \text{ pu}$

$$Y_{11} = \frac{1}{Z_{12}} + \frac{1}{Z_{12}} + \frac{1}{j0.3}$$

$$= \frac{2}{Z_{12}} + \frac{1}{j0.3} = \frac{2}{0.032 + j0.071} + \frac{1}{j0.3} = 10.552 - j26.74$$

$$Y_{12} = \frac{1}{Z_{12}} \times 2 = \frac{1 \times 2}{0.032 + j0.071} = 10.552 - j23.413$$

$$Y_{22} = \frac{1}{Z_{12}} + \frac{1}{Z_{12}} + \frac{1}{j0.3} + \frac{1}{j0.025}$$

$$= 10.5523 - j66.7463$$

$$Y_{23} = \frac{1}{j0.025} = -j40$$

$$Y_{33} = -j40$$

$$Y_{13} = 0$$



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$$Y_{bus} = \begin{bmatrix} 10.552 - j26.74 & 10.552 - j28.413 & 0 \\ -(10.552 - j23.413) & 10.552 - j66.743 & 40j \\ 0 & 40j & -40j \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = Y_{bus} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$= Y_{bus} \begin{bmatrix} 1 \\ 0.979 - j0.034 \\ 0.949 - j0.033 \end{bmatrix}$$

$$= \begin{bmatrix} -2.038912 - j26.30726 \\ -1.17085 - j4.327 \\ 0.04 + j1.2 \end{bmatrix}$$

$$\bar{I}_{21} = \frac{\bar{V}_2 - \bar{V}_1}{\bar{Z}_{12}} + \frac{\bar{V}_2 Y_{12}}{2}$$

$$= \frac{5.2568 - 11.7657j}{12.553 \angle -67^\circ}$$

$$= 0.3604 \angle 151.99^\circ \text{ pu}$$

$$\bar{I}_{23} = -2 \times \bar{I}_{21} = 1.1207 \angle -28.006^\circ \text{ pu}$$

$$V_3' = \frac{V_2}{1.03} = 0.9514 \angle -2^\circ$$

$$\bar{V}_3 = \bar{V}_3' - \bar{I}_{23} \times j0.025 = 0.9395 \angle -3.5358^\circ \text{ pu}$$

$$S_3 = \bar{V}_3 \bar{I}_3^* = (0.958 + j0.436) \text{ pu}$$

$$\text{Active power} = 0.958 \text{ pu}$$

$$\text{Reactive power} = 0.436 \text{ pu}$$



b) active power 20%.

bus 1  $\rightarrow$  slack

bus 2  $\rightarrow P, Q$

bus 3  $\rightarrow PQ$

Ybus matrix.

$$\begin{array}{c|c|c} & 1 & 2 & 3 \\ \hline 1 & \begin{array}{c} 10.552 \\ j26.74 \end{array} & \begin{array}{c} 10.552 \\ -23.413 \end{array} & 0 \\ \hline 2 & \begin{array}{c} -10.552 \\ j23.413 \end{array} & \begin{array}{c} 10.552 \\ -j66.746 \end{array} & j40 \\ \hline 3 & 0 & j40 & -40j \end{array}$$

$$\begin{array}{c} 2 \\ 3 \end{array} \left[ \begin{array}{c|c} 10.552 & 10.552 \\ j23.413 & -j66.746 \end{array} \right]$$

$$Y_r = \frac{1}{0.025} = -40j.$$

$$\bar{V}_2 = 0.986 \angle -2^\circ$$

$$\bar{V}_3 = 0.9399 \angle -3.8821^\circ$$

$$\bar{V}_1 = 1 \angle 0^\circ$$

$$\begin{array}{c} P_2 \\ P_3 \end{array} \left[ \begin{array}{cc} -B_{22} & -B_{23} \\ -B_{32} & -B_{33} \end{array} \right] \left[ \begin{array}{c} \Delta \delta_2 \\ \Delta \delta_3 \end{array} \right] = \left[ \begin{array}{c} \Delta P_2 / V_2 \\ \Delta P_2 / V_3 \end{array} \right]$$

Ignoring contribution of iff nonun transformer,  $a = 1 \angle 0^\circ$

$$\left[ \begin{array}{cc} -40j & 40j \\ 40j & -40j \end{array} \right]$$

$$B' = \left[ \begin{array}{cc} 81.8035 & -40 \\ -40 & 40 \end{array} \right]$$