

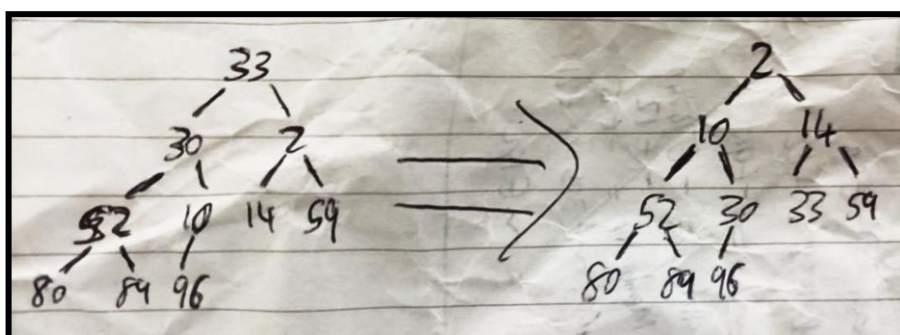
data keys: 33, 30, 2, 52, 10, 14, 59, 80, 89, 96

1. Binary Heap

Operation	Explanation	Heap (as list)
H1	Create a hole at 1 (root of tree)	[]
I33	Insert 33 at the first hole	[33]
H2	Create a hole at 2 (left child of root)	[33,]
X	Move hole up as next number is less than 33	
I30	Insert 30 at the second hole (now at root)	[30, 33]
H3	Create a hole at 3 (right child of root)	[30, 33,]
X	Move hole up as next number is less than 30	
I2	Insert 2 at the third hole (now at root)	[2, 30, 33]
H4	Create a hole at 4 (left child of hole 2)	[2, 30, 33,]
I52	Insert 52 at the fourth hole	[2, 30, 33, 52]
H5	Create a hole at 5 (right child of hole 2)	[2, 30, 33, 52,]
X	Move hole up as next number less than 33	
I10	Insert 10 at the fifth hole (now left child of root)	[2, 10, 30, 33, 52]
H6	Create a hole at 6 (left child of hole 3)	[2, 10, 30, 33, 52,]
X	Move hole up as next number less than 30	
I14	Insert 14 at the sixth hole (now right child of root)	[2, 10, 14, 30, 33, 52]
H7	Create a hole at 7 (right child of hole 3)	[2, 10, 14, 30, 33, 52,]
I59	Insert 59 at the seventh hole	[2, 10, 14, 30, 33, 52, 59]
H8	Create a hole at 8 (left child of hole 4)	[2, 10, 14, 30, 33, 52, 59,]
I80	Insert 80 at the eighth hole	[2, 10, 14, 30, 33, 52, 59, 80]
H9	Create a hole at 9 (right child of hole 4)	[2, 10, 14, 30, 33, 52, 59, 80,]
I89	Insert 89 at the ninth hole	[2, 10, 14, 30, 33, 52, 59, 80, 89]
H10	Create a hole at 10 (left child of hole 5)	[2, 10, 14, 30, 33, 52, 59, 80, 89,]
I96	Insert 96 at the tenth hole	[2, 10, 14, 30, 33, 52, 59, 80, 89, 96]

2. Heap Build

Operation	Explanation	Heap (as list)
	No operation yet – all values inserted	[33, 30, 2, 52, 10, 14, 59, 80, 89, 96]
X10	Swap 10 with 30 as 30 above and 10 smaller	[33, 10, 2, 52, 30, 14, 59, 80, 89, 96]
X2	Swap 2 with 33 as 33 above and 2 smaller	[2, 10, 33, 52, 30, 14, 59, 80, 89, 96]
X14	Swap 14 with 33 as 33 above and 14 smaller	[2, 10, 14, 52, 30, 33, 59, 80, 89, 96]



3. Heap Sort

Operation	Explanation	Heap (as list)	Removed
M2	Remove 2 from the root as smallest	[, 10, 14, 52, 30, 33, 59, 80, 89, 96]	[2]
L	Move Left child up (10) and hole down	[10, , 14, 52, 30, 33, 59, 80, 89, 96]	
R	Move Right child up (30) and hole down	[10, 30, 14, 52, , 33, 59, 80, 89, 96]	
X96	Move 96 up into the hole as it is the next smallest number down and gets rid of the hole	[10, 30, 14, 52, 96, 33, 59, 80, 89]	
M10	Remove 10 from the root as smallest	[, 30, 14, 52, 96, 33, 59, 80, 89]	[2, 10]
R	Move Right child up (14) and hole down	[14, 30, , 52, 96, 33, 59, 80, 89]	
X33	Move 33 up into the hole as it is the next smallest number down and gets rid of the hole	[14, 30, 33, 52, 96, 59, 80, 89]	
M14	Remove 14 from the root as smallest	[, 30, 33, 52, 96, 59, 80, 89]	[2, 10, 14]
L	Move Left child up (30) and hole down	[30, , 33, 52, 96, 59, 80, 89]	
L	Move Left child up (52) and hole down	[30, 52, 33, , 96, 59, 80, 89]	
X80	Move 80 up into the hole as it is the next smallest number down and gets rid of the hole	[30, 52, 33, 80, 96, 59, 89]	
M30	Remove 30 from the root as smallest	[, 52, 33, 80, 96, 59, 89]	[2, 10, 14, 30]
R	Move Right child up (33) and hole down	[33, 52, , 80, 96, 59, 89]	
X59	Move 59 up into the hole as it is the next smallest number down and gets rid of the hole	[33, 52, 59, 80, 96, 89]	
M33	Remove 33 from the root as smallest	[, 52, 59, 80, 96, 89]	[2, 10, 14, 30, 33]
L	Move Left child up (52) and hole down	[52, , 59, 80, 96, 89]	
L	Move Left child up (80) and hole down	[52, 80, 59, , 96, 89]	
X89	Move 89 up into the hole as it is the next smallest number down and gets rid of the hole	[52, 80, 59, 89, 96]	
M52	Remove 52 from the root as smallest	[, 80, 59, 89, 96]	[2, 10, 14, 30, 33, 52]
X59	Move 59 up into the hole as it is the next smallest number down and gets rid of the hole	[59, 80, 89, 96]	
M59	Remove 59 from the root as smallest	[, 80, 89, 96]	[2, 10, 14, 30, 33, 52, 59]
L	Move Left child up (52) and hole down	[80, , 89, 96]	
X89	Move 89 up into the hole as it is the next smallest number down and gets rid of the hole	[80, 89, 96]	
M80	Remove 80 from the root as smallest	[, 89, 96]	[2, 10, 14, 30, 33, 52, 59, 80]
L	Move Left child up (89) and hole down	[89, , 96]	
X96	Move 96 up into the hole as it is the next smallest number down and gets rid of the hole	[89, 96]	
M89	Remove 89 from the root as smallest	[, 96]	[2, 10, 14, 30, 33, 52, 59, 80, 89]
X96	Move 96 up into the hole as it is the next smallest number down and gets rid of the hole	[96]	
M96	Remove 96 from the root as smallest	[]	[2, 10, 14, 30, 33, 52, 59, 80, 89, 96]

4. AVL Tree

Operation	Explanation	AVL Tree (as list)
I33	Insert 33 as root	[33]
I30L33	Insert 30 left of 33 as less than 33	[33, 30]
I2L30	Insert 2 left of 30 as less than 30	[33, 30, 2]
R30	Rotate tree about 30 so 30 becomes the root	[30, 2, 33]
I52R33	Insert 52 right of 33 as bigger than 33	[30, 2, 33, 52]
I10R2	Insert 10 right of 2 as bigger than 2 but less than 30	[30, 2, 33, 10, 52]
I14R10	Insert 14 right of 10 as bigger than 10 but less than 2	[30, 2, 33, 10, 52, 14]
R10	Rotate tree about 10 so left child becomes 10 and its children are 2 on the left and 14 on the right now	[30, 10, 33, 2, 14, 52]
I59R52	Insert 59 right of 52 as bigger than 52	[30, 10, 33, 2, 14, 52, 59]
R52	Rotate tree about 52 so right child becomes 52 and its children are 33 on the left and 59 on the right now	[30, 10, 52, 2, 14, 33, 59]
I80R59	Insert 80 right of 59 as bigger than 59	[30, 10, 52, 2, 14, 33, 59, 80]
I89R80	Insert 89 right of 80 as bigger than 80	[30, 10, 52, 2, 14, 33, 59, 80, 89]
R80	Rotate tree about 80 so that right child of 52 is now 80 and its children are now 59 on the left and 89 on the right	[30, 10, 52, 2, 14, 33, 80, 59, 89]
I96R89	Insert 96 right of 89 as bigger than 89	[30, 10, 52, 2, 14, 33, 80, 59, 89, 96]
R33	Rotate tree about 33 twice so that 33 becomes the new root	[33, 30, 52, 10, 80, 2, 14, 59, 89, 96]
R33	33 should now be the root as there are 4 elements less than it and 5 more than it, tree can be balanced this way	
R14	Rotate tree about 14 twice so that 14 becomes the left child of 33 and its children are 10 on the left and 30 on the right now	[33, 14, 52, 10, 30, 80, 2, 59, 89, 96]
R59	Rotate tree about 59 twice so that 59 becomes the right child of 33 and its children are 52 on the left and 80 on the right now	[33, 14, 59, 10, 30, 52, 80, 2, 89, 96]
R89	Finally, rotate tree about 89 so that 89 becomes the right child of 59 and its children are 80 on the left and 96 on the right now	[33, 14, 59, 10, 30, 52, 89, 2, 80, 96]

5. Hash Table (1)

Operation	Explanation	Hash Table (as array)
P0	Probe index 0 as $h_1(33) = 33 \bmod 11 = 0$	[, , , , , , , , ,]
I33@0	Insert 33 at 0 as it is free	[33, , , , , , , , ,]
P8	Probe index 8 as $h_1(30) = 30 \bmod 11 = 8$	[33, , , , , , , , ,]
I30@8	Insert 30 at 8 as it is free	[33, , , , , , , 30, ,]
P2	Probe index 2 as $h_1(2) = 2 \bmod 11 = 2$	[33, , , , , , , 30, ,]
I2@2	Insert 2 at 2 as it is free	[33, , 2, , , , , , 30, ,]
P8	Probe index 8 as $h_1(52) = 52 \bmod 11 = 8$	[33, , 2, , , , , , 30, ,]
P9	Probe index 9 as index 8 isn't free and linear probing is used (+1 to previously probed index)	[33, , 2, , , , , , 30, ,]
P52@9	Insert 52 at 9 as it is free	[33, , 2, , , , , , 30, 52,]
P10	Probe index 10 as $h_1(10) = 10 \bmod 11 = 10$	[33, , 2, , , , , , 30, 52,]
I10@10	Insert 10 at 10 as it is free	[33, , 2, , , , , , 30, 52, 10]
P3	Probe index 3 as $h_1(14) = 14 \bmod 11 = 3$	[33, , 2, , , , , , 30, 52, 10]
I14@3	Insert 14 at 3 as it is free	[33, , 2, 3, , , , , , 30, 52, 10]
P4	Probe index 4 as $h_1(59) = 59 \bmod 11 = 4$	[33, , 2, 3, , , , , , 30, 52, 10]
I59@4	Insert 59 at 4 as it is free	[33, , 2, 3, 59, , , , , , 30, 52, 10]
P3	Probe index 3 as $h_1(80) = 80 \bmod 11 = 3$	[33, , 2, 3, 59, , , , , , 30, 52, 10]
P4	Probe index 4 as index 3 isn't free and linear probing is used (+1 to previously probed index)	[33, , 2, 3, 59, , , , , , 30, 52, 10]
P5	Probe index 4 as index 3 isn't free and linear probing is used (+1 to previously probed index)	[33, , 2, 3, 59, , , , , , 30, 52, 10]
I80@5	Insert 80 at 5 as it is free	[33, , 2, 3, 59, 80, , , , , , 30, 52, 10]
P1	Probe index 1 as $h_1(89) = 89 \bmod 11 = 1$	[33, 89, 2, 3, 59, 80, , , , , , 30, 52, 10]
I89@1	Insert 89 at 1 as it is free	[33, 89, 2, 3, 59, 80, , , , , , 30, 52, 10]
P8	Probe index 8 as $h_1(96) = 8 \bmod 11 = 8$	[33, 89, 2, 3, 59, 80, , , , , , 30, 52, 10]
P9	Probe index 9 as index 8 isn't free and linear probing is used (+1 to previously probed index)	[33, 89, 2, 3, 59, 80, , , , , , 30, 52, 10]
P10	Probe index 10 as index 9 isn't free and linear probing is used (+1 to previously probed index)	[33, 89, 2, 3, 59, 80, , , , , , 30, 52, 10]
P0	Probe index 0 as index 10 isn't free and linear probing is used (+1 to previously probed index, reset to 0 as mod 11 used [max index 10])	[33, 89, 2, 3, 59, 80, , , , , , 30, 52, 10]
P1	Probe index 1 as index 0 isn't free and linear probing is used (+1 to previously probed index)	[33, 89, 2, 3, 59, 80, , , , , , 30, 52, 10]
P2	Probe index 2 as index 1 isn't free and linear probing is used (+1 to previously probed index)	[33, 89, 2, 3, 59, 80, , , , , , 30, 52, 10]
P3	Probe index 3 as index 2 isn't free and linear probing is used (+1 to previously probed index)	[33, 89, 2, 3, 59, 80, , , , , , 30, 52, 10]
P4	Probe index 4 as index 3 isn't free and linear probing is used (+1 to previously probed index)	[33, 89, 2, 3, 59, 80, , , , , , 30, 52, 10]
P5	Probe index 5 as index 4 isn't free and linear probing is used (+1 to previously probed index)	[33, 89, 2, 3, 59, 80, , , , , , 30, 52, 10]
P6	Probe index 6 as index 5 isn't free and linear probing is used (+1 to previously probed index)	[33, 89, 2, 3, 59, 80, , , , , , 30, 52, 10]
I96@6	Finally, insert 96 at 6 as it is free	[33, 89, 2, 3, 59, 80, 96, , , 30, 52, 10]

6. Hash Table (2)

Operation	Explanation	Hash Table (as array)
P0	Probe index 0 as $h_1(33) = 33 \bmod 11 = 0$	[, , , , , , , , , ,]
I33@0	Insert 33 at 0 as it is free	[33, , , , , , , , , ,]
P8	Probe index 8 as $h_1(30) = 30 \bmod 11 = 8$	[33, , , , , , , , , ,]
I30@8	Insert 30 at 8 as it is free	[33, , , , , , , 30, ,]
P2	Probe index 2 as $h_1(2) = 2 \bmod 11 = 2$	[33, , , , , , , 30, ,]
I2@2	Insert 2 at 2 as it is free	[33, , 2, , , , , 30, ,]
P8	Probe index 8 as $h_1(52) = 52 \bmod 11 = 8$	[33, , 2, , , , , 30, ,]
P10	Since there is a collision, use double hashing substituting into the formula: $\text{new_bucket} = (h_1(x) + i \cdot h_2(x)) \bmod n$ where n is the size of the hash table (i.e. 11 in this case) and i is incremented every time there is a collision [can be considered as number of collisions so far] and $h_2(x)$ is the secondary hashing function giving $h_2(52) = (52 \bmod 3) + 1 = 2$ (Yusuf et al., Collision resolution techniques in Hash table: A review - 2021, Page 758)	[33, , 2, , , , , 30, ,]
I52@10	Insert 52 at 10 as it is free	[33, , 2, , , , , 30, , 52]
P10	Probe index 10 as $h_1(10) = 10 \bmod 11 = 10$	[33, , 2, , , , , 30, , 52]
P1	Probe index 1 as there is a collision and $\text{new_bucket} = (h_1(10) + i \cdot h_2(10)) \bmod 11 = (10 + 1 \cdot 2) \bmod 11 = 12 \bmod 11 = 1$	[33, , 2, , , , , 30, , 52]
I10@1	Insert 10 at 1 as it is free	[33, 10, 2, , , , , 30, , 52]
P3	Probe index 3 as $h_1(14) = 14 \bmod 11 = 3$	[33, 10, 2, , , , , 30, , 52]
I14@3	Insert 14 at 3 as it is free	[33, 10, 2, 14, , , , 30, , 52]
P4	Probe index 4 as $h_1(59) = 59 \bmod 11 = 4$	[33, 10, 2, 14, , , , 30, , 52]
I59@4	Insert 59 at 4 as it is free	[33, 10, 2, 14, 59, , , 30, , 52]
P3	Probe index 3 as $h_1(80) = 80 \bmod 11 = 3$	[33, 10, 2, 14, 59, , , 30, , 52]
P6	Probe index 6 as there is a collision and $\text{new_bucket} = (h_1(80) + i \cdot h_2(80)) \bmod 11 = (3 + 1 \cdot 3) \bmod 11 = 6 \bmod 11 = 6$	[33, 10, 2, 14, 59, , , 30, , 52]
I80@6	Insert 80 at 6 as it is free	[33, 10, 2, 14, 59, , 80, , 30, , 52]
P1	Probe index 1 as $h_1(89) = 89 \bmod 11 = 1$	[33, 10, 2, 14, 59, , 80, , 30, , 52]
P4	Probe index 4 as there is a collision and $\text{new_bucket} = (h_1(89) + i \cdot h_2(89)) \bmod 11 = (1 + 1 \cdot 3) \bmod 11 = 4 \bmod 11 = 4$	[33, 10, 2, 14, 59, , 80, , 30, , 52]
P7	Probe index 7 as there is a collision and $\text{new_bucket} = (h_1(89) + i \cdot h_2(89)) \bmod 11 = (1 + 2 \cdot 3) \bmod 11 = 7 \bmod 11 = 7$	[33, 10, 2, 14, 59, , 80, , 30, , 52]
I89@7	Insert 89 at 7 as it is free	[33, 10, 2, 14, 59, , 80, 89, 30, , 52]
P8	Probe index 8 as $h_1(96) = 96 \bmod 11 = 8$	[33, 10, 2, 14, 59, , 80, 89, 30, , 52]
P9	Probe index 9 as there is a collision and $\text{new_bucket} = (h_1(96) + i \cdot h_2(96)) \bmod 11 = (8 + 1 \cdot 1) \bmod 11 = 9 \bmod 11 = 9$	[33, 10, 2, 14, 59, , 80, 89, 30, , 52]
I96@9	Finally, insert 96 at 9 as it is free	[33, 10, 2, 14, 59, , 80, 89, 30, 96, 52]

References

Yusuf, A.D. et al. (2021) 'Collision resolution techniques in Hash table: A review', International Journal of Advanced Computer Science and Applications, 12(9), pp. 758–758.
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