Bir Matrisin Traspozu

$A_{m\times n}$ lik bir matris

$A=(a_{ij})_{m\times n}$ ise

$A^t=(a_{ji})_{n\times m}$ olarak

oluşturulan yeni matrise

A nın transpozu denir.

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix}_{2\times 4} \longrightarrow A^t=\begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \\ a_{14} & a_{24} \end{bmatrix}_{4\times 2}$$

Özellikler

- $(A^t)^t = A$
- $kA^t = (kA)^t$
- $(A+B)^t = A^t + B^t$
- $(AB)^t = B^t A^t$

Simetrik Matris

$A_{m\times m}$ matrisi

- $A = A^t$

şartını sağlıyorsa simetriktir.

Anti Simetrik Matris

$A_{m\times n}$ matrisi

$$A = -A^t$$

şartını sağlıyorsa anti simetriktir.

Bloklara Ayırma Yöntemi

Uygun boyutlu matrisler, Alt bloklara ayrılıp çarpılabilir.

$$A = \left[\begin{array}{cc|c} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ \hline a_{31} & a_{32} & a_{33} \end{array}\right] \rightarrow A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \rightarrow$$

$$A_1 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix}$$

$$A_3 = \begin{bmatrix} a_{31} & a_{32} \end{bmatrix}$$

$$A_4 = \begin{bmatrix} a_{33} \end{bmatrix}$$

örn

$$A=\begin{bmatrix} 1 & 2 & 1 \\ \hline 3 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix} \longrightarrow A=\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 3 & 4 & 0 \\ \hline 0 & 0 & 2 \end{array}\right] \longrightarrow \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$$

$$B=\left[\begin{array}{c|ccc} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right] \longrightarrow B=\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \longrightarrow \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix}$$

$$AB=\begin{bmatrix} \boxed{A_1\cdot B_1+A_2\cdot B_3}_{2\times 3} & A_1 B_2+A_2\cdot B_4 \\ \hline A_3\cdot B_1+A_4 B_3 & A_3 B_2+A_4 B_4 \end{bmatrix}$$

$$A_1B_1+A_2B_3=\underbrace{\underbrace{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}_{2\times 2}\cdot\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}}+\underbrace{\underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{2\times 1}\cdot\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}_{1\times 3}}$$

$$A_1B_1+A_2B_3=\begin{bmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \end{bmatrix}_{2\times 3}+\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times 3}$$

$$AB=\begin{bmatrix} \boxed{\begin{matrix} 9 & 12 & 15 \\ & & \end{matrix}} & 4 \\ & \end{bmatrix}$$

Temel Satır (Sütun) İşlemleri

A matrisi üzerinde yapılan aşağıdaki 3 işleme temel satır (sütun) işlemleri denir.

1) A matrisinin herhangi iki satırını (sütununu) değiştirmek
2) A matrisinin herhangi bir satırını (sütununu) $k \neq 0$ $k \in \mathbb{R}$ ile çarpmak
3) A matrisinin herhangi bir satırına (sütununa) $k \neq 0$ $k \in \mathbb{R}$ ile çarpılmış bir satırı (sütunu) eklemek.

1) $r_i \leftrightarrow r_j$
2) $k.r_i \rightarrow r_i$
3) $k.r_i + r_j \rightarrow r_j$

1) $c_i \leftrightarrow c_j$
2) $k.c_i \rightarrow c_i$
3) $k.c_i + c_j \rightarrow c_j$

Denk Matrisler

Temel satır (sütun) işlemleri ile birbirlerine dönüştürülebilen matrislere denk matrisler denir.

İlk 3 kurala sahip A nın formuna A nın basamak formu denir.

4 kurala sahip ise

Satırca indirgenmiş basamak form denir.

$$A = \begin{bmatrix} \boxed{1} & 0 & 1 & 0 \\ 0 & \boxed{1} & 3 & 0 \\ 0 & 0 & 0 & \boxed{1} \end{bmatrix}$$

Satırca indirgenmiş basamak form

$A_{m\times n}$ lik matris aşağıdaki özelliklere sahip bir matrise elementer işlemler ile dönüştürülebiliyorsa bu yeni forma A'nın satırca indirgenmiş basamak formu denir.

$$A=\begin{bmatrix}1 & 2\\ 1 & 0\end{bmatrix} \xrightarrow{-r_2+r_1} \begin{bmatrix}0 & 2\\ 1 & 0\end{bmatrix}$$

A'nın satırca indirgenmiş formu?

1) Sıfır satırı varsa en altta.

2) Solda sıfırdan farklı ilk eleman 1 olmalı (Pivot)

3) Tüm Pivotlar basamak halinde olmalı

4) Pivotun bulunduğu sütundaki tüm elemanlar sıfır olmalı.

$$\xrightarrow{r_1 \leftrightarrow r_2} \begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix} \xrightarrow{\frac{1}{2}\cdot r_2} \begin{bmatrix}\boxed{1} & 0\\ 0 & \boxed{1}\end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & 5 & -2 \\ 1 & 4 & 6 & -2 \\ -1 & -1 & -3 & 2 \end{bmatrix}$$

A nın satırca indirgenmiş formu?

$$A = \begin{bmatrix} \boxed{2} & 0 & 2 & 0 \\ 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\frac{1}{2} r_1} \begin{bmatrix} \boxed{1} & 0 & 1 & 0 \\ 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Basamak

$$\xrightarrow{-r_2 + r_1} \begin{bmatrix} \boxed{1} & 0 & 0 & 0 \\ 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

indirgenmiş

$$A=\begin{bmatrix} 1 & 3 & 5 & -2 \\ 1 & 4 & 6 & -2 \\ -1 & -1 & -3 & 2 \end{bmatrix}$$

A nın satırca indirgenmiş formu?

$$A=\begin{bmatrix} 1 & 2 & 3 \\ \boxed{1} & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 4 & 1 \end{bmatrix} \xrightarrow{\substack{-r_2+r_1 \\ -r_2+r_3 \\ -r_2+r_4}} \begin{bmatrix} 0 & 2 & 2 \\ \boxed{1} & 0 & 1 \\ 0 & 1 & -1 \\ 0 & \boxed{4} & 0 \end{bmatrix} \xrightarrow{\substack{r_1 \leftrightarrow r_2 \\ \frac{1}{4}\cdot r_4}} \begin{bmatrix} \boxed{1} & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & -1 \\ 0 & \boxed{1} & 0 \end{bmatrix} \xrightarrow{\substack{-r_4+r_3 \\ -2r_4+r_2}} \begin{bmatrix} \boxed{1} & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & \boxed{-1} \\ 0 & \boxed{1} & 0 \end{bmatrix}$$

$$\xrightarrow{\substack{r_3+r_1 \\ 2r_3+r_2}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{matrix} \\ \\ -r_3 \\ \\ \end{matrix} \xrightarrow{r_2 \leftrightarrow r_4} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A=\begin{bmatrix} 0 & \boxed{1} & 0 & 1 \\ \boxed{1} & 2 & 1 & 1 \\ 0 & 0 & \boxed{1} & 1 \end{bmatrix} \xrightarrow{r_1 \leftrightarrow r_2} \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{-r_3+r_1 \to r_1} \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{-2r_2+r_1 \to r_1} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

matrisini indirgeyin.

Basamak form

indirgenmiş

$$A=\begin{bmatrix}1 & 3 & 5 & -2\\ 1 & 4 & 6 & -2\\ -1 & -1 & -3 & 2\end{bmatrix}$$

A nın satırca indirgenmiş formu?

$$A=\begin{bmatrix}1 & 2 & 3\\ \boxed{1 \quad 0 \quad 1}\\ 1 & 1 & 0\\ 1 & 4 & 1\end{bmatrix}\begin{array}{l}-r_2+r_1\\ -r_2+r_3\\ -r_2+r_4\end{array}\begin{bmatrix}0 & 2 & 2\\ \boxed{1} & 0 & 1\\ 0 & 1 & -1\\ 0 & \boxed{4} & 0\end{bmatrix}\begin{array}{l}r_1\leftrightarrow r_2\\ \frac{1}{4}r_4\end{array}\begin{bmatrix}\boxed{1} & 0 & 1\\ 0 & 2 & 2\\ 0 & 1 & -1\\ 0 & \boxed{1} & 0\end{bmatrix}\xrightarrow[-2r_4+r_2]{-r_4+r_3}\begin{bmatrix}\boxed{1} & 0 & 1\\ 0 & 0 & 2\\ 0 & 0 & \boxed{-1}\\ 0 & \boxed{1} & 0\end{bmatrix}$$

$$\xrightarrow[2r_3+r_2]{r_3+r_1}\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}\begin{array}{l}r_2\leftrightarrow r_4\\ \\ -r_3\end{array}\longrightarrow\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}$$

$$\begin{bmatrix}2 & 1 & 1 & 0\\ 0 & 2 & 2 & 2\\ 4 & 4 & 4 & 2\end{bmatrix}\xrightarrow[\frac{1}{2}r_2]{-2r_1+r_3}\begin{bmatrix}2 & 1 & 1 & 0\\ 0 & \boxed{1} & 1 & 1\\ 0 & 2 & 2 & 2\end{bmatrix}\xrightarrow[-2r_2+r_3]{-r_2+r_1}\begin{bmatrix}2 & 0 & 0 & -1\\ 0 & 1 & 1 & 1\\ 0 & 0 & 0 & 0\end{bmatrix}\xrightarrow{\frac{1}{2}r_1}\begin{bmatrix}1 & 0 & 0 & -1/2\\ 0 & 1 & 1 & 1\\ 0 & 0 & 0 & 0\end{bmatrix}$$

indirgeyiniz!

$$A=\begin{bmatrix} \boxed{1} & 1 & 1 & 9 \\ 0 & \boxed{1} & -2 & 5 \\ 0 & 1 & 2 & 2 \\ 0 & 2 & 0 & 7 \end{bmatrix} \xrightarrow[\substack{-r_2+r_3 \\ -2r_2+r_4}]{-r_2+r_1} \begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & \boxed{4} & -3 \\ 0 & 0 & 4 & -3 \end{bmatrix} \xrightarrow{\frac{1}{4}r_3} \begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & 1 & -3/4 \\ 0 & 0 & 4 & -3 \end{bmatrix} \xrightarrow[-4r_3+r_4]{\substack{-3r_3+r_1 \\ 2r_3+r_2}} \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & -3/4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

indirgenmiş

A'yı indirgeyiniz.

$$A=\begin{bmatrix} 1 & -1 & 1 & 0 & -3 \\ -1 & 2 & 3 & 1 & 4 \\ 3 & 4 & 1 & 1 & 10 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1/5 & 13/5 \\ 0 & 0 & 1 & 1/5 & -2/5 \end{bmatrix}$$

indirgeyiniz.

$$A=\begin{bmatrix}1 & 3 & 5 & -2\\ 1 & 4 & 6 & -2\\ -1 & -1 & -3 & 2\end{bmatrix} \longrightarrow \begin{bmatrix}1 & 0 & 2 & -2\\ 0 & 1 & 1 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

A nın satırca indirgenmiş formu?

$$A=\begin{bmatrix}0 & 3 & 4\\ 2 & -1 & 6\\ -1 & 2 & 5\end{bmatrix} \longrightarrow \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Ayı indirgeyin

$$\xrightarrow{r_1-r_2} \begin{bmatrix}-1 & 2 & 0 & 3 & 3\\ 2 & 8 & -8 & 2 & 4\\ 2 & 0 & -2 & -1 & 0\end{bmatrix} \xrightarrow[\frac{1}{2}r_2]{-r_1} \begin{bmatrix}1 & -2 & 0 & -3 & -3\\ 1 & 4 & -4 & 1 & 2\\ 2 & 0 & -2 & -1 & 0\end{bmatrix} \xrightarrow[-2r_1+r_3]{-r_1+r_2} \begin{bmatrix}1 & -2 & 0 & -3 & -3\\ 0 & 6 & -4 & 4 & 5\\ 0 & 4 & -2 & 5 & 6\end{bmatrix} \xrightarrow{\frac{1}{4}r_3} \begin{bmatrix}1 & -2 & 0 & -3 & -3\\ 0 & 6 & -4 & 4 & 5\\ 0 & 1 & -\frac{1}{2} & \frac{5}{4} & \frac{3}{2}\end{bmatrix}$$

$$\xrightarrow[-6r_3+r_2]{2r_3+r_1} \begin{bmatrix}1 & 0 & -1 & -\frac{1}{2} & 0\\ 0 & 0 & -1 & -\frac{7}{2} & -4\\ 0 & 1 & -\frac{1}{2} & \frac{5}{4} & \frac{3}{2}\end{bmatrix} \xrightarrow{r_2 \leftrightarrow r_3} \begin{bmatrix}1 & 0 & -1 & -\frac{1}{2} & 0\\ 0 & 1 & -\frac{1}{2} & \frac{5}{4} & \frac{3}{2}\\ 0 & 0 & -1 & -\frac{7}{2} & -4\end{bmatrix}$$

$$\xrightarrow{-r_3} \begin{bmatrix}1 & 0 & -1 & -\frac{1}{2} & 0\\ 0 & 1 & -\frac{1}{2} & \frac{5}{4} & \frac{3}{2}\\ 0 & 0 & 1 & \frac{7}{2} & 4\end{bmatrix} \xrightarrow[\frac{1}{2}r_3+r_2]{r_3+r_1} \begin{bmatrix}1 & 0 & 0 & 3 & 4\\ 0 & 1 & 0 & 3 & \frac{7}{2}\\ 0 & 0 & 1 & \frac{7}{2} & 4\end{bmatrix}$$