Control Theory – 2B

CONTROL OF MULTIVARIABLE SYSTEM

Robotics & Automation pathway

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A. Contents

B. Introduction

This laboratory session involved analysis, modeling, and controlling multivariable processes, with a specific emphasis placed on the three-tank system. With a use of MATLAB and Simulink, both theoretical and practical controls in a system were considered, critical in managing complex processes in modern engineering. Multivariable processes, such as the three-tank system, present specific challenges with an interrelated configuration of in and out, and require high-tech controls for maximization of performance.

Control systems have a role of controlling process variable and ensuring performance targets under changing and external disturbances. It involves maintaining stability, achieving accuracy, and offering robustness to uncertain environment variation. engineers develop controls not only for stabilizing processes at and about a nominal working point but for allowing processes transition between state, follow a reference signal, and maximize important performance factors such as energy efficiency, output, or environment suitability. All these have to be achieved in despite possibly having a dynamic, unknown, or even changing disturbances, a fact that underlines a critical role for a control system in ensuring reliable and efficient operations.

The key operations of a control system include cancelling out disturbances for maintaining stability, allowing a system follow desired pathes or setpoints, controlling procedural phases such as startup and shut down, and copings with changing dynamics through changing settings. Others involve fault detection for minimizing damage or re-modeling processes, supervision of parts of a system for dynamically changing working conditions, coordination for coordination between interrelated processes, and improvement through training with operational experiences for optimized performance over times. All these demand a range of techniques, from traditional controllers to high-tech smart ones that utilize artificial intelligence for adaptable and predictive decision processes.

During this laboratory session, such approaches were utilized in an attempt to model, simulate, and analyze the three-tank system under numerous configurations. Because of its nonlinearities and complications wrought with its variable interdependencies, such a system warranted analysis in a simulation environment. Linearization, simulation in an open-loop and a closed-loop, and state feedback controllers, in fact, facilitated analysis of the system in terms of its controllability, observability, and performance in a laboratory environment. Responses to disturbances and techniques in pole placement for performance maximization facilitated a deeper consideration of robust control system engineering and its application.

The laboratory exercise emphasized the important role played by a control system in modern engineering practice. With theoretical underpinnings supplemented with real-life tools such as MATLAB and Simulink, it painted a critical examination of multivariable system dynamics and efficient controlling methodologies. Gains in such an exercise can directly apply to many industries and even academic studies, supporting the important role played by sophisticated controlling methodologies in achieving operational expertise.

C. State of the Art

1. Three-Tank System

Modern industrial processes entail concurrent manipulation of several variables in a single system, with any single variable having an impact on all output values. In such processes, high control over liquids is most critical, with direct impact over efficiency and quality of a product. As a complex multivariable system with high interdependencies and nonlinear behavior, three-tank is an ideal system with a key target for studies in controlling level of a liquid.

Designing three-tank controllers is no ordinary feat with its nonlinear behavior in tank-to-tank and flow behavior. Model predictive control (MPC) has become a key target for exploiting modern capabilities in its approaches in tackling such complexity, with comparative studies with traditional proportional-integral (PI) controllers bearing witness to its overall effectiveness. In experiments in a real three-tank pilot system with integration in an industrial ABB 800xA automation platform, for instance, settle times include 120 seconds for both tanks for MPC, in contrast with 200 seconds for tank one and 150 seconds for tank two for PI controllers. All such experimental values have testified that with its high accuracy and rapidity in controlling a system, MPC trounces traditional PI controllers.

2. Multivariable Control

, Multivariable control involves computerization of output and setpoint change in a range of control loops, actions previously performed manually by operators. Manual multivariable control involves such actions taken during shifts in operators. Techniques such as model predictive control (MPC) and model-less multivariable control (XMC), in contrast, allow computerization, and with them, efficiency in multivariable control systems is vastly increased.

Automated multivariable control, through multi-loop closing, possesses many advantages over single-loop computerization, including increased uniformity and punctuality, reduced alarm and constraint violations, and overall maximization of the process. Apart from that, computerized multivariable control carries with it considerable operational and financial savings, and forms an important part of current techniques for controlling processes.

D. Methods

1. Linearization

The linearization of a three-tank system utilized Taylor approximation in an attempt to obtain a state-space model in a linear form. There were three configurations with regard to its pumps and valve controls: single-pump with single valve (Single-Input Multiple-Output or SIMO), single-pump with two valves (Multiple-Input Multiple-Output or MIMO), and two pumps with two valves (MIMO). All three configurations were modelled and examined sequentially.

A multitank system nonlinear model is derived using Bernoulli's principle, and it is an idealized model for describing laminar outflow of an ideal fluid. Dynamics of a three cascade tank system can be modelled via the following equations:

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Here, H_1, H_2, H_3 represent the water levels in the upper, middle, and lower tanks, respectively, while q is the pump inflow rate. The parameters C_1, C_2, C_3 denote the resistance coefficients for the output orifices of the tanks. The geometric parameters of the tanks are a, b, c, w, R, and $H_{2\max}, H_{3\max}$ represent the maximum water levels for the middle and lower tanks.

The nonlinearities in the system arise primarily from the square-root term $C_i\sqrt{H_i}$ and the variable cross-sectional areas that depend on water levels H_2 and H_3. To address these nonlinearities, the system was linearized around an operating point defined by $H_0 = \begin{bmatrix} H_{10} \\ H_{20} \\ H_{30} \end{bmatrix}$ and $U_0 = \begin{bmatrix} q_0 \\ C_{10} \\ C_{20} \\ C_{30} \end{bmatrix}$.

Using Taylor expansion, the state-space form was expressed as $\dot{X} = A\Delta X + B\Delta U$, $Y = C\Delta X + D\Delta U$, where $\Delta X = \begin{bmatrix} H_1 - H_{10} \\ H_2 - H_{20} \\ H_3 - H_{30} \end{bmatrix}$ and $\Delta U = \begin{bmatrix} q - q_0 \\ C_1 - C_{10} \\ C_2 - C_{20} \\ C_3 - C_{30} \end{bmatrix}$.

For the pump-controlled system, the following state-space matrices were derived:

$$\dot{H}_1 = \frac{1}{aw}q - \frac{1}{aw}C_1\sqrt{H_1}$$

$$\dot{H}_2 = \frac{1}{cw + \frac{H_2}{H_{2\max}}bw}C_1\sqrt{H_1} - \frac{1}{cw + \frac{H_2}{H_{2\max}}bw}C_2\sqrt{H_2}$$

$$\dot{H}_3 = \frac{1}{w\left(R^2 - (H_{3\max} - H_3)^2\right)}C_2\sqrt{H_2} - \frac{1}{w\left(R^2 - (H_{3\max} - H_3)^2\right)}C_3\sqrt{H_3}$$

$$A_p = \begin{bmatrix} -\frac{C_1}{2aw\sqrt{H_{10}}} & 0 & 0 \\ \frac{C_1}{2(cw+\frac{H_{20}}{H_{2\max}}bw)\sqrt{H_{10}}} & -\frac{C_2}{2(cw+\frac{H_{20}}{H_{2\max}}bw)\sqrt{H_{20}}} & 0 \\ 0 & \frac{C_2}{2w(R^2-(H_{3\max}-H_{30})^2)\sqrt{H_{20}}} & -\frac{C_3}{2w(R^2-(H_{3\max}-H_{30})^2)\sqrt{H_{30}}} \end{bmatrix}$$

$$B_p = \begin{bmatrix} \frac{1}{aw} \\ 0 \\ 0 \end{bmatrix}, \quad C_p = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad D_p = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For the valve-controlled system, similar linearization steps were applied, resulting in distinct state-space matrices based on the three valve inputs (C_1, C_2, C_3). For the combined pump and valve-controlled system, the linearized model was expressed as $A_{pv} = A_v$, $B_{pv} = \begin{bmatrix} B_p & B_v \end{bmatrix}$, $C_{pv} = C_p$, and $D_{pv} = 0$.

In MATLAB/Simulink, open-loop simulations were conducted to visualize the system's behavior, and closed-loop simulations were performed to evaluate system performance under state feedback. Key parameters used for simulation include $C_1 = 1.0057 \times 10^{-4}$, $C_2 = 1.1963 \times 10^{-4}$, $C_3 = 9.8008 \times 10^{-5}$, $H_{10} = 0.1425\,m$, $H_{20} = 0.1007\,m$, $H_{30} = 0.15\,m$, and $q_0 = 3.7958 \times 10^{-5}\,m^3/s$.

2. Matlab and Simulink

In MATLAB script, the matrices AAA, BBB, CCC, and DDD were calculated using the state-space representation as shown in Equation 6. The MATLAB function `ss2tf` was employed to derive the transfer function from these state-space matrices. Simulink was utilized to design and simulate different block diagrams, enabling the visualization of the dynamics of the three-tank system.

Figure 2 illustrates the subsystem corresponding to the upper tank, labeled as Sub System 1. **Figure 3** represents the subsystem for the middle tank, referred to as Sub System 2, while **Figure 4** depicts Sub System 3, which corresponds to the lower tank. Finally, **Figure 5** shows the complete three-tank system modeled in Simulink, integrating all three subsystems to represent the entire process.

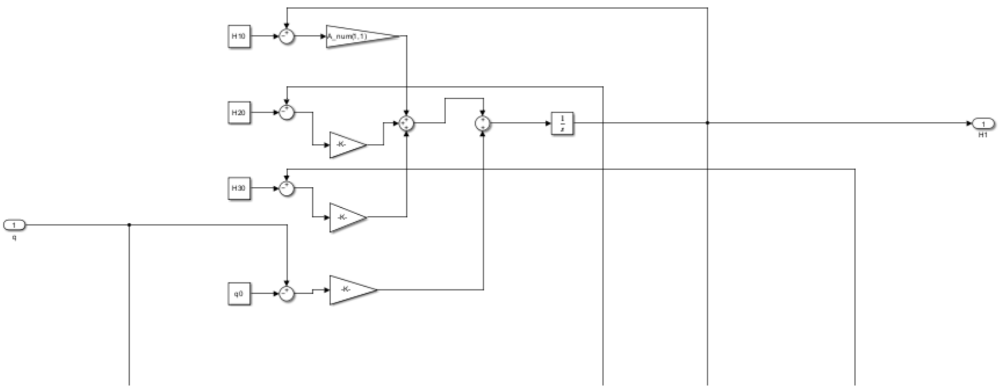

Figure 1 – The Sub System 1 – Upper Tank

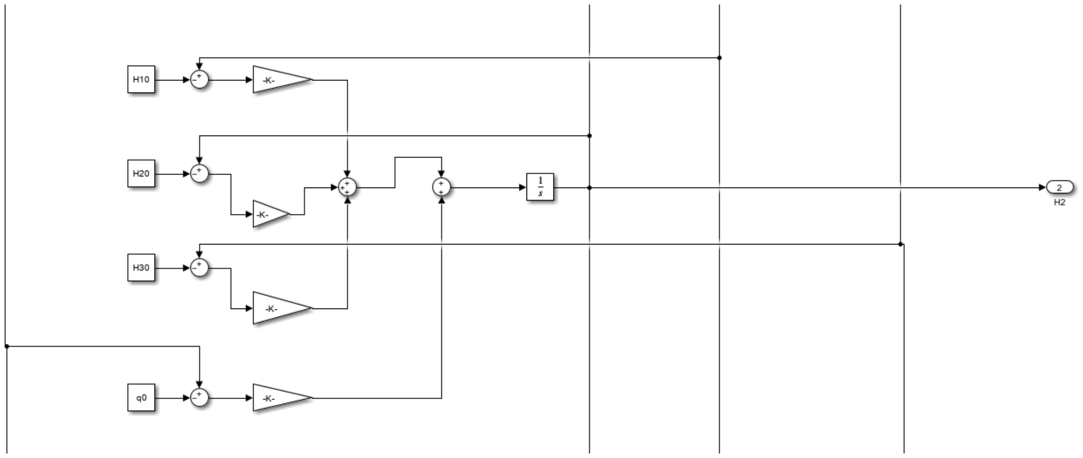

Figure 2 – The Sub System 2 – Middle Tank

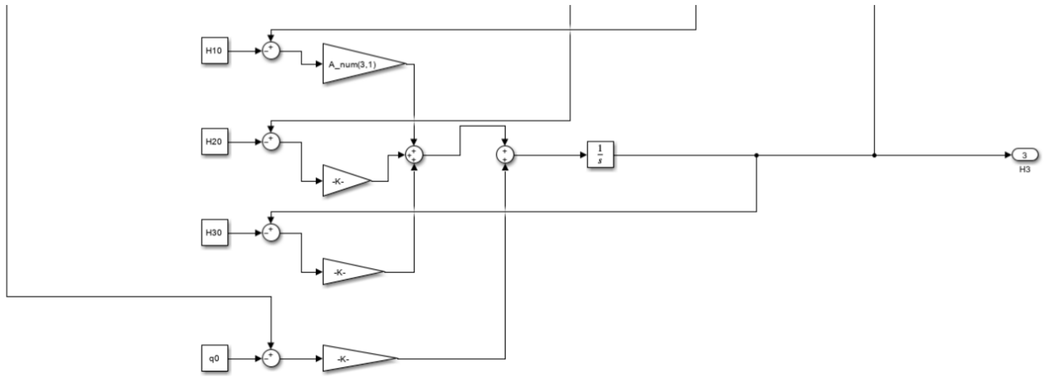

Figure 3 – The Sub System 3 – The Lower Tank

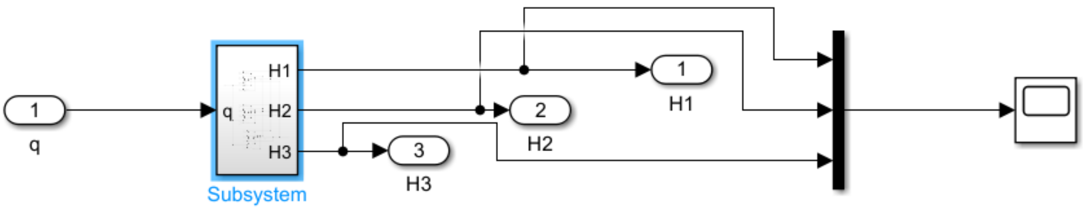

Figure 4 – The Three Tank System – Simulink model

3. State-Feedback Controller

Incorporating feedback in a closed-loop control system enhances its performance by dynamically adjusting the controller output to minimize error. Feedback not only mitigates the effects of disturbances but also improves the system's stability. By doing so, it ensures a more reliable and repeatable control loop.

To guarantee system stability, the poles were strategically set to -0.9, -0.5, and -0.7. This choice of poles stabilizes the system while providing a desired dynamic response. MATLAB was used to implement this adjustment. Place **Figure 6** after this paragraph to show the procedure for configuring new poles for the matrix A in the closed-loop system.

3. Results

Simulink simulations were conducted to observe the water levels in the three tanks over time. Initially, the heights of the three tanks, represented by the blue, orange, and yellow curves, reached their target levels between 5 and 25 seconds. This demonstrated the effectiveness of the initial controller in managing the dynamics of the multitank system.

After running the initial simulations and obtaining results, unintended changes were made to the Simulink model and code. These changes introduced errors, and despite significant efforts, it was not possible to identify or resolve the source of the problem in the modified version.

Before these changes, adjusting the poles of the state-feedback controller improved the system's performance. The desired water levels were reached more quickly, all within 10 seconds. This highlighted the ability to fine-tune the controller to optimize system response times and achieve superior performance. Unfortunately, due to the modifications to the Simulink model and code, the improved response could no longer be reproduced.

E. Conclusion

This laboratory exercise provided valuable insights into the control of multivariable systems using the three-tank system as a practical example. By employing MATLAB and its Simulink features, we explored key concepts such as system linearization and state-space representation, laying a strong foundation for real-world applications of control theory.

The exercise emphasized the significance of linearizing nonlinear systems for effective modeling and control. Through systematic analysis of the three configurations—pump-controlled (SIMO), valve-controlled (MIMO), and pump-valve-controlled (MIMO)—we developed and validated open-loop and closed-loop models. These models allowed us to evaluate system performance based on criteria such as time response, overshoot, and robustness against disturbances.

One notable challenge was the inability to refine the Simulink models outside the lab due to accessibility constraints. Despite this limitation, the hands-on experience with MATLAB scripts and Simulink simulations deepened our understanding of state-feedback controller design, pole placement, and the critical analysis of system controllability and observability.

In conclusion, this project served as a practical bridge between theoretical concepts and their implementation in multivariable control systems. The knowledge gained extends beyond the lab, providing a strong technical foundation applicable to more complex control challenges in real-world engineering scenarios. Moreover, this exercise highlighted the importance of iterative model refinement and thorough performance evaluation to ensure the robustness and reliability of control systems.