# TORSION AND BENDING LOADS

## A. Optimization of a drive shaft

### Analytical Analysis

Torque transmitted by the shaft when the engine is working at maximum power:

$$P = Tw \rightarrow T_{max} = \frac{P_{max}}{w} = \frac{132 * 10^3 \left(\frac{N.m}{s}\right)}{\frac{2\pi\left(\frac{rad}{rev}\right) * 1200\left(\frac{rad}{rev}\right)}{60\left(\frac{s}{min}\right)}} \rightarrow T_{max} = 1050.42\ N.m$$

Max shear stress ($\tau_{max}$) is created at the outer radius of the hollow shaft, for which polar moment of inertia (J) is needed.

$$J = \frac{\pi}{2}(R_o^4 - R_i^4) = \frac{\pi}{2}[(0.035m)^4 - (0.028m)^4] \rightarrow J = 1.392 * 10^{-6}\ m^4$$

Then, **the max shear stress** is:

$$\tau_{max} = \frac{T * R_o}{J} = \frac{(1050.42\ N.m) * (0.035m)}{1.392 * 10^{-6}\ m^4} = 26.41\frac{N}{m^2} \rightarrow \tau_{max} = 26.41\ MPa$$

For twist angle ($\varphi$), the shear modulus (G) is needed.

$$G = \frac{E}{2(1+\vartheta)} = \frac{200 * 10^9 Pa}{2(1+0.3)} = 76.92\ * 10^9 Pa \rightarrow G = 7.692\ * 10^{10}\left(\frac{N}{m^2}\right)$$

Then, **the twist angle** is:

$$\varphi = \frac{T * L}{J * G} = \frac{(1050.42\ N.m)(1.8m)}{(1.392 * 10^{-6}\ m^4)\left[7.692\ * 10^{10}\left(\frac{N}{m^2}\right)\right]} \rightarrow \varphi = 0.017\ rad\ (0.97\ deg)$$

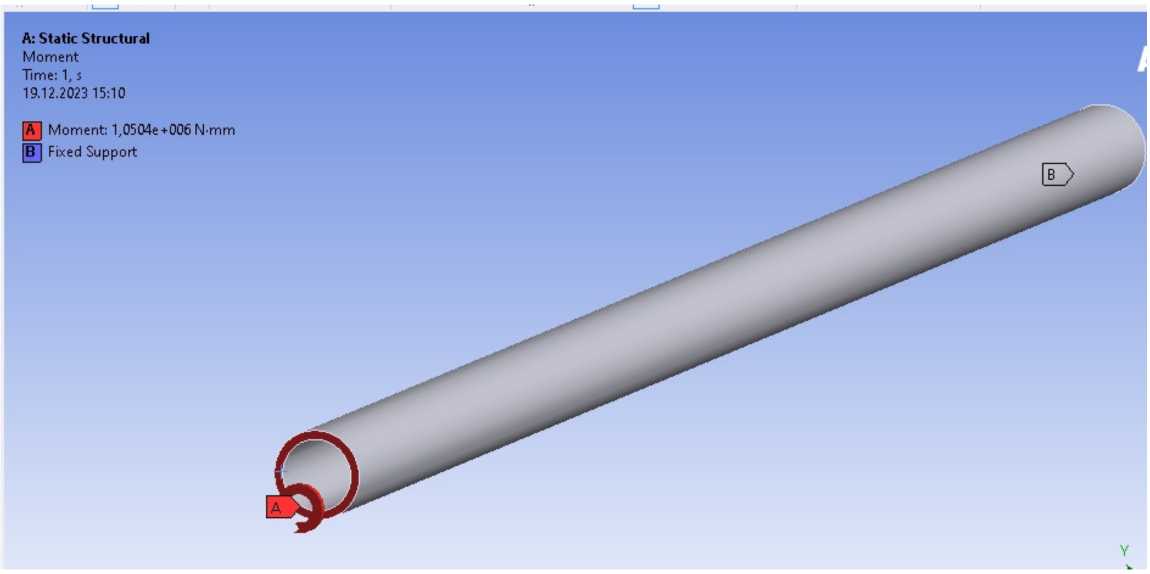
The max deformation ($\delta_{max}$) can also be found:

$$\delta_{max} = R_o * \varphi = (35mm) * 0.017\ rad = 0.6218\ mm$$

### Numerical Analysis {Using simulation software (ANSYS)}

Steel having $E$ = 200 $GPa$, $\nu$ = 0.3 is used in a *Structural Study* simulation with *5-mm-mesh*:

Simulation results: $\tau_{max}$= 26.73 MPa and $\varphi$=0.017 rad. These are very comparable with the analytical results.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Moment
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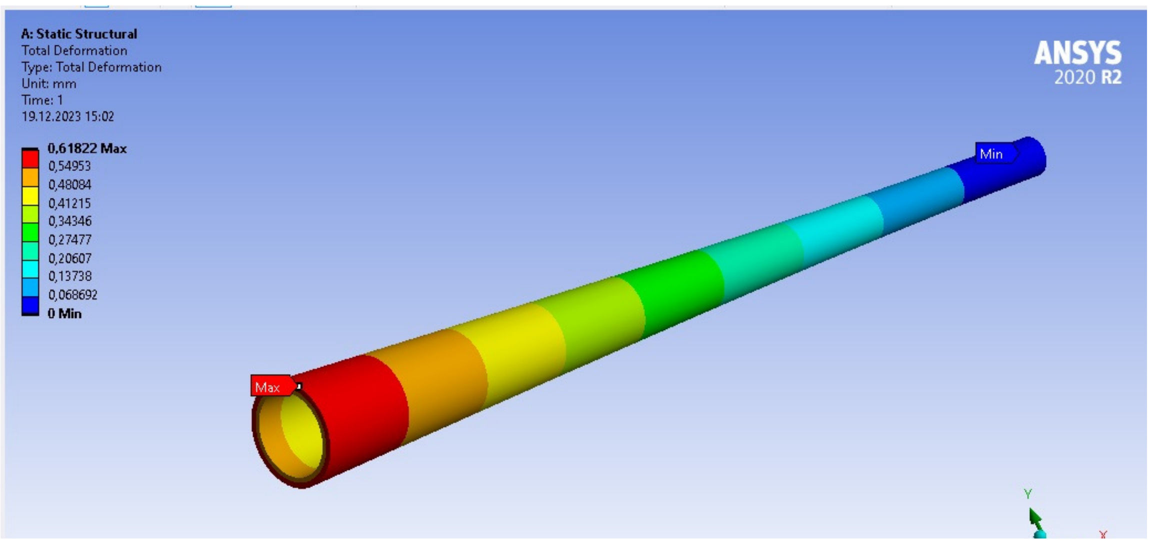
B Fixed Support
B
A
Y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Type: Total Deformation
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ANSYS
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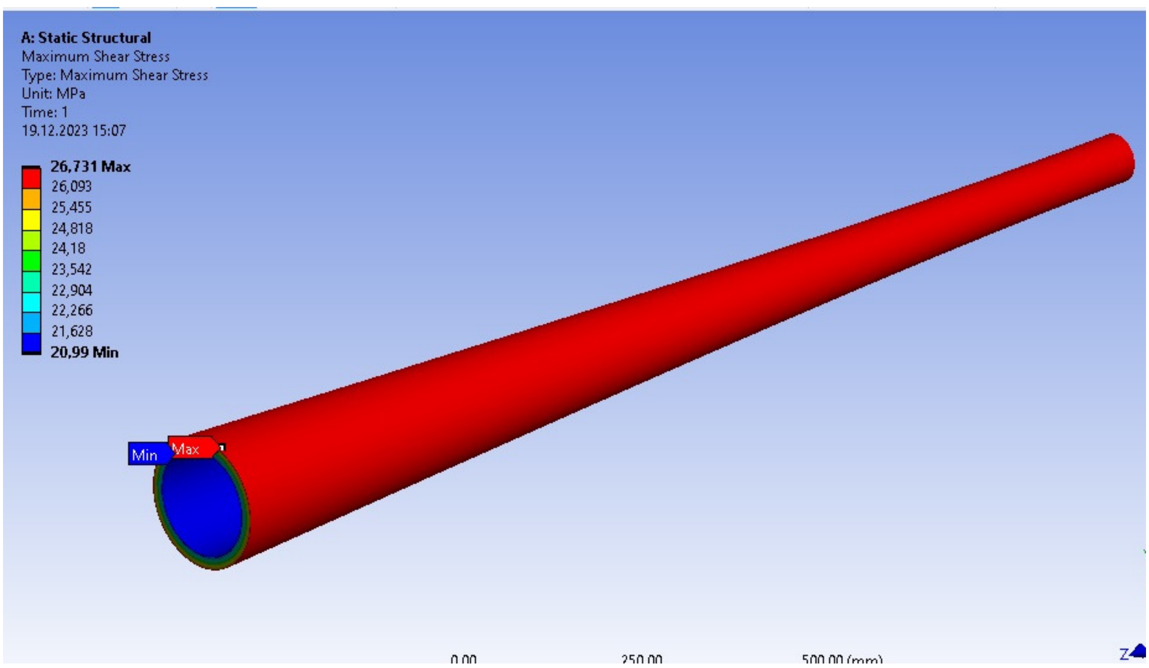
0 Min
Min
Max
Y
X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Maximum Shear Stress
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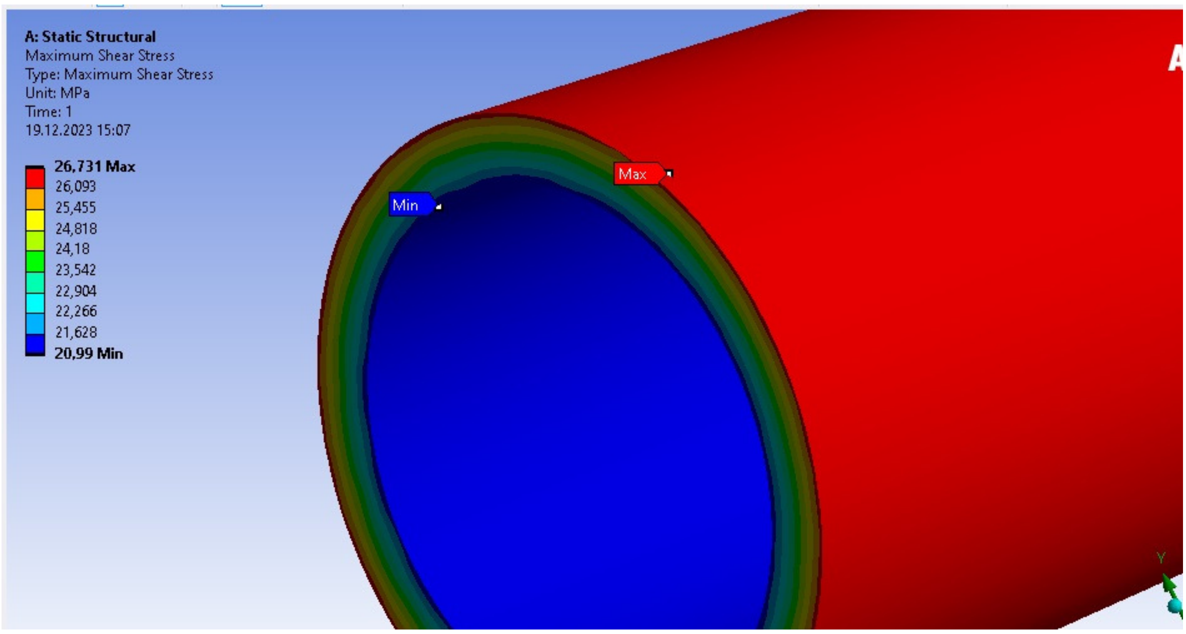
24,818
24,18
23,542
22,904
22,266
21,628
20,99 Min
Min
Max
0,00
250,00
500,00 (mm)
Z




Above figures show in order:

1) Fixed support on one side with torque ($T_{app}$) applied on the other side,
2) Total deformation (It is in tangential direction and gives $\delta_{max}$)
3) Maximum shear stress (created on $T_{app}$ side)
4) Maximum shear stress (created on $T_{app}$ side) in detailed view.

## Optimization (Using ANSYS)

HM-epoxy and EG-polyester materials are added as a new material in ANSYS Engineering Data part with the mechanical properties (E, ν and ρ) given in the question.

The following results are obtained:

<u>HM-epoxy</u>: $\tau_{max}$= 26.731 MPa, φ=0.037 rad.
<u>EG-polyester</u>: $\tau_{max}$= 26.753 MPa, φ=0.112 rad.

Knowing that $\tau_{all}$= 35 MPa and $\varphi_{all}$=0.04 rad, **HM-epoxy** seems to be a good alternative to steel. Comparing with the steel (ρ=8000kg/m$^3$), the weight of the shaft is reduced by 5 times.

## B. Design and optimization of a fixed Crane

For this part, the crane shown in Fig 1 is modelled. {m=10000kg is applied as 98100 N external load (shown in purple) in –y direction. The crane is supported on the lower face of the square base (shown in green colour)}. Length of the lateral extension is fixed as 15 m (radius of crane).

3×3 m2 square base is created as shown in the Fig 1 with smooth transition. Its height is optimized as 34 cm.

Wall thickness is optimized to be 22 cm. (107 MPa of P1 stress (max principle stress) was calculated in the first trial with 35cm, while 44.9 MPa of P1 stress was obtained in the final trial with 22cm.)

Because of high initial P1 stress values, a support (Fig 2) is added into the main column of the crane. Its wall thickness is 4mm, its inner length extends between internal opposite surfaces of the column. This support is only added to the column.

To reduce the amount of material various extursion-cuts are made as shown in figures. The figures below shown details of this design:

**Fig 1)** Fixed support and loading of the crane.

**Fig 2)** Isometric view with dashed hidden lines.

**Fig 3)** Support extrusion, added into the main column (shown with blue colour).

**Fig 4)** Various etrusion-cuts

**Fig 5)** Various etrusion-cuts

**Fig 6)** Various etrusion-cuts

**Fig 7)** Maximum principal stress ($1^{st}$ Principle Stress, P1) distribution a) in isometric view, b) in back isometric view

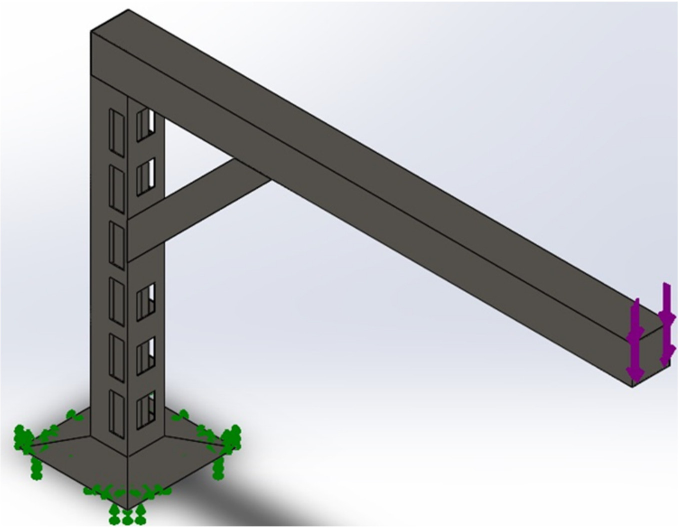


Fig 1

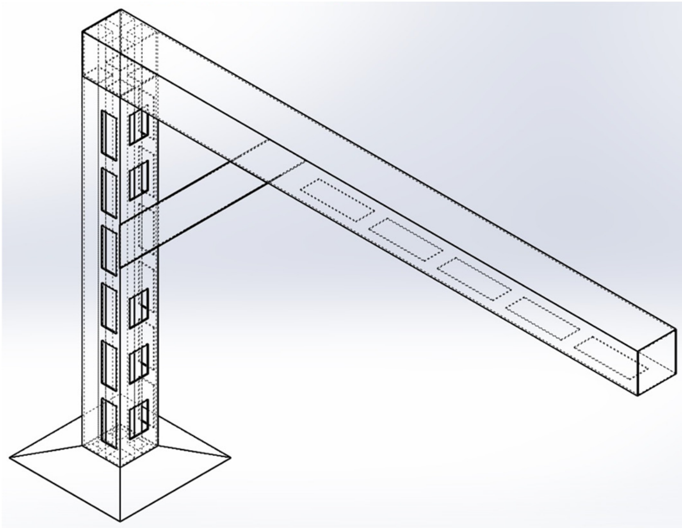


Fig 2

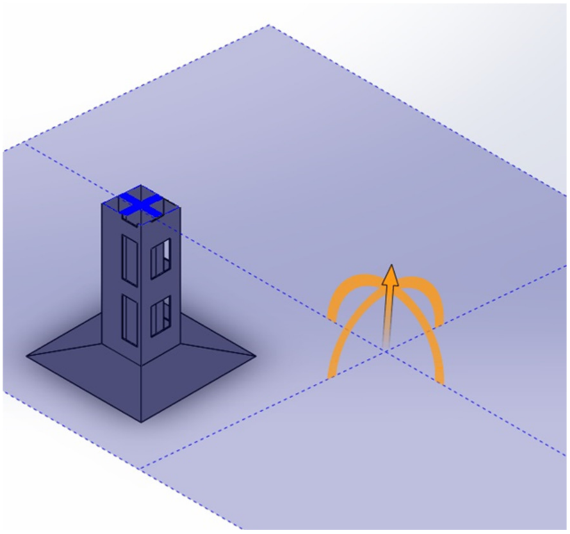


Fig 3

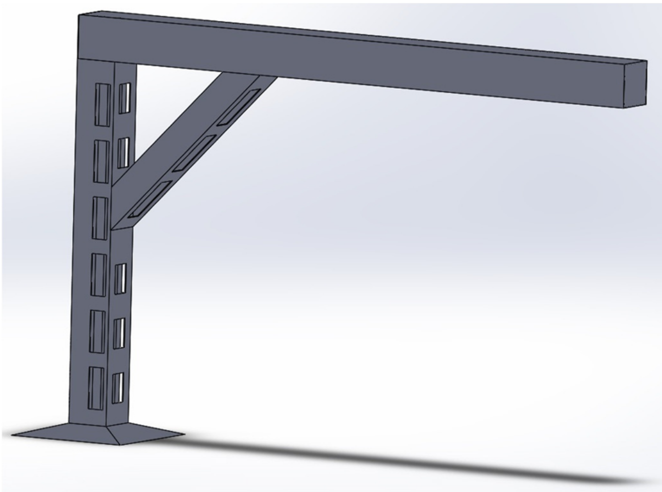


Fig 4

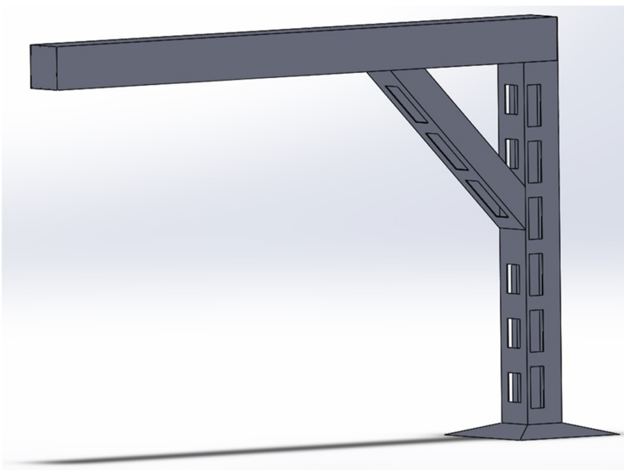


Fig 5

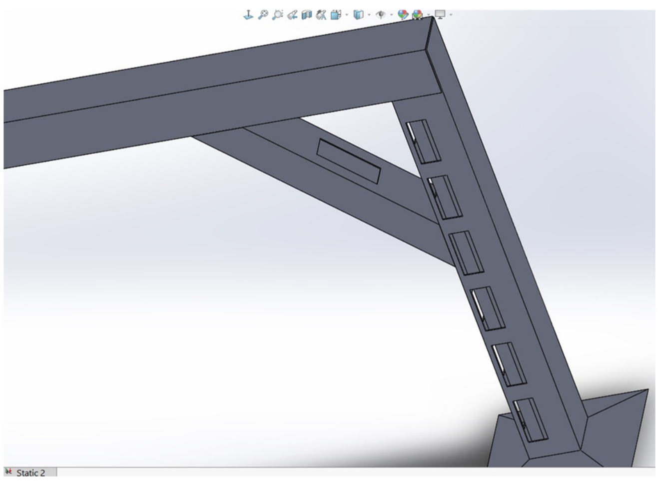


Fig 6

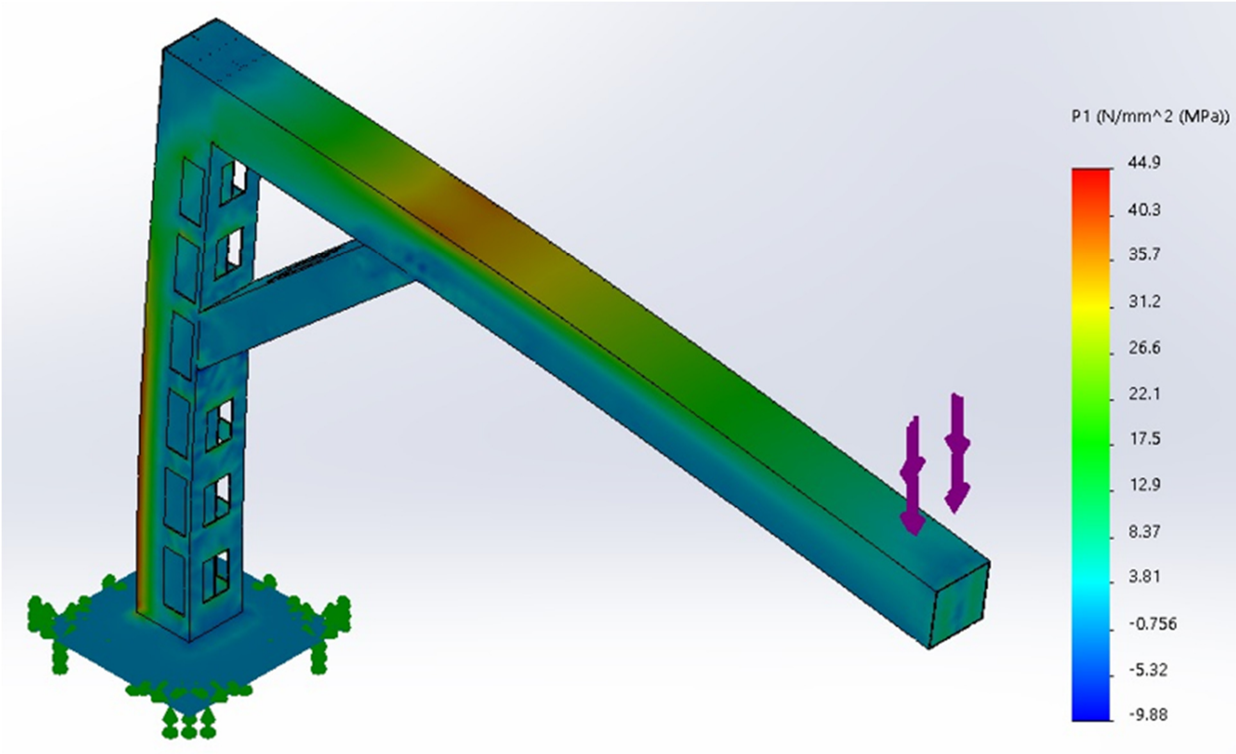

P1 (N/mm^2 (MPa))
44.9
40.3
35.7
31.2
26.6
22.1
17.5
12.9
8.37
3.81
-0.756
-5.32
-9.88

**Fig 7a**

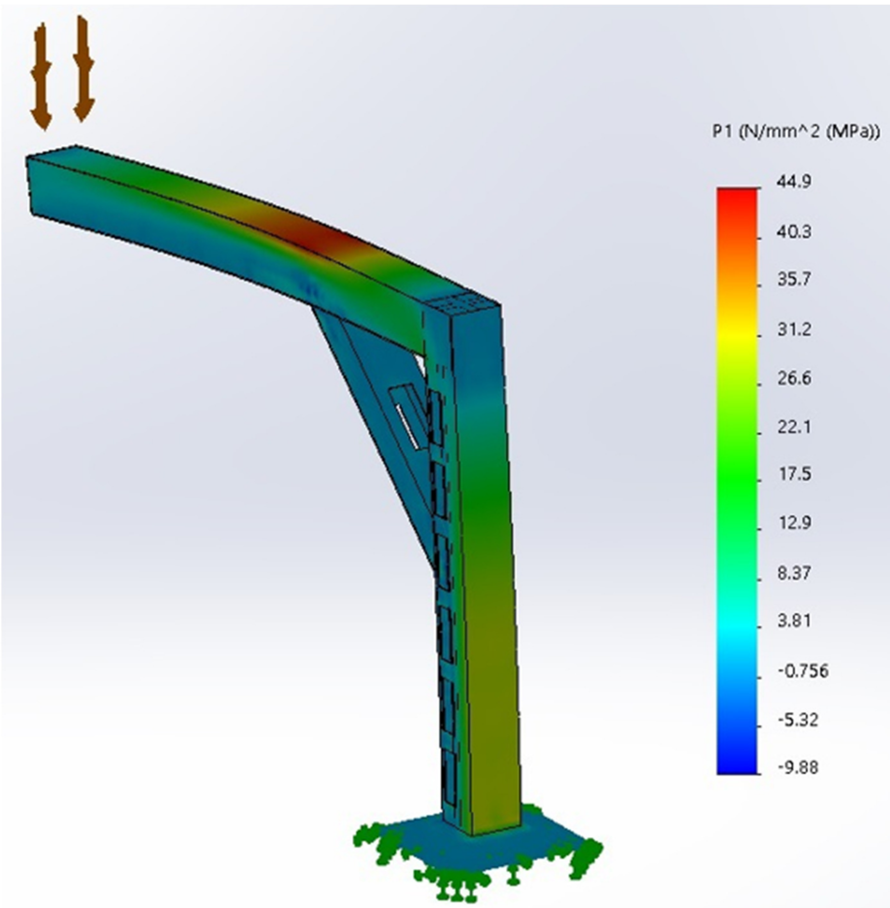

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**Fig 7b**