Collapsing „nebula” dust cloud Gregs solution first version (there are better ones!)

IAAC 2nd pr Task 4 d:

assume the original gas or dust cloud is begun to collapse because of its own mass which is M. Calculate a proper formula for the value of the original Rnebula if after the collapse of the cloud a properly functioning star will form from the mass of the cloud and the given pressure of the reaction.

Assume the following:

the gas is ideal

the cloud is a perfect sphere

the star has to be functioning

the star has to be atleast 1Msun

the gas can be H.

then for every gas particle while collapsing:

$$Ekin < Egrav.pot$$

and

$$Vparticle < Vescape$$

given that the collapse has a radius of r then the gravitational potential that the particle has is:

$$\int_0^{dR} Fg \times dr = \int_0^{dR} \frac{GM}{r^2} \times dr = -\frac{GM}{dR}$$

so we get that:

$$Ekin < -\frac{GM}{dR}$$

lets solve for the velocity terms:

we assume that no outer potetional energy change is made then:

$$vparticle = vkin$$

thus:

$$vkin < vescape = \sqrt{\frac{2GM}{r}}$$

Now if we assume an ideal gas enivoremt then we can assume for Kinetical Theory of Gas that:

$$Ekin = \frac{3}{2} Nk_{bt} = \frac{3}{2} R$$

where R is the ideal gas constant which equals to: R=8,314

and for solving vkin:

$$vkin = \sqrt{3 \times \frac{k_b T}{m}}$$

so eventually get the equations of:

$$\frac{3}{2} R_{gas} < -\frac{GM}{dR}$$

$$\sqrt{\frac{2GM}{r}} > \sqrt{3 \times \frac{k_b T}{m}}$$

Thus solving for terms dR and M that are the original mass and radius of the star we get:

$$dR > -\frac{3R_{gas}}{2GM}$$

and:

$$M > \frac{3kTr}{2Gm}$$

for getting the original term of R we have to find the anti-dervative of it during the collapse it takes time form t=zero and until dt then with integration:

$$dR > -\frac{3R_{gas}}{2GM}$$

then:

$$R > \int_{o}^{dt} -\frac{3R_{gas}}{2GM} \times dR = -\frac{3R_{gas}dt}{2GM}$$

Note for the next part that)according to Wikipedia) the nebula has to be atleast as heavy as the Jeans mass theorem says to collapse and create a living star like our sun. calculating with the theorem says:

$Mj \rightarrow \rho J = 1 \times 10^{-19} g/cm^3$ to be atleast 1Msun thus the given minimum of the mass is:

$$M > \rho J \times Vcloud = \rho J \times \frac{4}{3} \pi dR^3$$

but we also know that:

$$M > \frac{3kTr}{2Gm}$$

thus:

$$\rho J \times \frac{4}{3}\pi dR^3 = \frac{3kTr}{2Gm}$$

and we know that:

$$dR > -\frac{3R_{gas}}{2GM}$$

and we can rewrite dR as:

$$dR > \sqrt[3]{\frac{9kTr}{8\pi Gm\rho J}}$$

thus:

$$-\frac{3R_{gas}}{2GM} = \sqrt[3]{\frac{9kTr}{8\pi Gm\rho J}}$$

thus M is rewritten as:

$$M = -\frac{2Gm}{3R_g} \times \sqrt[3]{\frac{9kTr}{8\pi Gm\rho J}}$$

with algebraic theorems based on power rules as:

$$\sqrt[n]{a} = a^{1/n}$$

and

$$\frac{a^b}{a^c} = a^{b-c}$$

and

$$a^b \times a^c = a^{b+c}$$

we can simplify the term to this form:

$$M = -\frac{\sqrt[3]{\frac{G^2kTr}{3m\pi\rho J}}}{R_g}$$

note that mass cant be negative so we have to take that:

$$M = \left| -\frac{\sqrt[3]{\frac{G^2kTr}{3m\pi\rho J}}}{R_g} \right| = \frac{\sqrt[3]{\frac{G^2kTr}{3m\pi\rho J}}}{R_g}$$

this is also true to the radius:

$$R > \left|-\frac{3R_{gas}dt}{2GM}\right| = \frac{3R_{gas}dt}{2GM}$$

So finally we have to write down the correct term of the R using:

$$R > \frac{3R_{gas}dt}{2GM}$$

and

$$M = \frac{\sqrt[3]{\frac{G^2kTr}{3m\pi\rho J}}}{R_g}$$

thus:

$$R > \frac{3R_{gas}dt}{2G\frac{\sqrt[3]{\frac{G^2kTr}{3m\pi\rho J}}}{R_g}}$$

this chunky term can be simplified with algebra with power rules as and fraction rules as:

$$\frac{a}{\frac{b}{c}} = \frac{a \times c}{b}$$

thus:

$$R > \frac{3{R_{gas}}^2dt}{2G\sqrt[3]{\frac{G^2kTr}{3m\pi\rho J}}}$$

further power rule and fraction rule simplification:

$$R > \frac{{R_{gas}}^2dt \times \sqrt[3]{81\pi m\rho_J}}{2\sqrt[3]{G^5k_bT \times r}}$$

where

Rgas=the ideal gas constant

dt=the time under the collapse happens

m=the mass of the star which has to be bigger or equal then the mass of the Sun

pJ=the Jeans density needed for atleast a 1 Sun mass star to form from an atleast Hydrogen cloud

kb=The Bolztman constant

T=the temperature of space

r=atleast 1 radius of the Sun or bigger

Conclusion: with this term having the information of the newborn star we can calculate the original needed radius of the nebula or gas cloud that has to be gravitationally collapsed to create a star with stable core fusion.

Note that this is only MY APPROACH and much SIMPLIER solutions also can exist because I used a more kinetic approach rather then utilizing the Jeans length theorem which also can give you simplier results. Please feel free to correct any mistakes that my calculations might have, some of the simplifying procces arent written down because it would make the document a lot longer but feel free to ask why is it like that.