

# Adaptive Sliding Mode Attitude and Trajectory Tracking Control of Quadrotor UAV

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**Abstract**—A self-adaptive sliding mode control method based on Randomized Feedforward Neural Network(RFNN) is proposed to address the position and attitude tracking control issues of the rotor mode and flight mode of tiltrotor quadcopter unmanned aerial vehicles after changes in tilt angle. Firstly, the tilting quadcopter dynamics system is divided into fully actuated subsystems and underactuated subsystems. Considering the uncertainty of model parameters and external disturbances in drones, the equivalent controller obtained by sliding mode control method cannot be directly applied to drones. Therefore, a Randomized Feedforward Neural Network(RFNN) is used to estimate the equivalent controller. Then, to ensure the stability of the control system and reduce controller chattering, a new switching controller was adopted. Finally, according to Lyapunov theory, both subsystems can reach the sliding mode surface, and the effectiveness of the method was verified through comparative simulation.

**Keywords**—sliding mode control, RFNN, quadrotor UAV, trajectory and attitude control

## I. INTRODUCTION

Due to its advantages of low cost, convenient use, lightweight and portable, quadcopter has been extensively studied and utilized in all kinds of fields such as firefighting [1], transportation [2], reconnaissance [3] and detection [4] in recent years. One of the key technologies in the design of drone systems is the ability to effectively control the drone's tracking of predetermined trajectories and attitudes. However, drone systems have strong nonlinearity, strong coupling, Mult-variability, and time-varying characteristics, which make them highly susceptible to external environmental influences and have great uncertainty. This is also need to be solved in UAV trajectory and attitude control of the main problems.

The common control algorithms for quadcopter drones currently available include PID algorithm, backstepping algorithm, self-disturbance rejection control algorithm, sliding mode control algorithm, etc. Reference [5] based on the non-singular terminal sliding mode surface, an adaptive fault-tolerant finite-time attitude control scheme is proposed to solve the adaptive attitude finite-time tracking control problem of the quadrotor UAV under the condition of actuator malfunction, input fill to capacity and outside interference. Reference [6] established a dynamic model of quadrotor for unmanned aerial vehicles under unbalanced load conditions using Newton Euler equations. A second-order SMC and ESO are designed using PID sliding mode surface to solve the attitude control problem with external interference. The main objective of reference [7] is to alleviate external disturbances, parameter uncertainties, and actuator failures, and to propose a method for adaptive barrier FTSMC of quadrotor UAV. In

reference [8], model uncertainty and external disturbance are considered, and an adaptive control scheme is adopted to approximate them, and an adaptive fast arrival non-singular terminal SMC method is proposed. The model uncertainty caused by input delay and mass change will affect the stability of quadrotor aircraft and reduce the tracking accuracy of UAV flight control. Literature [9] proposed a solution to the tracking control problem with mass uncertainty and input time delay in the complex operation environment of quadrotor system. Literature [10] Problem of incomplete yaw angle working range in sliding mode control of a class of quadrotor UAV. In reference [11], a new approach is proposed for finite time attitude and altitude tracking with bounded disturbance in unmanned aerial vehicles systems. Using this method, the desired reference points can be tracked in finite time. Reference [12] proposes an adaptive sliding mode fault-tolerant control strategy for actuator faults and model uncertainties in quadrotor unmanned aerial vehicles. However, in the design of sliding mode control, the controller is often designed too idealized, resulting in the actual sliding mode motion state not being able to accurately reach the pre designed sliding mode surface, but crossing back and forth on both sides of the sliding mode surface, leading to chattering phenomenon. Tipping can cause increased energy consumption in control, hardware damage to the system, and other hazards. The methods for suppressing chattering are mostly improving the convergence law of sliding mode or the sliding surface.

Based on the existing results, this paper proposes a sliding mode control method based on RFNN to achieve trajectory and attitude control of unmanned aerial vehicles, and designs relevant simulations to verify the performance of trajectory tracking.

## II. PROBLEM FORMULATION

Firstly, establish the body Cartesian coordinate system with the UAV's barycenter as the coordinate origin  $R_b$ . Secondly, establish the geodetic Cartesian coordinate system  $R_E$ . Finally, the dynamics model of the quadrotor UAV is established by using Euler-Lagrange modeling method. The dynamic equation is:

$$\begin{cases} \ddot{x} = (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) u_1 / m + f_x \\ \ddot{y} = (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) u_1 / m + f_y \\ \ddot{z} = \cos \phi \cos \theta u_1 / m - g + f_z \\ \ddot{\phi} = u_2 / I_x + f_\phi \\ \ddot{\theta} = u_3 / I_y + f_\theta \\ \ddot{\psi} = u_4 / I_z + f_\psi \end{cases} \quad (1)$$

In the formula:  $f_x = -K_1\dot{x}/m + d_1$ ,  $f_y = -K_2\dot{y}/m + d_2$ ,  
 $f_\phi = qr(I_y - I_z)/I_x + I_r q \Omega_r / I_x - K_4 p^2 / I_x + d_4$ ,  
 $f_z = -K_3\dot{z}/m + d_3$ ,  
 $f_\theta = pr(I_z - I_x)/I_y - I_r p \Omega_r / I_y - K_5 q^2 / I_y + d_5$ ,  
 $f_\psi = pq(I_x - I_y)/I_z - K_6 r^2 / I_z + d_6$ ;  $[x, y, z]^T$  is the position vector of the drone's barycenter in the geodetic coordinate system.  $[\phi, \theta, \psi]^T$  represents the Euler angle vector of the aircraft in the geodetic frame of reference.  $[p, q, r]^T$  represents the angular acceleration vector of the drone in the ontological frame.  $m$  is the total mass of the UAV.  $g$  is the gravitational acceleration.  
 $\omega_r = -\omega_1 + \omega_2 - \omega_3 + \omega_4$ ,  $\omega_i$  represents the angular velocity of the  $i$  th rotor,  $i = 1, 2, 3, 4$ .  $I_x, I_y, I_z$  represents the moment of inertia of the aircraft along three coordinate axes.  $J_r$  is the moment of inertia of the aircraft rotor.  $C$  is the proportional coefficient.  $K_i$  ( $i = 1, 2, \dots, 6$ ) represents the resistance coefficient;  $d_i$  ( $i = 1, 2, \dots, 6$ ) is an unmodeled item in the system, mainly composed of model uncertainty and external interference terms.

### III. ADAPTIVE SLIDING MODE CONTROLLER DESIGN

According to the motion form and coupling mechanism of quadrotor UAV, they are divided into two subsystems: fully actuated subsystem  $(z, \psi)$  and underactuated subsystem  $(x, y, \phi, \theta)$ .

#### A. Controller Design for Fully Actuated Subsystem

Consider designing the following sliding mode manifold[13]:

$$s_i = c_i e_i + \dot{e}_i, i = z, \psi \quad (2)$$

In the formula:  $c_z, c_\psi > 0$ , tracking error  $e_z$  and  $e_\psi$  are defined as  $e_z = z_d - z, e_\psi = \psi_d - \psi$ .

Select the following convergence law:

$$\dot{s}_x = -k_i s - \eta_i \text{sig}^{\rho_i}(s_x), i = z, \phi, \theta, \psi \quad (3)$$

In the formula:  $0 < \rho < 1; k_i, \eta_i > 0$ .

Combining equations (1), (2), and (3), the integrated controller  $u_1, u_4$  can be designed accordingly as:

$$\begin{cases} u_1 = \frac{m}{\cos \phi \cos \theta} (-\hat{f}_z + g + \ddot{z}_d + c_z \dot{e}_z + \eta_z \text{sig}^{\rho_z}(s_z) + k_z s_z) \\ u_4 = \frac{1}{I_z} (-\hat{f}_\psi + \ddot{\psi}_d + c_\psi \dot{e}_\psi + \eta_\psi \text{sig}^{\rho_\psi}(s_\psi) + k_\psi s_\psi) \end{cases} \quad (4)$$

In the formula: the total interference term  $f_i, i = z, \psi$  is unknown,  $\hat{f}_i$  is the estimated value of  $f_i$ .

Using RFNN [14] to approximate the unknown coefficient  $c_i$  and the total interference term  $f_i, i = z, \psi$ , the following adaptive law is designed:

$$\dot{\hat{\beta}}_{f_i} = -s_i H_i / \gamma_1 \quad (5)$$

In the formula:  $\gamma_i > 0$ ,  $\beta_{f_i}$  are Output weights for the neural network,  $H_i$  represents the activation function. Above all is the numerical estimate derivative over time.

Select the following Lyapunov function:

$$V_i = (s_i^2 + \gamma_1 \tilde{\beta}_{f_i}^T \tilde{\beta}_{f_i}) / 2, i = z, \psi \quad (6)$$

Substitute the adaptive law into the derivative of equation (6), it can be concluded that:

$$\dot{V}_i = -k_i s_i^2 - \eta_i |s_i|^{\rho_i+1} \leq 0 \quad (7)$$

Due to  $V_i \geq 0$  and  $\dot{V}_i \leq 0$ , it can be inferred from the Barbalat lemma that the system state tracking error will converge to 0.

#### B. Controller Design for Underactuated Actuated Subsystem

Consider designing the following sliding mode manifold:

$$\begin{cases} s_\phi = c_1 \dot{e}_\phi + c_2 e_\phi + \dot{e}_\phi + c_3 e_\phi \\ s_\theta = c_4 \dot{e}_\theta + c_5 e_\theta + \dot{e}_\theta + c_6 e_\theta \end{cases} \quad (8)$$

In the formula: racking error  $e_x, e_y, e_\phi$  and  $e_\theta$  are defined as  $e_x = x_d - x, e_y = y_d - y, e_\phi = \phi_d - \phi, e_\theta = \theta_d - \theta$ .

Select the same convergence law as the fully driven subsystem mentioned above here. Combining equations (1), (8), and (4), the integrated controller  $u_2, u_3$  can be designed accordingly as:

$$\begin{cases} u_2 = I_x (c_1 \ddot{e}_y + c_2 \dot{e}_y + c_3 \ddot{e}_\phi + \ddot{\phi}_d - f_\phi + k_\phi s_\phi + \eta_\phi \text{sig}^{\rho_\phi}(s_\phi)) \\ u_3 = I_y (c_4 \ddot{e}_x + c_5 \dot{e}_x + c_6 \ddot{e}_\theta + \ddot{\theta}_d - f_\theta + k_\theta s_\theta + \eta_\theta \text{sig}^{\rho_\theta}(s_\theta)) \end{cases} \quad (9)$$

In the formula: the total interference term  $f_i, i = \phi, \theta$  is unknown,  $\hat{f}_i$  is the estimated value of  $f_i$ .

Using RFNN [14] to approximate the total interference term  $f_i, i = \phi, \theta$ , the following adaptive law is designed:

$$\dot{\hat{\beta}}_\phi = -s_\phi H_\phi / \gamma_2, \quad \dot{\hat{\beta}}_\theta = -s_\theta H_\theta / \gamma_3 \quad (10)$$

In the formula:  $\gamma_2, \gamma_3 > 0$ ,  $\beta_i, i = \phi, \theta$  are Output weights for the neural network,  $H_i$  is the activation function.

The above are all estimated derivatives of the above values over time.

Considering that the tracking errors  $e_i, i = y, \phi$  and  $e_i, i = x, \theta$  convergence of the underactuated subsystem are the same, taking  $e_i, i = y, \phi$  as an example.

Select the following Lyapunov function:

$$V_\phi = (s_\phi^2 + \gamma_2 \tilde{\beta}_\phi^T \tilde{\beta}_\phi) / 2 \quad (11)$$

Substitute the adaptive law into the derivative of equation (11), it can be concluded that:

$$\dot{V}_\phi = -k_\phi s_\phi^2 - \eta_\phi |s_\phi|^{\rho_\phi+1} \leq 0 \quad (12)$$

For  $i = \phi, \theta$ , due to  $V_i \geq 0$  and  $\dot{V}_i \leq 0$ , it can be inferred from the Barbalat lemma that the system state tracking error will converge to 0.

#### IV. SIMULATION

This section uses equation (1) as the simulation dynamic model for quadcopter drones, and conducts flight control simulation experiments using MATLAB/Simulink platform.

Parameters of quadcopter model:  $g = 9.8m/s^2$ ,  $m = 1.1kg$  is mass of quadcopter,  $l = 0.5m$  is the half length of pendulum,  $d_i$  is the disturbance term. The time-varying expected position trajectories are  $x_d = \sin(0.1t)$  and  $y_d = \cos(0.1 \times t)$ , the time-varying expected altitude trajectory is  $z_d = 0.5t$ , and the time-varying expected yaw angle is  $\psi_d = 0.1 \times \sin(0.5 \times t)$ . The initial values for roll angle and pitch angle are set to  $[\phi_0, \theta_0] = [0, 0]$ . In addition, in order to fully verify the robustness of the proposed method, time-varying trigonometric functions were used to simulate the unmodeled and external interference terms of the system during the simulation experiment, specifically:

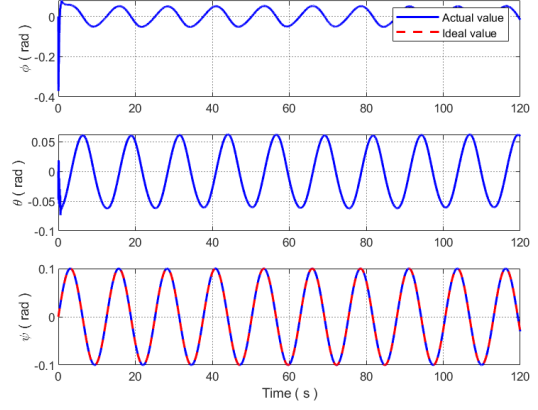
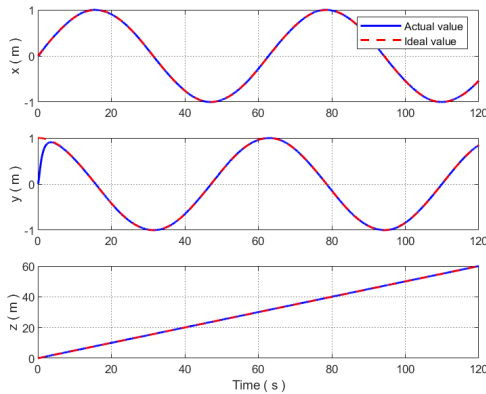
$$\begin{aligned} d_1 &= 0.6 \cos(0.5t) & d_2 &= 0.5 \sin(0.5t) \\ d_3 &= 0.5 \sin(0.5t) \cos(0.5t) & d_4 &= 0.5 \sin(0.5t) \\ d_5 &= 0.8 \cos(0.5t) & d_6 &= 0.7 \sin(0.5t) \cos(0.5t). \end{aligned}$$


Fig.1. Trajectory and attitude tracking.

Figure 1 on the left shows the tracking trajectory at position (x, y, z); The right side of Figure 1 shows the roll angle  $\phi$ , pitch angle  $\theta$  and Yaw angle  $\psi$ . From Figure 1, it can be seen that the control method proposed in this paper can ensure that the quadcopter drone can achieve the preset expected trajectory tracking flight well under model uncertainty and external disturbances, and the flight control system has strong robustness and adaptability.

#### V. CONCLUSION

Adaptive sliding mode control method based on RFNN is proposed to solve the control problem of quadrotor UAV under model uncertainty and outside interference. Quadrotor UAV is implemented under the model uncertainty and outside interference fast track the location of the gesture, improves the flight control system's robustness and adaptive ability.

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