SCHOOL OF MATHEMATICAL AND COMPUTER SCIENCES

Computer Science

F29FA1

Foundations I

Class Test 2015/16

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13 October 2015
Duration: 0.5 Hours

Answer ALL questions

1. Let $M = ((\lambda y'.((z(\lambda x.(yx)))y'))z)$.

(a) Remove as many parenthesis as possible from the term M given above without changing its meaning. (1)

(b) Give the subterms of M. (1)

(c) Give the result of $M[z := y']$. (1)

(d) η-reduce M to an η-normal form M_1 and then give the β-normal form of M_1. (1)

(e) β-reduce M to a β-normal form M_2 and then give the η-normal form of M_2. (1)

(f) Give the $\beta\eta$-normal form of M. (1)

2. Suppose that A, B, and C are three lambda terms and the only things you know about A, B and C are the following three facts:
1) $A \twoheadrightarrow_\beta B$, 2) $A \twoheadrightarrow_\beta C$, 3) Both B and C are in β-normal forms.
For each of the following statements state whether it is false, true or possibly true or possibly false or both. In each case, justify your answer.

(a) $B \twoheadrightarrow_\beta A$. (1)

(b) $A =_\alpha B$. (1)

(c) $B \rightarrow_\beta C$. (1)

(d) $B =_\beta C$. (1)

(e) A is not weakly β-normalising. (1)

3. Let $A \equiv \lambda x.x\Omega\mathbf{false}$ and $B \equiv \lambda xy.y\Omega$ where $\Omega \equiv (\lambda x.xx)(\lambda x.xx)$ and $\mathbf{false} \equiv \lambda xy.y$.

(a) State whether A is weakly β-normalising. Justify your answer. (1)

(b) Define strong normalisation and show that A is not strongly β-normalising. (1)

(c) Is B weakly β-normalising? Justify your answer. (1)

(d) Is AB weakly β-normalising? Justify your answer. (1)

END OF PAPER