Logic and Proof Revision Manual

In this revision manual, you will be asked to refer to Prof. Lawson's material that is posted online.

Boolean Algebra

You were given a formal definition for this in your course notes. Expressed informally, our special two-element Boolean algebra can be thought of as satisfying the following:

1- There are only two values to consider: 0 and 1
2- There are two binary operation + and $\cdot$ In this algebra 1+1≠2! That is because we do not have a symbol 2, the only symbols we can use are 0 and 1. In the notes the definitions for the operations are provided, so fill out the tables:

+	1	0
1		
0		

$\cdot$	1	0
1		
0		

3- There is one unary operator referred to as the not-operator so that
$\bar{0} = 1$ and $\bar{1} = 0$

We can then have variables such as x, y, z and $\bar{x}$, $\bar{y}$ and $\bar{z}$ which gives rise to 10 axioms. You do not have to memorize these axioms, as these will be given to you in the exam.

B1: $(x + y) + z = x + (y + z)$
B2: $x + y = y + x$
B3: $x + 0 = x$
B4: $(x \cdot y) \cdot z = x \cdot (y \cdot z)$
B5: $x \cdot y = y \cdot x$
B6: $x \cdot 1 = x$
B7: $x \cdot (y + z) = x \cdot y + x \cdot z$
B8: $x + (y \cdot z) = (x + y) \cdot (x + z)$
B9: $x + \bar{x} = 1$
B10: $x \cdot \bar{x} = 0$

There are some more equalities that follow directly, and which you may use in your work. But remember, you may be asked to prove any one of them. The proof for the first equality is provided further down.

$x + 1 = 1$
$x \cdot 1 \ = 1$
$x + x = x$
$x \cdot x \ = x$

Most of the axioms look familiar from other algebras that you may be familiar with, except perhaps axiom B8. Study it carefully and what it means.

Also note that you may need to apply the axioms from right to left, so that when you simplify the expressions
$x \cdot y + x \cdot z$, you may apply axiom B7 to get the equivalent $x \cdot (y + z)$
or
$(x + y) \cdot (x + z)$, you may apply axiom B8 to get the equivalent $x + (y \cdot z)$
Note that sometimes the · symbol may be left out so, $x \cdot y = xy$
State which one of the axioms was applied to get from the left-hand side to the right-hand side, remember to think of the missing · symbol:
1)
$ab + b(b + \bar{c}) + \bar{b}c = ab + bb + b\bar{c} + \bar{b}c$

2)
$bb = bb + 0$
$bb + 0 = bb + b\bar{b}$
$bb + b\bar{b} = b(b + \bar{b})$
$b(b + \bar{b}) = b \cdot 1$
$b \cdot 1 = b$

Similarly, you should show that
$b + b = b$

3) To show that $x + 1 = 1$, you use the following steps – make sure you justify each one :

$x + 1 = x + x + \bar{x}$
$x + x + \bar{x} = x + \bar{x}$
$x + \bar{x} = 1$

You should now solve some further examples using Boolean Algebra.