Logic and Proof Revision Manual

In this revision manual, you will be asked to refer to Prof. Lawson's material that is posted online.

http://www.ma.hw.ac.uk/~markl/teaching/LOGIC/

Propositional Logic – Chapter 4

1- Truth Tables

In this world of logic, values can be either true (T) or false (F). The variables we use are usually lower case p, q, r, s. There are connectors which operate on the variables. The two incomplete tables below show all the operators we covered.

p	q	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \rightarrow q$	$p \leftrightarrow q$	$p \downarrow q$
F	F						
F	T						
T	F						
T	T						

p	$\neg p$
F	
T	

a. Write down the names of each one of the connectors (or operators) that are shown in the tables, saying which ones are unary and which ones are binary.
b. Copy the tables in your own handwriting, using a ruler, and fill in the missing values in both tables. Unless you know these tables by heart, you will not be able to make any progress.
c. Write down the truth table for $p \rightarrow q$ and also for $\neg p \vee q$. Why can we say that these two expressions are equivalent? Remember this very important equivalence!
d. Write down the truth table for $p \wedge (q \vee r)$ and also for $(p \wedge q) \vee r$. This example shows the importance of the order in which we proceed to evaluate.
e. Write down the truth table for $\neg p \rightarrow (q \vee r)$
f. Write down the truth table for $(p \wedge q) \rightarrow q$ and explain why this is a tautology
g. Write down the truth table for $(p \wedge \neg p) \vee (q \wedge \neg q)$ and explain why this is a contradiction.

Here you can use an online truth table generator
http://turner.faculty.swau.edu/mathematics/materialslibrary/truth/

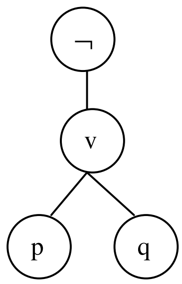
2- Propositions and well formed formulas

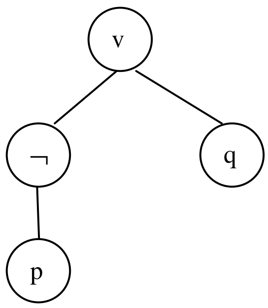
You must also understand the terminology: what is a proposition (it is not a sentence!), and what is a well-formed formula WFF and translating English sentences into WFF and vice versa. What is a conjunction? What is a disjunction? You can use this additional reference for practice.

http://www.bu.edu/linguistics/UG/course/lx502/_docs/lx502-propositional%20logic.pdf

3- Parse Trees

You should know how to generate the parse tree for a given well-formed formula, understanding the difference between different representations. Here are two simple examples: which wff is being represented?





4- Equivalences

There are 8 lemmas that you should know to help you prove equivalences of wffs. You also have

$T \vee p = T$	$\neg p \wedge p = F$
$T \wedge p = p$	$\neg p \vee p = T$

Using these, you have an alternative to truth tables, which is perhaps the easiest way of proving equivalence.

For example, use a truth table to show that

$\neg(p \vee (\neg p \wedge q))$ and $(\neg p \wedge \neg q)$ are logically equivalent.

Starting with the left hand side, and applying the lemmas

$\neg(p \vee (\neg p \wedge q))$ [DeMorgan's Law]
$\Leftrightarrow \neg p \wedge \neg(\neg p \wedge q)$ [DeMorgan's Law]
$\Leftrightarrow \neg p \wedge (p \vee \neg q)$ [Distributive Law]
$\Leftrightarrow (\neg p \wedge p) \vee (\neg p \wedge \neg q)$ [why?]
$\Leftrightarrow F \vee (\neg p \wedge \neg q)$ [why?]
$\Leftrightarrow \neg p \wedge \neg q$

To show $(p \wedge q) \to (p \vee q)$ is a **tautology**

$\Leftrightarrow \neg(p \wedge q) \vee (p \vee q)$ [why?]
$\Leftrightarrow (\neg p \vee \neg q) \vee (p \vee q)$ [DeMorgan's Law]

$\Leftrightarrow \; (\neg p \vee p) \vee (\neg\, q \vee q)$ [Associative and Commutative Laws]
$\Leftrightarrow \; T \vee T$
$\Leftrightarrow T$

5- Different forms of logic expressions

Some terms first: $p \wedge q$ is called a 'conjunction',
$p \vee q$ is called a 'disjunction'

And the very important equivalences
$p \rightarrow q \Leftrightarrow \neg\, p \vee q$
$p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

An expression can be in **conjunctive normal** form CNF $(p \vee \neg\, q) \wedge (p \vee q)$
$(p \vee \neg\, q \vee s) \wedge (\neg\, \mathrm{p} \vee \mathrm{q} \vee s)$

An expression can be in **disjunctive normal** form DNF $(p \wedge \neg\, q) \vee (p \wedge q)$
$(\neg p \wedge \neg\, q \wedge \mathrm{r}) \vee (p \wedge q \wedge \neg\, \mathrm{r})$

You can use the equivalences to change over from one form to the other. Another easy way is to do this using a truth table.

To get the CNF of the exclusive or expression, start with the truth table.

p	q	p ⊕ q
T	T	F
T	***F***	***T****
F	***T***	***T****
F	F	F

You then select the rows in which that expression evaluates to True, in this case, there are 2 rows, and you build the CNF expression $(\boldsymbol{p} \wedge \neg\, \boldsymbol{q}) \vee (\neg \boldsymbol{p} \wedge \boldsymbol{q})$

Another example is the expression $(p \vee q) \rightarrow \neg\, r$ and again you start by building the truth table.

p	q	r	$(p \vee q) \rightarrow \neg\, r$
T	**T**	**T**	F
T	***T***	***F***	***T****
T	F	T	F
T	***F***	***F***	***T****
F	T	T	F
F	***T***	***F***	***T****
F	***F***	***T***	***T****
F	***F***	***F***	***T****

Here you have 5 rows that evaluate to true, so your CNF expression will have 5 brackets accordingly

$(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (p \wedge q \wedge r) \vee (p \wedge q \wedge r)$

Now go back to your notes and see how you can get the DNF for an expression! There, you look at the rows that evaluate to F. You can also try this link:
http://www.ocf.berkeley.edu/~fricke/projects/quinto/dnf.html

6- Truth Trees

(Do not confuse truth tables, parse trees and truth trees.)
You were given plenty of materials on this topic in class, and were told that you cannot solve truth tree questions without knowing the following decomposition:

$\neg\neg p$ p	$p \wedge q$ p q	$\neg(p \wedge q)$ $\neg p \qquad \neg q$
$p \vee q$ $p \qquad q$	$p \rightarrow q$ $\neg p \qquad q$	$\neg(p \rightarrow q)$ p $\neg q$
$\neg(p \vee q)$ $\neg p$ $\neg q$	$p \leftrightarrow q$ $p \qquad \neg p$ $q \qquad \neg q$	$\neg(p \leftrightarrow q)$ $p \qquad \neg p$ $\neg q \qquad q$

Using truth trees, you can solve different kinds of questions.

a- You determine if a wff is a tautology (or contingent)
b- You can determine if a wff is a contradiction (or contingent)
c- You can determine if an argument is valid

Your notes included one link and here are two further links to sentential logic truth tree solvers online so you may check your answers:

http://gablem.com/logic
http://www.umsu.de/logik/trees/