Questions

1. For each term A of the terms below, give its translation $\omega(A)$ from M to Λ showing all the steps, their number and underlining all the parts you are working on, just like we did in the above example:

   (a) $(\lambda xz.xz)$. (1)

   $\underline{\omega(\lambda xz.xz)} =^{0}$

   $\underline{\omega(\lambda xz.xz)} =^{3}$

   $\lambda\underline{\omega_{[X]}(\lambda z.xz)} =^{3}$

   $\lambda\lambda\underline{\omega_{[Z,X]}(xz)} =^{2}$

   $\lambda\lambda\underline{\omega_{[Z,X]}(x)}\omega_{[Z,X]}(z) =^{1}$

   $\lambda\lambda 2\underline{\omega_{[Z,X]}(z)} =^{1}$

   $\lambda\lambda 21$

   (b) $(\lambda xy.xy)$. (1)

   $\underline{\omega(\lambda xy.xy)} =^{0}$

   $\underline{\omega(\lambda xy.xy)} =^{3}$

   $\lambda\underline{\omega_{[X]}(\lambda y.xy)} =^{3}$

   $\lambda\lambda\underline{\omega_{[Y,X]}(xy)} =^{2}$

   $\lambda\lambda\underline{\omega_{[Y,X]}(x)}\omega_{[Y,X]}(y) =^{1}$

   $\lambda\lambda 2\underline{\omega_{[Y,X]}(y)} =^{1}$

   $\lambda\lambda 21$

(c) $xz(\lambda xy.xy)$. (1)

$\omega(xz(\lambda xy.xy)) =^{0}$

$\omega_{[x,y,z]}(xz(\lambda xy.xy)) =^{3}$

$\lambda\omega_{[x,x,y,z]}(xz(\lambda y.xy)) =^{3}$

$\lambda\lambda\omega_{[y,x,x,y,z]}(xz(xy)) =^{2}$

$\lambda\lambda\omega_{[y,x,x,y,z]}(x)\ \omega_{[y,x,x,y,z]}(x\ z(xy) =^{2}$

$\lambda\lambda 2\omega_{[y,x,x,y,z]}(z)\ \omega_{[y,x,x,y,z]}(xy) =^{2}$

$\lambda\lambda 25\omega_{[y,x,x,y,z]}(x)\ \omega_{[y,x,x,y,z]}(y) =^{1}$

$\lambda\lambda 252\omega_{[y,x,x,y,z]}(y) =^{1}$

$\lambda\lambda 2521$

(d) $(\lambda xy.xy)xz$. (1)

$\omega((\lambda xy.xy)xz) =^{0}$

$\omega_{[x,y,z]}((\lambda xy.xy)xz) =^{3}$

$\lambda\omega_{[x,x,y,z]}((\lambda y.xy)xz) =^{3}$

$\lambda\lambda\omega_{[y,x,x,y,z]}((xy)xz) =^{2}$

$\lambda\lambda\omega_{[y,x,x,y,z]}(xy)\omega_{[y,x,x,y,z]}(xz) =^{2}$

$\lambda\lambda\omega_{[y,x,x,y,z]}(x)\omega_{[y,x,x,y,z]}(y)\omega_{[y,x,x,y,z]}(xz) =^{2}$

$\lambda\lambda 2\omega_{[y,x,x,y,z]}(y)\omega_{[y,x,x,y,z]}(xz) =^{2}$

$\lambda\lambda 21\omega_{[y,x,x,y,z]}(x)\omega_{[y,x,x,y,z]}(z) =^{1}$

$\lambda\lambda 212\omega_{[y,x,x,y,z]}(z) =^{1}$

$\lambda\lambda 2125$

2. Give a translation function f from M to M that will translate terms in M to terms in M so for example:
   $f((\lambda x.x)y) = y\ [x]x$
   $f((\lambda x.(\lambda y.xy)z)(\lambda z.z)) = [z]z\ [x]\ z\ [y]\ y\ x.$ (1)
   $f(v) =$ v
   $f(\lambda v.A) =$ [v]A$^{|}$
   $f(AB) =$ <B$^{|}$> A$^{|}$

3. Use your translation function f of Question 2, to translate all the terms in Question 1 above into terms of **M** . That is, give $f(\lambda xz.xz)$ and $f(\lambda xy.xy)$ and $f(xz(\lambda xy.xy))$ and $f((\lambda xy.xy)xz)$ showing all the steps. (2)

   - $f(\lambda xz.xz) = .. = .. = ...$
   - $f(\lambda xy.xy) = .. = .. = ...$
   - $f(xz(\lambda xy.xy)) = .. = .. = ...$
   - $f((\lambda xy.xy)xz) = .. = .. = ...$

4. For each of BEXP and IEXP write a printing function printBEXP (respectively printIEXP) that prints its elements nicely just like we wrote printLEXP which prints nicely the elements of LEXP. (2)

```
(*Prints a term in item lambda calculus*)
fun printIEXP (IID v) =
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fun printIEXP (IID v) =
    print v
  | printIEXP (ILAM (v,e)) =
    (print "[";
     print v;
     print "]";
     printIEXP e)
  | printIEXP (IAPP(e1,e2)) =
    (print "<";
     printIEXP e2;
     print ">";
     printIEXP e1);
```

```
(*Prints a term in classical lambda calculus with de Bruijn indices*)
fun printBEXP (BID n) =
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```
fun printBEXP (BID n) =
    print (Int .toString (n))
  | printBEXP (BLAM (e)) =
    (print "(\\";
     printBEXP e;
     print ")")
  | printBEXP (BAPP(e1,e2)) =
    (print "(";
     printBEXP e1;
     print " ";
     printBEXP e2;
     print ")");
```

5. For each term below, write it in LEXP and print it using printLEXP, write its translation by f into IEXP and print it using printIEXP, and its translation by ω into BEXP and print it using printBEXP,

   (a) $(\lambda xz.xz)$. (1.5)

```
printLEXP at101 gives
   (\x.(\z.(x z)))val it = () : unit

printIEXP aIt101 gives
  [x][z]<z>xval it = () : unit

printBEXP aBt101 gives
   (\(\(2 1)))val it = () : unit
```

   (b) $(\lambda xy.xy)$. (1.5)

```
printLEXP bt101 gives
   (\x.(\y.(x y)))val it = () : unit

printIEXP bIt101 gives
   [x][y]<x>yval it = () : unit

printBEXP bBt101 gives
   (\(\(2 1)))val it = () : unit
```

(c) $xz(\lambda xy.xy)$. (1.5)

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printLEXP ct101 gives
   ((x z) (\x.(\y.(x y))))val it = () : unit
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(d) $(\lambda xy.xy)xz$. (1.5)

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