



SCHOOL OF MATHEMATICAL AND COMPUTER SCIENCES

Computer Science

F29FA1

Foundations I

Class Test 2016/17

11 October 2016
Duration: 0.5 Hour

Answer ALL questions

1. Let $A \equiv x(\lambda x.x)((\lambda xy.xy)(\lambda z.zz)y)$.

- (a) Insert as many parenthesis as possible in A without changing its meaning. (1)
- (b) Give the subterms of A , each subterm on a separate line. (2)
- (c) Is A β -normalising? If yes, β -reduce A until there are no β -redexes left, showing all the β -reduction steps. If not, explain why not. (1)
- (d) Give the $\beta\eta$ -normal form of A if it exists, otherwise, say why it does not exist. (1)
- (e) Give the term $A[x := (\lambda x.xx)(\lambda x.xx)]$. (1)
- (f) Is $A[x := (\lambda x.xx)(\lambda x.xx)]$ β -normalising? If yes, give the β -normal form showing all the reduction steps you used to reach it. If not, give a detailed proof why it is not. (2)
- (g) Give the term $A[x := \lambda x.x]$. (1)
- (h) Is $A[x := \lambda x.x]$ β -normalising? If yes, give the β -normal form showing all the reduction steps you used to reach it. If not, give a detailed proof why it is not. (2)
- (i) Give three terms A , B and C such that $B \rightarrow_\beta C$ and $A[x := B] \rightarrow_\beta^3 A[x := C]$ (i.e., B β -reduces to C in one step whereas $A[x := B]$ β -reduces to $A[x := C]$ in three steps. (2)
- (j) For each of the reduction paths below, state whether it is standard or not and also whether it follows the leftmost β -reduction strategy or not. In each case, justify your answer. (2)
 - 1. $(\lambda z.(\lambda x.(\lambda y.xy)z)(\lambda z.z))x \rightarrow_\beta (\lambda z.(\lambda y.(\lambda z.z)y)z)x \rightarrow_\beta (\lambda z.(\lambda z.z)z)x \rightarrow_\beta (\lambda z.z)x$.
 - 2. $(\lambda z.(\lambda x.(\lambda y.xy)z)(\lambda z.z))x \rightarrow_\beta (\lambda z.(\lambda x.xz)(\lambda z.z))x \rightarrow_\beta (\lambda z.(\lambda z.z)z)x \rightarrow_\beta (\lambda z.z)x$.

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