HERIOT WATT UNIVERSITY

SCHOOL OF MATHEMATICAL AND COMPUTER SCIENCES

Department of Computer Science

F29FA

FOUNDATIONS I

MOCK TEST — 2020/21

Duration: 0.5 Hour

ANSWER ALL QUESTIONS

1. Let $M \equiv (\lambda xyz.xz(yz))(\lambda xy.x)((\lambda z.zz)(\lambda z.zz))(\lambda x.x)x$.
$K \equiv \lambda xy.x$.
$S \equiv \lambda xyz.xz(yz)$.
$I \equiv \lambda x.x$.
$B \equiv \lambda xy.y(xxy)$.
$C \equiv BB$.
$\Omega \equiv (\lambda z.zz)(\lambda z.zz)$.
$F \equiv \lambda x.yzx$.

(a) Give $F[y := xI]$ showing all the substitution steps. (2.5)

(b) Give the meaning of the following terms:

- K. (0.5)
- S. (1)

(c) Insert the full parenthesis in $S(KI)$. Note here that you should write S, K and I in full. (1.5)

(d) Write M using the *minimum* number of symbols you need from K, S, I, B, C, Ω and x (you may not need all these symbols and your answer should have the minimum number of symbols needed, also you cannot use anything else in M except these symbols; for example C is written using only the B symbol). (1)

(e) Which of the terms Ω, SK and $K\Omega$ is a subterm of M. Explain why. (1.5)

(f) Give all the subterms of SK. (1.5)

(g) Is M strongly β-normalising? Prove your answer always underlying the redex you are working on. (1.5)

(h) Is M β-normalising? If yes, β-reduce M until there are no β-redexes left, showing all the β-reduction steps, underlining at each stage the redex you are contracting, and always keeping the term as compact as possible. If the term is not β-normalising, give a detailed proof why it is not. (2)

(i) Does M have a β-normal form? If yes, give the β-normal form. If not, say why not. (0.5)

(j) Show that for any term A we have $CA \twoheadrightarrow_\beta A(CA)$. (1.5)

END OF PAPER