SCHOOL OF MATHEMATICAL AND COMPUTER SCIENCES

Computer Science

F29FA1

Foundations I

Class Test 2014/15

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13 October 2014
Duration: 0.5 Hour

Answer ALL questions

1. **(a)** Remove as many parenthesis as possible from the following expression without changing its meaning: $(\lambda x.(\lambda y.(\lambda z.((((((xy)z)(\lambda x.x))(\lambda x.((xz)(yz))))(\lambda x.(((xx)y)y)))))))$. (1)

(b) Give the subterms of $\lambda xyz.xyz(\lambda x.x)$. (1)

(c) Find λ-terms A, B and C such that: A, B and C are all in β-normal form, BC does not have a β-normal form and $A(BC)$ has a β-normal form. (2)

(d) If for all A, B, we have $A =_\beta B$, would the lambda calculus be allowed to be a model of computation? (1)

(e) Do you know of a computable function which cannot be represented in the λ-calculus? If so give it. Otherwise say why it is not the case. (1)

(f) Let $K = \lambda xy.x$. Show that $KAB =_\beta A$.
Hint: you can assume that $y \notin FV(A)$ and hence $A[y := B] \equiv A$. (2)

(g) Assume $v \notin FV(A)$ and $v \notin FV(B)$. Show that if $Av =_{\beta\eta} Bv$ then $A =_{\beta\eta} B$. (3)

(h) Is $(\lambda xyz.xyz)(\lambda x.xx)(\lambda x.xx)x(\lambda x.xx)(\lambda x.xx)$ β-normalising? If yes, give the β-normal form showing all the reduction steps you used to reach it. If not, give a detailed proof why it is not. (3)

(i) Give the $\beta\eta$-normal form of $\lambda z.(z(\lambda y.((\lambda x.x)(\lambda x.xx)(\lambda y.y))y))z$. (1)

END OF PAPER